Mathematics

By a group of supervisors

Interactive E-learning Application

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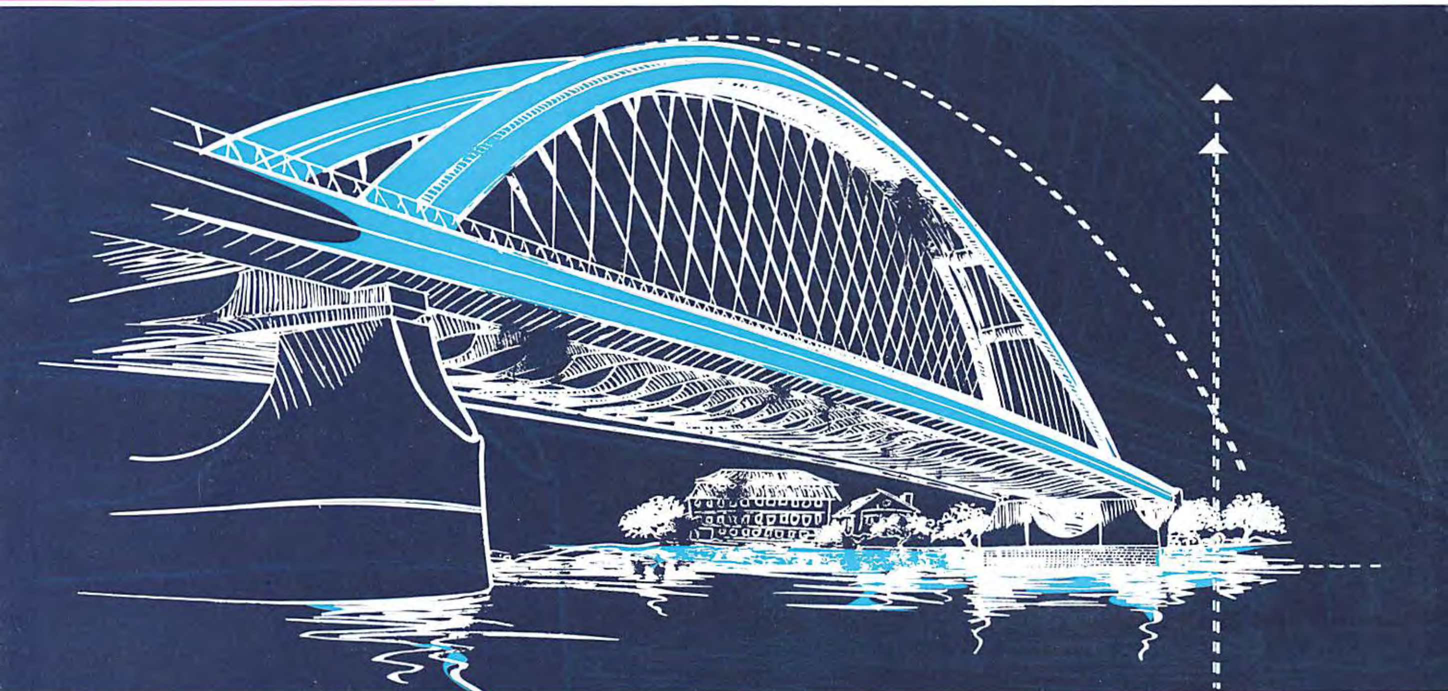


The Main Book

FIRST TERM

1 SEC.

2023

EL-MOASSER

CONTENTS

First Algebra and Trigonometry

UNIT 1 Algebra, relations and functions.

UNIT 2 Trigonometry.

Second Geometry

UNIT 3 Similarity.

UNIT 4 The triangle proportionality theorems.

First

Algebra and Trigonometry

UNIT 1 | Algebra, relations and functions.

UNIT 2 | Trigonometry.

Unit One

Algebra, relations and functions.

Unit Lessons

- Pre-requirements on unit one.

Lesson 1 An introduction in complex numbers.

Lesson 2 Determining the types of roots of a quadratic equation.

Lesson 3 Relation between the two roots of the second degree equation and the coefficients of its terms.

Lesson 4 Forming the quadratic equation whose two roots are known.

Lesson 5 Sign of a function.

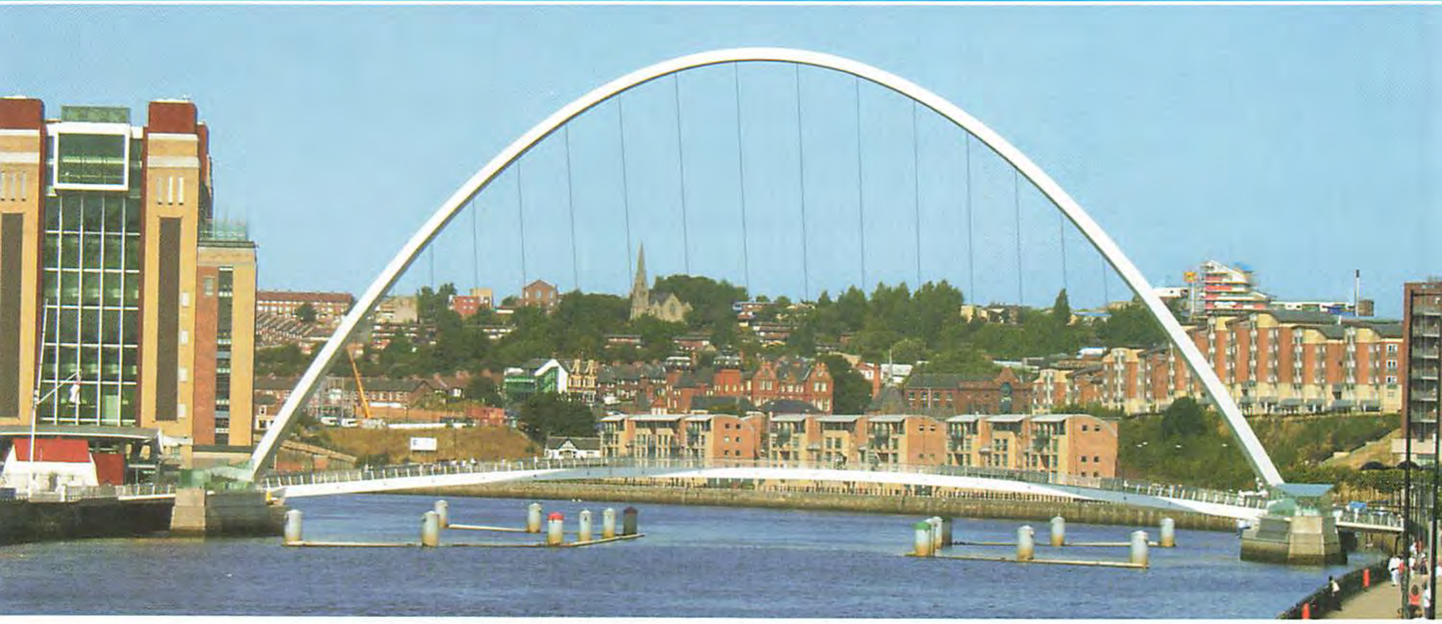
Lesson 6 Quadratic inequalities in one variable.

Learning outcomes

By the end of this unit, the student should be able to :

- Solve a quadratic equation in one variable algebraically and graphically.
- Use the quadratic equation in one variable to solve some life applications.
- Recognize an introduction in complex numbers (Definition of the complex number, integer powers of i and equality of two complex numbers).
- Carry out operations on the complex numbers.
- Recognize the two conjugate numbers in the complex numbers.
- Recognize the discriminant of the quadratic equation in one variable.
- Investigate the type of the two roots of the quadratic equation in one variable given the coefficients of its terms.
- Find the sum and the product of the two roots of a quadratic equation in one variable.
- Find some of the coefficients of terms of the quadratic equation in one variable in terms of one of the two roots or both of them.
- Form the quadratic equation in one variable whose roots are given.
- Form the quadratic equation in one variable given another quadratic equation in one variable.
- Investigate the sign of a function (constant - linear - quadratic).
- Solve quadratic inequalities in one variable.

Pre-requirements on unit one



First Solving the quadratic equation in one variable algebraically

1 By factorization

Example 1

Find in $\mathbb{R}$ the solution set of each of the following equations :

1 $x^2 - 5x - 6 = 0$

2 $4x^2 = 25$

Solution

1 $\because x^2 - 5x - 6 = 0$ $\quad \therefore (x - 6)(x + 1) = 0$ "factorizing the trinomial"

$\therefore$ Either $x - 6 = 0$ or $x + 1 = 0$

$\therefore x = 6$ or $x = -1$

$\therefore$ The solution set $= \{6, -1\}$

Remember that

The quadratic equation in one variable has at most two solutions in $\mathbb{R}$

2 $\because 4x^2 = 25$ $\quad \therefore 4x^2 - 25 = 0$

$\therefore (2x - 5)(2x + 5) = 0$ "factorizing the difference between two squares"

$\therefore$ Either $2x - 5 = 0$ or $2x + 5 = 0$

$\therefore x = \frac{5}{2}$ or $x = -\frac{5}{2}$

$\therefore$ The solution set $= \left\{\frac{5}{2}, -\frac{5}{2}\right\}$

Another solution

$\because 4x^2 = 25$ $\quad \therefore x^2 = \frac{25}{4}$ $\quad \therefore x = \pm\sqrt{\frac{25}{4}}$

$\therefore x = \pm \frac{5}{2}$ $\quad \therefore$ The solution set $= \left\{\frac{5}{2}, \frac{-5}{2}\right\}$

2 By the general formula

To find the roots of the quadratic equation : $a x^2 + b x + c = 0$ where $a \neq 0$

use the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 2

Find the solution set of each of the following equations in $\mathbb{R}$:

1 $x^2 - 2x - 6 = 0$

2 $x + \dfrac{5}{x} = 4$, $x \neq 0$

Solution

1 The expression : $x^2 - 2x - 6$ is difficult to be factorized , so we use the general formula.

$\because a = 1$, $b = -2$, $c = -6$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times (-6)}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{4 + 24}}{2} = \frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$$

$\therefore$ The solution set $= \{1 + \sqrt{7} , 1 - \sqrt{7}\}$

2 $\because x + \dfrac{5}{x} = 4$ "By multiplying both sides of the equation by x"

$\therefore x^2 + 5 = 4x$

$\therefore x^2 - 4x + 5 = 0$ "Notice putting the equation in the form : $a x^2 + b x + c = 0$"

$\therefore a = 1$, $b = -4$, $c = 5$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 5}}{2} = \frac{4 \pm \sqrt{-4}}{2}$$

, $\because \sqrt{-4} \notin \mathbb{R}$ $\therefore$ There is no real roots of the equation : $x^2 - 4x + 5 = 0$

$\therefore$ The solution set $= \varnothing$

TRY TO SOLVE

Find in $\mathbb{R}$ the solution set of each of the following equations :

1 $x^2 - 5x + 6 = 0$

2 $5x^2 + 2x = 4$

3 $3x^2 = 27$

4 $x(x - 4) = 3$

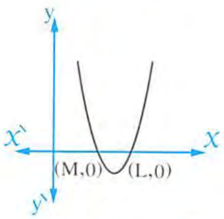
Second Solving the quadratic equation in one variable graphically

To solve the quadratic equation in one variable graphically , we do the following :

1. Put the equation on the form : $a X^2 + b X + c = 0$
2. Let $f(X) = a X^2 + b X + c$
3. Graph the function f
4. Determine the points of intersection of the curve with the X-axis , then the X-coordinates of these intersection points are the solutions of the equation $f(X) = 0$ i.e. $a X^2 + b X + c = 0$

According to that , we have three cases

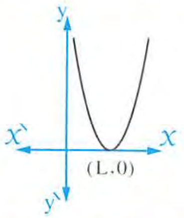
1 The curve intersects X-axis at two points



There are two solutions in $\mathbb{R}$

The S.S. = $\{L , M\}$

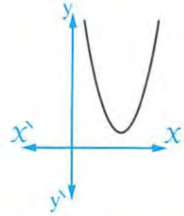
2 The curve touches X-axis at one point



There is a unique solution in $\mathbb{R}$

The S.S. = $\{L\}$

3 The curve does not intersect X-axis



There is no solution in $\mathbb{R}$

The S.S. = $\varnothing$

Example 3

Find graphically in $\mathbb{R}$ the S.S. of the equation :

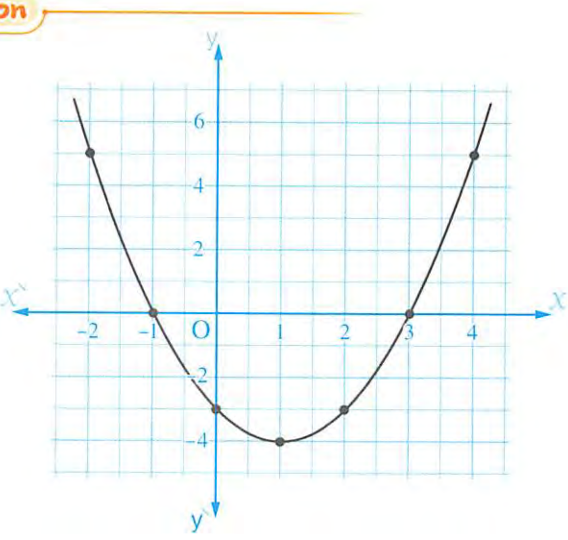
$X^2 - 2X - 3 = 0$ using the interval $[-2 , 4]$

Solution

Let $f(X) = X^2 - 2X - 3$

X	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

From the graph , the S.S. = $\{3 , -1\}$



Remark

In case of the interval is not given , then we can graph the function by finding the vertex of the curve which is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$, and then we find some points to the right of it , and the same number of points to the left of it.

Example 4

Solve graphically in $\mathbb{R}$ the equation :

$4X(X-1)-5=0$, then verify the result algebraically "given that $\sqrt{6} \simeq 2.4$"

Solution

$\because 4X(X-1)-5=0 \qquad \therefore 4X^2-4X-5=0$

First Graphically :

Let $f(X)=4X^2-4X-5$

- **Find the vertex point of the curve :**

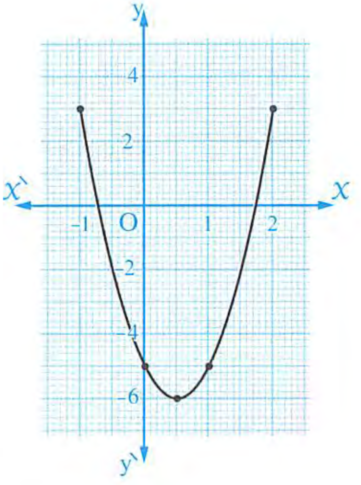
$\because$ The X-coordinate of the vertex point $=\frac{-b}{2a}=\frac{4}{8}=\frac{1}{2}$

$, f\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)^2-4\left(\frac{1}{2}\right)-5=-6$

$\therefore$ The vertex point of the curve is $\left(\frac{1}{2}, -6\right)$

- **Form the following table :**

X	-1	0	$\frac{1}{2}$	1	2
y	3	-5	-6	-5	3



- **From the graph we notice that :**

The roots are -0.7 and 1.7 approximately.

Second Algebraically :

$\because X=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ where $a=4$, $b=-4$, $c=-5$

$\therefore X=\frac{-(-4)\pm\sqrt{(-4)^2-4\times4\times(-5)}}{2\times4}=\frac{4\pm\sqrt{96}}{8}=\frac{4\pm4\sqrt{6}}{8}=\frac{1\pm\sqrt{6}}{2}\simeq\frac{1\pm2.4}{2}$

$\therefore$ The two roots of the equation are 1.7 and -0.7 approximately.

TRY TO SOLVE

Solve graphically in $\mathbb{R}$ the equation :

$X^2-4X+4=0$, taking $X\in[0, 4]$, then verify the result algebraically.

Lesson 1

An introduction in complex numbers

Introduction

- There are many problems that can not be solved by the use of real numbers alone. For example , we are unable to solve the equation $x^2 = -1$ There is no real number "a" such that $a^2 = -1$ Thus we must extend the set of real numbers $\mathbb{R}$ to a new set of numbers to enable us to find the solution of the equation $x^2 = -1$

  This new set is called THE SET OF COMPLEX NUMBERS , and before studying the set of complex numbers in details , we will firstly recognize the imaginary number "i".

The imaginary number "i"

The imaginary number "i" is defined as the number whose square is – 1

i.e. $\boxed{i^2 = -1}$

Thus we can solve the equation : $x^2 = -1$ as follows :

$\because x^2 = -1$ $\qquad \therefore x^2 = i^2$

$\therefore x = \pm\sqrt{i^2}$ $\qquad \therefore x = \pm i$

$\therefore$ The solution set $= \{i , -i\}$

Notice that

- $i \times i = i^2 = -1$
- $-i \times -i = i^2 = -1$

Remarks

- The number "i" does not belong to the set of real numbers.

  i.e. $i \notin \mathbb{R}$, so it will not be represented by a point on the real number line.
- The numbers $3i , -2i , \sqrt{5}\,i$, ... are imaginary numbers.
- If a is a real positive number, then $\sqrt{-a} = \sqrt{a}\,i$

For example :

$\sqrt{-2}=\sqrt{2\,i^2}=\sqrt{2}\,i$, $\sqrt{-3}=\sqrt{3\,i^2}=\sqrt{3}\,i$, $\sqrt{-25}=\sqrt{25\,i^2}=5\,i$ and so on ...

- The operations on the square roots can not be generalized on the imaginary numbers.

If a and b are two negative real numbers , then $\sqrt{a}\times\sqrt{b}\neq\sqrt{a\,b}$

For example $\sqrt{-1}\times\sqrt{-1}\neq\sqrt{-1\times-1}$

because $\sqrt{-1}\times\sqrt{-1}=\sqrt{i^2}\times\sqrt{i^2}=i\times i=i^2=-1$

but $\sqrt{-1\times-1}=\sqrt{(-1)^2}=\sqrt{1}=1$

Integer powers of "i"

The number "i" satisfies the rules of powers that you have studied in the preparatory stage and since $i^2=-1$, then :

- $i^3=i^2\times i=-1\times i=-i$
- $i^4=i^2\times i^2=-1\times-1=1$
- $i^5=i^4\times i=1\times i=i$
- $i^6=i^4\times i^2=1\times-1=-1$ and so on.

From this we find that :

- The integer powers of "i" give one of the values i , – 1 , – i or 1
- This values are repeated if the power is increased by 4

Generally : For each $n\in\mathbb{Z}$,

- $i^{4n}=(i^4)^n=1^n=1$
- $i^{4n+1}=i^{4n}\times i=1\times i=i$
- $i^{4n+2}=i^{4n}\times i^2=1\times-1=-1$
- $i^{4n+3}=i^{4n}\times i^3=1\times-i=-i$
- $i^{4n+4}=i^{4n}\times i^4=1\times1=1$ ... and so on.

In another way

To find i^n where n is an integer → **We find the remainder of the division $n\div4$, if :**

- The remainder = 0 **then** $i^n=1$
- The remainder = 1 **then** $i^n=i$
- The remainder = 2 **then** $i^n=i^2=-1$
- The remainder = 3 **then** $i^n=i^3=-i$

For example :

- $i^{16}=1$ «because $16\div4=4$ without remainder»
- $i^{63}=-i$ «because $63\div4=15$ with remainder 3»
- $i^{42}=-1$ «because $42\div4=10$ with remainder 2»
- $i^{101}=i$ «because $101\div4=25$ with remainder 1»
- i^{4n+23} where $n\in\mathbb{Z}=-i$ «because $(4n+23)\div4=n+5$ with remainder 3»

Remark

We can express "1" using the imaginary number i to integer powers from the multiples of 4 , and this helps in simplyfying some of imaginary numbers , for example : $i^{-19} = \frac{1}{i^{19}} = \frac{i^{20}}{i^{19}} = i$

The complex number

The complex number is the number that can be written in the form **a + b i**

, where a and b are two real numbers and $i^2 = -1$

- a is called the real part.
- b is called the imaginary part.

Examples for complex numbers : $2 - i$, $7 + 13i$, $5i - 4$, $\sqrt{2} + \sqrt{3}\,i$

Remarks

For any complex number $Z = a + b\,i$, then :

1. If $b = 0$, then $Z = a$ and we say that Z is a real number.

Such as $Z = 5$ is a real number and it is a complex number whose imaginary number = 0

2. If $a = 0$, then $Z = b\,i$ and we say that Z is an imaginary number. (where $b \neq 0$)

Such as $Z = 2i$ is an imaginary number and it is a complex number.

From the previous , every real number is a complex number whose imaginary number = zero and so the set of real numbers is a subset of set of complex numbers that can be defined as the following.

The set of complex numbers

The set of complex numbers $\mathbb{C}$ is defined as $\mathbb{C} = \{a + b\,i : a \in \mathbb{R} , b \in \mathbb{R} , i^2 = -1\}$

Example 1

Find the solution set of each of the following equations in the set of complex numbers :

1. $2x^2 + 18 = 0$
2. $x^2 + x + 1 = 0$

Solution

1 $\because 2x^2 + 18 = 0$ $\therefore 2x^2 = -18$ $\therefore x^2 = -9$

$\therefore x = \pm\sqrt{-9}$ $\therefore x = \pm\sqrt{9i^2}$ $\therefore x = \pm 3i$

$\therefore$ The solution set $= \{3i , -3i\}$

2 $\because a = 1 , b = 1 , c = 1$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}\,i}{2} = \frac{-1}{2} \pm \frac{\sqrt{3}}{2} i$$

$\therefore$ The solution set $= \left\{\frac{-1}{2} + \frac{\sqrt{3}}{2} i , \frac{-1}{2} - \frac{\sqrt{3}}{2} i\right\}$

TRY TO SOLVE

Find the solution set of each of the following equations in the set of complex numbers :

1 $5x^2 + 180 = 0$ **2** $x^2 - 2x + 5 = 0$

Equality of two complex numbers

Two complex numbers are equal if and only if the two real parts are equal and the two imaginary parts are equal.

i.e. If $(a + b\,i)$ and $(c + d\,i)$ are two complex numbers and if $a = c , b = d$

, then $a + b\,i = c + d\,i$

and vice versa If $a + b\,i = c + d\,i$, then $a = c , b = d$

Notice that Order in complex numbers whose imaginary part not equal to zero has no meaning , we do not know which is greater $(5 + 3i)$ or $(-4 + 7i)$?

Example 2

Find the values of x and y which satisfy each of the following where $x \in \mathbb{R}$, $y \in \mathbb{R}$, $i^2 = -1$:

1 $(2x - 3) + 5i = 7 + (3 - 2y)i$ **2** $x + y\,i = \sqrt{-4} + i^{22}$

3 $x - 3y + (2x + y)i = 6 + 5i$

Solution

1 $\because 2x - 3 = 7$ $\therefore 2x = 10$ $\therefore x = 5$

$, \because 3 - 2y = 5$ $\therefore -2y = 2$ $\therefore y = -1$

2 $x + y\,i = 2\,i + i^{4(5)+2}$ $\therefore x + y\,i = 2\,i + i^2 = 2\,i + (-1)$

$\therefore x + y\,i = -1 + 2\,i$ $\therefore x = -1$, $y = 2$

3 $\therefore x - 3\,y = 6$ (1)

, $2\,x + y = 5$ (2)

Multiply the equation (2) by 3

$\therefore 6\,x + 3\,y = 15$ (3)

By adding (1) and (3) : $\therefore 7\,x = 21$ $\therefore x = 3$

By substituting in (2) : $\therefore y = -1$

TRY TO SOLVE

Find the values of x and y which satisfy each of the following :

1 $x + y\,i = 3\,i^{-1} + 4$

2 $4\,x - y + (2\,x + y)\,i = 5 + 7\,i$

Adding and subtracting complex numbers

- When adding or subtracting two complex numbers , we add or subtract real parts together and add or subtract imaginary parts together.

Example 3

Find the result of each of the following in the simplest form :

1 $(3 + 7\,i^{13}) + (5 - 9\,i)$

2 $\left(2 - \sqrt{-16}\right) - (5 - i)$

Solution

1 $\because i^{13} = i$ $\therefore$ The expression $= (3 + 7\,i) + (5 - 9\,i) = (3 + 5) + (7\,i - 9\,i) = 8 - 2\,i$

2 $\because \sqrt{-16} = 4\,i$

$\therefore$ The expression $= (2 - 4\,i) - (5 - i) = (2 - 4\,i) + (-5 + i) = (2 - 5) + (-4\,i + i) = -3 - 3\,i$

Multiplying complex numbers

Two complex numbers can be multiplied just as the algebraic expressions , considering $i^2 = -1$

Example 4

Find the result of each of the following in the simplest form :

1. $(4 + 3i)(2 - 5i)$
2. $(5 - 2i)(5 + 2i)$
3. $(3 + 2i)^2$
4. $(1 - i)^4$

Solution

1 $(4 + 3i)(2 - 5i) = 4(2 - 5i) + 3i(2 - 5i)$

$= 8 - 20i + 6i - 15i^2$

$= 8 - 20i + 6i + 15$ (where $i^2 = -1$)

$= (8 + 15) + (-20i + 6i) = 23 - 14i$

Notice that You can solve directly by using multiplication by inspection as follows :

$$-20i$$

$(4 + 3i)(2 - 5i) = 8 - 14i - 15i^2$ (where $i^2 = -1$)

$+6i$ $\qquad = 8 - 14i + 15 = 23 - 14i$

2 $(5 - 2i)(5 + 2i) = 25 - 4i^2$

$= 25 + 4$ (where $i^2 = -1$)

$= 29$

Remember that

$(a + b)(a - b) = a^2 - b^2$

3 $(3 + 2i)^2 = 9 + 12i + 4i^2$

$= 9 + 12i - 4$ (where $i^2 = -1$)

$= 5 + 12i$

Remember that

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

4 $(1 - i)^4 = \left((1 - i)^2\right)^2 = (1 - 2i + i^2)^2 = (1 - 2i - 1)^2$

$= (-2i)^2 = 4i^2 = -4$

Remark

$(1 \pm i)^{2n} = (\pm 2\, i)^n$ where $n \in \mathbb{Z}$

- **Proof :** $(1 \pm i)^{2\,n} = [(1 \pm i)^2]^n = [1 \pm 2\, i - 1]^n = (\pm 2\, i)^n$
- This remark is used to simplify some complex numbers as the following :

1. $(1 + i)^{200} = (2\, i)^{100} = 2^{100}\, i^{100} = 2^{100}$
2. $(3 - 3\, i)^4 = 3^4\,(1 - i)^4 = 3^4\,(-2\, i)^2 = 3^4 \times 2^2\, i^2 = -324$

TRY TO SOLVE

Find the result of each of the following in the simplest form :

1. $(\sqrt{4} + \sqrt{-25}) + (-3 - 4\, i)$
2. $(2 - i)\,(2 + \sqrt{-1})$
3. $(2 + 3\, i^{21})\,(5 + i^{31})$
4. $i\,(5 - 3\, i)$
5. $(1 - i)^{32}$

The two conjugate numbers

The two numbers $a + b\, i$ and $a - b\, i$ are called conjugate numbers.

Note : Take care that the complex number and its conjugate differ only in the sign of their imaginary parts.

For example : The two numbers $3 + 4\, i$, $3 - 4\, i$ are conjugate numbers.

Remarks

- The conjugate of the number $2\, i - 5$ is the number $-2\, i - 5$ not $2\, i + 5$
- The conjugate of the number $2\, i$ is $-2\, i$
- The conjugate of the number 3 is 3
- The sum of the two conjugate numbers is always a real number , and the product of the two conjugate numbers is always a real number.

For example The complex number $3 + 4\, i$ its conjugate is $3 - 4\, i$, then :

* Their sum $= (3 + 4\, i) + (3 - 4\, i) = (3 + 3) + (4\, i - 4\, i) = 6 \in \mathbb{R}$
* Their product $= (3 + 4\, i)\,(3 - 4\, i) = 9 - 16\, i^2 = 9 + 16 = 25 \in \mathbb{R}$

TRY TO SOLVE

Write the conjugate of $5 - 4\, i$, then find :

1. The sum of the number and its conjugate.
2. The product of the number and its conjugate.

Example 5

Simplify to the simplest form :

1 $\dfrac{4-3\,i}{i}$ **2** $\dfrac{10}{3+i}$

3 $\dfrac{3+2\,i}{2-5\,i}$ **4** $\dfrac{(2+i)\,(1-i)}{(1+i)\,(3-2\,i)}$

Solution

Notice : To simplify the fraction whose denominator is a complex number , we multiply its two terms by the conjugate of denominator.

1 $\dfrac{4-3\,i}{i} \times \dfrac{-i}{-i} = \dfrac{-4\,i+3\,i^2}{-i^2} = \dfrac{-4\,i-3}{-(-1)} = -3-4\,i$

2 $\because$ The conjugate of the denominator is $(3-i)$

$$\therefore \frac{10}{3+i} = \frac{10\,(3-i)}{(3+i)\,(3-i)} = \frac{10\,(3-i)}{9-i^2} = \frac{10\,(3-i)}{9+1} = \frac{10\,(3-i)}{10} = 3-i$$

3 $\dfrac{3+2\,i}{2-5\,i} = \dfrac{(3+2\,i)\,(2+5\,i)}{(2-5\,i)\,(2+5\,i)} = \dfrac{6+15\,i+4\,i+10\,i^2}{4-25\,i^2}$

$$= \frac{6+19\,i-10}{4+25} = \frac{-4+19\,i}{29} = \frac{-4}{29} + \frac{19}{29}\,i$$

4 $\dfrac{(2+i)\,(1-i)}{(1+i)\,(3-2\,i)} = \dfrac{2-2\,i+i-i^2}{3-2\,i+3\,i-2\,i^2} = \dfrac{2-i+1}{3+i+2} = \dfrac{3-i}{5+i}$

$$, \frac{3-i}{5+i} = \frac{(3-i)\,(5-i)}{(5+i)\,(5-i)} = \frac{15-8\,i-1}{25-i^2} = \frac{14-8\,i}{26} = \frac{2\,(7-4\,i)}{26} = \frac{7}{13} - \frac{4}{13}\,i$$

TRY TO SOLVE

Simplify to the simplest form :

1 $\dfrac{2+i}{3-4\,i}$ **2** $\dfrac{(2+i)\,(3+i)}{(2-i)\,(3-i)}$

Example 6

If $x = \frac{7-i}{2-i}$ and $y = \frac{13-i}{4+i}$

Prove that : x and y are conjugate numbers , then prove that : $x^2 + y^2 = 16$

Solution

$\because x = \frac{7-i}{2-i} = \frac{(7-i)(2+i)}{(2-i)(2+i)} = \frac{14+7i-2i-i^2}{4-i^2} = \frac{14+5i+1}{4+1} = \frac{15+5i}{5} = 3+i$

$, y = \frac{13-i}{4+i} = \frac{(13-i)(4-i)}{(4+i)(4-i)} = \frac{52-13i-4i+i^2}{16-i^2} = \frac{52-17i-1}{16+1} = \frac{51-17i}{17} = 3-i$

$\therefore x$ and y are conjugate numbers " **Notice that** the signs of the imaginary parts are different."

$, x^2 = (3+i)^2 = 9+6i+i^2 = 8+6i$

$, y^2 = (3-i)^2 = 9-6i+i^2 = 8-6i$

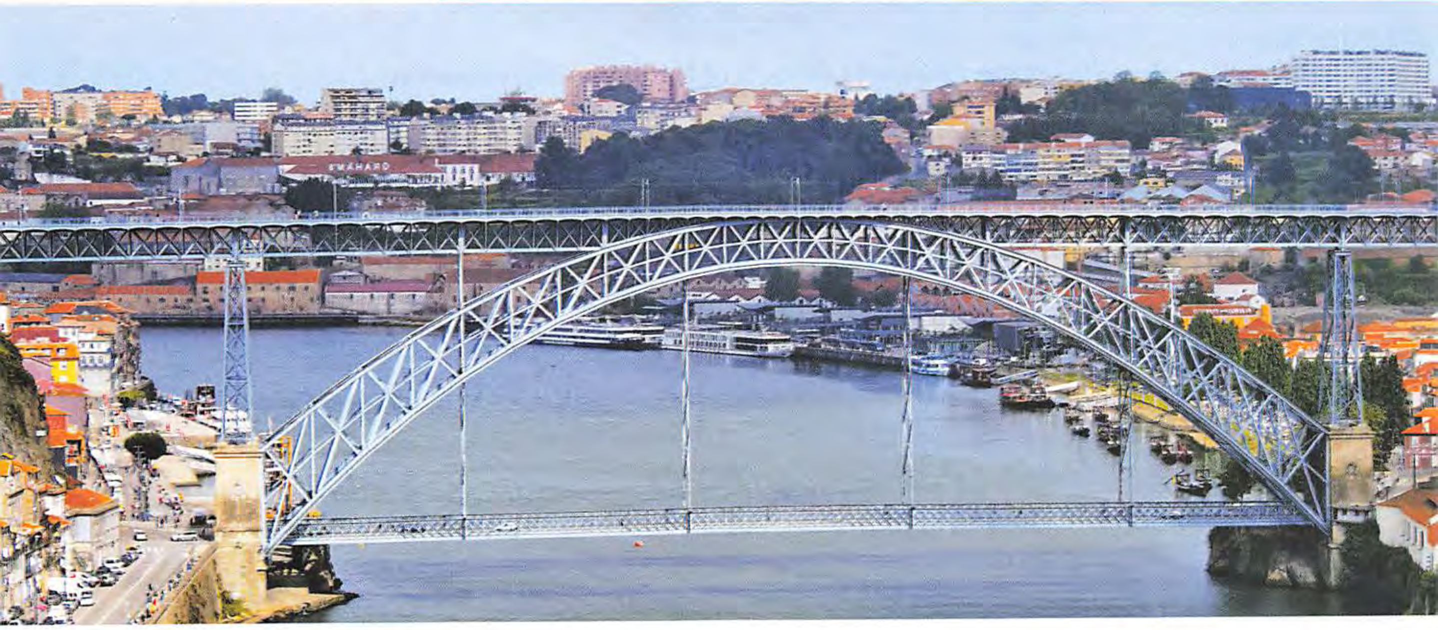
$\therefore x^2 + y^2 = (8+6i) + (8-6i) = (8+8) + (6i-6i) = 16$

TRY TO SOLVE

Prove that a and b are conjugate numbers if : $a = \frac{1-2i}{1-3i}$ and $b = \frac{2-i}{3-i}$

Lesson **2**

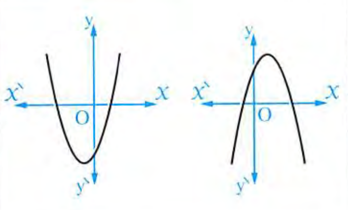
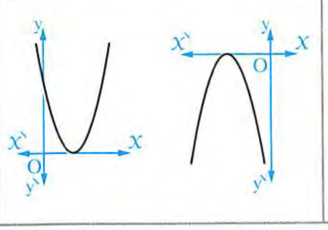
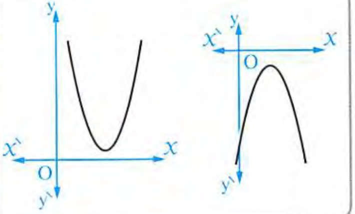
Determining the types of roots of a quadratic equation



- You have previously studied how to solve the second degree equation (the quadratic equation) in one variable in $\mathbb{R}$, and you have known that when solving it , we have two solutions at most.
- In this lesson , we will determine the types of the two roots of the quadratic equation without solving it.

Discriminant

- Using the formula in solving the quadratic equation : $a x^2 + b x + c = 0$, where $a \neq 0$, we get two roots : $\dfrac{-b+\sqrt{b^2-4ac}}{2a}$, $\dfrac{-b-\sqrt{b^2-4ac}}{2a}$
- Both of these two roots include the expression : $\sqrt{b^2-4ac}$
- The expression : $b^2 - 4ac$ is called the discriminant of the quadratic equation because it is used to determine the types of roots of the quadratic equation as follows :

Discriminant	positive $(b^2 - 4ac) > 0$	equal to zero $b^2 - 4ac = 0$	negative $(b^2 - 4ac) < 0$
Type of the two roots	Two different real roots	Two equal real roots	Two complex and non real roots
A sketch for the function related to the equation			

Example 1

Determine the type of the two roots of each of the following equations :

1 $x^2 - 3x + 5 = 0$ **2** $x^2 + 10x + 25 = 0$ **3** $3x^2 + 10x = 4$

Solution

1 $\because a = 1$, $b = -3$, $c = 5$

$\therefore$ The discriminant $= b^2 - 4ac = (-3)^2 - 4 \times 1 \times 5 = -11$ (negative quantity)

$\therefore$ The two roots are complex and non real.

2 $\because a = 1$, $b = 10$, $c = 25$

$\therefore$ The discriminant $= b^2 - 4ac = (10)^2 - 4 \times 1 \times 25 = 0$

$\therefore$ The two roots are real and equal.

3 $\because 3x^2 + 10x - 4 = 0$ $\therefore a = 3$, $b = 10$, $c = -4$

$\therefore$ The discriminant $= b^2 - 4ac = (10)^2 - 4 \times 3 \times (-4) = 148$ (positive quantity)

$\therefore$ The two roots are different and real.

TRY TO SOLVE

Determine the type of the two roots of each of the following equations :

1 $x^2 - 7x + 10 = 0$ **2** $x^2 + 4x + 5 = 0$ **3** $4x^2 - 12x = -9$

Example 2

Prove that the two roots of the equation : $7x^2 - 11x + 5 = 0$ are two complex and non real roots , then use the formula to find these two roots.

Solution

$\because a = 7$, $b = -11$, $c = 5$

$\therefore$ The discriminant $= b^2 - 4ac = (-11)^2 - 4 \times 7 \times 5 = -19 < 0$

$\therefore$ The two roots are complex and non real roots.

, $\because x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{11 \pm \sqrt{-19}}{14} = \dfrac{11 \pm \sqrt{19}\,i}{14}$

$\therefore$ The two roots of the equation are $\dfrac{11 + \sqrt{19}\,i}{14}$, $\dfrac{11 - \sqrt{19}\,i}{14}$

TRY TO SOLVE

If $x^2 - 4x + 5 = 0$, then prove that the two roots are complex and not real , then use the general formula to find these two roots.

Example 3

If the two roots of the equation : $x^2 - kx + 2k - 4x + 5 = 0$ are equal , then find the real values of k and find these two roots.

Solution

Put the equation on the general form $\therefore x^2 - (k+4)x + (2k+5) = 0$

$\therefore$ The discriminant $= (k+4)^2 - 4 \times 1 \times (2k+5) = k^2 + 8k + 16 - 8k - 20 = k^2 - 4$

$\because$ The two roots of the equation are equal $\therefore$ The discriminant $= 0$

$\therefore k^2 - 4 = 0$ $\therefore k^2 = 4$ $\therefore k = \pm 2$

$\therefore$ at $k = 2$ $\therefore$ The equation is $x^2 - 6x + 9 = 0$ $\therefore (x-3)^2 = 0$ $\therefore x = 3$

at $k = 2$ the two roots are equal , each one $= 3$

, at $k = -2$ $\therefore$ The equation is $x^2 - 2x + 1 = 0$ $\therefore (x-1)^2 = 0$ $\therefore x = 1$

at $k = -2$ the two roots are equal , each one $= 1$

TRY TO SOLVE

Find the real value of k which makes the two roots of the equation :
$4x^2 - 8x + k = 0$ equal and find these two roots.

Example 4

1 **Find the real values of m which satisfy that the equation : $x^2 - (2m-1)x + m^2 = 0$ has no real roots (i.e. has no solutions in $\mathbb{R}$)**

2 **Find the real values of k which satisfy that the equation : $x^2 + 2(k-1)x + k^2 = 0$ has two real roots (i.e. has solutions in $\mathbb{R}$)**

Solution

1 $\because$ The equation does not have real roots $\therefore b^2 - 4ac < 0$

$\therefore (2m-1)^2 - 4m^2 < 0$ $\therefore 4m^2 - 4m + 1 - 4m^2 < 0$

$\therefore -4\,m < -1$ $\therefore m > \frac{1}{4}$

$\therefore$ The equation has no real roots if $m \in \left]\frac{1}{4}, \infty\right[$

2 $\because$ The equation has two real roots $\therefore$ The two roots are either different or equal

$\therefore b^2 - 4\,a\,c \geq 0$ $\therefore 4\,(k-1)^2 - 4 \times 1 \times k^2 \geq 0$

$\therefore 4\,k^2 - 8\,k + 4 - 4\,k^2 \geq 0$ $\therefore -8\,k \geq -4$ $\therefore k \leq \frac{1}{2}$

$\therefore$ The equation has two real roots if $k \in \left]-\infty, \frac{1}{2}\right]$

TRY TO SOLVE

If the equation : $m^2 x^2 + (2\,m - 2)\,x + 1 = 0$ has no roots in $\mathbb{R}$, find the real values of m

Example 5

Prove that for all real values of a , there is no real roots for the equation :
$4\,x^2 - 12\,a\,x + 9\,a^2 + 4 = 0$

Solution

The discriminant $= (-12\,a)^2 - 4\,(4)\,(9\,a^2 + 4)$

$= 144\,a^2 - 144\,a^2 - 64 = -64$ (is negative quantity for all values of a)

$\therefore$ There is no real roots of the equation.

Remark

If the coefficients a , b and c in the quadratic equation : $a\,x^2 + b\,x + c = 0$ are rational numbers and the discriminant is a perfect square , then the roots are real rational numbers.

For example :

1 The equation : $3\,x^2 - 5\,x - 2 = 0$

- The terms coefficients are : $3 , -5 , -2$ (rational numbers)
- The discriminant = 49 (perfect square number)

$\therefore$ The roots are real rational

To verify that

By substitution in the general formula , the roots are $2 , -\frac{1}{3}$ (real rational)

2 The equation : $x^2 - 2\sqrt{5}\,x + 1 = 0$

- The terms coefficients are : $1 , -2\sqrt{5} , 1$ (the middle term coefficient is irrational real)
- The discriminant = 16 (perfect square number)

$\therefore$ The roots are real irrational

To verify that

By substitution in the general formula. the roots are $\sqrt{5} + 2 , \sqrt{5} - 2$ (real irrational)

Notice that in the equation $x^2 - 2\sqrt{5}\,x + 1 = 0$

although the discriminant is perfect square number , the roots are real irrational because the coefficient of the middle term is irrational.

Example 6

If a and b are rational numbers ,

prove that the two roots of the equation : $a\,x^2 + (a^2 + b^2)\,x + a\,b^2 = 0$ are rational.

Solution

$\because$ The discriminant $= (a^2 + b^2)^2 - 4 \times a \times a\,b^2 = a^4 + 2\,a^2\,b^2 + b^4 - 4\,a^2\,b^2$

$= a^4 - 2\,a^2\,b^2 + b^4 = (a^2 - b^2)^2$ is a perfect square

$\therefore$ The coefficients are rational numbers and the discriminant is a perfect square

$\therefore$ The two roots of the equation are rational.

TRY TO SOLVE

If a is a rational number , prove that the two roots of the equation :
$15\,x^2 - (10 + 3\,a)\,x + 2\,a = 0$ are rational.

Remark

If the discriminant of the quadratic equation (of real coefficients) isn't positive , then the two roots of the quadratic equation are two conjugate complex numbers.

For example :

The equation $x^2 - 2\,x + 2 = 0$

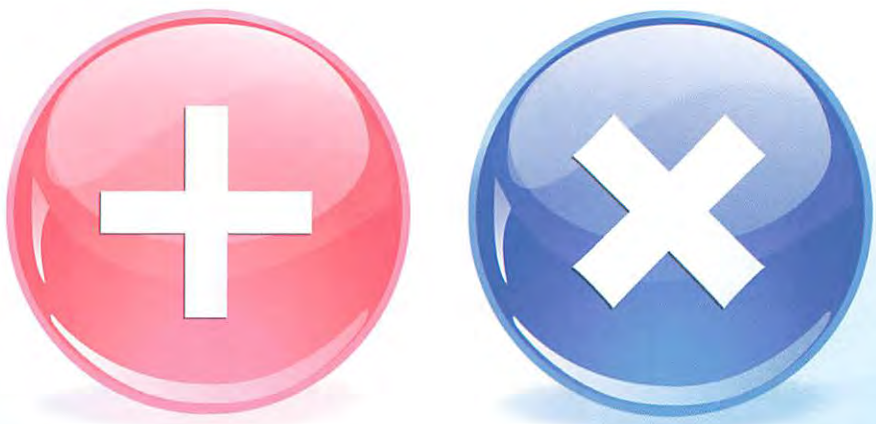
- The terms coefficients are : 1 , -2 , 2 (real numbers)
- The discriminant $= -4$ (not positive)

$\therefore$ The roots are conjugate complex and to verify that substitute in the general formula the roots are :

$1 + i$, $1 - i$ (conjugate complex)

Lesson 3

Relation between the two roots of the second degree equation and the coefficients of its terms



We know that the two roots of the quadratic equation : $a x^2 + b x + c = 0$, $a \neq 0$ are :

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad , \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a} \text{, then :}$$

1 The sum of the two roots $= \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-2b}{2a} = \frac{-b}{a}$

i.e. The sum of the two roots $= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

2 The product of the two roots $= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{b^2 - (b^2 - 4ac)}{4a^2}$

$$= \frac{b^2 - b^2 + 4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

i.e. The product of the two roots $= \frac{\text{Absolute term}}{\text{Coefficient of } x^2}$

In a symbolic form , we write :

If L and M are the two roots of the quadratic equation : $a x^2 + b x + c = 0$, then :

1 $L + M = \frac{-b}{a}$ **2** $L M = \frac{c}{a}$

Example 1

Without solving the equation , find the sum and the product of the two roots of each of the following equations :

1 $2x^2 + 5x - 12 = 0$

2 $6x^2 - 11x = 10$

Solution

1 $\because a = 2 \quad , \quad b = 5 \quad , \quad c = -12$

$\therefore$ The sum of the two roots $= \frac{-b}{a} = \frac{-5}{2}$

, the product of the two roots $= \frac{c}{a} = \frac{-12}{2} = -6$

Check the solution with noticing that the two roots are $\frac{3}{2}$ and -4

2 $\because 6x^2 - 11x - 10 = 0$ $\quad \therefore a = 6 \quad , \quad b = -11 \quad , \quad c = -10$

$\therefore$ The sum of the two roots $= \frac{-b}{a} = \frac{-(-11)}{6} = \frac{11}{6}$

, the product of the two roots $= \frac{c}{a} = \frac{-10}{6} = \frac{-5}{3}$

TRY TO SOLVE

If $3x^2 + 5 = 4x$, find the sum and product of the two roots.

Example 2

1 **If the sum of the two roots of the equation :** $2x^2 + kx + 1 = 0$ is $\frac{-3}{2}$, then find the value of k , and solve the equation in the set of complex numbers.

2 **If the product of the two roots of the equation :** $2x^2 - 4x + k = 0$ is $4\frac{1}{2}$, then find the value of k , and solve the equation in the set of complex numbers.

Solution

1 $\because$ The sum of the two roots $= \frac{-3}{2}$ $\quad \therefore \frac{-k}{2} = \frac{-3}{2}$

$\therefore \quad k = 3$ $\quad \therefore$ The equation is $2x^2 + 3x + 1 = 0$

$\therefore (2x + 1)(x + 1) = 0$ $\quad \therefore \quad x = -\frac{1}{2}$ or $x = -1$

2 $\because$ The product of the two roots $= 4\frac{1}{2} = \frac{9}{2}$ $\therefore \frac{k}{2} = \frac{9}{2}$ $\therefore$ $k = 9$

$\therefore$ The equation is $2x^2 - 4x + 9 = 0$ $\therefore a = 2$, $b = -4$, $c = 9$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 2 \times 9}}{2 \times 2} = \frac{4 \pm \sqrt{-56}}{4} = \frac{4 \pm \sqrt{56}\, i}{4} = \frac{4 \pm 2\sqrt{14}\, i}{4} = 1 \pm \frac{\sqrt{14}}{2} i$$

$\therefore$ $x = 1 + \frac{\sqrt{14}}{2} i$ or $x = 1 - \frac{\sqrt{14}}{2} i$

TRY TO SOLVE

1 If the sum of the two roots of the equation : $2x^2 - ax + 6 = 0$ is $3\frac{1}{2}$, then find the value of a , and solve the equation in the set of complex numbers.

2 If the product of the two roots of the equation : $x^2 + 3x + a = 0$ is 5 , then find the value of a , and solve the equation in the set of complex numbers.

Example 3

1 **If $x = -3$ is one of the two roots of the equation :** $2x^2 + kx - 3 = 0$, then find the other root , and find the value of k

2 **If $x = 6$ is one of the two roots of the equation :** $x^2 - 5x + k = 0$, then find the other root , and find the value of k

3 **If -1 and 5 are the two roots of the equation :** $ax^2 + bx - 5 = 0$, then find the value of each of a and b

Solution

1 $\because$ The product of the two roots $= \frac{c}{a} = \frac{-3}{2}$ $\therefore -3 \times$ the other root $= \frac{-3}{2}$

$\therefore$ The other root $= \frac{-3}{2} \times \frac{1}{-3}$ $\therefore$ The other root $= \frac{1}{2}$

$\because$ The sum of the two roots $= \frac{-b}{a} = \frac{-k}{2}$,

$\because$ The two roots are -3 , $\frac{1}{2}$ $\therefore -3 + \frac{1}{2} = \frac{-k}{2}$

$\therefore \frac{-5}{2} = \frac{-k}{2}$ $\therefore$ $k = 5$

Another solution :

$\because X = -3$ is one of the roots of the equation : $2X^2 + kX - 3 = 0$, then it satisfies it.

$\therefore 2(-3)^2 + k(-3) - 3 = 0$

$\therefore 18 - 3k - 3 = 0$ $\quad\quad \therefore k = 5$

$\therefore$ The equation is : $2X^2 + 5X - 3 = 0$ $\quad\quad \therefore (2X - 1)(X + 3) = 0$

$\therefore 2X - 1 = 0$, then $X = \frac{1}{2}$ $\quad\quad$ or $X + 3 = 0$, then $X = -3$

$\therefore$ The other root $= \frac{1}{2}$

2 $\because$ The sum of the two roots $= \frac{-b}{a} = \frac{-(-5)}{1} = 5$

$\therefore 6 +$ the other root $= 5$ $\quad\quad \therefore$ The other root $= -1$

$\because$ The product of the two roots $= \frac{c}{a} = \frac{k}{1} = k$,

$\because$ The two roots are 6 , -1 $\quad\quad \therefore 6 \times (-1) = k$ $\quad\quad \therefore k = -6$

*** Try to solve this example by another method as in 1**

3 $\because$ The product of the two roots $= \frac{c}{a}$

$\therefore -1 \times 5 = \frac{-5}{a}$ $\quad\quad \therefore -5 = \frac{-5}{a}$ $\quad\quad \therefore a = 1$

$\because$ The sum of the two roots $= \frac{-b}{a}$ $\quad\quad \therefore -1 + 5 = \frac{-b}{1}$

$\therefore 4 = -b$ $\quad\quad \therefore b = -4$

Another solution :

$\because -1$ is a root of the equation. $\quad\quad \therefore a(-1)^2 + b(-1) - 5 = 0$

$\therefore a - b = 5$ $\quad\quad (1)$

$\because 5$ is a root of the equation. $\quad\quad \therefore a(5)^2 + b(5) - 5 = 0$

$\therefore 25a + 5b = 5$ "Divide by 5"

$\therefore 5a + b = 1$ $\quad\quad (2)$

Adding the equations (1) and (2) : $\therefore 6a = 6$ $\quad\quad \therefore a = 1$

By substituting in (1) : $\therefore 1 - b = 5$ $\quad\quad \therefore b = -4$

TRY TO SOLVE

Find the other root of each of the following equations , then find the value of k :

1 If $X = -1$ is one of the two roots of the equation : $X^2 + kX - 7 = 0$

2 If $X = \frac{5}{3}$ is one of the two roots of the equation : $9X^2 - 9X + k = 0$

Example 4

If $\left(1+\sqrt{2}\,i\right)$ is one of the two roots of the equation : $x^2-2x+c=0$ where $c \in \mathbb{R}$, then find :

1 The other root. 2 The value of c

Solution

$\because$ The sum of the two roots $=\frac{-(-2)}{1}=2$

$\therefore \left(1+\sqrt{2}\,i\right)$ + the other root $=2$

$\therefore$ The other root $=2-\left(1+\sqrt{2}\,i\right)$

i.e. The other root $=1-\sqrt{2}\,i$

Notice that

$\because$ Coefficients of the terms are real and one of the two roots is non real complex number

$\therefore$ The other root is the conjugate of the given root.

i.e. It equals $\left(1-\sqrt{2}\,i\right)$

$\because$ The product of the two roots $=c$ $\therefore \left(1-\sqrt{2}\,i\right)\left(1+\sqrt{2}\,i\right)=c$

$\therefore 1^2-\left(\sqrt{2}\,i\right)^2=c$ $\therefore 1-2\,i^2=c$

$\therefore 1+2=c$ $\therefore c=3$

Another solution :

$\because \left(1+\sqrt{2}\,i\right)$ is one of the two roots of the given equation.

$\therefore$ It satisfies the equation. $\therefore \left(1+\sqrt{2}\,i\right)^2-2\left(1+\sqrt{2}\,i\right)+c=0$

$\therefore 1+2\sqrt{2}\,i+\left(\sqrt{2}\,i\right)^2-2-2\sqrt{2}\,i+c=0$ $\therefore 1+\cancel{2\sqrt{2}\,i}-2-2-\cancel{2\sqrt{2}\,i}+c=0$

$\therefore -3+c=0$ $\therefore c=3$

i.e. $x^2-2x+3=0$

We can use the general formula to find the required other root.

TRY TO SOLVE

If $\left(\sqrt{2}+i\right)$ is one of the two roots of the equation : $x^2-2\sqrt{2}\,x+c=0$ where $c \in \mathbb{R}$, then find :

1 The other root. 2 The value of c

Remarks

In the quadratic equation : $a x^2 + b x + c = 0$

1 If $a = 1$, then $L + M = -b$ and $L M = c$

i.e. The sum of the two roots = the additive inverse of the coefficient of x , the product of the two roots = the absolute term.

2 If $b = 0$, then $L + M = 0$, i.e. $L = -M$

i.e. One of the two roots of the equation is the additive inverse of the other.

3 If $a = c$, then $L M = 1$, i.e. $L = \frac{1}{M}$

i.e. One of the two roots of the equation is the multiplicative inverse of the other.

Example 5

1 **Find the value of k , if one of the roots of the equation :** $3 x^2 + (k - 3) x + 7 = 0$ is the additive inverse of the other root.

2 **Find the value of k , if one of the roots of the equation :** $2 k x^2 + 7 x + k^2 + 1 = 0$ is the multiplicative inverse of the other.

Solution

1 $\because$ One of the roots is the additive inverse of the other

$\therefore b = 0$ $\qquad \therefore k - 3 = 0$ $\qquad \therefore k = 3$

2 $\because$ One of the roots is the multiplicative inverse of the other

$\therefore a = c$ $\qquad \therefore k^2 + 1 = 2 k$

$\therefore k^2 - 2 k + 1 = 0$ $\qquad \therefore (k - 1)^2 = 0$ $\qquad \therefore k = 1$

TRY TO SOLVE

Complete :

1 If one of the two roots of the equation : $x^2 + (k - 5) x - 9 = 0$ is the additive inverse of the other , then k = ...

2 If one of the two roots of the equation : $x^2 + 3 x + c = 0$ is the multiplicative inverse of the other , then c = ...

Example 6

Find the value of d , if one of the two roots of the equation : $x^2 + d x - 50 = 0$ is double the additive inverse of the other root.

Solution

Let one of the two roots = L $\therefore$ The other root = – 2 L

, $\because$ the product of the two roots = $\frac{\text{absolute term}}{\text{coefficient of } x^2}$ $\therefore L(-2L) = \frac{-50}{1}$

$\therefore -2L^2 = -50$ $\therefore L^2 = 25$ $\therefore L = \pm 5$

, $\because$ the sum of the two roots = $\frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$ $\therefore L + (-2L) = \frac{-d}{1}$

$\therefore -L = -d$ $\therefore L = d$ $\therefore d = \pm 5$

TRY TO SOLVE

Find the value of k , if one of the two roots of the equation : $x^2 - k x + 12 = 0$ is three times the other root.

Example 7

Find the satisfying condition which makes one of the two roots of the equation : $a x^2 + b x + c = 0$ equal to the additive inverse of twice the other root.

Solution

Let one of the two roots be L $\therefore$ The other root = – 2 L

, $\because$ the sum of the two roots = $\frac{-b}{a}$

$\therefore L + (-2L) = \frac{-b}{a}$ $\therefore L = \frac{b}{a}$ (1)

$\because$ The product of the two roots = $\frac{c}{a}$ $\therefore L \times (-2L) = \frac{c}{a}$ $\therefore L^2 = \frac{-c}{2a}$ (2)

By substituting from (1) in (2) : $\therefore \left(\frac{b}{a}\right)^2 = \frac{-c}{2a}$

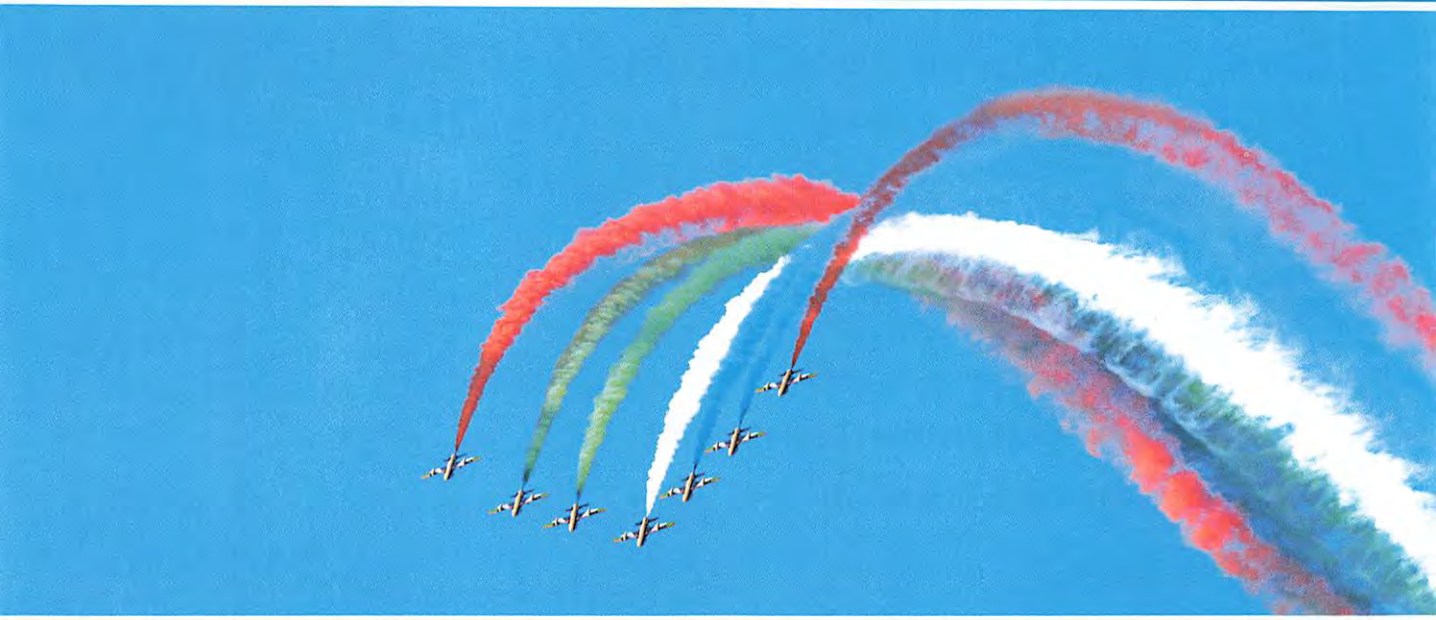
$\therefore \frac{b^2}{a^2} = \frac{-c}{2a}$ $\therefore \frac{b^2}{a} = \frac{-c}{2}$ $\therefore 2b^2 + ac = 0$ (That is the required condition)

TRY TO SOLVE

Find the satisfying condition which makes one of the two roots of the equation : $a x^2 + b x + c = 0$ equal to four times the other root.

Lesson 4

Forming the quadratic equation whose two roots are known



Let L and M be the two roots of the quadratic equation : $a\,x^2 + b\,x + c = 0$

By multiplying the two sides by $\frac{1}{a}$ where $a \neq 0$, the equation becomes in the form :

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ **i.e.** $x^2 - \left(\frac{-b}{a}\right)x + \frac{c}{a} = 0$ (1)

But $\frac{-b}{a} = L + M$, $\frac{c}{a} = LM$

By substituting in (1) , we get the quadratic equation whose roots are L , M which is :

$x^2 - (L + M)\,x + L\,M = 0$ (2)

i.e. x^2 – (the sum of the two roots) x + the product of the two roots = 0

And by factorizing the trinomial in the left side of the equation (2) , we get another form of the last equation which is $(x - L)(x - M) = 0$

Example 1

Form the quadratic equation whose roots are :

1 $\frac{3}{2}$, $\frac{5}{4}$ 2 $3 + \sqrt{2}$, $3 - \sqrt{2}$ 3 $\frac{-1+i}{i}$, $\frac{2}{1+i}$

Solution

1 The sum of the two roots $= \frac{3}{2} + \frac{5}{4} = \frac{11}{4}$, the product of them $= \frac{3}{2} \times \frac{5}{4} = \frac{15}{8}$

, $\because$ the equation is x^2 – (the sum of the two roots) x + the product of the two roots = 0

$\therefore$ The equation is $x^2 - \frac{11}{4}\,x + \frac{15}{8} = 0$ (by multiplying by 8)

$\therefore$ The equation is $8\,x^2 - 22\,x + 15 = 0$

2 The sum of the two roots $= 3 + \sqrt{2} + 3 - \sqrt{2} = 6$

, the product of the two roots $= \left(3 + \sqrt{2}\right)\left(3 - \sqrt{2}\right) = 7$

$\therefore$ The equation is $x^2 - 6x + 7 = 0$

3 $\because \frac{-1 + i}{i} = \frac{(-1 + i)\,i}{i \times i} = \frac{-i + i^2}{i^2} = \frac{-i - 1}{-1} = 1 + i$

, $\frac{2}{1 + i} = \frac{2\,(1 - i)}{(1 + i)\,(1 - i)} = \frac{2 - 2i}{1 - i^2} = \frac{2 - 2i}{2} = 1 - i$

$\therefore$ The sum of the two roots $= 1 + i + 1 - i = 2$

, the product of the two roots $= (1 + i)\,(1 - i) = 2$

$\therefore$ The equation is $x^2 - 2x + 2 = 0$

TRY TO SOLVE

Form the quadratic equation whose roots are :

1 -4 , 7

2 $3 - 2i$, $\frac{4 + 7i}{2 + i}$

Forming a quadratic equation from the roots of another equation

Example 2

If the two roots of the equation : $x^2 - 5x - 6 = 0$ are L , M , find the equation whose roots are L + 7 , M + 7

Solution

The required in this example is forming an equation using a given equation where there is a certain relation between the roots of the two equations. There are many methods for solving this example and we will mention them in the following :

The first method

1 Find the two roots of the given equation.

2 Find the two roots of the required equation.

3 Form the required equation.

$\because x^2 - 5x - 6 = 0$ $\qquad\qquad$ $\therefore (x - 6)\,(x + 1) = 0$

$\therefore 6$, -1 are the two roots of the given equation.

Let L = 6 , M = – 1 , the two roots of the required equation be D , E

$\therefore$ D = L + 7 = 6 + 7 = 13 , E = M + 7 = – 1 + 7 = 6

$\therefore$ D + E = 13 + 6 = 19 , DE = 13 × 6 = 78

$\therefore$ The required equation is $x^2 - 19x + 78 = 0$

The second method

Let D and E be the two roots of the required equation

$\therefore$ D = L + 7 , E = M + 7 $\qquad \therefore$ D + E = L + 7 + M + 7 = L + M + 14

, $\because$ L + M = 5 (from the given equation) $\qquad \therefore$ D + E = 5 + 14 = 19

, DE = (L + 7) (M + 7) = LM + 7 (L + M) + 49

, $\because$ LM = – 6 (from the given equation) $\qquad \therefore$ DE = – 6 + 7 × 5 + 49 = 78

$\therefore$ The required equation is $x^2 - 19x + 78 = 0$

The third method

Let D and E be the two roots of the required equation

$\therefore$ D = L + 7 , E = M + 7 $\qquad \therefore$ L = D – 7 , M = E – 7

$\because$ L is one of the two roots of the given equation : $x^2 - 5x - 6 = 0$

$\therefore L^2 - 5L - 6 = 0$

, $\because$ L = D – 7 $\qquad \therefore (D-7)^2 - 5(D-7) - 6 = 0$

$\therefore D^2 - 14D + 49 - 5D + 35 - 6 = 0 \qquad \therefore D^2 - 19D + 78 = 0$

i.e. D is a root of the equation : $x^2 - 19x + 78 = 0$ (which is the required equation)

Remark

The third method is used only if the relation between the first root of the given equation and the first root of the required equation is the same relation between the second root of the given equation and the second root of the required equation.

Remember the following identities

1. $L^2 + M^2 = (L+M)^2 - 2LM$
2. $(L-M)^2 = (L+M)^2 - 4LM$
3. $L^3 + M^3 = (L+M)\left[(L+M)^2 - 3LM\right]$
4. $L^3 - M^3 = (L-M)\left[(L+M)^2 - LM\right]$
5. $\frac{1}{M} + \frac{1}{L} = \frac{L+M}{LM}$
6. $\frac{L}{M} + \frac{M}{L} = \frac{L^2+M^2}{LM} = \frac{(L+M)^2 - 2LM}{LM}$

Example 3

If L , M are the two roots of the equation : $x^2 - 7x + 9 = 0$ where L > M , find the numerical value of each of the following expressions :

1 $L^2 + M^2$

2 $L^2 + 3LM + M^2$

3 $L - M$

4 $L^3 - M^3$

Solution

$\because$ L , M are the two roots of the equation : $x^2 - 7x + 9 = 0$ $\therefore$ $L + M = 7$ and $LM = 9$

1 $L^2 + M^2 = (L + M)^2 - 2LM = (7)^2 - 2 \times 9 = 49 - 18 = 31$

2 $L^2 + 3LM + M^2 = (L^2 + 2LM + M^2) + LM = (L + M)^2 + LM = (7)^2 + 9 = 49 + 9 = 58$

3 $(L - M)^2 = (L + M)^2 - 4LM = (7)^2 - 4 \times 9 = 49 - 36 = 13$

$\therefore L - M = \sqrt{13}$, where L > M

4 $L^3 - M^3 = (L - M)\left[(L + M)^2 - LM\right]$

by substituting from (3) :

$\therefore L^3 - M^3 = \sqrt{13}\,(7^2 - 9) = \sqrt{13}\,(49 - 9) = 40\sqrt{13}$

Example 4

If the two roots of the equation : $x^2 - 8x + 5 = 0$ are L and M , form the equation whose roots are $\frac{1}{L}$ and $\frac{1}{M}$

Solution

$\because$ L and M are the two roots of the given equation. $\therefore$ $L + M = 8$ and $LM = 5$

$\because$ $\frac{1}{L}$ and $\frac{1}{M}$ are the two roots of the required equation.

$\therefore$ The sum of the two roots $= \frac{1}{L} + \frac{1}{M} = \frac{M + L}{LM} = \frac{8}{5}$

, the product of the two roots $= \frac{1}{L} \times \frac{1}{M} = \frac{1}{LM} = \frac{1}{5}$

$\therefore$ The required equation is $x^2 - \frac{8}{5}x + \frac{1}{5} = 0$

i.e. $5x^2 - 8x + 1 = 0$

Example 5

If L and M are the two roots of the equation :

$x^2 - 5x + 9 = 0$, find the equation whose roots are L^2 and M^2

Solution

$\because$ L and M are the two roots of the given equation. $\therefore$ $L + M = 5$ and $LM = 9$

$\because$ L^2 and M^2 are the two roots of the required equation.

$\therefore$ The sum of the two roots $= L^2 + M^2 = (L + M)^2 - 2\,LM = 5^2 - 2 \times 9 = 7$

, the product of the two roots $= L^2 \times M^2 = (LM)^2 = 9^2 = 81$

$\therefore$ The required equation is $x^2 - 7x + 81 = 0$

Example 6

If L and M are the two roots of the equation :

$3x^2 + 5x - 7 = 0$, find the equation whose roots are $L + \frac{1}{M}$, $M + \frac{1}{L}$

Solution

$\because$ L and M are the two roots of the given equation.

$\therefore$ $L + M = -\frac{5}{3}$ and $LM = \frac{-7}{3}$

, $\because$ $L + \frac{1}{M}$, $M + \frac{1}{L}$ are the two roots of the required equation.

$\therefore$ The sum of the two roots $= L + \frac{1}{M} + M + \frac{1}{L} = L + M + \frac{L + M}{LM}$

$$= \frac{-5}{3} + \frac{\frac{-5}{3}}{\frac{-7}{3}} = \frac{-5}{3} + \frac{5}{7} = \frac{-35 + 15}{21} = -\frac{20}{21}$$

, the product of the two roots $= \left(L + \frac{1}{M}\right)\left(M + \frac{1}{L}\right) = LM + \frac{1}{LM} + 2$

$$= \frac{-7}{3} - \frac{3}{7} + 2 = \frac{-49 - 9 + 42}{21} = \frac{-16}{21}$$

$\therefore$ The required equation is $x^2 - \frac{-20}{21}x + \frac{-16}{21} = 0$

i.e. $21x^2 + 20x - 16 = 0$

TRY TO SOLVE

If L , M are the two roots of the equation :

$2x^2 - 3x - 1 = 0$, find the equation whose roots are L^2 , M^2

Example 7

If $\frac{2}{L}$, $\frac{2}{M}$ are the two roots of the equation : $x^2 - 6x + 4 = 0$,

find the equation whose roots are L , M

Solution

$\because \frac{2}{L}$, $\frac{2}{M}$ are the two roots of the given equation.

$\therefore \frac{2}{L} \times \frac{2}{M} = 4$ $\quad \therefore \frac{4}{LM} = 4$ $\quad \therefore LM = 1$

, $\frac{2}{L} + \frac{2}{M} = 6$ $\quad \therefore \frac{2L + 2M}{LM} = 6$ $\quad \therefore \frac{2(L+M)}{1} = 6$ $\quad \therefore L + M = \frac{6}{2} = 3$

, $\because$ L and M are the two roots of the required equation , $L + M = 3$, $LM = 1$

$\therefore$ The required equation is $x^2 - 3x + 1 = 0$

TRY TO SOLVE

If $\frac{1}{L}$ and $\frac{1}{M}$ are the two roots of the equation : $6x^2 - 5x + 1 = 0$,

find the equation whose roots are L and M

Example 8

If the difference between the two roots of the equation : $x^2 - kx + 4k = 0$ equals three times the product of the two roots of the equation : $x^2 - 3x - k = 0$, find the value of k

Solution

Let L and M be the two roots of the equation : $x^2 - kx + 4k = 0$

$\therefore L + M = k$, $LM = 4k$

, $\because$ the difference between L and M equals three times the product of the two roots of the equation : $x^2 - 3x - k = 0$ $\quad \therefore L - M = -3k$

$\because (L - M)^2 = (L + M)^2 - 4\ LM$ (from the previous identities)

$\therefore (-3\ k)^2 = k^2 - 4\ (4\ k)$ $\quad\therefore 9\ k^2 = k^2 - 16\ k$ $\quad\therefore 8\ k^2 + 16\ k = 0$

$\therefore 8\ k\ (k + 2) = 0$ $\quad\therefore k = 0$ or $k + 2 = 0$ $\quad\therefore k = -2$

Another solution :

By using the law of the difference between the two roots :

$\because\ L - M = \frac{\pm\sqrt{\text{the discriminant}}}{a} = \frac{\pm\sqrt{b^2 - 4\ ac}}{a}$ and from the equation :

$x^2 - k\ x + 4\ k = 0$, we found that :

$L - M = \pm\sqrt{k^2 - 16\ k}$ (1)

, $\because$ L – M equals three times the product

of the two roots of : $x^2 - 3\ x - k = 0$

$\therefore L - M = -3\ k$ (2)

, from (1) , (2) :

$\therefore \pm\sqrt{k^2 - 16\ k} = -3\ k$, by squaring both sides

$\therefore k^2 - 16\ k = 9\ k^2$ $\quad\therefore 8\ k^2 + 16\ k = 0$ $\quad\therefore k = 0$ or $k = -2$

Remark

It is possible to deduce the law of the difference between the two roots from the general formula with the same method used for finding the sum of the two roots in the previous lesson.

TRY TO SOLVE

If the difference between the two roots of the equation : $x^2 + k\ x + 2\ k = 0$ equals twice the product of the two roots of the equation : $6\ x^2 + 5\ x + k = 0$, find the value of k

Lesson 5

Sign of a function

Investigating the sign of a function

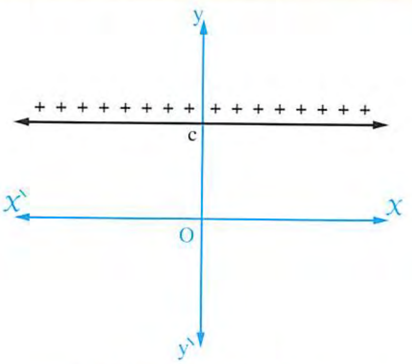
Investigating the sign of a function f in the variable X is to determine the values of X at which the values of the function f are as follows :

- Positive , i.e. $f(X) > 0$
- Negative , i.e. $f(X) < 0$
- Equal to zero , i.e. $f(X) = 0$

First The sign of the constant function

The following figures represent the two functions :

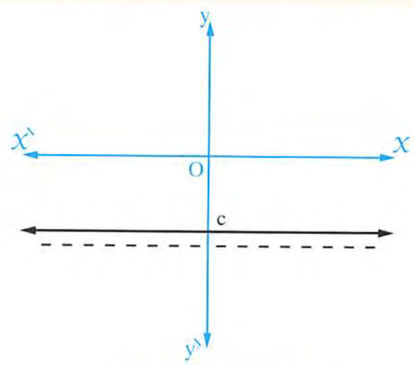
$f : f(X) = c$ (where c is positive)



We notice that

The function is positive for all $X \in \mathbb{R}$

$f : f(X) = c$ (where c is negative)



We notice that

The function is negative for all $X \in \mathbb{R}$

From the previous , we deduce that :

The sign of the constant function $f : f(X) = c$, $c \in \mathbb{R}^*$ is the same sign of c $\forall\ X \in \mathbb{R}$

Notice that

The symbol $\forall$ means "for every"

For example :

- If $f(X) = 5$, then the sign of the function f is positive for all $X \in \mathbb{R}$
- If $f(X) = -3$, then the sign of the function f is negative for all $X \in \mathbb{R}$

TRY TO SOLVE

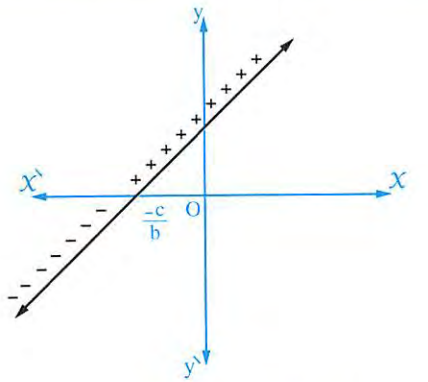
Determine the sign of each of the following two functions :

1 $f : f(X) = 10$ 2 $f : f(X) = -\frac{2}{5}$

Second The sign of the first degree function (linear function)

The following figures represent the two functions :

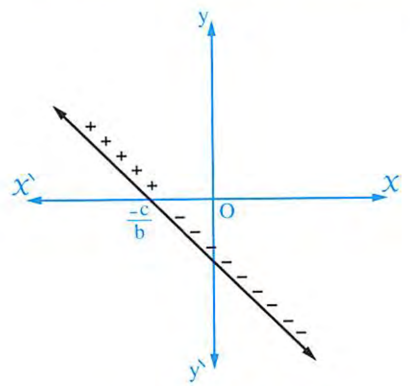
$f : f(X) = b\,X + c$ (b is positive)



We notice that the sign of the function :

- is the same as the sign of b (**positive**) at $X > \frac{-c}{b}$
- is opposite to the sign of b (**negative**) at $X < \frac{-c}{b}$
- equals **zero** at $X = \frac{-c}{b}$

$f : f(X) = b\,X + c$ (b is negative)



We notice that the sign of the function :

- is the same as the sign of b (**negative**) at $X > \frac{-c}{b}$
- is opposite to the sign of b (**positive**) at $X < \frac{-c}{b}$
- equals **zero** at $X = \frac{-c}{b}$

From the previous , we deduce that :

To find the sign of the linear function $f : f(X) = \text{b}\,X + \text{c}$, $\text{b} \neq 0$, we put $f(X) = 0$

$\therefore \text{b}\,X + \text{c} = 0$ $\qquad \therefore X = \frac{-\text{c}}{\text{b}}$

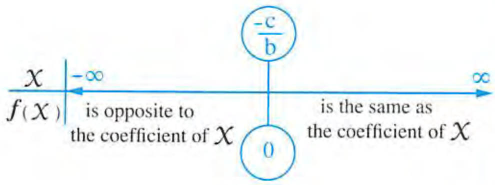
$\therefore$ The sign of the function f :

1 Is the same as the sign of b at $X > \frac{-\text{c}}{\text{b}}$

2 Is opposite to the sign of b at $X < \frac{-\text{c}}{\text{b}}$

3 $f(X) = 0$ at $X = \frac{-\text{c}}{\text{b}}$

And we illustrate this on the opposite number line.



Example 1

Determine the sign of each of the following two functions using the number line :

1 $f : f(X) = 3X + 6$ $\qquad$ **2** $f : f(X) = 1 - \frac{1}{2}X$

Solution

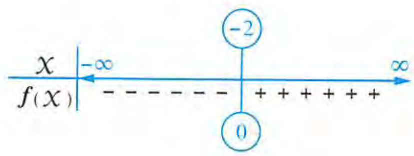
1 $\because f(X) = 3X + 6$ $\qquad$ put $f(X) = 0$

$\therefore 3X + 6 = 0$ $\qquad \therefore X = -2$

$\therefore$ The sign of the function f is :

- positive at $X > -2$
- negative at $X < -2$
- $f(X) = 0$ at $X = -2$

We illustrate the solution on the opposite number line.



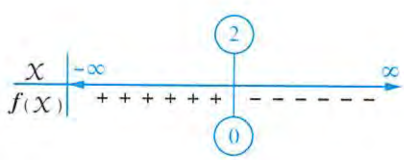
2 $\because f(X) = -\frac{1}{2}X + 1$ $\qquad$ put $f(X) = 0$

$\therefore -\frac{1}{2}X = -1$ $\qquad \therefore X = 2$

$\therefore$ The sign of the function f is :

- negative at $X > 2$
- positive at $X < 2$
- $f(X) = 0$ at $X = 2$

We illustrate the solution on the opposite number line.



TRY TO SOLVE

Determine the sign of each of the following two functions :

1 $f : f(X) = -3X + 6$ **2** $f : f(X) = 2 + \frac{1}{2}X$

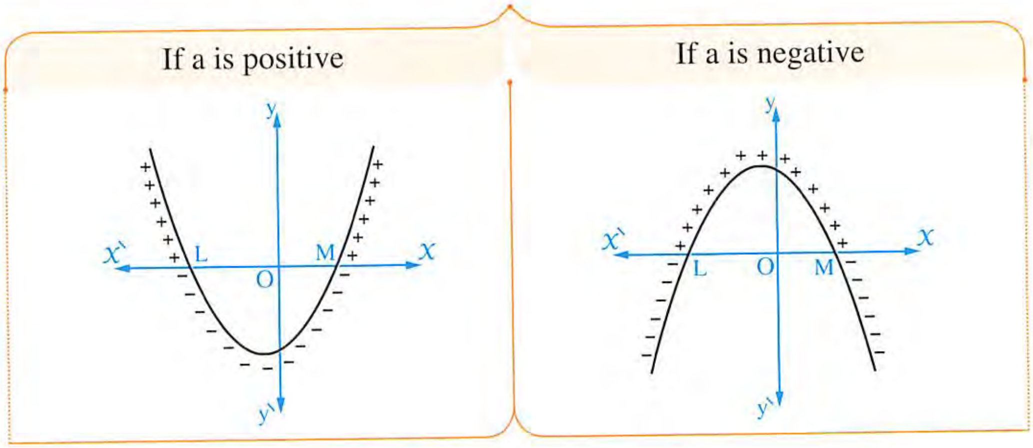
Third The sign of the second degree function (quadratic function)

To determine the sign of the quadratic function $f : f(X) = aX^2 + bX + c$, $a \neq 0$

, we have to obtain the discriminant of the equation : $aX^2 + bX + c = 0$, there are three cases :

1 The discriminant : $b^2 - 4ac > 0$

The equation has two real roots , let them be L , M where L < M



The sign of the function is as follows :

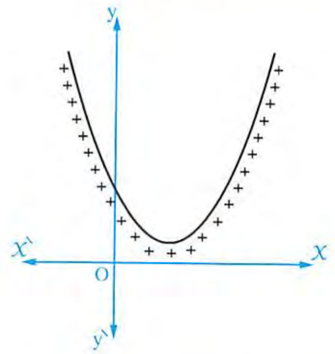
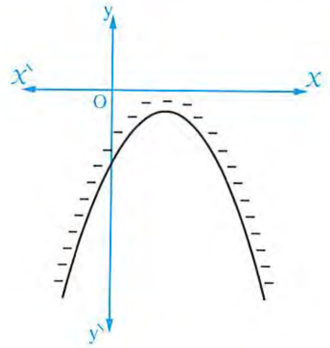
- Is the same as the sign of a at $X \in \mathbb{R} - [L , M]$
- Is opposite to the sign of a at $X \in]L , M[$
- Equals zero at $X \in \{L , M\}$

And we illustrate this on the opposite number line.

X	$-\infty$		L		M		∞
$f(X)$		the same as the sign of a	0	opposite to the sign of a	0	the same as the sign of a	

2 The discriminant : $b^2 - 4ac < 0$

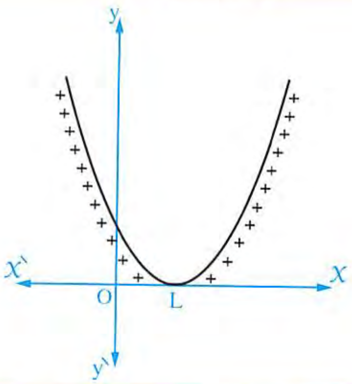
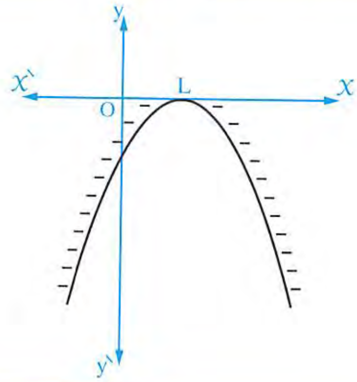
There is no real roots for the equation and thus the sign of the function is as follows :

If a is positive	If a is negative
	

The sign of the function is the same as the sign of a $\forall\ x \in \mathbb{R}$

3 The discriminant : $b^2 - 4ac = 0$

There are two equal roots for the equation , let each of them be L

If a is positive	If a is negative
	

The sign of the function is as follows :

- Is the same as a at $x \neq L$
- Is equal to zero at $x = L$

We can illustrate this on the opposite number line.

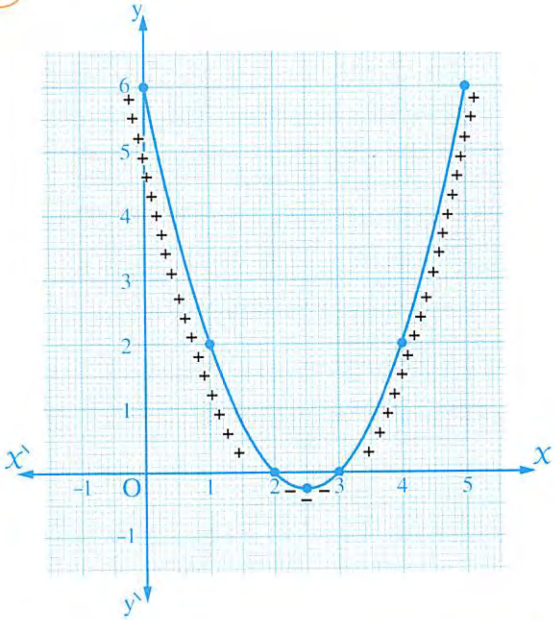
x	$-\infty$		L		∞
$f(x)$		is the same as the sign of a	0	is the same as the sign of a	

Example 2

Draw the graph of the function : $f : f(X) = X^2 - 5X + 6$ in the interval $[0, 5]$, from the graph determine the sign of the function f in $\mathbb{R}$

Solution

X	0	1	2	2.5	3	4	5
$f(X)$	6	2	0	−0.25	0	2	6



From the graph, we notice that the sign of f is :

- Positive at $X \in \mathbb{R} - [2, 3]$
- Negative at $X \in]2, 3[$
- $f(X) = 0$ at $X \in \{2, 3\}$

Remark

If the required is investigating the sign of the function in the given interval, then the sign of f is :

- Positive at $X \in [0, 2[\cup]3, 5]$ or $[0, 5] - [2, 3]$
- Negative at $X \in]2, 3[$
- $f(X) = 0$ at $X \in \{2, 3\}$

Remember that

In the previous example :

- The domain of the function f is the set of the real numbers $\mathbb{R}$
- The range of the function f is $[-0.25, \infty[$
- The vertex of the curve is $(2.5, -0.25)$ and the function has a minimum value equals -0.25
- The symmetry axis equation is $X = 2.5$

Example 3

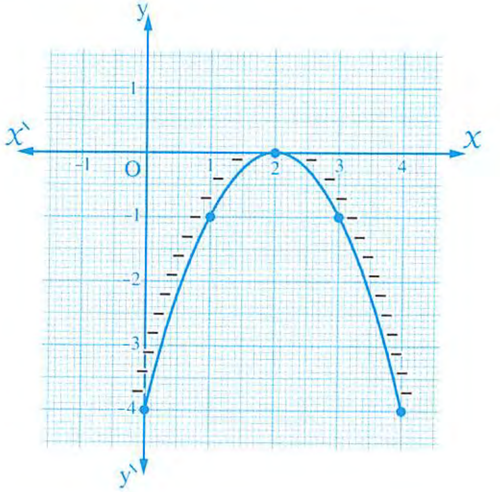
Draw the graph of the function :

$f : f(X) = -X^2 + 4X - 4$ in the interval $[0, 4]$

, from the graph determine the sign of the function f in $\mathbb{R}$

Solution

X	0	1	2	3	4
$f(X)$	-4	-1	0	-1	-4



From the graph , we notice that :

- $f(X) = 0$ at $X = 2$
- The sign of f is negative at $X \in \mathbb{R} - \{2\}$

Example 4

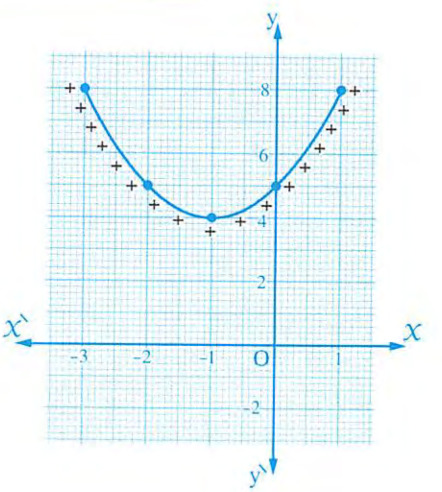
Draw the graph of the function :

$f : f(X) = X^2 + 2X + 5$ in the interval $[-3, 1]$

, from the graph determine the sign of the function f in $\mathbb{R}$

Solution

X	-3	-2	-1	0	1
$f(X)$	8	5	4	5	8



From the graph , we notice that the sign of the function f is positive $\forall X \in \mathbb{R}$

TRY TO SOLVE

Draw the graph of the function :

$f : f(X) = X^2 - 2X - 3$ in the interval $[-2, 4]$, from the graph determine the sign of f in $\mathbb{R}$

Example 5

Determine the sign of each of the following functions , showing that on the number line :

1 $f : f(X) = X^2 + 2X - 3$

2 $f : f(X) = X^2 - 3X + 5$

3 $f : f(X) = 4X^2 - 12X + 9$

4 $f : f(X) = 9 + 2X - X^2$

Solution

1 $\because$ The discriminant $= b^2 - 4ac = 4 - 4 \times 1 \times (-3) = 4 + 12 = 16$ (> zero)

$\therefore$ The equation $X^2 + 2X - 3 = 0$ has two roots.

By factorization $\therefore (X + 3)(X - 1) = 0$

$\therefore X = -3$ or $X = 1$

$\because$ a (coefficient of X^2) $= 1 > 0$

$\therefore$ The sign of the function f is :

- positive at $X \in \mathbb{R} - [-3, 1]$
- negative at $X \in]-3, 1[$
- $f(X) = 0$ at $X \in \{-3, 1\}$

X	$-\infty$		-3		1		∞
$f(X)$		+ + + + +	0	− − − − −	0	+ + + + +	

2 $\because$ The discriminant $= b^2 - 4ac = 9 - 4 \times 1 \times 5 = 9 - 20 = -11$ (< zero)

$\therefore$ The equation : $X^2 - 3X + 5 = 0$ has no real roots

, $\because a = 1 > 0$

$\therefore$ The sign of the function f is positive $\forall X \in \mathbb{R}$

X	$-\infty$		∞
$f(X)$		+ + + + + + + + + + + + + + + + +	

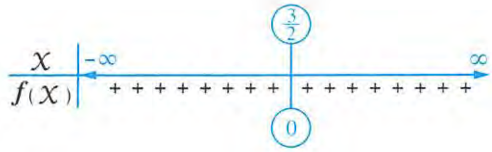
3 $\because$ The discriminant $= b^2 - 4ac = 144 - 4 \times 4 \times 9 = 144 - 144 = 0$

$\therefore$ The equation : $4X^2 - 12X + 9 = 0$ has two equal roots

By factorization : $\therefore (2X - 3)^2 = 0$ $\quad \therefore X = \frac{3}{2}$

, $\because$ a = 4 > 0

$\therefore$ The sign of the function f is :



- positive at $X \in \mathbb{R} - \left\{\frac{3}{2}\right\}$
- $f(X) = 0$ at $X = \frac{3}{2}$

4 $\because$ The discriminant = $b^2 - 4ac = 4 - 4 \times (-1) \times 9 = 40$ (> zero)

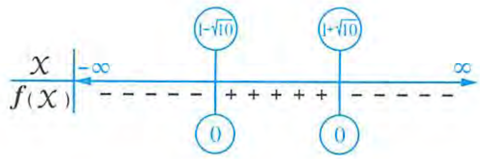
$\therefore$ The equation : $9 + 2X - X^2 = 0$ has two roots.

By using the general formula

$$\therefore X = \frac{-2 \pm \sqrt{40}}{-2} = \frac{-2 \pm 2\sqrt{10}}{-2} = 1 \pm \sqrt{10}$$

, $\because$ a (coefficient of X^2) = $-1 < 0$ $\therefore$ The sign of the function f is :

- negative at $X \in \mathbb{R} - \left[1 - \sqrt{10}, 1 + \sqrt{10}\right]$
- positive at $X \in \left]1 - \sqrt{10}, 1 + \sqrt{10}\right[$
- $f(X) = 0$ at $X \in \left\{1 - \sqrt{10}, 1 + \sqrt{10}\right\}$



TRY TO SOLVE

Determine the sign of each of the following functions :

1 $f : f(X) = X^2 - X - 6$

2 $f : f(X) = -X^2 - 4X - 4$

3 $f : f(X) = X^2 - 4X + 5$

Example 6

If $f : f(X) = X - 1$, $g : g(X) = X^2 + X - 6$

, find the interval at which the two functions f , g are positive together , also the interval at which f , g are negative together.

Solution

$\because f(X) = X - 1$ $\therefore f(X) = 0$ at $X = 1$

, f is positive at $X > 1$ i.e. In the interval $]1, \infty[$

, f is negative at $X < 1$ i.e. In the interval $]-\infty, 1[$

, $\because$ g $(X) = X^2 + X - 6$,

We get the two roots of the equation $X^2 + X - 6 = 0$ as follows :

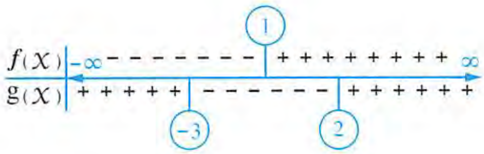
$(X - 2)(X + 3) = 0$ $\quad\quad\therefore X = 2$ or $X = -3$

$\therefore$ g $(X) = 0$ at $X \in \{2, -3\}$

, g is positive at $X \in \mathbb{R} - [-3, 2]$

, g is negative at $X \in]-3, 2[$

By noticing the opposite figure we find :



- f , g are positive together in the interval

  $]2, \infty[$ which is the interval representing $]1, \infty[\cap \mathbb{R} - [-3, 2]$

- f , g are negative together at $]-3, 1[$ which is equal to $]-\infty, 1[\cap]-3, 2[$

TRY TO SOLVE

Determine the sign of each of the functions : $f_1 : f_1(X) = 2 - X$ and $f_2 : f_2(X) = X^2 - 9X + 18$ and when their signs are negative together.

Example 7

Prove that for all values of $X \in \mathbb{R}$ the two roots of the equation : $X^2 + 2kX + k - 2 = 0$ are real and different.

Solution

$\because X^2 + 2kX + k - 2 = 0$

$\therefore a = 1$, $b = 2k$, $c = k - 2$

$\therefore$ The discriminant $= b^2 - 4ac = (2k)^2 - 4(k - 2) = 4k^2 - 4k + 8$

and the two roots are real and different if the discriminant is positive ,

thus we investigate the sign of the function

$f : f(k) = 4k^2 - 4k + 8$ as follows :

$\because$ The discriminant $= b^2 - 4ac = (-4)^2 - 4 \times 4 \times 8 = 16 - 128 = -112$ (< zero)

$\therefore$ The equation $4k^2 - 4k + 8 = 0$ has no real roots , $\because a > 0$

$\therefore$ The sign of the function f is positive for all the values of $k \in \mathbb{R}$

$\therefore$ The discriminant of the equation $x^2 + 2kx + k - 2 = 0$ is positive $\forall x \in \mathbb{R}$

Thus the two roots of the equation $x^2 + 2kx + k - 2 = 0$ are real and different $\forall x \in \mathbb{R}$

Another solution :

$\because$ The discriminant of the equation : $x^2 + 2kx + k - 2 = 0$ is $4k^2 - 4k + 8$

$\because 4k^2 - 4k + 8 = 4k^2 - 4k + 1 + 7 = (2k - 1)^2 + 7$ is positive $\forall k \in \mathbb{R}$

$\therefore$ The two roots of the equation $x^2 + 2kx + k - 2 = 0$ are real and different $\forall x \in \mathbb{R}$

Using the Technology

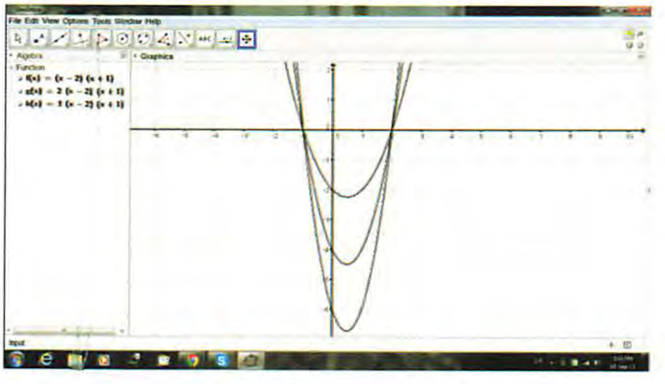
By using the program GeoGebra , draw in one graph the functions defined with the following rules :

1 $f(x) = (x - 2)(x + 1)$

2 $g(x) = 2(x - 2)(x + 1)$

3 $k(x) = 3(x - 2)(x + 1)$

You will get the opposite graph.



From the graph , we notice that the three curves are open upwards and intersect the x-axis at the points $(2, 0)$, $(-1, 0)$ and the solution set of each equation which is related to each function is $\{2, -1\}$

• Try to investigate the sign of each of the previous functions.

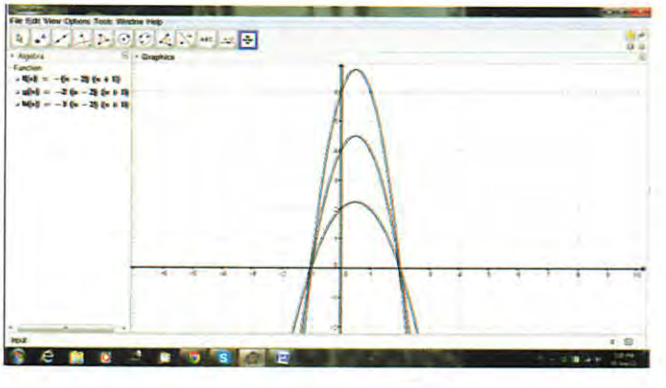
Also, by using the same program draw in one graph the functions defined with the following rules :

1 $f(x) = -(x - 2)(x + 1)$

2 $g(x) = -2(x - 2)(x + 1)$

3 $k(x) = -3(x - 2)(x + 1)$

You will get the opposite graph.



From the graph , we notice that the three curves are open downwards and intersect the X-axis at the previous points (2 , 0) , (– 1 , 0) , the solution set of each equation which is related to each function is the same solution set $\{2 , -1\}$

• Try to investigate the sign of each of the previous functions.

Conclusion

If L , M are the roots of the quadratic equation , then we can form the rule of the function which is related to the quadratic equation on the form :

$f(X) = a(X - L)(X - M)$ where $a \in \mathbb{R} - \{0\}$

• The curve is open upwards if $a > 0$

• The curve is open downwards if $a < 0$

Lesson 6

Quadratic inequalities in one variable

Preface

- You have studied before inequalities of first degree in one variable as :

  $X + 3 > 5$, $4 - 2X \leq 2$
- Solving an inequality means finding all values of the unknown which satisfy this inequality.
- When solving an inequality in $\mathbb{R}$, the solution set is an interval.

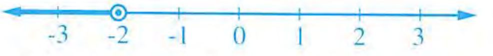
For example :

When solving the inequality : $-2X + 6 > 10$ in $\mathbb{R}$

, we find that : $-2X > 4$ $\therefore X < -2$

$\therefore$ The solution set is the real numbers which are less than -2

i.e. The solution set = $]-\infty , -2[$



- In this lesson , you will learn how to solve the inequalities of second degree in one unknown (quadratic inequalities) in $\mathbb{R}$, as the following inequalities :

  $X^2 - 5X + 6 > 0$, $X^2 + X \geq 2$, $X(X - 6) < -5$

Solving the quadratic inequalities in $\mathbb{R}$

To solve the quadratic inequality in $\mathbb{R}$, follow the following steps :

1. Write the quadratic function related to the inequality.
2. Study the sign of this quadratic function.
3. Determine the intervals which satisfy the inequality.

Example 1

Find in $\mathbb{R}$ the solution set of the inequality : $x^2 - 5x + 6 > 0$

Solution

First : Write the quadratic function related to the inequality as follows :

$f(x) = x^2 - 5x + 6$

Second : Study the sign of f as follows :

$\because$ The discriminant $= b^2 - 4ac = 25 - 4 \times 1 \times 6 = 1$ (> zero)

$\therefore$ The equation : $x^2 - 5x + 6 = 0$ has two different roots.

By factorizing :

$\therefore (x - 2)(x - 3) = 0$

$\therefore x = 2$ or $x = 3$

x	$-\infty$		2		3		∞
$f(x)$		+ + + + +	0	− − − − −	0	+ + + + +	

Third : Determine the intervals which satisfy $x^2 - 5x + 6 > 0$ (positive)

$\therefore$ The solution set $=]-\infty, 2[\cup]3, \infty[$ or $\mathbb{R} - [2, 3]$

Notice that

From the previous example :

The solution set of the inequality : $x^2 - 5x + 6 < 0$ in $\mathbb{R}$ is $]2, 3[$

TRY TO SOLVE

Find in $\mathbb{R}$ the solution set of each of the following inequalities :

1. $x^2 - 2x - 8 > 0$
2. $x^2 - 2x - 8 < 0$

Example 2

Find in $\mathbb{R}$ the solution set of the inequality : $(x + 5)(x - 1) \geq x + 5$

Solution

$\because (x + 5)(x - 1) \geq x + 5$ $\quad \therefore x^2 + 4x - 5 \geq x + 5$ $\quad \therefore x^2 + 3x - 10 \geq 0$

First : Write the quadratic function related to the inequality as follows :

$f(X) = X^2 + 3X - 10$

Second : Study the sign of the function f as follows :

$\because$ The discriminant $= b^2 - 4ac = 9 - 4 \times 1 \times (-10) = 49$ (> zero)

$\therefore$ The equation $X^2 + 3X - 10 = 0$ has two different roots

By factorizing :

$\therefore (X - 2)(X + 5) = 0$

$\therefore X = 2$ or $X = -5$

X	$-\infty$		-5		2		∞
$f(X)$		+ + + +	0	− − − − − − −	0	+ + + +	

Third : Determine the intervals which satisfy that : $X^2 + 3X - 10 \geq 0$

$\therefore$ The solution set =

$]-\infty, -5] \cup [2, \infty[$ or $\mathbb{R} -]-5, 2[$

Notice that

From the previous example :

The solution set of the inequality : $(X + 5)(X - 1) \leq X + 5$ in $\mathbb{R}$ is $[-5, 2]$

TRY TO SOLVE

Find in $\mathbb{R}$ the solution set of each of the following inequalities :

1 $2X^2 + 5X \geq 3$

2 $X(X + 6) < 4X + 15$

Example 3

Find in $\mathbb{R}$ the solution set of each of the following inequalities :

1 $X^2 - 3X + 5 < 0$

2 $X^2 + 2X + 4 > 0$

3 $4X - X^2 - 4 < 0$

4 $X^2 - 6X + 9 \leq 0$

Solution

1 By putting $f(X) = X^2 - 3X + 5$ and investigating the sign of the function f, we find that :

The discriminant $= b^2 - 4ac = 9 - 4 \times 1 \times 5 = -11 < 0$

$\therefore$ The equation : $X^2 - 3X + 5 = 0$ has no real roots.

$\because a = 1 > 0$

$\therefore$ The sign of the function f is positive for every $x \in \mathbb{R}$

$\therefore$ The solution set of the inequality : $x^2 - 3x + 5 < 0$ is $\varnothing$

2 By putting $f(x) = x^2 + 2x + 4$ and investigating the sign of the function f , we find that :

The discriminant $= b^2 - 4ac = 4 - 4 \times 1 \times 4 = -12 < 0$

$\therefore$ The equation : $x^2 + 2x + 4 = 0$ has no real roots

, $\because a = 1 > 0$

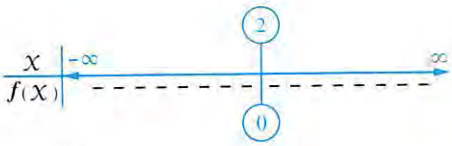
$\therefore$ The sign of the function f is positive for every $x \in \mathbb{R}$

$\therefore$ The solution set of the inequality : $x^2 + 2x + 4 > 0$ is $\mathbb{R}$

3 By putting $f(x) = 4x - x^2 - 4$ and investigating the sign of f , we find that :

The discriminant $= b^2 - 4ac = 16 - 4 \times (-1) \times (-4) = 0$

$\therefore$ The equation : $4x - x^2 - 4 = 0$ has two equal roots



By factorization : $\therefore (x - 2)^2 = 0$ $\therefore x = 2$

$\because a = -1 < 0$

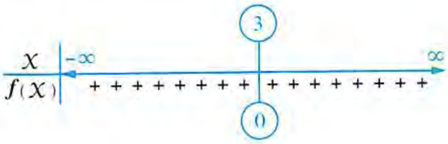
$\therefore$ The function is negative at $x \in \mathbb{R} - \{2\}$, $f(x) = 0$ at $x = 2$

$\therefore$ The solution set of the inequality : $4x - x^2 - 4 < 0$ is $\mathbb{R} - \{2\}$

4 By putting $f(x) = x^2 - 6x + 9$ and investigating the sign of f , we find that :

The discriminant $= b^2 - 4ac = 36 - 4 \times 1 \times 9 = 0$

$\therefore$ The equation : $x^2 - 6x + 9 = 0$ has two equal roots



By factorization : $\therefore (x - 3)^2 = 0$ $\therefore x = 3$

$\because a = 1 > 0$

$\therefore$ The function is positive at $x \in \mathbb{R} - \{3\}$, $f(x) = 0$ at $x = 3$

$\therefore$ The solution set of the inequality : $x^2 - 6x + 9 \leq 0$ is $\{3\}$

TRY TO SOLVE

Find in $\mathbb{R}$ the solution set of each of the following inequalities :

1 $x^2 + x + 12 > 0$

2 $-x^2 + x - 1 > 0$

3 $x^2 - 2x + 1 > 0$

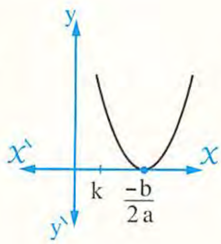
4 $10x - x^2 - 25 \leq 0$

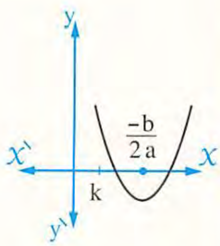
Remarks

If the quadratic equation $a\,x^2 + b\,x + c = 0$

where f is the related function with it , then :

1 Conditions that each of the two roots of the equation is greater than a real number k are :





- $b^2 - 4\,a\,c \geq 0$
- $a\,f(k) > 0$
- $\frac{-b}{2\,a} > k$

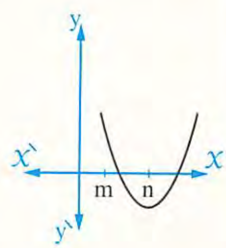
For example :

If each of the two roots of the equation $x^2 - 5\,x + m = 0$ is greater than 2 , then :

- $25 - 4\,m \geq 0$ $\qquad \therefore\ m \leq 6\frac{1}{4}$
- $4 - 5\,(2) + m > 0$ $\qquad \therefore\ m > 6$
- $\frac{5}{2} > 2$ "satisfied for all values of m"

, then to satisfy the 3 conditions : $6 < m \leq 6\frac{1}{4}$

2 Conditions that only one of the two roots of the equation lies between the two real numbers m , n is : $f(m) \times f(n) <$ zero



For example :

If only one root of the equation $x^2 - b\,x + 12 = 0$ is belong to the interval $]1\ ,4[$

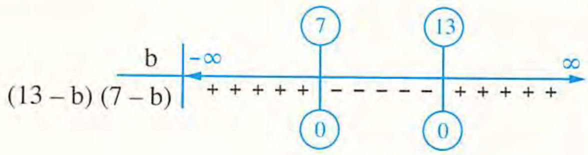
, then $f(1) \times f(4) < 0$

$\therefore (1 - b + 12)(16 - 4\,b + 12) < 0$

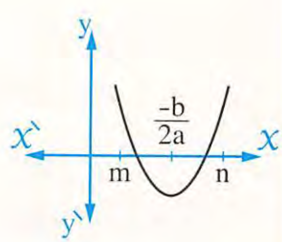
$\therefore (13 - b)(28 - 4\,b) < 0$

$\therefore (13 - b)(7 - b) < 0$

$\therefore\ b \in\ ]7\ ,13[$



3 Conditions that the two roots of the equation are lying between the two real numbers m , n where m < n are :



- $b^2 - 4\,a\,c \geq 0$
- $a\,f(m) > 0$
- $a\,f(n) > 0$
- $m < \frac{-b}{2\,a} < n$

For example :

If the two roots of the equation $4x^2 - 2x + h = 0$

are elements of the interval $]-1 , 1[$, then :

- $4 - 4 \times 4 \times h \geq 0$ $\quad \therefore h \leq \frac{1}{4}$ ①
- $4 f(-1) > 0$ $\quad \therefore 4 \times (4 + 2 + h) > 0$ $\quad \therefore h > -6$ ②
- $4 f(1) > 0$ $\quad \therefore 4 (4 - 2 + h) > 0$ $\quad \therefore h > -2$ ③
- $-1 < \frac{2}{2 \times 4} < 1$ satisfies for all values of h ④

From ① , ② , ③ and ④ $\quad \therefore -2 \leq h \leq \frac{1}{4}$

Unit Two

Trigonometry.

Unit Lessons

Lesson 1 — Directed angle.

Lesson 2 — Systems of measuring angle (Degree measure - radian measure).

Lesson 3 — Trigonometric functions.

Lesson 4 — Related angles.

Lesson 5 — Graphing trigonometric functions.

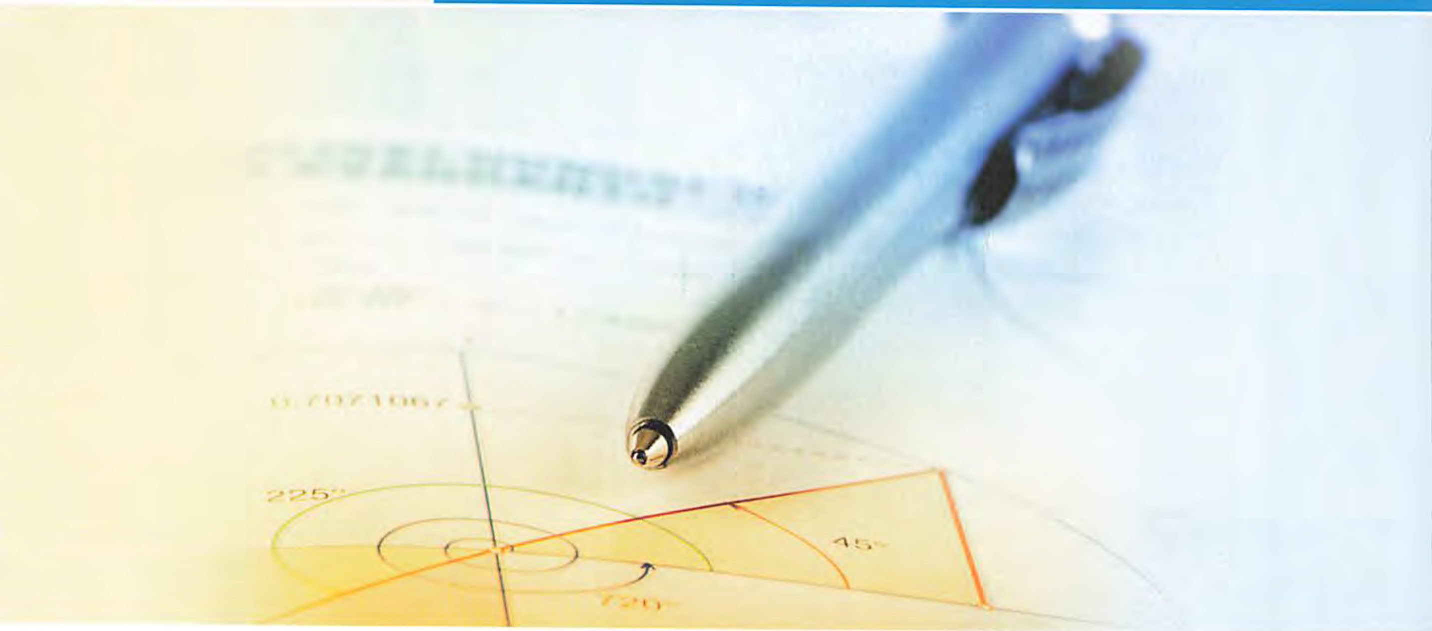
Lesson 6 — Finding the measure of an angle given the value of one of its trigonometric ratios.

Learning outcomes

By the end of this unit, the student should be able to :

- Recognize the directed angle.
- Recognize the positive measure and negative measure of the directed angle.
- Recognize the standard position of the directed angle.
- Recognize the concept of the equivalent angles.
- Determine the quadrant that the directed angle in its standard position lies.
- Recognize the radian measure of a central angle in a circle.
- Convert a degree measure of an angle into a radian measure and vice versa.
- Recognize signs of trigonometric functions in each quadrant.
- Find trigonometric functions of some related angles of a special angle.
- Use calculator to find trigonometric ratios.
- Use calculator to carry out special arithmetic operations of converting degree measure into radian measure and vice versa.
- Graph trigonometric functions (Sine - Cosine).
- Use computer to graph trigonometric functions.
- Solve life applications using trigonometric functions.
- Find the measure of an angle given one of its trigonometric ratios.

Lesson 1 Directed angle

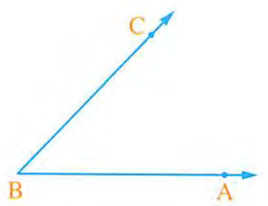


- We have studied that the angle is the union of two rays with a common starting point.

In the opposite figure :

If $\overrightarrow{BA}$, $\overrightarrow{BC}$ are two rays with a common starting point B , then

$\overrightarrow{BA} \cup \overrightarrow{BC} = \angle ABC$ and the two rays $\overrightarrow{BA}$, $\overrightarrow{BC}$ are called the sides of the angle and the point B is the vertex of the angle.



- As we knew ordering the sides of the angle is not important.

  We can write $\angle$ ABC or $\angle$ CBA to express the same angle.

- In this lesson , we will study a new concept which is "*directed angle*" and some related subjects.

Directed angle

If we take into account the order of the angle sides , such that one of them is the initial side and the other is the terminal side , then the angle is written as "*an ordered pair*" whose first projection is the initial side and the second projection is the terminal side.

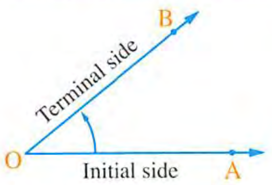
The angle in this case is called "*directed angle*" , its agreed to draw an arrow between its two sides comes out of the initial side to the terminal side.

Definition of the directed angle

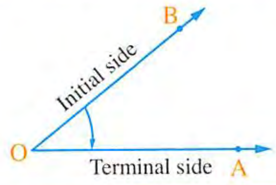
The directed angle is an ordered pair of two rays called the sides of the angle with a common starting point called the vertex.

If $\overrightarrow{OA}$, $\overrightarrow{OB}$ are the two sides of an angle whose vertex is "O" , then :

The ordered pair $(\overrightarrow{OA}, \overrightarrow{OB})$ represents the directed angle $\angle$ AOB , whose initial side is $\overrightarrow{OA}$, and terminal side is $\overrightarrow{OB}$



The ordered pair $(\overrightarrow{OB}, \overrightarrow{OA})$ represents the directed angle $\angle$ BOA whose initial side is $\overrightarrow{OB}$, and terminal side is $\overrightarrow{OA}$

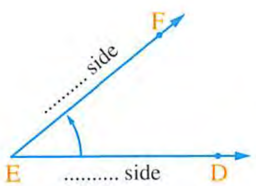


From the previous , we deduce that :

directed angle $\angle$ AOB $\neq$ directed angle $\angle$ BOA because $(\overrightarrow{OA}, \overrightarrow{OB}) \neq (\overrightarrow{OB}, \overrightarrow{OA})$

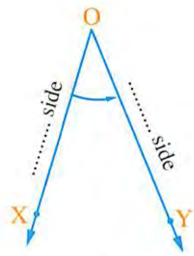
Check your understanding *Complete :*

1



$(\overrightarrow{ED}, \overrightarrow{EF})$ represents the directed angle $\angle$ ....

2



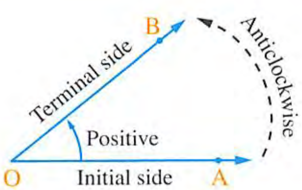
(.... , ...) represents the directed angle $\angle$ XOY

Positive and negative measures of a directed angle

The measure of the directed angle is

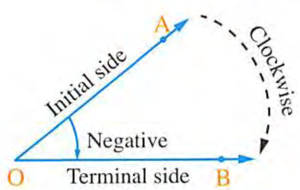
Positive

If the direction of the rotation from the initial side to the terminal side is *anticlockwise*



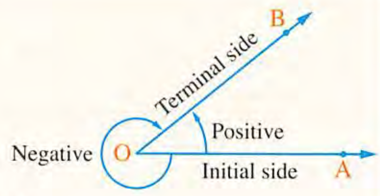
Negative

If the direction of the rotation from the initial side to the terminal side is *clockwise*



Remark

Each non zero directed angle has two measures, one is positive and the other is negative such that the sum of the absolute values of the two measures equals 360°

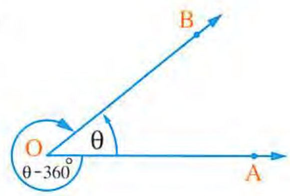


i.e. | Positive measure of the directed angle |

+ | Negative measure of the same directed angle | = 360°

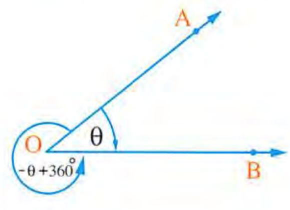
So that :

1 If the positive measure of the directed angle = θ , then the negative measure of the same directed angle = θ – 360°



For example : The negative measure of the directed angle of measure 210° = 210° – 360° = – 150°

2 If the negative measure of the directed angle = – θ , then the positive measure of the same angle = – θ + 360°



For example : The positive measure of the directed angle of measure (– 120°) = – 120° + 360° = 240°

TRY TO SOLVE

Find :

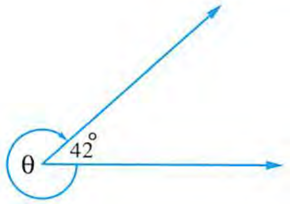
1 The positive measure of the directed angle whose measure is (– 170°)

2 The negative measure of the directed angle whose measure is 320°

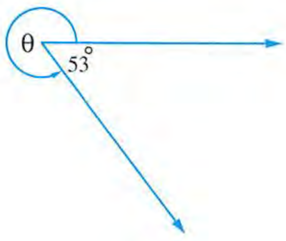
Example 1

Find the measure of the directed angle θ in each of the following figures :

1



2



Solution

1 ∵ The rotation direction is clockwise

∴ The measure of the angle is negative

∴ $\theta = 42° - 360° = -318°$

2 ∵ The rotation direction is anticlockwise

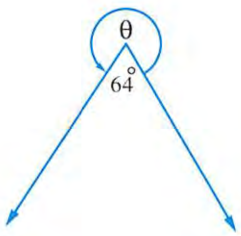
∴ The measure of the angle is positive

∴ $\theta = -53° + 360° = 307°$

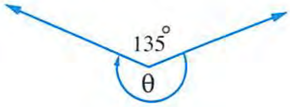
TRY TO SOLVE

Find the measure of the directed angle θ in each of the following figures :

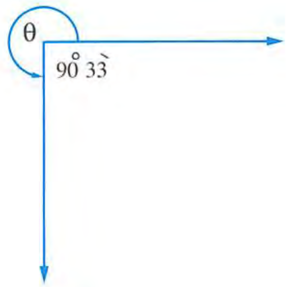
1



2



3



The standard position of the directed angle

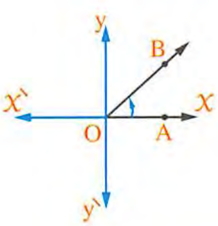
A directed angle is in the standard position if the following two conditions are satisfied :

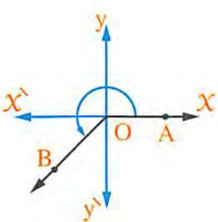
1 Its initial side lies on the positive direction of the X-axis.

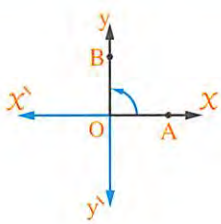
2 Its vertex is the origin point of an orthogonal coordinate plane.

So that :

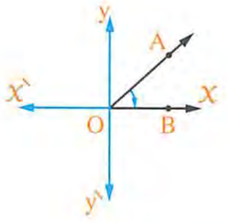
- All the following directed angles are **in the standard position** because they verfiy the two conditions :

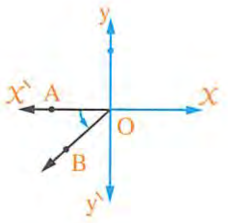


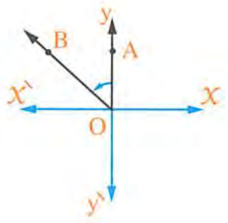




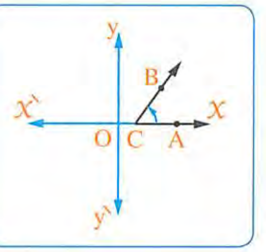
- All the following directed angles are **not in the standard position** because the initial side does not lie on $\overrightarrow{OX}$







- The directed angle in the opposite figure is **not in the standard position** because its vertex is not the origin point O



Equivalent angles

- **If we notice the directed angles in the standard position in the following figures :**

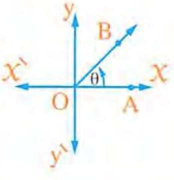


Fig. (1)

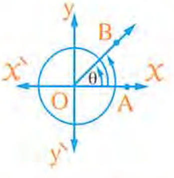


Fig. (2)

Fig. (3)

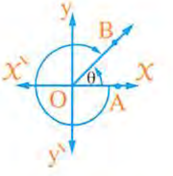


Fig. (4)

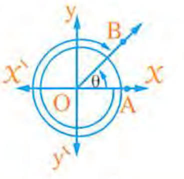


Fig. (5)

We notice the following :

1. The angles in the five figures have the same terminal side $\overrightarrow{OB}$

2. The measure of the angle in fig. (1) = θ ,

   The measure of the angle in fig. (2) = $\theta + 360°$,

   The measure of the angle in fig. (3) = $\theta + 2 \times 360°$,

   The measure of the angle in fig. (4) = $-(360° - \theta) = \theta - 360°$,

   The measure of the angle in fig. (5) = $-(2 \times 360° - \theta) = \theta - 2 \times 360°$

From this , we can conclude :

If θ is the measure of a directed angle in the standard position, then the angles whose measures are :

$(\theta \pm 360°)$, $(\theta \pm 2 \times 360°)$, $(\theta \pm 3 \times 360°)$ ... , $(\theta \pm n \times 360°)$, such that n is an positive integer have common terminal side.

These angles that have common terminal side are called "equivalent angles".

Definition of the equivalent angles

Several directed angles in the standard position are said to be equivalent when they have one common terminal side.

Example 2

Determine two angles , one with positive measure and the other with negative measure having common terminal side for :

1 $100°$

2 $-250°$

Solution

1 An angle with positive measure $= 100° + 360° = 460°$

An angle with negative measure $= 100° - 360° = -260°$

2 An angle with positive measure $= -250° + 360° = 110°$

An angle with negative measure $= -250° - 360° = -610°$

Notice that

There are an infinite number of other positive and negative measures of angles having common terminal side.

Example 3

Determine the smallest positive measure for each of the angles whose measures are as follows :

1 $-62°$

2 $-225°$

3 $530°$

4 $-790°$

Solution

1 The smallest positive measure $= -62° + 360° = 298°$

2 The smallest positive measure $= -225° + 360° = 135°$

3 The smallest positive measure $= 530° - 360° = 170°$

4 The smallest positive measure $= -790° + 3 \times 360° = 290°$

TRY TO SOLVE

1 **Determine a negative measure for each of :**

(1) 72°　　　　**(2)** 1150°

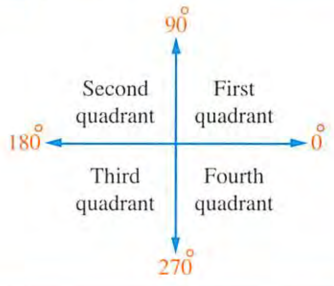
2 **Determine the smallest positive measure for each of :**

(1) – 115°　　　　**(2)** 405°

Angle position in the orthogonal coordinate plane

We know that the orthogonal coordinate plane is divided into four quadrants as in the opposite figure.

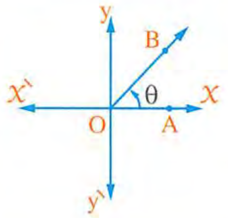
The position of the directed angle is determined by its terminal side when it is in its standard position.



If we draw the directed angle ∠ AOB in the standard position of positive measure θ , then :

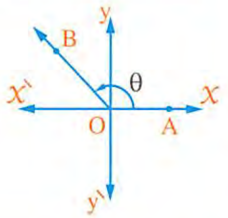
The terminal side $\overrightarrow{OB}$ lies in a quadrant as follows :

First quadrant



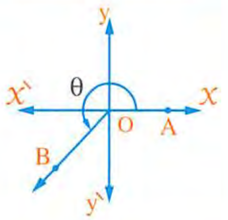
∠ AOB lies in **the first quadrant** $0° < \theta < 90°$

Second quadrant



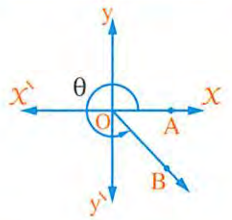
∠ AOB lies in **the second quadrant** $90° < \theta < 180°$

Third quadrant



∠ AOB lies in **the third quadrant** $180° < \theta < 270°$

Fourth quadrant



∠ AOB lies in **the fourth quadrant** $270° < \theta < 360°$

Remark

If the terminal side lies on one of the two axes, then the angle is called "quadrantal angle".

i.e. The angles whose measures are 0° , 90° , 180° , 270° , 360° are quadrantal angles.

Example 4

Determine the quadrant in which each of the directed angles whose measures are as follows lies :

1 $213°$	2 $132°$	3 $-310°$	4 $-12°$
5 $270°$	6 $964°$	7 $-1070°$	

Solution

1 $\because 180° < 213° < 270°$ $\therefore$ The angle lies in the third quadrant.

2 $\because 90° < 132° < 180°$ $\therefore$ The angle lies in the second quadrant.

3 The smallest positive measure $= -310° + 360° = 50°$

$\because 0° < 50° < 90°$

$\therefore$ The angle of measure $50°$ lies in the first quadrant

$\therefore$ The angle of measure $-310°$ also lies in the first quadrant.

Notice that

To determine the quadrant which the directed angle lies in, we have to find the smallest positive measure of it.

4 The smallest positive measure $= -12° + 360° = 348°$

$\because 270° < 348° < 360°$ $\therefore$ The angle of measure $348°$ lies in the fourth quadrant.

$\therefore$ The angle of measure $-12°$ also lies in the fourth quadrant.

5 $270°$ is a quadrantal angle.

6 The smallest positive measure $= 964° - 2 \times 360° = 244°$

, $\because 180° < 244° < 270°$ $\therefore$ The angle of measure $244°$ lies in the third quadrant.

$\therefore$ The angle of measure $964°$ also lies in the third quadrant.

7 The smallest positive measure $= -1070° + 3 \times 360° = 10°$

, $\because 0° < 10° < 90°$ $\therefore$ The angle of measure $10°$ lies in the first quadrant.

$\therefore$ The angle of measure $-1070°$ also lies in the first quadrant.

TRY TO SOLVE

Determine the quadrant in which each of the directed angles whose measures are as follows lies :

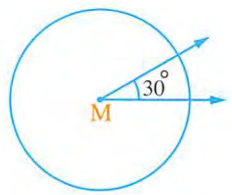
1 $67°$ 2 $-220°$ 3 $875°$ 4 $-2020°$

Lesson 2

Systems of measuring angle (Degree measure - Radian measure)

Degree measure system

It depends on dividing the circle into 360 equal arcs in length, then the central angle whose sides pass through the two ends of one of the arcs, its measure equals one degree which is symbolized by 1°, and the central angle which subtends between its sides 30 arcs of this arcs, its measure equals 30° and so on.



The unit of measurement of the degree measure

The degree is the unit of measuring the angle in the degree measure which is divided into 60 equal parts, each part is called a minute , and it is symbolized by $1'$, also the minute is divided into 60 equal parts , each part is called a second and it is symbolized by $1''$

i.e. $1^\circ = 60'$, $1' = 60''$

In this type of measuring angle , the protractor is used as an instrument for measuring angles in degrees.

Remember that

Calculator can be used to convert parts of degrees and minutes into minutes and seconds and vice versa

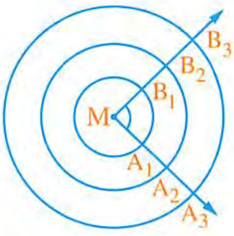
Such as

* $37\frac{3}{8}^\circ = 37^\circ\ 22'\ 30''$ — $37\frac{3}{8}$ [°'''] [=] $37^\circ\ 22'\ 30''$
* $70^\circ\ 37'\ 30'' = 70\frac{5}{8}^\circ$ — 70 [°'''] 37 [°'''] 30 [°'''] [=] [SHIFT] [S↔D] $70\frac{5}{8}$

Radian measure system

This measure depends on the following geometrical fact :

In the concentric circles , the ratio of the length of the arc of any central angle, and the length of the radius of its corresponding circle equals constant quantity.



i.e. $$\frac{\text{length of } \overset{\frown}{A_1 B_1}}{MA_1} = \frac{\text{length of } \overset{\frown}{A_2 B_2}}{MA_2} = \frac{\text{length of } \overset{\frown}{A_3 B_3}}{MA_3} = \text{constant quantity}$$

and this constant is the radian measure of the angle.

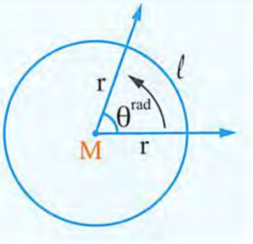
i.e. The radian measure of a central angle in a circle

$$= \frac{\text{length of the arc which the central angle subtends}}{\text{length of the radius of this circle}}$$

Definition

If θ^{rad} is the radian measure of a central angle in a circle of radius length r subtends an arc of length ℓ , then

$$\theta^{rad} = \frac{\ell}{r}$$



and since the radius length of the circle r is constant , then the radian measure of the central angle varies directly as the length of the subtended arc.

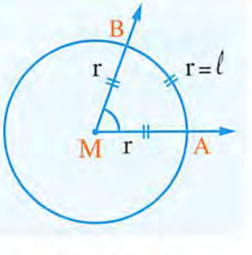
The unit of measurement of the radian measure

The radian angle is the unit of measuring the angle in the radian measure , and we can define the radian angle as follows which is denoted by (1^{rad}) and is read as one radian.

Definition

The radian angle is a central angle in a circle subtends an arc of length equals the length of the radius of the circle.

Notice : $\theta^{rad} = \frac{\ell}{r}$ $\quad \therefore \theta^{rad} = \frac{r}{r} = 1^{rad}$

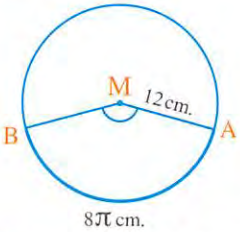


For example : The measure of the central angle that subtends an arc whose length equals double the length of the radius of its circle = 2^{rad}

Example 1

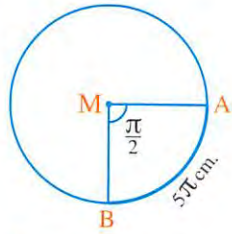
In each of the following circles , find the required under each figure approximating to the nearest tenth :

1



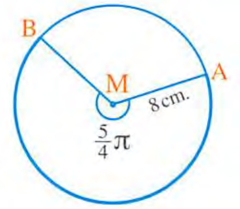
Find : m (∠ AMB) in radian measure.

2



Find : The radius length of circle M

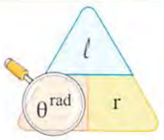
3



Find : The length of $\widehat{AB}$ the greater.

Solution

1



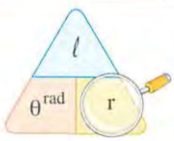
$\theta^{rad} = \frac{\ell}{r}$

$\theta^{rad} = ?$, $\ell = 8\pi$ cm. , $r = 12$ cm.

$\therefore$ m (∠ AMB) in radian measure $= \frac{\ell}{r} = \frac{8\pi}{12}$

$= \frac{2}{3}\pi \simeq 2.1^{rad}$

2



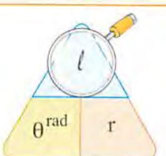
$r = \frac{\ell}{\theta^{rad}}$

$r = ?$, $\ell = 5\pi$ cm. , $\theta^{rad} = \frac{\pi}{2}$

$\therefore$ The radius length $= \frac{\ell}{\theta^{rad}} = \frac{5\pi}{\frac{\pi}{2}}$

$= 5\pi \times \frac{2}{\pi} = 10$ cm.

3



$\ell = \theta^{rad} \times r$

$\ell = ?$, $\theta^{rad} = \frac{5}{4}\pi$, $r = 8$ cm.

$\therefore$ The length of $\widehat{AB}$ the greater $= \theta^{rad} \times r$

$= \frac{5}{4}\pi \times 8 = 10\pi \simeq 31.4$ cm.

Remark

If the length of the radius of a circle is the unit , then the circle is called "the unit circle" , where $\theta^{rad} = \ell$

For example : In the unit circle , the central angle that subtends an arc of length $\frac{1}{2}\pi$ unit length has a radian measure $= \frac{1}{2}\pi \simeq 1.57^{rad}$

TRY TO SOLVE

1 Find the radian measure of the central angle which subtends an arc of length 15 cm. if the radius length of the circle is 10 cm.

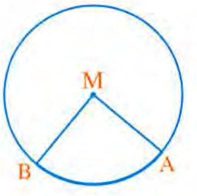
2 Find the length of the arc in a circle of radius length 8 cm. if the measure of the central angle subtended by it is $\frac{7\pi}{12}$ approximating the result to the nearest hundredth.

3 Find the length of the radius of the circle in which a central angle of measure $\frac{9\pi}{8}$ is drawn subtending an arc of length 24 cm. to the nearest tenth.

The relation between the radian measure and the degree measure

You have known that , in a circle : $\frac{\text{Measure of the arc}}{\text{Measure of the circle}} = \frac{\text{Length of this arc}}{\text{Circumference of the circle}}$

i.e. In the opposite figure : $\frac{m(\overset{\frown}{AB})}{360^\circ} = \frac{\text{Length of } \overset{\frown}{AB}}{2\pi r}$



$, \because m(\angle AMB) = m(\overset{\frown}{AB})$ $\quad \therefore \frac{m(\angle AMB)}{180^\circ} = \frac{\text{Length of } \overset{\frown}{AB}}{\pi r}$

Assuming that : $m(\angle AMB)$ equals x° in degrees and equals θ^{rad} in radians and the length of $\overset{\frown}{AB} = \ell$

$\therefore \frac{x^\circ}{180^\circ} = \frac{\ell}{\pi r}$ $\quad , \because \theta^{rad} = \frac{\ell}{r}$

$\therefore \frac{x^\circ}{180^\circ} = \frac{\theta^{rad}}{\pi}$ and from it $\theta^{rad} = x^\circ \times \frac{\pi}{180^\circ}$, $x^\circ = \theta^{rad} \times \frac{180^\circ}{\pi}$

Example 2

1 Find the radian measure of the angle whose degree measure is $75^\circ\ 32'\ 15''$ approximating the result to the nearest thousandth.

2 Find the degree measure of the angle whose radian measure is 2.38^{rad}

Solution

1 $\because \theta^{rad} = x^\circ \times \frac{\pi}{180^\circ}$ $\quad \therefore \theta^{rad} = 75^\circ\ 32'\ 15'' \times \frac{\pi}{180^\circ} \simeq 1.318^{rad}$

2 $\because x^\circ = \theta^{rad} \times \frac{180^\circ}{\pi}$ $\quad \therefore x^\circ = 2.38^{rad} \times \frac{180^\circ}{\pi} \simeq 136^\circ\ 21'\ 50''$

TRY TO SOLVE

1 Convert the measure of the angle 1.2^{rad} into degrees.

2 Convert the measure of the angle $72^\circ\ 30'$ into radians approximating the result to the nearest hundredth.

Enrichment information

There is another unit of measuring angles called (Grad) which equals $\frac{1}{200}$ of the measure of the straight angle.

If x , θ , y are the measures of three angles respectively in degrees , radian and grade , then $\frac{x^\circ}{180^\circ} = \frac{\theta^{rad}}{\pi} = \frac{y^{grad}}{200}$

Remarks

1 If the radian measure of an angle equals π (radian) , then its degree measure

$= \pi \times \frac{180^\circ}{\pi} = 180^\circ$ **i.e.** π in radians is equivalent to 180° in degrees.

For example : $\frac{3}{5}\pi$ is equivalent to $\frac{3}{5} \times 180^\circ = 108^\circ$

2 If the degree measure of an angle is known , and it is required to convert it into radian measure in terms of π , then we use the relation : $\theta^{rad} = x^\circ \times \frac{\pi}{180^\circ}$ without substituting with π

For example : • 18° is equivalent to $18^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{10}$

• 135° is equivalent to $135^\circ \times \frac{\pi}{180^\circ} = \frac{3}{4}\pi$

Example 3

Determine the quadrant in which the directed angle of each of the angles whose measures are as follows lies :

1 2.02^{rad} **2** -7.3^{rad} **3** $\frac{5}{4}\pi$

Solution

To determine the quadrant in which the directed angle lies, we find its degree measure :

1 $\because x^\circ = \theta^{rad} \times \frac{180^\circ}{\pi} = 2.02 \times \frac{180^\circ}{\pi} \simeq 115^\circ\ 44'\ 15''$

$\therefore$ The angle whose measure is 2.02^{rad} is equivalent to $115^\circ\ 44'\ 15''$ in degrees.

$\because$ The angle of measure $115^\circ\ 44'\ 15''$ lies in the second quadrant

$\therefore$ The angle of measure 2.02^{rad} lies in the second quadrant.

2 $\because x^\circ = -7.3^{rad} \times \frac{180^\circ}{\pi} \simeq -418^\circ\ 15'\ 33''$

$\because$ The angle of measure $-418^\circ\ 15'\ 33''$ is equivalent to

$-418^\circ\ 15'\ 33'' + 2 \times 360^\circ = 301^\circ\ 44'\ 27''$

$\because$ The angle of measure $301^\circ\ 44'\ 27''$ lies in the fourth quadrant

$\therefore$ The angle of measure -7.3^{rad} lies in the fourth quadrant.

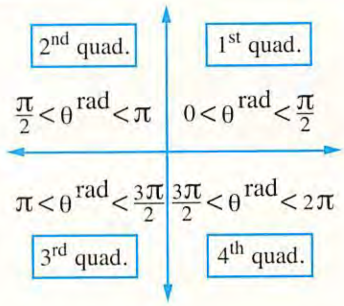
3 $\because \frac{5\pi}{4}$ is equivalent to $\frac{5}{4} \times 180^\circ = 225^\circ$

$\because$ The angle whose measure is 225° lies in the third quadrant.

$\therefore$ The angle whose measure is $\frac{5\pi}{4}$ lies in the third quadrant.

Remark

It is possible to determine the quadrant in which the directed angle - whose radian measure is known in terms of π - lies without converting to degrees using the opposite figure :



For example :

By using the opposite figure we can determine in which quadrant the angle whose measure is $\frac{5}{4}\pi$ in the last example lies where

$\pi < \frac{5\pi}{4} < \frac{3\pi}{2}$

$\therefore$ The angle whose measure is $\frac{5}{4}\pi$ lies in the third quadrant.

TRY TO SOLVE

Find the quadrant that each of the following angles lies in :

1. The angle of measure $\frac{5\pi}{3}$
2. The angle of measure -0.3π
3. The angle of measure 5.7^{rad}
4. The angle of measure -6.4^{rad}

Example 4

Find the length of the arc subtended by the central angle whose measure is $152^\circ\ 26'\ 17''$ drawn in a circle of radius length 10.5 cm. approximating the result to the nearest cm.

Solution

$\because \theta^{rad} = x^\circ \times \frac{\pi}{180^\circ} = 152^\circ\ 26'\ 17'' \times \frac{\pi}{180^\circ} \simeq 2.6605^{rad}$

$\therefore \ell = \theta^{rad} \times r = 2.6605 \times 10.5 \simeq 28$ cm.

Example 5

Find each of the radian measure and the degree measure of the central angle subtending an arc of length 12.6 cm. in a circle of radius length 7.2 cm.

Solution

$\theta^{rad} = \frac{\ell}{r} = \frac{12.6}{7.2} = 1.75^{rad}$

, $x^\circ = 1.75^{rad} \times \frac{180^\circ}{\pi} \simeq 100^\circ\ 16'\ 3''$

Example 6

Find the circumference of the circle that has an inscribed angle of measure 30° subtending an arc of length 5 cm.

Solution

$\because$ The measure of the inscribed angle = 30°

$\therefore$ The measure of the corresponding central angle = 60°

$\therefore \theta^{rad} = 60^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{3}$ $\qquad$ $\therefore r = \frac{\ell}{\theta^{rad}} = 5 \div \left(\frac{\pi}{3}\right) = \frac{15}{\pi}$ cm.

$\therefore$ The circumference of the circle = $2\pi r = 2\pi \times \frac{15}{\pi} = 30$ cm.

Example 7

Two angles , the sum of their radian measures = $3\frac{1}{7}^{rad}$, and the difference between their degree measures = 30° , find the measure of each of them in degrees and in radians.

Solution

$\because 3\frac{1}{7}^{rad} = \frac{22}{7} \times \frac{180^\circ}{\pi} = 180^\circ$ assuming the two angles are A , B such that : m (∠ A) > m (∠ B)

$\therefore$ m (∠ A) + m (∠ B) = 180° , m (∠ A) – m (∠ B) = 30°

By adding :

$\therefore$ 2 m (∠ A) = 210°

$\therefore$ m (∠ A) = 105°

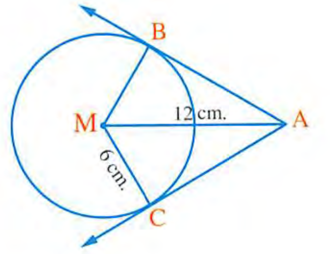
$\therefore$ m (∠ B) = 75°

$\therefore$ m (∠ A) in radians = $105^\circ \times \frac{\pi}{180^\circ} \simeq 1.83^{rad}$

$\therefore$ m (∠ B) in radians = $75^\circ \times \frac{\pi}{180^\circ} \simeq 1.31^{rad}$

Example 8

In the opposite figure : $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle M whose radius length is 6 cm. If AM = 12 cm. , find the length of the major arc $\overset{\frown}{BC}$ to the nearest integer.



Solution

$\because$ $\overrightarrow{AC}$ is a tangent to the circle M $\quad\therefore$ $\overline{MC} \perp \overrightarrow{AC}$

In Δ AMC :

$\because$ m ($\angle$ ACM) = 90° , MC = $\frac{1}{2}$ AM $\quad\therefore$ m ($\angle$ CAM) = 30°

$\therefore$ m ($\angle$ AMC) = 60°

, $\because$ $\overrightarrow{MA}$ bisects $\angle$ BMC $\quad\therefore$ m ($\angle$ BMC) = 120°

$\therefore$ m ($\angle$ BMC) the reflex = 360° – 120° = 240°

, $\because$ $\theta^{rad} = x^\circ \times \frac{\pi}{180^\circ}$

$\therefore$ $\theta^{rad} = 240^\circ \times \frac{\pi}{180^\circ} = \frac{4\pi}{3}$

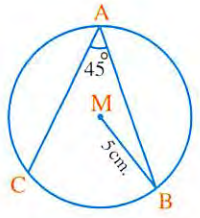
, $\because$ $\ell = \theta^{rad} \times r$

$\therefore$ The length of $\overset{\frown}{BC}$ the major = $\frac{4\pi}{3} \times 6 = 8\pi \simeq 25$ cm.

TRY TO SOLVE

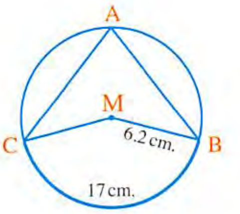
Find the required under each figure :

1



The length of $\overset{\frown}{BC}$

2



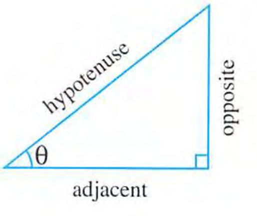
m ($\angle$ A)

Lesson 3

Trigonometric functions

We have studied before the basic trigonometric ratios of an acute angle and we have known that :

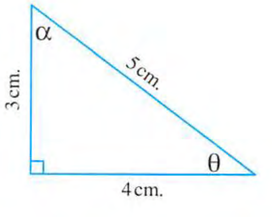
In any right-angled triangle :



$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$, $\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

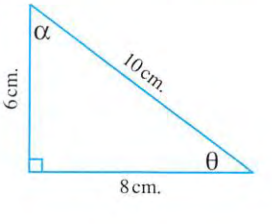
, $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$

In the opposite figure :



$\sin \theta = \frac{3}{5}$	$\cos \theta = \frac{4}{5}$	$\tan \theta = \frac{3}{4}$
$\sin \alpha = \frac{4}{5}$	$\cos \alpha = \frac{3}{5}$	$\tan \alpha = \frac{4}{3}$

and if we draw another triangle similar to the previous triangle , we find that :



$\sin \theta = \frac{6}{10} = \frac{3}{5}$	$\cos \theta = \frac{8}{10} = \frac{4}{5}$	$\tan \theta = \frac{6}{8} = \frac{3}{4}$
$\sin \alpha = \frac{8}{10} = \frac{4}{5}$	$\cos \alpha = \frac{6}{10} = \frac{3}{5}$	$\tan \alpha = \frac{8}{6} = \frac{4}{3}$

From the previous , we deduce that :

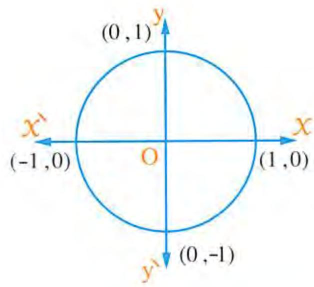
1 $\sin \theta$, $\cos \theta$, $\tan \theta$ in the two triangles are equal.

i.e. The trigonometric ratio of the angle is constant and does not depend on the area of the triangle.

2 $\sin \theta \neq \sin \alpha$, $\cos \theta \neq \cos \alpha$, $\tan \theta \neq \tan \alpha$ in any of the two triangles.

i.e. The trigonometric ratio is changed by the change of the angle which is known by "The trigonometric functions"

The unit circle

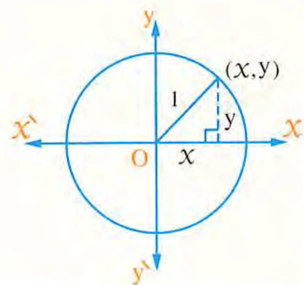


In the orthogonal coordinate system the circle of centre at the origin point and radius equals the unit of length is called a unit circle.

Notice from the previous figure :

- The unit circle intersects the x-axis at two points which are $(1 , 0)$, $(-1 , 0)$
- The unit circle intersects the y-axis at two points which are $(0 , 1)$, $(0 , -1)$

Remark

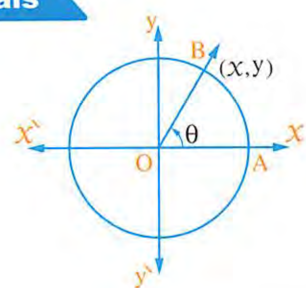


If the point $(x , y) \in$ the unit circle , then

* $x^2 + y^2 = 1$ from Pythagoras' theorem.

* $x \in [-1 , 1]$, $y \in [-1 , 1]$

The basic trigonometric functions and their reciprocals



If we draw the directed angle AOB in the standard position and its terminal side intersects the unit circle at the point B (x , y) and if m $(\angle AOB) = \theta$, then we can define the following :

First The basic trigonometric functions of the angle of measure θ are :

1 Cosine of the angle = X - coordinate of the point B **i.e.** $\cos\theta = X$

2 Sine of the angle = y - coordinate of the point B **i.e.** $\sin\theta = y$

3 Tangent of the angle = $\frac{y\text{ - coordinate of the point B}}{X\text{ - coordinate of the point B}}$ **i.e.** $\tan\theta = \frac{y}{X} = \frac{\sin\theta}{\cos\theta}$, where $X \neq 0$

Notice that The point B (X , y) can be written as $(\cos\theta , \sin\theta)$

Second The reciprocals of the basic trigonometric functions of the angle of measure θ are :

1 The secant of the angle (sec) = $\frac{1}{X\text{ - coordinate of the point B}}$

i.e. $\sec\theta = \frac{1}{X} = \frac{1}{\cos\theta}$, where $X \neq 0$

2 The cosecant of the angle (csc) = $\frac{1}{y\text{ - coordinate of the point B}}$

i.e. $\csc\theta = \frac{1}{y} = \frac{1}{\sin\theta}$, where $y \neq 0$

3 The cotangent of the angle (cot) = $\frac{X\text{ - coordinate of the point B}}{y\text{ - coordinate of the point B}}$

i.e. $\cot\theta = \frac{X}{y} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$, where $y \neq 0$

Example 1

Find all trigonometric functions for an angle of measure θ which is drawn in the standard position and its terminal side intersects the unit circle at the point A in each of the following :

1 $A\left(\frac{3}{5} , \frac{4}{5}\right)$

2 $A(-1 , 0)$

3 $A\left(-\frac{1}{2} , y\right)$, where $y > 0$

4 $A(-X , X)$ where $X > 0$

Solution

1 $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$, $\tan\theta = \frac{4}{5} \div \frac{3}{5} = \frac{4}{3}$

, $\sec\theta = \frac{5}{3}$, $\csc\theta = \frac{5}{4}$, $\cot\theta = \frac{3}{4}$

2 $\cos\theta = -1$, $\sin\theta = 0$, $\tan\theta = \frac{0}{-1} = 0$

, $\sec\theta = -1$, $\csc\theta = \frac{1}{0}$ (undefined) , $\cot\theta = \frac{-1}{0}$ (undefined)

3 $\because x^2 + y^2 = 1$ $\therefore \left(-\frac{1}{2}\right)^2 + y^2 = 1$

$\therefore y^2 = 1 - \frac{1}{4} = \frac{3}{4}$ $\therefore y = \pm\frac{\sqrt{3}}{2}$

, $\because y > 0$ $\therefore y = \frac{\sqrt{3}}{2}$ $\therefore A\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$

$\therefore \cos\theta = -\frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$, $\tan\theta = \frac{\sqrt{3}}{2} \div -\frac{1}{2} = -\sqrt{3}$

, $\sec\theta = -2$, $\csc\theta = \frac{2}{\sqrt{3}}$, $\cot\theta = \frac{-1}{\sqrt{3}}$

4 $\because x^2 + y^2 = 1$ $\therefore (-x)^2 + x^2 = 1$

$\therefore 2x^2 = 1$ $\therefore x^2 = \frac{1}{2}$

$\therefore x = \pm\frac{1}{\sqrt{2}}$, $\because x > 0$

$\therefore x = \frac{1}{\sqrt{2}}$ $\therefore A\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\therefore \cos\theta = \frac{-1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$, $\tan\theta = \frac{1}{\sqrt{2}} \div \frac{-1}{\sqrt{2}} = -1$

, $\sec\theta = -\sqrt{2}$, $\csc\theta = \sqrt{2}$, $\cot\theta = -1$

TRY TO SOLVE

Find all trigonometric functions of an angle θ drawn in the standard position whose terminal side intersects the unit circle at the point B for each of the following :

1 $B\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

2 $B(0, x)$, where $x < 0$

3 $B(-y, -y)$, where $y > 0$

Remark

The equivalent angles have the same trigonometric functions :

i.e. For all values of $n \in \mathbb{Z}$ (set of integers) , then

- $\cos(\theta + 2n\pi) = \cos\theta = x$, $\sec(\theta + 2n\pi) = \sec\theta = \frac{1}{x}$, where $x \neq 0$
- $\sin(\theta + 2n\pi) = \sin\theta = y$, $\csc(\theta + 2n\pi) = \csc\theta = \frac{1}{y}$, where $y \neq 0$
- $\tan(\theta + 2n\pi) = \tan\theta = \frac{y}{x}$, where $x \neq 0$, $\cot(\theta + 2n\pi) = \cot\theta = \frac{x}{y}$, where $y \neq 0$

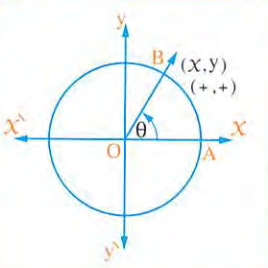
For example :

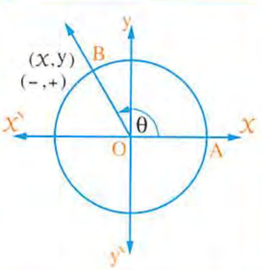
- $\cos 420° = \cos(60° + 360°) = \cos 60°$
- $\sec 840° = \sec(120° + 2 \times 360°) = \sec 120°$
- $\tan(-1500°) = \tan(300° - 5 \times 360°) = \tan 300°$

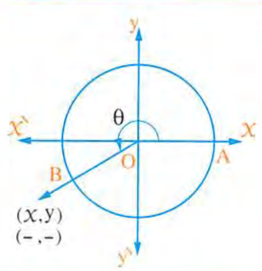
Signs of trigonometric functions

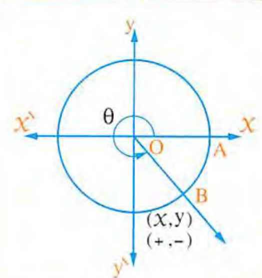
If $\angle$ AOB the directed is in its standard position and its terminal side intersects the unit circle at the point B (x , y) and m $(\angle \text{AOB}) = \theta$, then

$\angle$ AOB lies in one of the quadrants as follows :





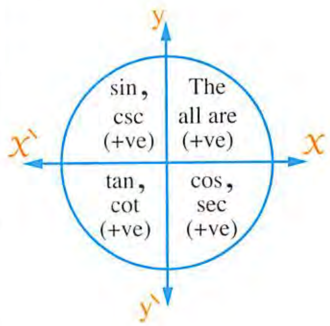




First quadrant	Second quadrant	Third quadrant	Fourth quadrant
$\theta \in]0 , \frac{\pi}{2}[$	$\theta \in]\frac{\pi}{2} , \pi[$	$\theta \in]\pi , \frac{3\pi}{2}[$	$\theta \in]\frac{3\pi}{2} , 2\pi[$
$x > 0 , y > 0$	$x < 0 , y > 0$	$x < 0 , y < 0$	$x > 0 , y < 0$
all the trigonometric functions are positive.	$\sin\theta$, $\csc\theta$ are positive and the other functions are negative.	$\tan\theta$, $\cot\theta$ are positive and the other functions are negative.	$\cos\theta$, $\sec\theta$ are positive and the other functions are negative.

- **We can summarize the previous results in the figure and in the following table :**

Quadrant	The interval that θ belongs to	sign of cos , sec	sign of sin , csc	sign of tan , cot
First	$]0 , \frac{\pi}{2}[$	+	+	+
Second	$]\frac{\pi}{2} , \pi[$	–	+	–
Third	$]\pi , \frac{3\pi}{2}[$	–	–	+
Fourth	$]\frac{3\pi}{2} , 2\pi[$	+	–	–



For example :

- **tan 320° is negative , because :**

The angle of measure 320° lies in the fourth quadrant 270° < 320° < 360°

- **sin 160° is positive , because :**

The angle of measure 160° lies in the second quadrant 90° < 160° < 180°

Remark

The trigonometric functions of the equivalent angles have the same sign.

Example 2

Determine the sign of each of the following trigonometric ratios :

1 sin 970°

2 $\cos \frac{7\pi}{3}$

3 tan (– 200°)

4 $\csc \left(-\frac{8}{5}\pi\right)$

Solution

1 sin 970° = sin (250° + 2 × 360°) = sin 250°

, ∵ 180° < 250° < 270°

i.e. This angle lies in the third quadrant.

∴ sin 250° is negative. ∴ sin 970° is negative.

2 $\cos \frac{7}{3}\pi = \cos\left(\frac{7}{3} \times 180°\right) = \cos 420° = \cos (60° + 360°) = \cos 60°$

, $\because 0° < 60° < 90°$

i.e. This angle lies in the first quadrant.

$\therefore \cos 60°$ is positive. $\therefore \cos \frac{7}{3}\pi$ is positive.

3 $\tan (-200°) = \tan (-200° + 360°) = \tan 160°$

, $\because 90° < 160° < 180°$

i.e. This angle lies in the second quadrant.

$\therefore \tan 160°$ is negative. $\therefore \tan (-200°)$ is negative.

4 $\csc\left(-\frac{8}{5}\pi\right) = \csc\left(-\frac{8}{5} \times 180°\right) = \csc (-288°) = \csc (-288° + 360°) = \csc 72°$

, $\because 0° < 72° < 90°$

i.e. This angle lies in the first quadrant.

$\therefore \csc 72°$ is positive. $\therefore \csc\left(-\frac{8}{5}\pi\right)$ is positive.

TRY TO SOLVE

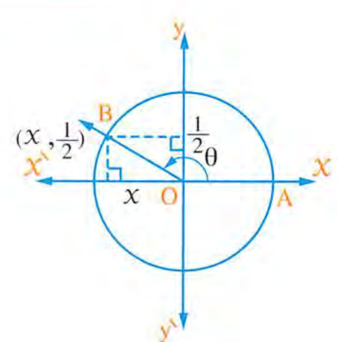
Determine the sign of each of the following trigonometric ratios :

1 $\cos 620°$ **2** $\sec (-30°)$ **3** $\cot \frac{11}{3}\pi$

Example 3

If B $\left(x , \frac{1}{2}\right)$ is the point of intersection of the terminal side of the directed angle of measure θ in its standard position with the unit circle where 90° < θ < 180° , find the value of each of : cos θ and tan θ

Solution



$\because 90° < \theta < 180°$ $\therefore$ B lies in the second quadrant

, $\because$ for any point (x , y) on the unit circle , we get $x^2 + y^2 = 1$

$\therefore x^2 + \left(\frac{1}{2}\right)^2 = 1$

$\therefore x^2 = 1 - \frac{1}{4} = \frac{3}{4}$ $\therefore x = \pm \frac{\sqrt{3}}{2}$

, $\because$ the point B $\left(x , \frac{1}{2}\right)$ lies in the second quadrant. $\therefore x = -\frac{\sqrt{3}}{2}$

$\therefore B = \left(\frac{-\sqrt{3}}{2} , \frac{1}{2}\right)$ $\therefore \cos\theta = \frac{-\sqrt{3}}{2}$, $\tan\theta = \frac{y}{x} = -\frac{1}{\sqrt{3}}$

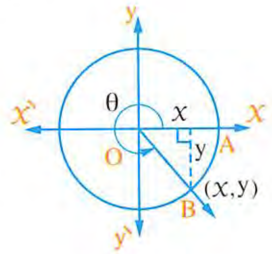
Example 4

If $\theta \in \left]\frac{3\pi}{2} , 2\pi\right[$, $\cos\theta = \frac{5}{13}$, then find all trigonometric functions of θ

Solution

Let m (∠ AOB) = θ where θ is in the 4th quadrant and the point B is (x , y)

$\therefore x = \cos\theta = \frac{5}{13}$, $y = \sin\theta$ where $\sin\theta < 0$



$\because x^2 + y^2 = 1$ $\therefore \left(\frac{5}{13}\right)^2 + \sin^2\theta = 1$

$\therefore \sin^2\theta = 1 - \frac{25}{169} = \frac{144}{169}$ $\therefore \sin\theta = -\frac{12}{13}$ $\therefore B = \left(\frac{5}{13} , \frac{-12}{13}\right)$

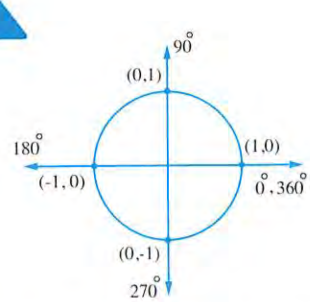
, then we get : $\tan\theta = \frac{y}{x} = \frac{-12}{5}$,

$\csc\theta = \frac{1}{y} = -\frac{13}{12}$, $\sec\theta = \frac{1}{x} = \frac{13}{5}$ and $\cot\theta = \frac{x}{y} = -\frac{5}{12}$

The trigonometric ratios of some special angles

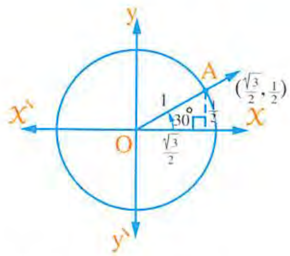
First The quadrantal angles (0° , 360° , 90° , 180° or 270°) :

The opposite figure illustrate the points of intersection of the terminal sides of the quadrantal angles with the unit circle, from which we can deduce the trigonometric ratios for these angles as shown in the following table :



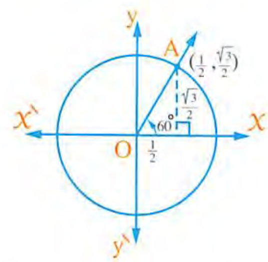
θ° in degree	θ in radian	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
0° or 360°	0 or 2 π	0	1	0	undefined	1	undefined
90°	$\frac{\pi}{2}$	1	0	undefined	1	undefined	0
180°	π	0	−1	0	undefined	− 1	undefined
270°	$\frac{3\pi}{2}$	−1	0	undefined	− 1	undefined	0

Second The angles of measures 30° , 60° and 45° :



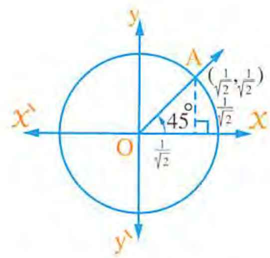
θ in degree = 30°

θ in radian = $\frac{\pi}{6}$



θ in degree = 60°

θ in radian = $\frac{\pi}{3}$



θ in degree = 45°

θ in radian = $\frac{\pi}{4}$

The previous figures show the points of intersection of the terminal side of each of the angles of measures 30° , 60° and 45° in the standard position with the unit circle , from which we can deduce the trigonometric ratios of these angles as shown in the following table :

$\theta°$ in degree	θ in radian	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1

Example 5

Find the value of :

$4\sin 30° \sin 90° - \cos 0° \sec 60° + 5\tan 45° + 10\cos^2 45° \sin 270° - \tan 30° \sin 180°$

Solution

The expression $= 4 \times \frac{1}{2} \times 1 - 1 \times 2 + 5 \times 1 + 10 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times (-1) - \frac{1}{\sqrt{3}} \times 0$

$= 2 - 2 + 5 - 5 - 0 = 0$

Example 6

Prove that : $\sin^2 60° + \sin^2 45° + \sin^2 30° = \cos^2\frac{\pi}{6}\sin\frac{\pi}{2} - \frac{1}{3}\tan^2\frac{\pi}{3}\cos\pi + \cos^2\frac{\pi}{3}\sin\frac{3\pi}{2}$

Solution

The left hand side $= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2}$

The right hand side $= \cos^2 30° \sin 90° - \frac{1}{3} \tan^2 60° \cos 180° + \cos^2 60° \sin 270°$

$= \left(\frac{\sqrt{3}}{2}\right)^2 \times 1 - \frac{1}{3} \times \left(\sqrt{3}\right)^2 \times (-1) + \left(\frac{1}{2}\right)^2 \times (-1) = \frac{3}{4} + 1 - \frac{1}{4} = \frac{3}{2}$

$\therefore$ The two sides are equal.

Example 7

Find the value of X which satisfies : $X \sin \frac{\pi}{6} \cos^2 \frac{\pi}{4} = \cos^2 30° \sin \frac{\pi}{2}$

Solution

$\because X \sin 30° \cos^2 45° = \cos^2 30° \sin 90°$

$\therefore X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 \times 1$

$\therefore \frac{1}{4} X = \frac{3}{4}$

$\therefore X = 3$

Example 8

If $0° < X < 90°$, find the value of X that satisfies :

$\sin X \sec^2 45° = \tan^2 60° - 2 \cos 360°$

Solution

$\because \sin X \sec^2 45° = \tan^2 60° - 2 \cos 360°$

$\therefore \sin X \times \left(\sqrt{2}\right)^2 = \left(\sqrt{3}\right)^2 - 2 \times 1$

$\therefore 2 \times \sin X = 3 - 2 = 1$

$\therefore \sin X = \frac{1}{2}$

$\therefore X = 30°$

TRY TO SOLVE

1 **Find the value of :**

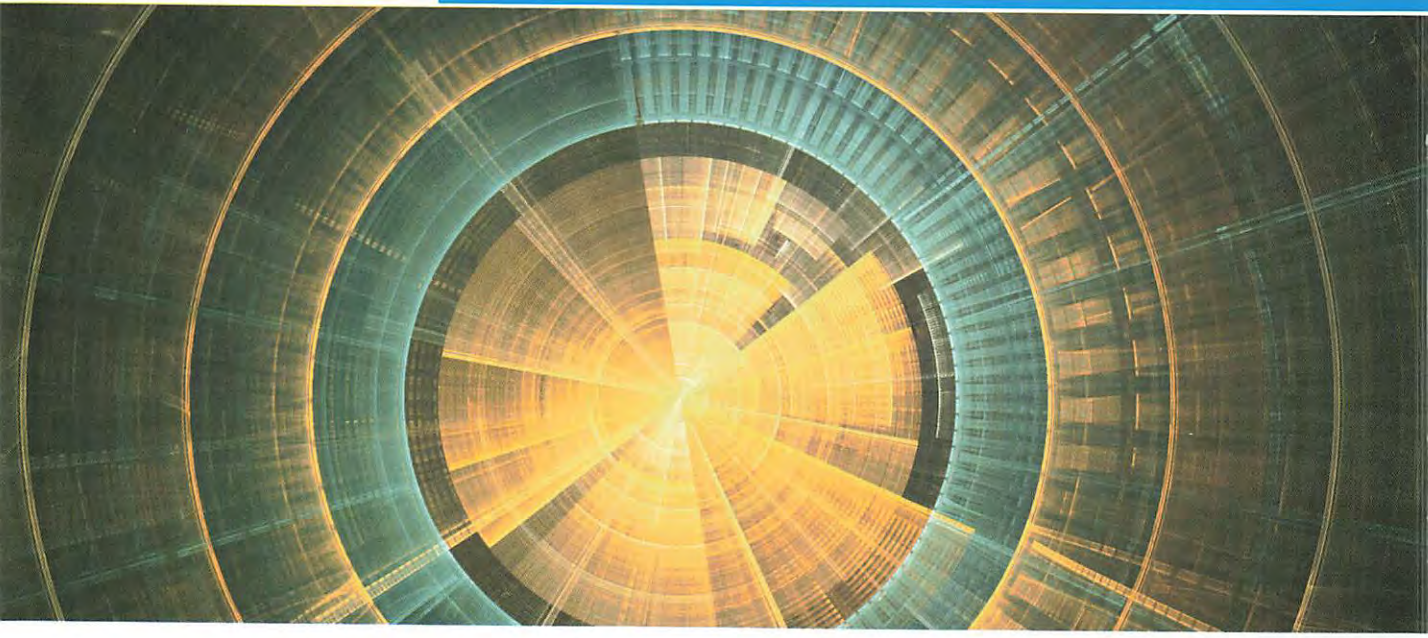
$\cos 90° \csc 30° + \sec^2 45° \sin 30° - \cos 270° \sin 180°$

2 **If $0° \le X \le 90°$, find the value of X which satisfies :**

$\cos X = \sin 30° \cos 60° + \cos 30° \sin 60°$

Lesson 4

Related angles



Definition of the related angles

They are two angles the difference between their measures or the sum of their measures equals a whole number of right angles.

For example : The two angles of measures 30° , 210° are two related angles.

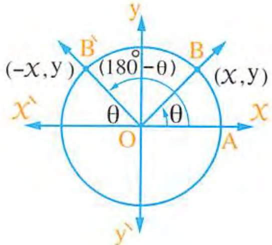
because : 210° – 30°= 180° **i.e.** Two right angles.

The relation between trigonometric functions of related angles

If the terminal side of the directed angle ∠ AOB in its standard position intersects the unit circle at the point B (X , y) and m (∠ AOB) = θ such that 0° < θ < 90° , then :

1 Relation between trigonometric functions of related angles of measures θ , (180° – θ) :

If B̀ (– X , y) is the image of the point B (X , y) by reflection in the y-axis , then m (∠ AOB̀) the directed = (180° – θ) thus :



$\sin(180° - \theta) = \sin\theta$, $\csc(180° - \theta) = \csc\theta$

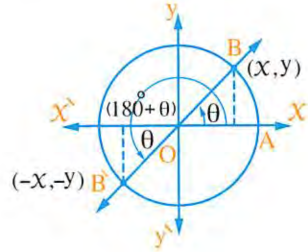
$\cos(180° - \theta) = -\cos\theta$, $\sec(180° - \theta) = -\sec\theta$

$\tan(180° - \theta) = -\tan\theta$, $\cot(180° - \theta) = -\cot\theta$

For example :
- $\sin 150° = \sin(180° - 30°) = \sin 30° = \frac{1}{2}$
- $\cos 120° = \cos(180° - 60°) = -\cos 60° = -\frac{1}{2}$
- $\cot 135° = \cot(180° - 45°) = -\cot 45° = -1$

2 Relation between trigonometric functions of related angles of measures θ , (180° + θ) :

If $\grave{B}$ $(-X , -y)$ is the image of the point B (X , y) by reflection in the origin point , then m ($\angle$ AO$\grave{B}$) the directed = (180° + θ) thus :



$\sin(180° + \theta) = -\sin\theta$, $\csc(180° + \theta) = -\csc\theta$

$\cos(180° + \theta) = -\cos\theta$, $\sec(180° + \theta) = -\sec\theta$

$\tan(180° + \theta) = \tan\theta$, $\cot(180° + \theta) = \cot\theta$

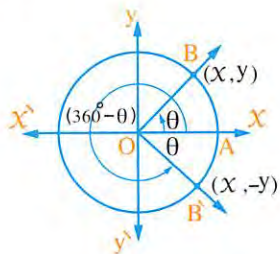
For example : • $\sin 225° = \sin(180° + 45°) = -\sin 45° = \frac{-1}{\sqrt{2}}$

• $\sec 210° = \sec(180° + 30°) = -\sec 30° = \frac{-2}{\sqrt{3}}$

• $\tan 240° = \tan(180° + 60°) = \tan 60° = \sqrt{3}$

3 Relation between trigonometric functions of related angles of measures θ , (360° – θ) :

If $\grave{B}$ $(X , -y)$ is the image of the point B (X , y) by reflection in the X-axis , then m ($\angle$ AO$\grave{B}$) the directed = (360° – θ) thus :



$\sin(360° - \theta) = -\sin\theta$, $\csc(360° - \theta) = -\csc\theta$

$\cos(360° - \theta) = \cos\theta$, $\sec(360° - \theta) = \sec\theta$

$\tan(360° - \theta) = -\tan\theta$, $\cot(360° - \theta) = -\cot\theta$

For example : • $\sin 300° = \sin(360° - 60°) = -\sin 60° = \frac{-\sqrt{3}}{2}$

• $\tan 315° = \tan(360° - 45°) = -\tan 45° = -1$

• $\sec 330° = \sec(360° - 30°) = \sec 30° = \frac{2}{\sqrt{3}}$

Note

The angle of measure (– θ) is equivalent to the angle of measure (360° – θ)

From this , we can deduce :

The relation between trigonometric functions of related angles of measures θ , (– θ) as follows :

$\sin(-\theta) = -\sin\theta$, $\csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$, $\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$, $\cot(-\theta) = -\cot\theta$

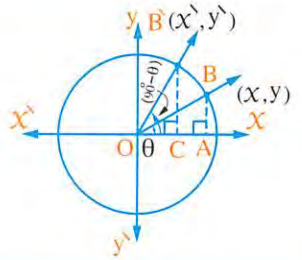
For example : • $\sin(-45°) = -\sin 45° = \frac{-1}{\sqrt{2}}$ • $\cos(-60°) = \cos 60° = \frac{1}{2}$

• $\cot(-30°) = -\cot 30° = -\sqrt{3}$

4 Relation between trigonometric functions of related angles of measures θ , $(90° - \theta)$:

In the opposite figure :

The terminal side of the directed angle of measure $(90° - \theta)$ in the standard position intersects the unit circle at the point $\grave{B}(\grave{x}, \grave{y})$



From the figure geometry , we find that :

$\triangle C\grave{B}O \equiv \triangle AOB$

$\therefore C\grave{B} = AO$, then $\grave{y} = x$ **i.e.** $\sin(90° - \theta) = \cos\theta$

, $CO = AB$, then $\grave{x} = y$ **i.e.** $\cos(90° - \theta) = \sin\theta$

, $\because \tan(90° - \theta) = \frac{\grave{y}}{\grave{x}} = \frac{x}{y}$ $\therefore \tan(90° - \theta) = \cot\theta$

Similarly , it is possible to deduce the relations between the reciprocals of the trigonometric functions of the two angles of measures θ , $(90° - \theta)$ as follows :

$\sin(90° - \theta) = \cos\theta$, $\csc(90° - \theta) = \sec\theta$

$\cos(90° - \theta) = \sin\theta$, $\sec(90° - \theta) = \csc\theta$

$\tan(90° - \theta) = \cot\theta$, $\cot(90° - \theta) = \tan\theta$

For example : • $\sin 70° = \sin(90° - 20°) = \cos 20°$

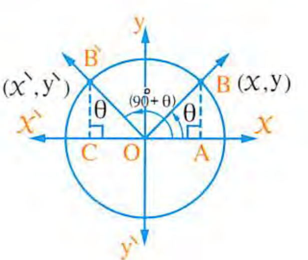
• $\frac{\sin 40°}{\cos 50°} = \frac{\sin(90° - 50°)}{\cos 50°} = \frac{\cos 50°}{\cos 50°} = 1$

• $\tan 10° - \cot 80° = \tan(90° - 80°) - \cot 80° = \cot 80° - \cot 80° = 0$

5 Relation between trigonometric functions of related angles of measures θ , $(90° + \theta)$:

In the opposite figure :

The terminal side of the directed angle of measure $(90° + \theta)$ in the standard position intersects the unit circle at the point $\grave{B}(\grave{x}, \grave{y})$



From the figure geometry , we find that :

$\Delta\, CO\grave{B} \equiv \Delta\, ABO$

$\therefore C\grave{B} = AO$, then $\grave{y} = x$ — i.e. $\sin(90° + \theta) = \cos\theta$

$, OC = AB$, then $\grave{x} = -y$ — i.e. $\cos(90° + \theta) = -\sin\theta$

$, \because \tan(90° + \theta) = \frac{\grave{y}}{\grave{x}} = \frac{x}{-y}$ $\quad \therefore \tan(90° + \theta) = -\cot\theta$

Similarly , it is possible to deduce the relations between the reciprocals of the trigonometric functions of the two angles of measures θ , $(90° + \theta)$ as follows :

$\sin(90° + \theta) = \cos\theta$, $\csc(90° + \theta) = \sec\theta$

$\cos(90° + \theta) = -\sin\theta$, $\sec(90° + \theta) = -\csc\theta$

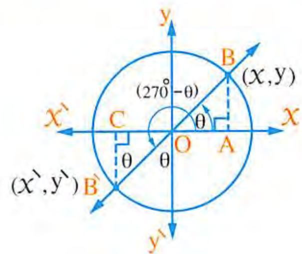
$\tan(90° + \theta) = -\cot\theta$, $\cot(90° + \theta) = -\tan\theta$

For example :
- $\sin 120° = \sin(90° + 30°) = \cos 30° = \frac{\sqrt{3}}{2}$
- $\cos 150° = \cos(90° + 60°) = -\sin 60° = \frac{-\sqrt{3}}{2}$
- $\cot 135° = \cot(90° + 45°) = -\tan 45° = -1$

6 Relation between trigonometric functions of related angles of measures θ , $(270° - \theta)$:

In the opposite figure :

The terminal side of the directed angle of measure $(270° - \theta)$ in the standard position intersects the unit circle at the point $\grave{B}\,(\grave{x} , \grave{y})$



From the figure geometry , we find that :

$\Delta\, CO\grave{B} \equiv \Delta\, ABO$

$\therefore C\grave{B} = AO$, then $\grave{y} = -x$ — i.e. $\sin(270° - \theta) = -\cos\theta$

$, CO = AB$, then $\grave{x} = -y$ — i.e. $\cos(270° - \theta) = -\sin\theta$

$, \because \tan(270° - \theta) = \frac{\grave{y}}{\grave{x}} = \frac{-x}{-y} = \frac{x}{y}$ $\quad \therefore \tan(270° - \theta) = \cot\theta$

Similarly , it is possible to deduce the relations between the reciprocals of the trigonometric functions of the two angles of measures θ , $(270° - \theta)$ as follows :

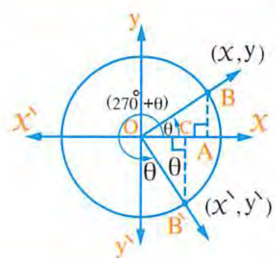
$\sin(270° - \theta) = -\cos\theta$	,	$\csc(270° - \theta) = -\sec\theta$
$\cos(270° - \theta) = -\sin\theta$	,	$\sec(270° - \theta) = -\csc\theta$
$\tan(270° - \theta) = \cot\theta$	,	$\cot(270° - \theta) = \tan\theta$

For example :

- $\sin 225° = \sin(270° - 45°) = -\cos 45° = \frac{-1}{\sqrt{2}}$
- $\tan 240° = \tan(270° - 30°) = \cot 30° = \sqrt{3}$
- $\csc 210° = \csc(270° - 60°) = -\sec 60° = -2$

7 Relation between trigonometric functions of related angles of measures θ , $(270° + \theta)$:

In the opposite figure :



The terminal side of the directed angle of measure $(270° + \theta)$ in the standard position intersects the unit circle at the point $\grave{B}(\grave{x}, \grave{y})$

From the figure geometry , we find that :

$\Delta\, CO\grave{B} \equiv \Delta\, ABO$

$\therefore C\grave{B} = AO$, then $\grave{y} = -x$ — i.e. $\sin(270° + \theta) = -\cos\theta$

, $CO = AB$, then $\grave{x} = y$ — i.e. $\cos(270° + \theta) = \sin\theta$

, $\because \tan(270° + \theta) = \frac{\grave{y}}{\grave{x}} = \frac{-x}{y}$ $\quad\therefore \tan(270° + \theta) = -\cot\theta$

Similarly , it is possible to deduce the relations between the reciprocals of the trigonometric functions of the two angles of measures θ , $(270° + \theta)$ as follows :

$\sin(270° + \theta) = -\cos\theta$	,	$\csc(270° + \theta) = -\sec\theta$
$\cos(270° + \theta) = \sin\theta$	,	$\sec(270° + \theta) = \csc\theta$
$\tan(270° + \theta) = -\cot\theta$	,	$\cot(270° + \theta) = -\tan\theta$

For example :

- $\sin 300° = \sin(270° + 30°) = -\cos 30° = \frac{-\sqrt{3}}{2}$
- $\sec 330° = \sec(270° + 60°) = \csc 60° = \frac{2}{\sqrt{3}}$
- $\cot 315° = \cot(270° + 45°) = -\tan 45° = -1$

We can summarize all the previous as follows (Where θ is the measure of an acute angle) :

First

We determine the quadrant in which the given angle lies

$(90° + \theta)$, $(180° - \theta)$	(θ) , $(90° - \theta)$
$(180° + \theta)$, $(270° - \theta)$	$(270° + \theta)$, $(360° - \theta)$

Second

We put the sign of the given trigonometric function according to the quadrant which is we determined.

Third

In the case of angles of measures θ , $(180° - \theta)$, $(180° + \theta)$, $(360° - \theta)$ or $(-\theta)$, the trigonometric function is written as it is and convert the angle of any form to θ

In the case of angles of measures $(90° - \theta)$, $(90° + \theta)$, $(270° - \theta)$ or $(270° + \theta)$, the trigonometric function is changed as the following :

- sin $\rightleftarrows$ cos
- tan $\rightleftarrows$ cot
- csc $\rightleftarrows$ sec

and convert the angle of any form to θ

For example :

$\cos(180° + \theta)$

$(180° + \theta)$ lies in the third quadrant

The function of cosine in the third quadrant is negative (–ve)

The function as it is because the measure of the angle is $(180° + \theta)$

$-\cos\theta$

$\therefore \cos(180° + \theta) = -\cos\theta$

For example :

$\sin(90° + \theta)$

$(90° + \theta)$ lies in the second quadrant

The function of sine in the second quadrant is positive (+ve)

The function is changed because the measure of the angle is $(90° + \theta)$

$+\cos\theta$

$\therefore \sin(90° + \theta) = \cos\theta$

Finding a trigonometric function of an angle whose measure is given ($\propto$)

First If $0° < \propto < 360°$ **i.e.** $\propto \in]0, 2\pi[$

1 We determine the quadrant in which the angle lies , then determine the sign of the trigonometric function.

2 We convert the trigonometric function of $\propto$ into the same trigonometric function of the angle θ and $\theta \in]0, \frac{\pi}{2}[$ as follows :

- Put $\propto$ in the form $(180° - \theta)$ if $\propto$ lies in the 2nd quadrant.
- Put $\propto$ in the form $(180° + \theta)$ if $\propto$ lies in the 3rd quadrant.
- Put $\propto$ in the form $(360° - \theta)$ if $\propto$ lies in the 4th quadrant.

Second If $\propto > 360°$ **i.e.** $\propto > 2\pi$

1 Put $\propto$ in the form of $(2n\pi + \theta)$ where $\theta \in]0, 2\pi[$, n is a positive integer , then the trigonometric function of the angle $\propto$ is the same of the angle θ

2 Find the trigonometric function of the angle θ as in the first.

Third If $\propto$ is (– ve) **i.e.** $\propto < 0°$

We follow one of the following two methods :

The first method

Apply the rule of the trigonometric function of the angle whose measure is negative , that is : $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$, $\tan(-\theta) = -\tan\theta$ and so on , then we find the trigonometric function of the angle θ as in the first and the second.

The second method

Add to $\propto$ an integer number of 2π (i.e. add to $\propto$ the measures $360°$ n or 2π n where $n \in \mathbb{Z}^+$) to get a positive angle $\theta \in]0, 2\pi[$, then we get the trigonometric function of the angle θ , the result is the same trigonometric function of the negative angle $\propto$

Example 1

Find the value of each of the following :

1 $\sin 240°$ 2 $\cos \frac{5\pi}{3}$ 3 $\cos 570°$ 4 $\tan(-150°)$

Solution

1 $\sin 240° = \sin(180° + 60°) = -\sin 60° = -\frac{\sqrt{3}}{2}$

2 $\cos \frac{5\pi}{3} = \cos\left(\frac{5 \times 180°}{3}\right) = \cos 300° = \cos(360° - 60°) = \cos 60° = \frac{1}{2}$

or $\cos \frac{5\pi}{3} = \cos\left(2\pi - \frac{\pi}{3}\right) = \cos \frac{\pi}{3} = \frac{1}{2}$

3 $\cos 570° = \cos(360° + 210°) = \cos 210° = \cos(180° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

4 $\tan(-150°) = -\tan 150° = -\tan(180° - 30°) = -(-\tan 30°) = \tan 30° = \frac{1}{\sqrt{3}}$

Example 2

Find the value of each of the following in two different methods :

1 $\sin 120°$ 2 $\cot 135°$ 3 $\cos(-240°)$ 4 $\sec \frac{15\pi}{4}$

Solution

1 $\sin 120° = \sin(180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}$

or $\sin 120° = \sin(90° + 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

2 $\cot 135° = \cot(180° - 45°) = -\cot 45° = -1$

or $\cot 135° = \cot(90° + 45°) = -\tan 45° = -1$

3 $\cos(-240°) = \cos 240° = \cos(180° + 60°) = -\cos 60° = -\frac{1}{2}$

or $\cos(-240°) = \cos 240° = \cos(270° - 30°) = -\sin 30° = -\frac{1}{2}$

4 $\sec \frac{15\pi}{4} = \sec\left(\frac{15 \times 180°}{4}\right) = \sec 675° = \sec(360° + 315°) = \sec 315°$

$= \sec(360° - 45°) = \sec 45° = \sqrt{2}$

or $\sec \frac{15\pi}{4} = \sec 315° = \sec(270° + 45°) = \csc 45° = \sqrt{2}$

Example 3

Without using the calculator , find the value of the following :

$\cos(-150°)\sin 600° + \cos\frac{2\pi}{3}\sin 330° - \sec\left(\frac{-5\pi}{4}\right)\tan 900°$

Solution

$\because \cos(-150°) = \cos 150° = \cos(180° - 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

, $\sin 600° = \sin(360° + 240°) = \sin 240° = \sin(180° + 60°) = -\sin 60° = -\frac{\sqrt{3}}{2}$

, $\cos\frac{2\pi}{3} = \cos 120° = \cos(180° - 60°) = -\cos 60° = -\frac{1}{2}$

, $\sin 330° = \sin(360° - 30°) = -\sin 30° = -\frac{1}{2}$

, $\sec\left(\frac{-5\pi}{4}\right) = \sec\frac{5\pi}{4} = \sec 225° = \sec(180° + 45°) = -\sec 45° = -\sqrt{2}$

, $\tan 900° = \tan(720° + 180°) = \tan 180° =$ zero

$\therefore$ The expression $= \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right) + \left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right) - \left(-\sqrt{2}\right)$ (zero)

$= \frac{3}{4} + \frac{1}{4} +$ zero $= 1$

TRY TO SOLVE

Without using the calculator :

1 **Find the value of :** $\cos 210° \sin 510° - \sin 330° \cos(-330°)$

2 **Prove that :** $\sin 600° \cos(-390°) + \sin 150° \cos(-240°) = -1$

Example 4

If the directed angle of measure θ is in the standard position , and its terminal side passes through the point $\left(\frac{5}{13}, \frac{12}{13}\right)$, find the following trigonometric functions :

1 $\sin(90° - \theta)$

2 $\cos(180° + \theta)$

3 $\sec(90° + \theta)$

4 $\csc(270° - \theta)$

5 $\tan(360° - \theta)$

6 $\cot(-\theta)$

Solution

$\because x^2 + y^2 = \left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2 = \frac{25}{169} + \frac{144}{169} = 1$

$\therefore$ The point $\left(\frac{5}{13} , \frac{12}{13}\right) \in$ unit circle

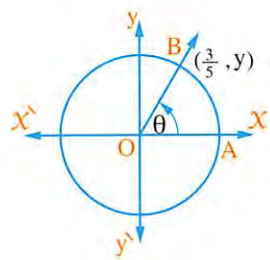
1. $\sin(90° - \theta) = \cos\theta = \frac{5}{13}$
2. $\cos(180° + \theta) = -\cos\theta = -\frac{5}{13}$
3. $\sec(90° + \theta) = -\csc\theta = -\frac{13}{12}$
4. $\csc(270° - \theta) = -\sec\theta = -\frac{13}{5}$
5. $\tan(360° - \theta) = -\tan\theta = -\frac{12}{5}$
6. $\cot(-\theta) = -\cot\theta = -\frac{5}{12}$

Example 5

If θ is the measure of an acute positive angle in its standard position and determines the point B $\left(\frac{3}{5} , y\right)$ on the unit circle , find :

1. $\tan(90° - \theta) + \sec(90° - \theta)$
2. $\cot(270° + \theta) - \tan(90° + \theta) - \sin(180° + \theta)$

Solution



$\because x^2 + y^2 = 1$ for any point on the unit circle.

$\therefore \frac{9}{25} + y^2 = 1$ $\qquad$ $\therefore y^2 = \frac{16}{25}$

$\therefore y = \frac{4}{5}$, where $y > 0$ $\qquad$ $\therefore B = \left(\frac{3}{5} , \frac{4}{5}\right)$

1. $\tan(90° - \theta) + \sec(90° - \theta) = \cot\theta + \csc\theta = \frac{3}{4} + \frac{5}{4} = \frac{8}{4} = 2$
2. $\cot(270° + \theta) - \tan(90° + \theta) - \sin(180° + \theta)$

$= -\tan\theta - (-\cot\theta) - (-\sin\theta)$

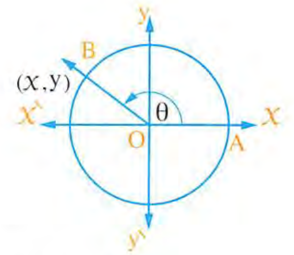
$= -\tan\theta + \cot\theta + \sin\theta = -\frac{4}{3} + \frac{3}{4} + \frac{4}{5} = \frac{13}{60}$

Example 6

If $\cos\theta = \frac{-4}{5}$ where $\theta \in]90° , 180°[$, find the value of each of the following :

1. $\sin(180° - \theta)$
2. $\sec(360° - \theta)$
3. $\cos(-\theta)$
4. $\tan(\theta - 180°)$

Solution



Let m ($\angle$ AOB) = θ , where $\theta \in]90°, 180°[$

as shown in the opposite figure and B (x , y)

$\therefore x = \cos\theta = \frac{-4}{5}$, $y = \sin\theta$, where $y > 0$

$\because x^2 + y^2 = 1$ $\therefore \left(\frac{-4}{5}\right)^2 + y^2 = 1$

$\therefore y^2 = 1 - \frac{16}{25} = \frac{9}{25}$ $\therefore y = \frac{3}{5}$ $\therefore B = \left(-\frac{4}{5}, \frac{3}{5}\right)$

1 $\sin(180° - \theta) = \sin\theta = \frac{3}{5}$

2 $\sec(360° - \theta) = \sec\theta = -\frac{5}{4}$

3 $\cos(-\theta) = \cos\theta = -\frac{4}{5}$

4 $\tan(\theta - 180°) = \tan(\theta - 180° + 360°) = \tan(180° + \theta) = \tan\theta = -\frac{3}{4}$

TRY TO SOLVE

If the terminal side of the directed angle of measure θ in its standard position intersects the unit circle at the point $\left(x, \frac{12}{13}\right)$ such that $90° < \theta < 180°$, find the value of :

$13\cos(360° - \theta) + \tan 225° + \sec^2 300° + 12\tan(270° - \theta)$

Note

We can find the values of the trigonometric functions of an angle directly if we draw the angle in its standard position and we draw the right-angled triangle that represents it by using the value of the given trigonometric function concerning the signs according to the quadrant in which the angle lies as follows :

In the 1st quadrant | In the 2nd quadrant | In the 3rd quadrant | In the 4th quadrant

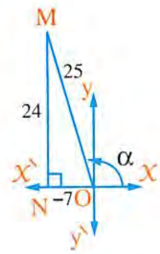
Example 7

If $\cos\alpha = \frac{-7}{25}$ where α is the smallest positive angle , $\tan\beta = \frac{3}{4}$

, where β is the greatest positive angle where $0° \le \beta \le 360°$

Find the value of : $\cos(180° + \alpha)\sin(\beta - 90°) + \sin(360° - \alpha)\sin(180° - \beta)$

Solution



$\because \cos\alpha < 0$

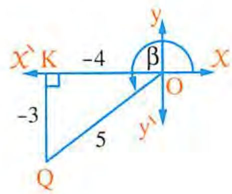
$\therefore \alpha$ lies in the 2nd or 3rd quadrant.

, $\because \alpha$ is the smallest positive angle.

$\therefore \alpha$ lies in the 2nd quadrant.

, $\because \cos\alpha = \frac{-7}{25}$ $\qquad \therefore (MN)^2 = (25)^2 - (7)^2 = 576$

$\therefore MN = 24$ length unit.



, $\because \tan\beta > 0$

$\therefore \beta$ lies in the 1st or 3rd quadrant.

, $\because \beta$ is the greatest positive angle. $\qquad \therefore \beta$ lies in the 3rd quadrant.

, $\because \tan\beta = \frac{3}{4}$ $\qquad \therefore (OQ)^2 = (3)^2 + (4)^2 = 25$

$\therefore OQ = 5$ length unit.

$\therefore$ The expression $= \cos(180° + \alpha)\sin(\beta - 90°) + \sin(360° - \alpha)\sin(180° - \beta)$

$$= -\cos\alpha \sin(270° + \beta) + (-\sin\alpha)\sin\beta$$

$$= (-\cos\alpha)(-\cos\beta) - \sin\alpha\sin\beta$$

$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$= \frac{-7}{25} \times \left(\frac{-4}{5}\right) - \frac{24}{25} \times \frac{-3}{5} = \frac{28}{125} + \frac{72}{125} = \frac{100}{125} = \frac{4}{5}$$

Remark

If $\sin\alpha = \cos\beta$ or $\tan\alpha = \cot\beta$ or $\csc\alpha = \sec\beta$

, then $\alpha + \beta = 90°$ such that α , β are the two measures of two acute positive angles.

For example : If $\tan 23° = \cot\alpha$, then $23° + \alpha = 90°$ i.e. $\alpha = 67°$

Example 8

If $\sin(3\theta + 28°) = \cos(2\theta - 13°)$, find one value of θ where $0° < \theta < 90°$

Solution

$\because \sin(3\theta + 28°) = \cos(2\theta - 13°)$

$\therefore 3\theta + 28° + 2\theta - 13° = 90°$ $\qquad \therefore 5\theta + 15° = 90°$

$\therefore 5\theta = 75°$ $\qquad \therefore \theta = 15°$

Notice that

There are other values for θ such as $\theta = 49°$ or $\theta = 87°$ that satisfy the equation and to find these values we have to generalize the previous remark to get a general solution for this kind of equations.

Generalizing the previous remark

1 If $\sin \alpha = \cos \beta$, then $\sin \alpha = \sin (90° - \beta)$

$\therefore \alpha = 90° - \beta$ or $\alpha + 90° - \beta = 180°$

$\therefore \alpha + \beta = 90°$ | $\therefore \alpha - \beta = 90°$

We can add the multiplies of (360°) to the angle 90°

An Important Alert

On solving , we must start by sine angle α

2 In the same way , we can deduce the same rules if $\csc \alpha = \sec \beta$

3 If $\tan \alpha = \cot \beta$, then :

$\tan \alpha = \tan (90° - \beta)$ or $\tan \alpha = \tan (270° - \beta)$

$\therefore \alpha = 90° - \beta$ | $\therefore \alpha = 270° - \beta$

$\therefore \alpha + \beta = 90°$ | $\therefore \alpha + \beta = 270°$

We can add the multiplies of (360°) to the angles 90° and 270°

So , the general solution for any two angles α , β could be written as follows :

The general solution to solve the equations in the form : $\sin \alpha = \cos \beta$ or $\csc \alpha = \sec \beta$ or $\tan \alpha = \cot \beta$

1 If $\sin \alpha = \cos \beta$

, then $\alpha \pm \beta = 90° + 360° n$ **i.e.** $\alpha \pm \beta = \frac{\pi}{2} + 2\pi n$ where $n \in \mathbb{Z}$

i.e. The measure of angle of sine $\pm$ the measure of angle of cosine $= 90° + 360° n$

2 If $\csc \propto = \sec \beta$

, then $\propto \pm \beta = 90° + 360° n$ i.e. $\propto \pm \beta = \frac{\pi}{2} + 2\pi n$ where $n \in \mathbb{Z}$

, $\propto \neq n\pi$, $\beta \neq (2n+1)\frac{\pi}{2}$

3 If $\tan \propto = \cot \beta$

, then $\propto + \beta = 90° + 180° n$ i.e. $\propto + \beta = \frac{\pi}{2} + \pi n$ where $n \in \mathbb{Z}$

, $\propto \neq (2n+1)\frac{\pi}{2}$, $\beta \neq n\pi$

Example 9

Find the general solution of the equation :
$\cos 2\theta = \sin 4\theta$, then find the values of θ where $\theta \in]0 , \frac{\pi}{2}[$

Solution

$\because \cos 2\theta = \sin 4\theta$

$\therefore \sin 4\theta = \cos 2\theta$

$\therefore \propto = 4\theta , \beta = 2\theta$

$\therefore 4\theta \pm 2\theta = \frac{\pi}{2} + 2\pi n$

$\therefore$ Either $6\theta = \frac{\pi}{2} + 2\pi n$

$\therefore \theta = \frac{\pi}{12} + \frac{\pi}{3} n$

or $2\theta = \frac{\pi}{2} + 2\pi n$

$\therefore \theta = \frac{\pi}{4} + \pi n$

$\therefore$ The general solution is $\frac{\pi}{12} + \frac{\pi}{3} n$ or $\frac{\pi}{4} + \pi n$ where $n \in \mathbb{Z}$

at $n = 0$: $\therefore \theta = \frac{\pi}{12} \in]0 , \frac{\pi}{2}[$ or $\theta = \frac{\pi}{4} \in]0 , \frac{\pi}{2}[$

at $n = 1$: $\therefore \theta = \frac{\pi}{12} + \frac{\pi}{3} = \frac{5}{12}\pi \in]0 , \frac{\pi}{2}[$ or $\theta = \frac{\pi}{4} + \pi = \frac{5}{4}\pi \notin]0 , \frac{\pi}{2}[$

at $n = 2$: $\therefore \theta = \frac{\pi}{12} + \frac{2\pi}{3} = \frac{3}{4}\pi \notin]0 , \frac{\pi}{2}[$

$\therefore$ The values of θ are $\frac{\pi}{12} , \frac{\pi}{4} , \frac{5\pi}{12}$ i.e. $15° , 45° , 75°$

TRY TO SOLVE

Find the general solution of the equation : $\sin 3\theta = \cos \theta$, then find all the values of θ where $\theta \in]0 , \frac{\pi}{2}[$ which satisfy the equation.

Example 10

Find the solution set of each of the following equations :

1. $2 \sin \theta - 1 = 0$ where $\theta \in]0, \frac{\pi}{2}[$
2. $2 \cos\left(\frac{\pi}{2} - \theta\right) + \sqrt{3} = 0$ where $\theta \in]0, 2\pi[$
3. $4 \cos^2 \theta - 3 = 0$ where $\theta \in]0, 2\pi[$

Solution

1 $\because 2 \sin \theta - 1 = 0$ $\quad \therefore \sin \theta = \frac{1}{2}$ (positive)

$\therefore \theta$ lies in the 1st or 2nd quadrant. $\quad \because$ The acute angle whose sine $= \frac{1}{2}$ is $30°$

$\therefore \theta = 30°$ or $\theta = 180° - 30° = 150°$ $\left(\text{refused because } \theta \in]0, \frac{\pi}{2}[\right)$

$\therefore$ The S.S $= \{30°\}$

2 $\because 2 \cos\left(\frac{\pi}{2} - \theta\right) + \sqrt{3} = 0$ $\quad \therefore 2 \sin \theta = -\sqrt{3}$

$\therefore \sin \theta = \frac{-\sqrt{3}}{2}$ (negative) $\quad \therefore \theta$ lies in the 3rd or 4th quadrant.

, $\because$ the acute angle whose sine $= \frac{\sqrt{3}}{2}$ is $60°$

$\therefore \theta = 180° + 60° = 240°$ or $\theta = 360° - 60° = 300°$

$\therefore$ The S.S $= \{240°, 300°\}$

3 $\because 4 \cos^2 \theta - 3 = 0$ $\quad \therefore 4 \cos^2 \theta = 3$

$\therefore \cos^2 \theta = \frac{3}{4}$ $\quad \therefore \cos \theta = \pm \frac{\sqrt{3}}{2}$

$\therefore$ Either $\cos \theta = \frac{\sqrt{3}}{2}$ (positive) $\quad \therefore \theta$ lies in the 1st or 4th quadrant.

, $\because$ the acute angle whose cosine $= \frac{\sqrt{3}}{2}$ is $30°$

$\therefore \theta = 30°$ or $\theta = 360° - 30° = 330°$

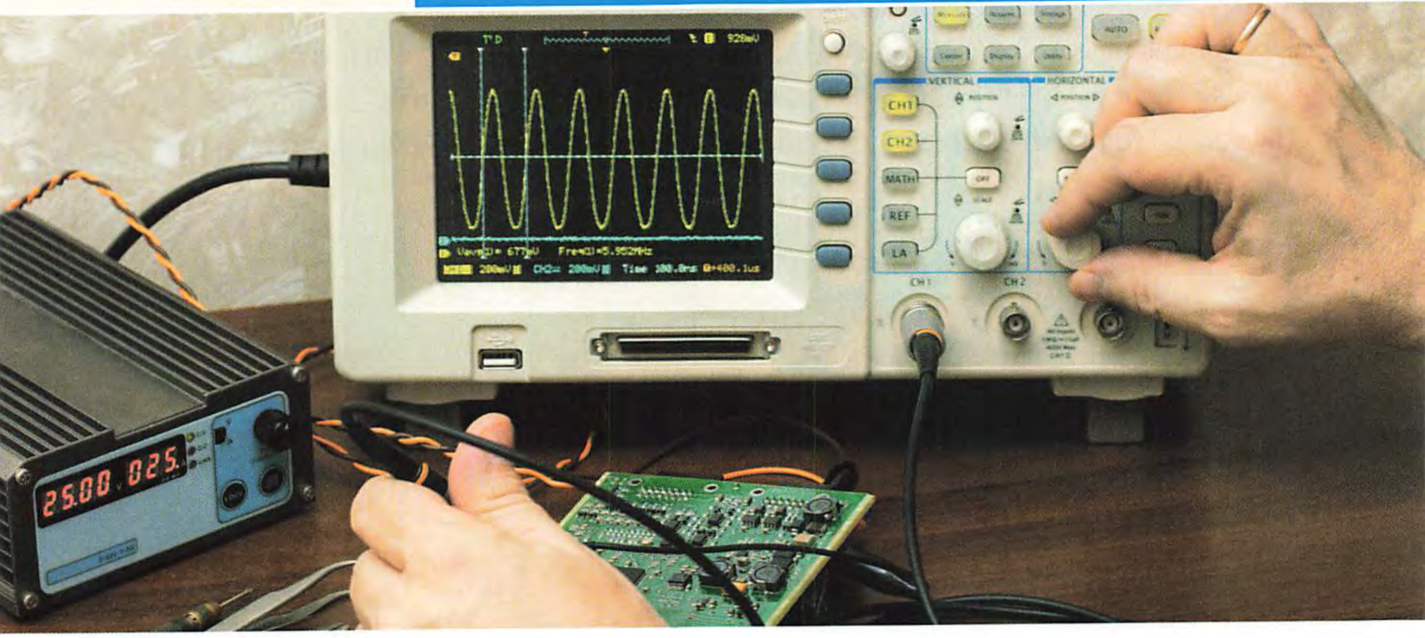
or $\cos \theta = \frac{-\sqrt{3}}{2}$ (negative) $\quad \therefore \theta$ lies in the 2nd or 3rd quadrant.

$\therefore \theta = 180° - 30° = 150°$ or $\theta = 180° + 30° = 210°$

$\therefore$ The S.S $= \{30°, 150°, 210°, 330°\}$

Lesson 5

Graphing trigonometric functions

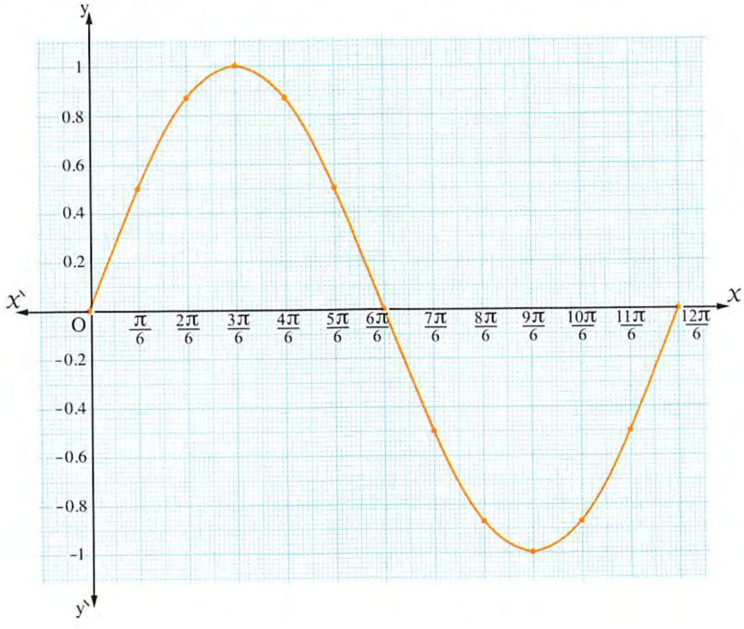


First Sine function : $f : f(\theta) = \sin\theta$

To represent the function $f : f(\theta) = \sin\theta$ graphically ,
we form the following table for some special values of θ , where $\theta \in [0 , 2\pi]$ and the corresponding values of $\sin\theta$

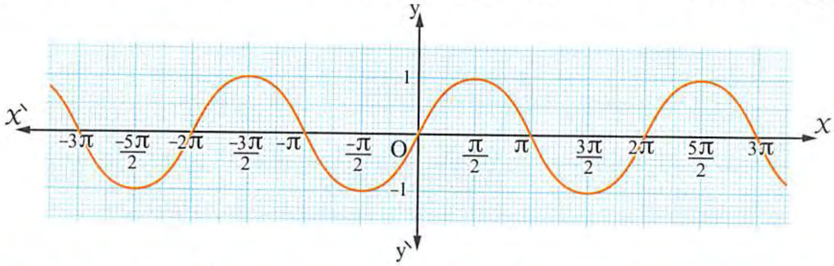
θ	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	2π
$\sin\theta$	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Represent all of the points that we get in the table on the coordinate axes and join them to get the curve of the function f on the interval $[0 , 2\pi]$



We notice that : The function is periodic and its period is 2π (**i.e.** 360°) where the curve of this function repeats itself on the intervals $[0, 2\pi]$, $[2\pi, 4\pi]$, $[4\pi, 6\pi]$, ... and also on the intervals $[-2\pi, 0]$, $[-4\pi, -2\pi]$, $[-6\pi, -4\pi]$, ...

The general form of the curve of the sine function is as shown in the following graph :



From the previous, we can deduce the properties of the sine function $f : f(\theta) = \sin\theta$:

1. The domain of the sine function is $]-\infty, \infty[$
2. • The maximum value of the function is 1 and it happens when $\theta = \frac{\pi}{2} + 2n\pi$, $n \in \mathbb{Z}$
   • The minimum value of the function is -1 and it happens when $\theta = \frac{3\pi}{2} + 2n\pi$, $n \in \mathbb{Z}$
3. The range of the function $= [-1, 1]$
4. The function is periodic and its period is 2π (**i.e.** 360°)

Example 1

Graph the function where $y = 2\sin\theta$, where $\theta \in [0, 2\pi]$, then from the graph find the maximum and minimum values of the function, its range and its period.

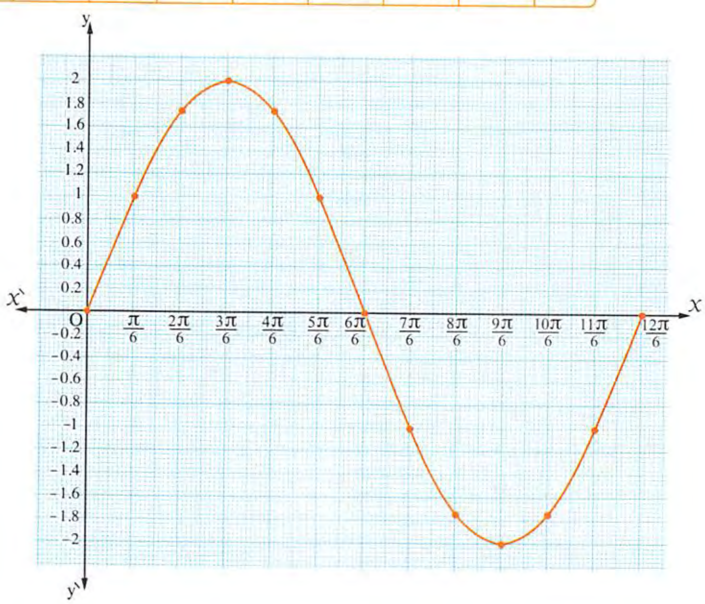
Solution

θ	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	2π
y	0	1	1.7	2	1.7	1	0	-1	-1.7	-2	-1.7	-1	0

• The maximum value of the function = 2, the minimum value of the function = -2

• The range of the function $= [-2, 2]$

• The period of the function $= 2\pi$ (**i.e.** 360°)



TRY TO SOLVE

Represent graphically the function $f : f(\theta) = 3 \sin \theta$, where $\theta \in [0 , 2\pi]$, then from the graph find :

1 The maximum and minimum values of the function.

2 The range of the function.

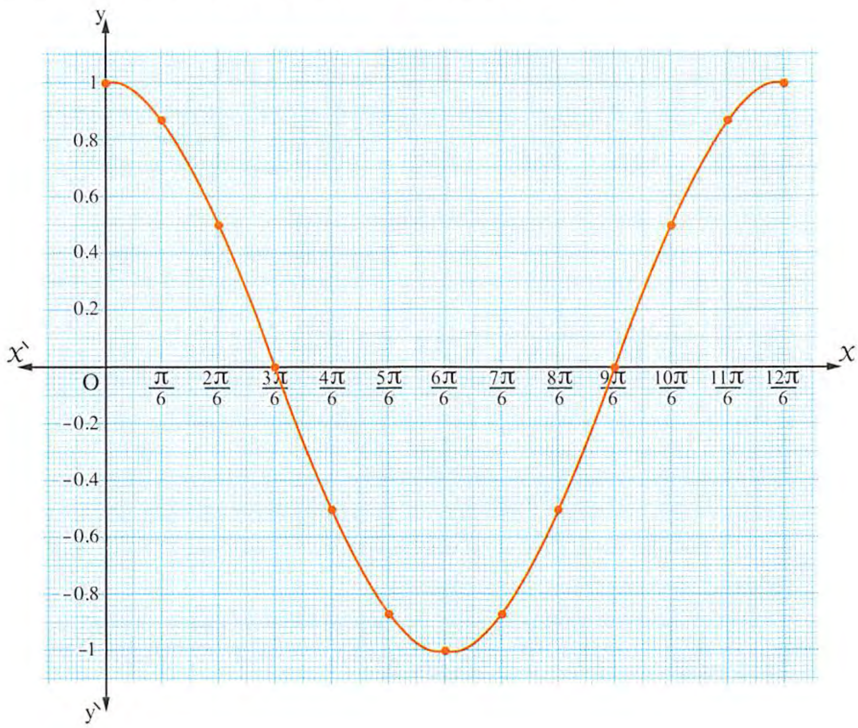
3 The period of the function.

Second Cosine function : $f : f(\theta) = \cos \theta$

To represent the function $f : f(\theta) = \cos \theta$ graphically , we form the following table for some special values of θ on the interval $[0 , 2\pi]$ and the corresponding values of $\cos \theta$

θ	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	2π
$\cos \theta$	1	0.87	0.5	0	− 0.5	− 0.87	− 1	− 0.87	− 0.5	0	0.5	0.87	1

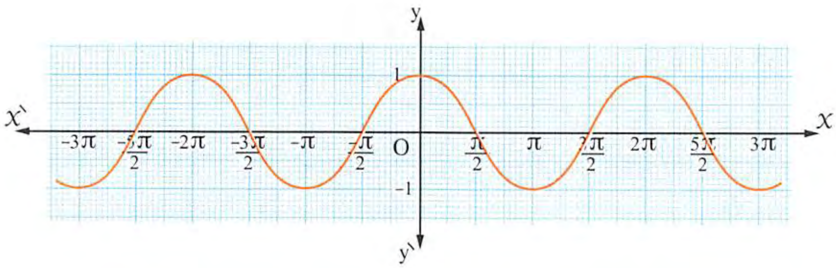
Represent all of the points that we get in the table on the coordinate axis and join them to get the curve of the function f on the interval $[0 , 2\pi]$



We notice that :

The function is periodic and its period is 2π (**i.e.** 360°) where the curve of this function repeats itself on the intervals $[0 , 2\pi]$, $[2\pi , 4\pi]$, $[4\pi , 6\pi]$, ... and also on the intervals $[-2\pi , 0]$, $[-4\pi , -2\pi]$, $[-6\pi , -4\pi]$, ...

The general form of the curve of the cosine function is as shown in the following graph :



From the previous, we can deduce the properties of the cosine function $f : f(\theta) = \cos\theta$:

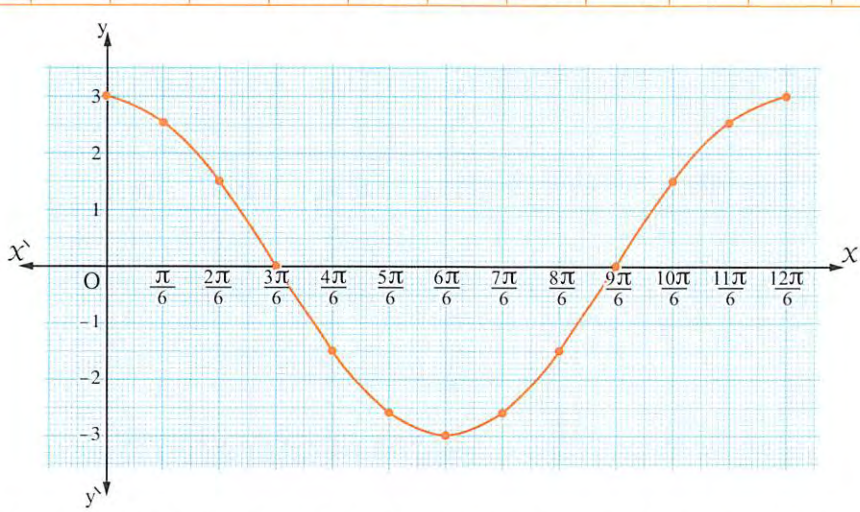
1. The domain of the cosine function is $]-\infty , \infty[$
2. - The maximum value of the function equals 1 and it happens when $\theta = 2n\pi$, where $n \in \mathbb{Z}$
   - The minimum value of the function equals -1 and it happens when $\theta = \pi + 2\pi n$, where $n \in \mathbb{Z}$
3. The range of the function $= [-1 , 1]$
4. The function is periodic and its period is 2π (**i.e.** 360°)

Example 2

Graph the function where $y = 3\cos\theta$, where $\theta \in [0 , 2\pi]$, and from the graph find the maximum and minimum values of the function , its range and its period.

Solution

θ	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	2π
y	3	2.6	1.5	0	– 1.5	– 2.6	– 3	– 2.6	– 1.5	0	1.5	2.6	3



- The maximum value of the function = 3 , the minimum value of the function = – 3
- The range of the function = $[-3 , 3]$
- The period of the function = 2π (**i.e.** 360°)

TRY TO SOLVE

Represent graphically the function $f : f(\theta) = 2\cos\theta$, where $\theta \in [0 , 2\pi]$, then from the graph find :

1. The maximum and minimum values of the function.
2. The range of the function.
3. The period of the function.

Note

Each of the two functions : $y = a \sin b\,\theta$, $y = a \cos b\,\theta$ is periodic , its period is $\frac{2\pi}{|b|}$ and its range is $[-a , a]$ where a is positive.

For example : The function $f : f(X) = 3 \sin 5X$ its range $[-3 , 3]$ and its period $\frac{2\pi}{5}$, If range of the function $f : f(X) = a \sin 5X$ is $[-3 , 3]$, then $a = \pm 3$

Using the technology

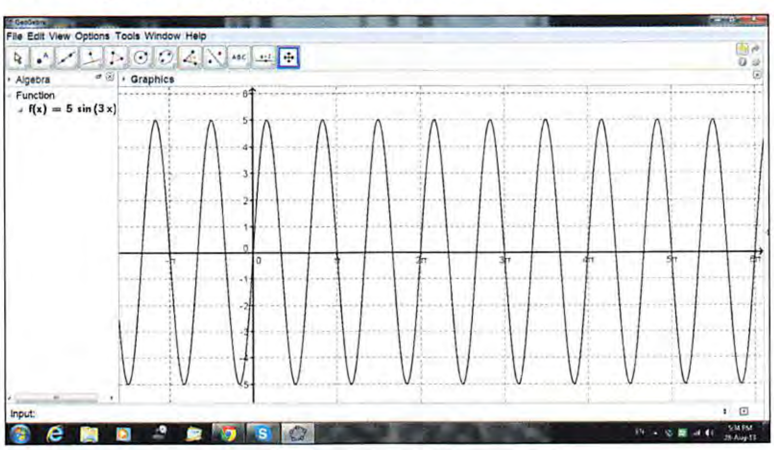
Use a graph program on your computer to graph the function where $y = 5 \sin 3\theta$, and from the graph , find :

- The range of the function.
- The maximum and minimum values of the function.
- The period of the function.

Solution

We will use GeoGebra Program that we can download for free from the website "www.geogebra.org"

1. Write in the "input" bar the form of the function "$y = 5 \sin (3X)$"
2. Press "enter" and the graph will appear as follows :



- The range of the function = $[-5 , 5]$
- The maximum value = 5 , the minimum value = -5
- The period of the function = $\frac{2\pi}{|b|} = \frac{2\pi}{3}$ i.e. 120°

Note It is possible to graph the function $y = 5 \sin 3\theta$ (in the previous example) where : $0° \le \theta \le 120°$ without using the computer as follows :

$\because 0° \le \theta \le 120°$

$\therefore 0° \le 3\theta \le 360°$

Substituting in 3θ with some values of special angles :

$0°, 30°, 60°, 90°, 120°, \dots, 360°$

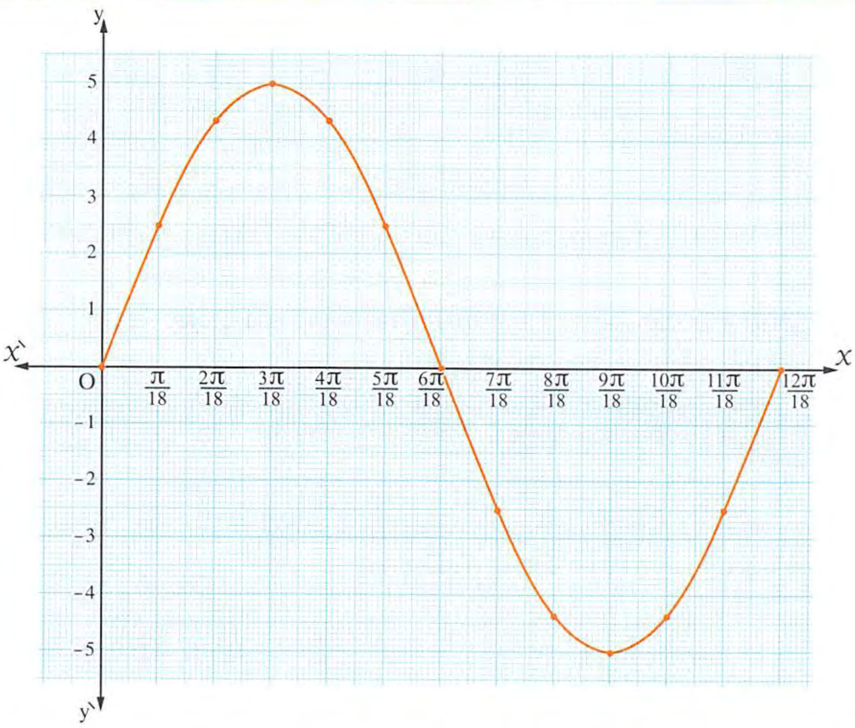
We get the values of θ by dividing by 3 , which are :

$0°, 10°, 20°, 30°, 40°, \dots, 120°$

which is equivalent to : $0, \frac{\pi}{18}, \frac{2\pi}{18}, \frac{3\pi}{18}, \dots, \frac{12\pi}{18}$

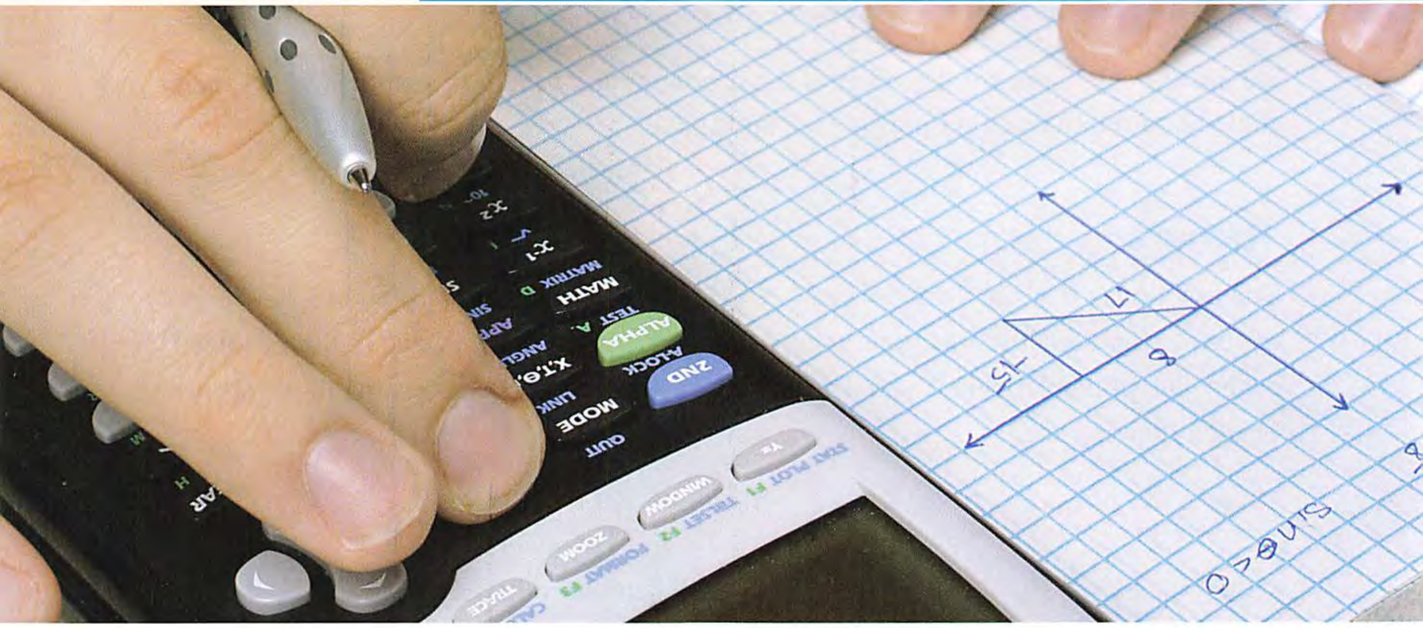
Then we form the following table :

θ	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$	$\frac{7\pi}{18}$	$\frac{8\pi}{18}$	$\frac{9\pi}{18}$	$\frac{10\pi}{18}$	$\frac{11\pi}{18}$	$\frac{12\pi}{18}$
$y = 5 \sin 3\theta$	0	2.5	4.3	5	4.3	2.5	0	−2.5	−4.3	−5	−4.3	−2.5	0



The graph represents one period of the function where $y = 5 \sin 3\theta$ which could be repeated to get the graph that appears when we represent it by using computer.

Lesson 6 Finding the measure of an angle given the value of one of its trigonometric ratios



* We have studied that if $y = \sin \theta$, then it is possible to find the value of y if the value of θ is known

i.e. If $\theta = 30^\circ$, then $y = \sin 30^\circ = \frac{1}{2}$

* There is an inverse form is used to find the value of θ if the value of y is known , which is $\theta = \sin^{-1} y$

i.e. If $y = \frac{1}{2}$, then $\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

Example 1

Find the measure of the positive acute angle θ which satisfies each of the following :

1 $\sin \theta = 0.6438$

2 $\cos \theta = 0.4517$

Solution

1 Using the keys of the calculator in the following succession from the left :

SHIFT | Sin ($\sin^{-1}$) | 0 | . | 6 | 4 | 3 | 8 | = | °,,,

, then the number $40^\circ\ 4'\ 32.75''$ will appear on the display. $\therefore \theta \simeq 40^\circ\ 4'\ 33''$

2 Using the keys of the calculator in the following succession from the left :

SHIFT | cos ($\cos^{-1}$) | 0 | . | 4 | 5 | 1 | 7 | = | °,,,

, then the number $63^\circ\ 8'\ 49.9''$ will appear on the display. $\therefore \theta \simeq 63^\circ\ 8'\ 50''$

Notice that

We use the calculator for the value of the trigonometric function is neither for a special angle nor a relative angle for a special angle.

Remark

The functions : $\theta = \sin^{-1} x$, $\theta = \cos^{-1} x$, $\theta = \tan^{-1} x$ are known as inverse functions of the basic trigonometric functions , these functions give a unique value of the variable θ for each value of the variable x and determine θ in a certain range according to the properties of each function so,

For example :

$\sin^{-1}(-\frac{1}{2}) = -30^\circ$ i.e. (unique value $\in [-\frac{\pi}{2}, \frac{\pi}{2}]$)

, $\cos^{-1}(\frac{1}{2}) = 60^\circ$ i.e. (unique value $\in [0, \pi]$)

So , as calculating θ where

$\theta = \sin^{-1} a$, $\theta = \cos^{-1} a$ or $\theta = \tan^{-1} a$ we use the calculator directly and the solution is a unique value but as calculating θ where $0 < \theta < 360^\circ$

, $\sin \theta = a$, $\cos \theta = a$ or $\tan \theta = a$ we do the steps as the following example.

Example 2

If $0^\circ < \theta < 360^\circ$, find θ which satisfies each of the following :

1 $\cos \theta = 0.8177$ **2** $\cot \theta = -8.6421$

Solution

1 $\because \cos \theta = 0.8177 > 0$ (positive)

$\therefore \theta$ lies in the 1^{st} or 4^{th} quadrant.

We find the acute angle whose cosine is 0.8177 by writing $\cos^{-1} 0.8177$ using the keys of the calculator in the following succession from the left :

SHIFT $\cos^{-1}$ COS 0 . 8 1 7 7 = °'''

$\therefore \cos^{-1} 0.8177 \simeq 35^\circ\, 8'\, 41''$

$\therefore$ The 1^{st} quadrant : $\theta \simeq 35^\circ\, 8'\, 41''$, the 4^{th} quadrant : $\theta \simeq 360^\circ - (35^\circ\, 8'\, 41'') = 324^\circ\, 51'\, 19''$

2 $\because \cot\theta = -8.6421 < 0$ (negative)

$\therefore$ θ lies in the 2^{nd} or 4^{th} quadrant.

We find the acute angle whose cotan is $|-8.6421|$ by writing $\cot^{-1} 8.6421$ using the keys of the calculator in the following succession from the left :

SHIFT $\tan^{-1}$ [tan] [8] [.] [6] [4] [2] [1] [x^{-1}] [=] [°′″]

$\therefore \cot^{-1} 8.6421 \simeq 6^\circ\ 36'\ 2''$

$\therefore$ The 2^{nd} quadrant : $\theta \simeq 180^\circ - (6^\circ\ 36'\ 2'') = 173^\circ\ 23'\ 58''$

, the 4^{th} quadrant : $\theta \simeq 360^\circ - (6^\circ\ 36'\ 2'') = 353^\circ\ 23'\ 58''$

TRY TO SOLVE

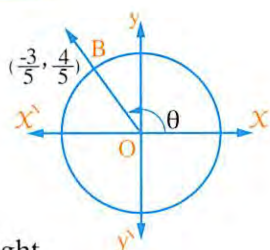
Find θ where $0^\circ < \theta < 360^\circ$ which satisfies :

1 $\sin\theta = 0.8$ **2** $\cot\theta = 0.4695$ **3** $\csc\theta = -2.9115$

Example 3

If the terminal side of the positive directed angle of measure θ in its standard position intersects the unit circle at the point B $\left(-\frac{3}{5}, \frac{4}{5}\right)$, find θ where $0^\circ < \theta < 360^\circ$

Solution



$\because$ The point B $\left(-\frac{3}{5}, \frac{4}{5}\right)$ lies in the 2^{nd} quadrant.

$\therefore$ The directed angle of measure θ lies in the 2^{nd} quadrant.

$\therefore \sin\theta = y = \frac{4}{5}$ $\qquad \therefore \theta = \sin^{-1}\frac{4}{5}$

and use the keys of the calculator in the following succession from left to right to find $\sin^{-1}\frac{4}{5}$:

SHIFT $\sin^{-1}$ [sin] [$\frac{4}{5}$] [=] [°′″]

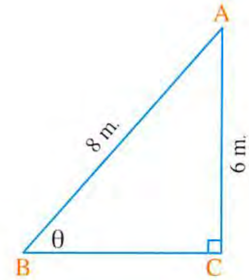
$\therefore \sin^{-1}\frac{4}{5} \simeq 53^\circ\ 7'\ 48''$

$\therefore \theta = 180^\circ - (53^\circ\ 7'\ 48'') = 126^\circ\ 52'\ 12''$

Example 4

A ladder of length 8 m. rests on a vertical wall and a horizontal ground. If the height of the ladder on the ground surface equals 6 m. , find in radian the measure of the angle of inclination of the ladder on the ground.

Solution



The ladder makes with the vertical wall and the horizontal ground a right-angled triangle , let Δ ABC be right at $\angle$ C , m ($\angle$ CBA) = θ

$\therefore \sin\theta = \frac{AC}{AB} = \frac{6}{8} = \frac{3}{4}$, where $0° < \theta < 90°$

$\therefore \theta = \sin^{-1}\frac{3}{4}$

and use the keys of the calculator in the following succession from left to right to find $\sin^{-1}\frac{3}{4}$:

$\therefore \theta \simeq 48° \, 35' \, 25''$ $\qquad \therefore \theta^{rad} = 48° \, 35' \, 25'' \times \frac{\pi}{180°} \simeq 0.848^{rad}$

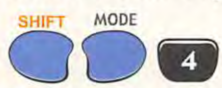
$\therefore$ The measure of the inclination angle of the ladder on the ground $\simeq 0.848^{rad}$

Note

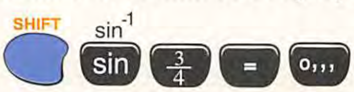
In the previous example :

$\theta = \sin^{-1}\frac{3}{4}$, we can get θ in radian directly using the calculator as follows :

1 Press , in succession , from left to right to convert the calculator from degree (Deg) system into radian (Rad) system.



2 Find θ in radian directly by pressing in succession from left to right

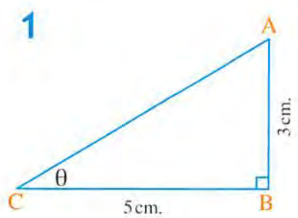


$\therefore \theta^{rad} \simeq 0.848$

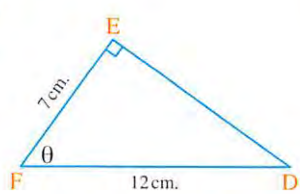
TRY TO SOLVE

Find θ in radian in each of the following right-angled triangles :

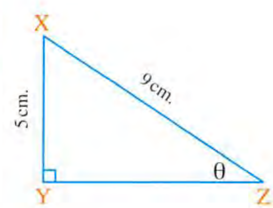
1



2



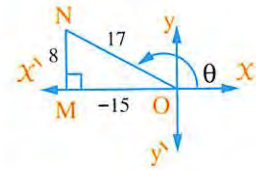
3



Example 5

If $\sin\theta = \frac{8}{17}$ where $90° < \theta < 180°$, find θ to the nearest second , then find the other trigonometric functions of the angle of measure θ

Solution



$\because \sin\theta = \frac{8}{17}$ $\therefore \theta = \sin^{-1}\frac{8}{17} \simeq 28^\circ\ 4'\ 21''$

, $\because 90^\circ < \theta < 180^\circ$ $\therefore \theta$ lies in the 2^{nd} quadrant.

$\therefore \theta = 180^\circ - 28^\circ\ 4'\ 21'' = 151^\circ\ 55'\ 39''$

$\because \sin\theta = \frac{8}{17}$ $\therefore$ let MN = 8 unit length , ON = 17 unit length.

, then (using Pythagoras theorem) OM = 15 unit length with a negative sign.

$\therefore \cos\theta = \frac{OM}{ON} = \frac{-15}{17}$, $\tan\theta = \frac{MN}{OM} = \frac{8}{-15} = \frac{-8}{15}$

, $\csc\theta = \frac{ON}{MN} = \frac{17}{8}$, $\sec\theta = \frac{ON}{OM} = \frac{17}{-15} = \frac{-17}{15}$, $\cot\theta = \frac{OM}{MN} = \frac{-15}{8}$

TRY TO SOLVE

If $\sin\theta = \frac{-1}{3}$, $270^\circ < \theta < 360^\circ$

1 Find : θ to the nearest second.

2 Find the value of each of : $\cos\theta$, $\tan\theta$, $\sec\theta$

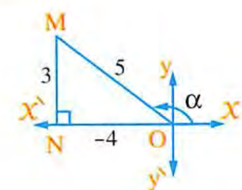
Example 6

If $\sin\alpha = \frac{3}{5}$ where $90^\circ < \alpha < 180^\circ$, $\tan\beta = \frac{-12}{5}$ where $\beta \in]\frac{3\pi}{2}, 2\pi[$

, $\sin\theta = \sin(180^\circ - \alpha)\cos(\beta - 180^\circ)\cos\alpha$

, find θ to the nearest minute where $0^\circ < \theta < 90^\circ$

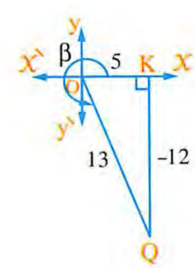
Solution



$\because (ON)^2 = (5)^2 - (3)^2 = 16$

$\therefore$ ON = 4 unit length with a negative sign.

, $\because (OQ)^2 = (12)^2 + (5)^2 = 169$ $\therefore$ OQ = 13 unit length.



$\therefore \sin\theta = \sin(180^\circ - \alpha)\cos(\beta - 180^\circ)\cos\alpha$

$= \sin\alpha\cos(180^\circ + \beta)\cos\alpha$

$= (\sin\alpha)(-\cos\beta)(\cos\alpha)$

$= \frac{3}{5} \times \frac{-5}{13} \times \frac{-4}{5} = \frac{12}{65}$

, $\because 0^\circ < \theta < 90^\circ$ $\therefore \theta$ lies in the 1^{st} quadrant.

Using the calculator , we find that : $\theta \simeq 10^\circ\ 38'$

Example 7

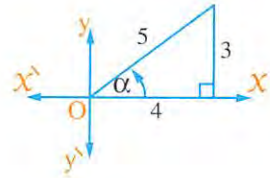
If $5 \sin (180^\circ - \alpha) = 3$ where $0^\circ < \alpha < 90^\circ$, $5 \cot (90^\circ + \beta) - 12 = 0$ where $90^\circ < \beta < 180^\circ$

Find the value of θ where : $\cos \theta = \cos (90^\circ + \alpha) \tan (270^\circ + \beta) \tan (270^\circ - \alpha)$

, where $\theta \in]0 , 2\pi[$

Solution

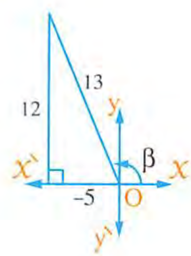
$\because 5 \sin (180^\circ - \alpha) = 3$ $\qquad \therefore 5 \sin \alpha = 3$



$\therefore \sin \alpha = \frac{3}{5}$ where α lies in the 1^{st} quadrant

, $\because 5 \cot (90^\circ + \beta) = 12$ $\qquad \therefore 5 (-\tan \beta) = 12$

$\therefore \tan \beta = \frac{-12}{5}$ where β lies in the 2^{nd} quadrant.



$\cos \theta = \cos (90^\circ + \alpha) \tan (270^\circ + \beta) \tan (270^\circ - \alpha)$

$= (-\sin \alpha) \times (-\cot \beta) \times \cot \alpha$

$= \frac{3}{5} \times -\frac{5}{12} \times \frac{4}{3} = -\frac{1}{3}$

, $\because \cos \theta < 0$

$\therefore \theta \in$ the 2^{nd} quadrant $\qquad$ or $\qquad \theta \in$ the 3^{rd} quadrant

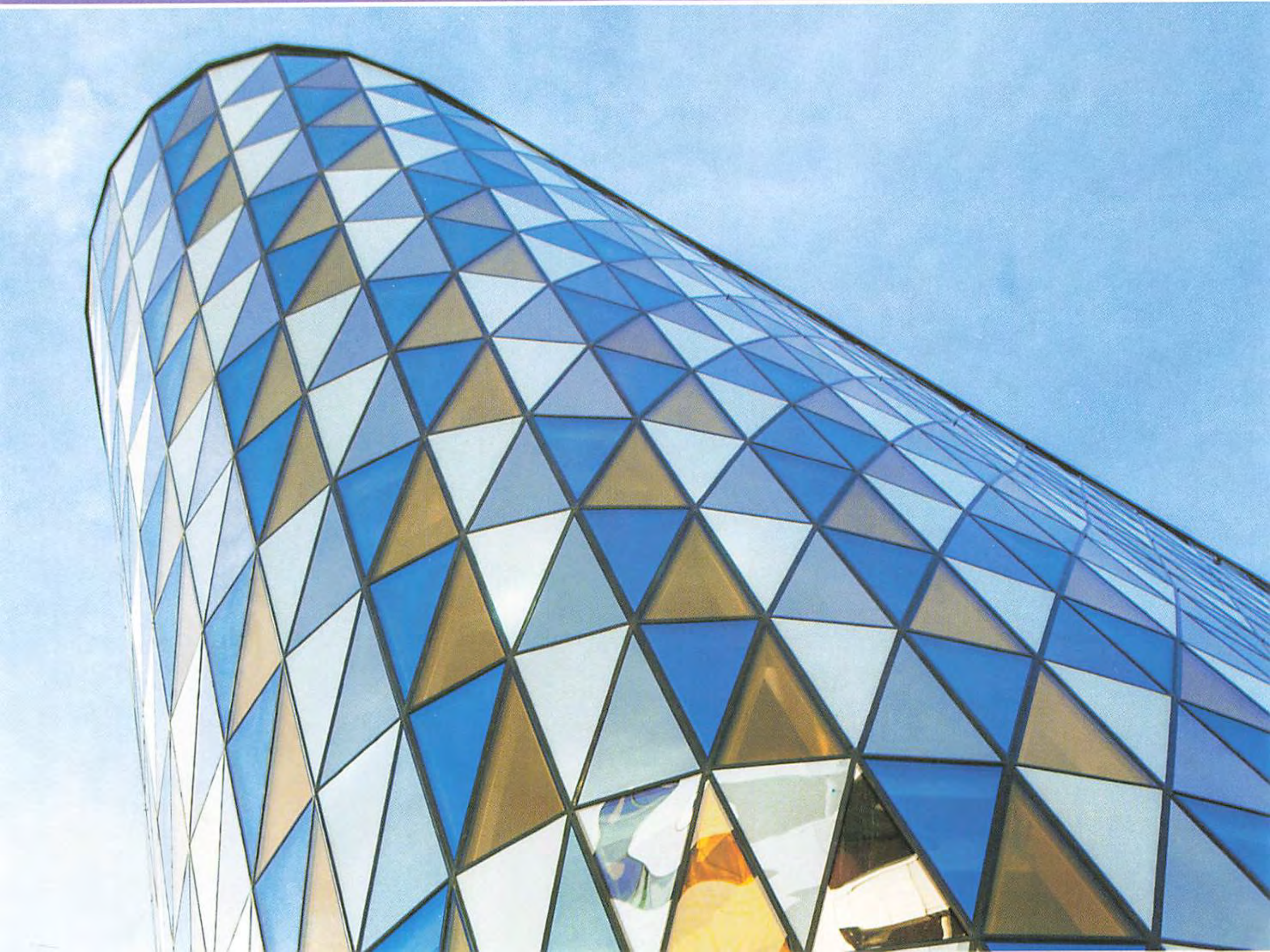
, $\because$ acute angle whose cosine $= \frac{1}{3}$ is $70^\circ\, 32'$

$\therefore \theta = 180^\circ - 70^\circ\, 32' = 109^\circ\, 28'$ $\qquad$ or $\qquad \theta = 180^\circ + 70^\circ\, 32' = 250^\circ\, 32'$

Second Geometry

UNIT 3 Similarity.

UNIT 4 The triangle proportionality theorems.



Unit Three

Similarity.

Unit Lessons

Lesson **1** Similarity of polygons.

Lesson **2** Similarity of triangles.

Lesson **3** The relation between the areas of two similar polygons.

Lesson **4** Applications of similarity in the circle.

Learning outcomes

By the end of this unit, the student should be able to :

- Revise what he / she has previously studied in the preparatory stage on similarity.
- Use the scale factor of similarity to find lengths of sides of similar polygons.
- Recognize similarity postulate "If two angles of one triangle are congruent to their two corresponding angles of another triangle, then the two triangles are similar".
- Know that : If a line is drawn parallel to one side of a triangle and intersects the other two sides or the lines containing them, then the resulting triangle is similar to the original triangle.
- Know that : In any right-angled triangle, the altitude to the hypotenuse separates the triangle into two triangles which are similar to each other and to the original triangle.
- Solve problems and mathematics applications on cases of similarity of two triangles.
- Recognize and prove the theorem : (If the side lengths of two triangles are in proportion, then the two triangles are similar).
- Recognize and prove the theorem : (If an angle of one triangle is congruent to an angle of another triangle and lengths of the sides including those angles are in proportion, then the triangles are similar).
- Use similarity of triangles in indirect measurements.
- Recognize and prove the theorem : (The ratio of the areas of the surfaces of two similar triangles equals the square of the ratio of the lengths of any two corresponding sides of the two triangles).
- Recognize and prove the theorem : (The ratio of the areas of the surfaces of two similar polygons equals the square of the ratio of the lengths of any two corresponding sides of the two polygons).
- Recognize and deduce the relation between two intersecting chords in a circle.
- Recognize and deduce the relation between two secants to a circle from a point outside it.
- Recognize the relation between the length of a tangent to a circle and the two parts of a secant where the tangent and the secant are drawn from the same point outside the circle.
- Model and solve life applications problems by using similarity of polygons in a circle.

Lesson 1

Similarity of polygons

Definition

Two polygons M_1 and M_2 (of same number of sides) are said to be similar if the following two conditions satisfied together :

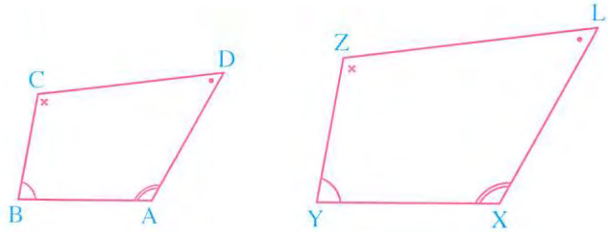
1. Their corresponding angles are congruent.
2. The lengths of their corresponding sides are proportional.

   In this case , we shall write :

   The polygon $M_1 \sim$ the polygon M_2

   That means the polygon M_1 is similar to the polygon M_2

In the opposite figure , if :



1. $m(\angle A) = m(\angle X)$

   $, m(\angle B) = m(\angle Y)$

   $, m(\angle C) = m(\angle Z)$

   $, m(\angle D) = m(\angle L)$

2. $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$

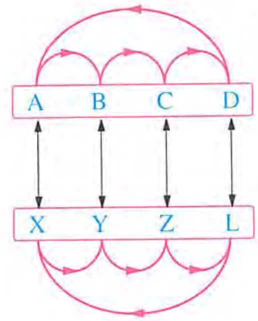
   Then the polygon ABCD $\sim$ the polygon XYZL

Remark 1

On writing the similar polygons , it is prefer to write them according to the order of their corresponding vertices to make it easy to deduce the equal angles in measure and write the proportion of corresponding side lengths.

For example :

If the polygon ABCD ~ the polygon XYZL , then :



1 m (∠ A) = m (∠ X) , m (∠ B) = m (∠ Y)
, m (∠ C) = m (∠ Z) , m (∠ D) = m (∠ L)

2 $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$

Remark 2

If the polygon ABCD ~ the polygon XYZL , then :

$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$ = K (similarity ratio or scale factor of similarity) , K > 0

If the scale factor of similarity of polygon ABCD to polygon XYZL = K

∴ The scale factor of similarity of polygon XYZL to polygon ABCD = $\frac{1}{K}$

Remark 3

Let K be the similarity ratio of polygon M_1 to polygon M_2 :

- If K > 1 , then polygon M_1 is an enlargement of polygon M_2 , where K is called the enlargement ratio.
- If 0 < K < 1 , then polygon M_1 is a shrinking to polygon M_2, where K is called the shrinking ratio.
- If K = 1 , then polygon M_1 is congruent to polygon M_2

In general , you can use the similarity ratio in calculation of the dimensions of similar figures.

Remark 4

In order that two polygons are similar , the two conditions should be verified together and verifying one of them only is not enough to be similar.

For example :

- All rectangles are not similar because although their corresponding angles are equal in measure (each = 90°) , but the lengths of their corresponding sides may be not proportional.
- Also all rhombuses are not similar because although the lengths of their corresponding sides are proportional , but their corresponding angles may be different in measure.

Remark 5

The congruent polygons are similar but it's not necessary that similar polygons are congruent.

Remark 6

If each of two polygons is similar to a third polygon , then they are similar.

i.e. If polygon $M_1 \sim$ polygon M_3 , polygon $M_2 \sim$ polygon M_3 , then polygon $M_1 \sim$ polygon M_2

Remark 7

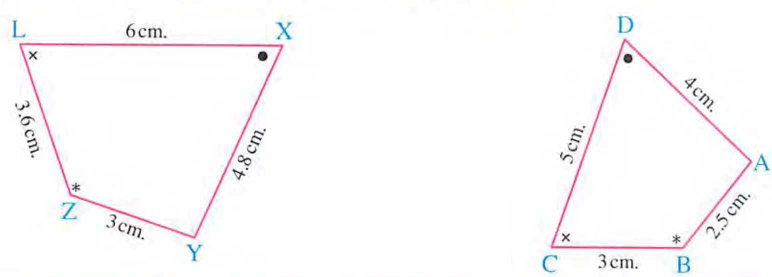
All regular polygons of the same number of sides are similar.

For example : • All equilateral triangles are similar. • All squares are similar.

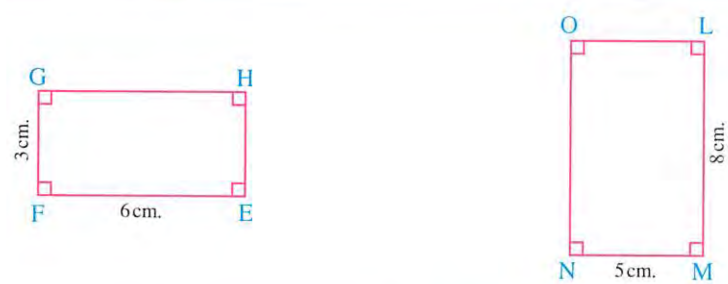
Example 1

Show which of the following pairs of polygons are similar , showing the reason and if they are similar , determine the similarity ratio :

1



2



Solution

1 The two polygons ABCD , YZLX are similar :

Because : m ($\angle$ B) = m ($\angle$ Z) , m ($\angle$ C) = m ($\angle$ L) , m ($\angle$ D) = m ($\angle$ X)

$\therefore$ m ($\angle$ A) = m ($\angle$ Y) , $\frac{AB}{YZ} = \frac{BC}{ZL} = \frac{CD}{LX} = \frac{DA}{XY}$, $\frac{2.5}{3} = \frac{3}{3.6} = \frac{5}{6} = \frac{4}{4.8}$

$\therefore$ The similarity ratio = $\frac{5}{6}$

2 The two polygons LMNO , EFGH are not similar :

Although : m (∠ L) = m (∠ E) , m (∠ M) = m (∠ F) , m (∠ N) = m (∠ G)

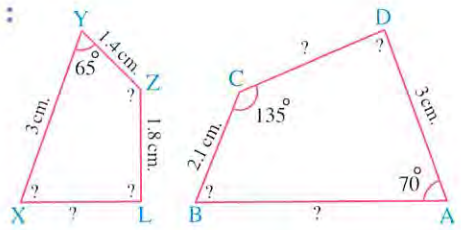
, m (∠ O) = m (∠ H) (Corresponding angles are congruent)

But : $\frac{LM}{EF} \neq \frac{MN}{FG}$ **Because :** $\frac{8}{6} \neq \frac{5}{3}$

Example 2

In the opposite figure :

If the two polygons ABCD and XYZL are similar , find :



1 The scale factor of similarity of polygon ABCD to polygon XYZL

2 The lengths of the unknown sides and measures of the unknown angles in each of the two polygons.

Solution

∵ The polygon ABCD ~ the polygon XYZL

∴ $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$ = the scale factor of similarity.

∴ $\frac{AB}{3} = \frac{2.1}{1.4} = \frac{CD}{1.8} = \frac{3}{LX}$ ∴ The scale factor of similarity $= \frac{2.1}{1.4} = \frac{3}{2}$ (First req.)

∴ $AB = \frac{3 \times 2.1}{1.4} = 4.5$ cm. , $CD = \frac{1.8 \times 2.1}{1.4} = 2.7$ cm.

, $LX = \frac{1.4 \times 3}{2.1} = 2$ cm.

, ∵ the polygon ABCD ~ the polygon XYZL

∴ m (∠ A) = m (∠ X) , m (∠ B) = m (∠ Y) , m (∠ C) = m (∠ Z)

, m (∠ D) = m (∠ L)

∴ m (∠ X) = 70° , m (∠ B) = 65° , m (∠ Z) = 135°

, ∵ the sum of measures of the interior angles of a quadrilateral = 360°

∴ m (∠ D) = m (∠ L) = 360° – (70° + 65° + 135°) = 90° (Second req.)

Remark

In the previous example , we notice that :

$\because$ The polygon ABCD $\sim$ the polygon XYZL

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$ = the scale factor of similarity

$= \frac{AB + BC + CD + DA}{XY + YZ + ZL + LX}$ (from proportion properties)

$\therefore \frac{\text{Perimeter of the polygon ABCD}}{\text{Perimeter of the polygon XYZL}} = \frac{12.3}{8.2} = \frac{3}{2}$ = the scale factor of similarity

i.e. **The ratio between the perimeters of two similar polygons = the ratio between the lengths of two corresponding sides of them.**

Example 3

Two similar polygons , the lengths of sides of one of them are 3 cm. , 5 cm. , 6 cm. , 8 cm. , 10 cm. and the perimeter of the other equals 48 cm. Find the lengths of the sides of the second polygon.

Solution

Let the polygon $\grave{A}\grave{B}\grave{C}\grave{D}\grave{E}$ $\sim$ the polygon ABCDE

$\therefore \frac{\text{The perimeter of the polygon } \grave{A}\grave{B}\grave{C}\grave{D}\grave{E}}{\text{The perimeter of the polygon ABCDE}} = \frac{\grave{A}\grave{B}}{AB} = \frac{\grave{B}\grave{C}}{BC} = \frac{\grave{C}\grave{D}}{CD} = \frac{\grave{D}\grave{E}}{DE} = \frac{\grave{E}\grave{A}}{EA}$

$, \because \frac{\text{the perimeter of the polygon } \grave{A}\grave{B}\grave{C}\grave{D}\grave{E}}{\text{the perimeter of the polygon ABCDE}} = \frac{48}{3 + 5 + 6 + 8 + 10} = \frac{48}{32} = \frac{3}{2}$

$\therefore \frac{\grave{A}\grave{B}}{AB} = \frac{\grave{B}\grave{C}}{BC} = \frac{\grave{C}\grave{D}}{CD} = \frac{\grave{D}\grave{E}}{DE} = \frac{\grave{E}\grave{A}}{EA} = \frac{3}{2}$

$\therefore \frac{\grave{A}\grave{B}}{3} = \frac{\grave{B}\grave{C}}{5} = \frac{\grave{C}\grave{D}}{6} = \frac{\grave{D}\grave{E}}{8} = \frac{\grave{E}\grave{A}}{10} = \frac{3}{2}$

$\therefore \grave{A}\grave{B} = 4.5$ cm. , $\grave{B}\grave{C} = 7.5$ cm. , $\grave{C}\grave{D} = 9$ cm. , $\grave{D}\grave{E} = 12$ cm. , $\grave{E}\grave{A} = 15$ cm. (The req.)

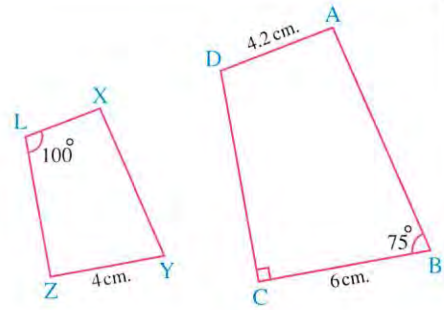
TRY TO SOLVE

In the opposite figure :

The polygon ABCD ~ the polygon XYZL

1 Calculate : m ($\angle$ X) , the length of $\overline{XL}$

2 If the perimeter of the polygon ABCD equals 25.8 cm. , calculate the perimeter of the polygon XYZL



Example 4

ABC is a triangle in which : AB = 4 cm. , BC = 5 cm. , AC = 8 cm.

Find the side lengths of another similar triangle if :

1 The scale factor of similarity = 2.4

2 The scale factor of similarity = 0.7

Solution

1 $\because$ The scale factor of similarity = 2.4 > 1

$\therefore$ The required triangle is an enlargement for Δ ABC

Let Δ XYZ ~ Δ ABC

$\therefore \frac{XY}{AB} = \frac{YZ}{BC} = \frac{ZX}{CA}$ = the scale factor of similarity.

$\therefore \frac{XY}{4} = \frac{YZ}{5} = \frac{ZX}{8} = 2.4$

$\therefore$ XY = 4 × 2.4 = 9.6 cm. , YZ = 5 × 2.4 = 12 cm. ,

ZX = 8 × 2.4 = 19.2 cm. (The req.)

2 $\because$ The scale factor of similarity = 0.7 < 1

$\therefore$ The required triangle is a shrinking for Δ ABC

Let Δ XYZ ~ Δ ABC

$\therefore \frac{XY}{AB} = \frac{YZ}{BC} = \frac{ZX}{CA}$ = the scale factor of similarity.

$\therefore \frac{XY}{4} = \frac{YZ}{5} = \frac{ZX}{8} = 0.7$

$\therefore$ XY = 4 × 0.7 = 2.8 cm. , YZ = 5 × 0.7 = 3.5 cm. , ZX = 8 × 0.7 = 5.6 cm. (The req.)

Lesson 2

Similarity of triangles

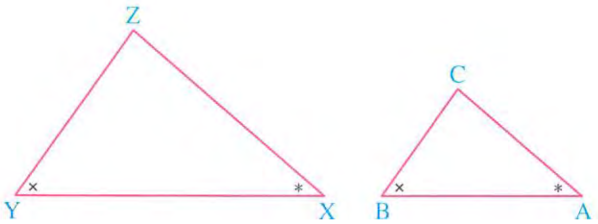
Cases of similarity of triangles

First case

Postulate (A. A. similarity postulate)

If two angles of one triangle are congruent to their two corresponding angles of another triangle , then the two triangles are similar.

In the opposite figure :



If $\angle A \equiv \angle X$

, $\angle B \equiv \angle Y$

, then $\Delta ABC \sim \Delta XYZ$

and we deduce that : $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$

Remarks

1. The two right-angled triangles are similar if the measure of an acute angle in one of them equals the measure of an acute angle in the other.
2. The two isosceles triangles are similar if the measure of an angle in one of them equals the measure of the corresponding angle in the other.
3. Any two equilateral triangles are similar.

Example 1

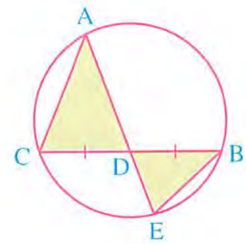
In the opposite figure :

$\overline{AE}$ and $\overline{BC}$ are two intersecting chords at D in a circle

, where D is the midpoint of $\overline{BC}$

Prove that : 1 $\Delta ADC \sim \Delta BDE$

2 $(BD)^2 = AD \times DE$



Solution

In $\Delta\Delta$ ADC and BDE :

$\because$ m ($\angle$ A) = m ($\angle$ B) "inscribed angles subtended by $\overset{\frown}{CE}$"

, m ($\angle$ ADC) = m ($\angle$ BDE) "V.O.A" $\therefore \Delta ADC \sim \Delta BDE$ (Q.E.D.1)

$\therefore \frac{AD}{BD} = \frac{DC}{DE}$ $\therefore BD \times DC = AD \times DE$

, but DC = BD "given"

$\therefore (BD)^2 = AD \times DE$ (Q.E.D.2)

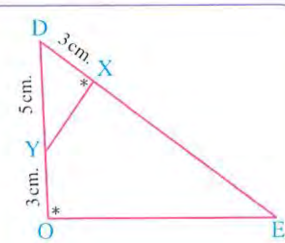
TRY TO SOLVE

In the opposite figure :

DEO is a triangle , m ($\angle$ O) = m ($\angle$ DXY)

, DX = YO = 3 cm. and DY = 5 cm.

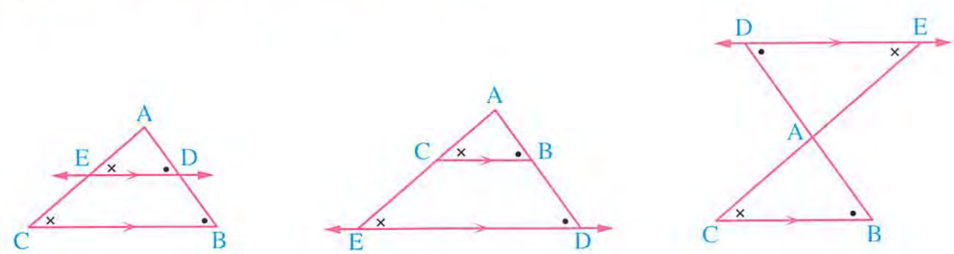
Find the length of : $\overline{XE}$



Corollary 1

If a line is drawn parallel to one side of a triangle and intersects the other two sides or the lines containing them , then the resulting triangle is similar to the original triangle.

In each of the following figures :



If $\overleftrightarrow{DE}$ // $\overline{BC}$ and intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ at D and E respectively , then $\Delta ABC \sim \Delta ADE$

Example 2

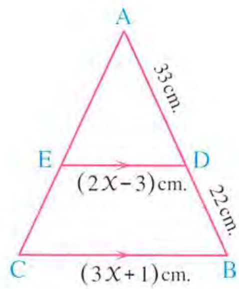
In the opposite figure :

$\overline{DE} \parallel \overline{BC}$, AD = 33 cm. , DB = 22 cm.

, DE = $(2X - 3)$ cm. and BC = $(3X + 1)$ cm.

1 **Prove that :** $\Delta ADE \sim \Delta ABC$

2 **Find the value of :** X



Solution

$\because \overline{DE} \parallel \overline{BC}$ $\therefore \Delta ADE \sim \Delta ABC$ (First req.)

$\therefore \frac{AD}{AB} = \frac{DE}{BC}$ $\therefore \frac{33}{55} = \frac{2X - 3}{3X + 1}$ $\therefore \frac{3}{5} = \frac{2X - 3}{3X + 1}$

$\therefore 9X + 3 = 10X - 15$ $\therefore X = 18$ (Second req.)

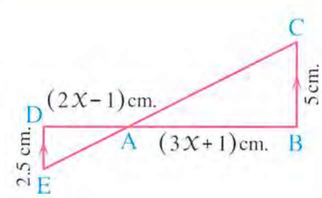
TRY TO SOLVE

In the opposite figure :

$\overline{CE} \cap \overline{BD} = \{A\}$, $\overline{BC} \parallel \overline{DE}$, BC = 5 cm. and DE = 2.5 cm.

1 **Prove that :** $\Delta ABC \sim \Delta ADE$

2 **Find the value of :** X



Corollary 2

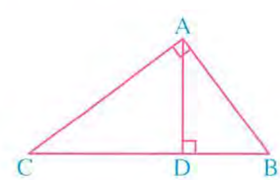
In any right-angled triangle , the altitude to the hypotenuse separates the triangle into two triangles which are similar to each other and to the original triangle.

In the opposite figure :

If ΔABC is a right-angled triangle at A and $\overline{AD} \perp \overline{BC}$

, then $\Delta DBA \sim \Delta DAC \sim \Delta ABC$

and it is left to the student to prove this corollary by using the previous postulate and its remarks.



Remarks on the previous figure :

1 From similarity of ΔΔ DBA and ABC , we get $\frac{DB}{AB} = \frac{BA}{BC}$

$\therefore (AB)^2 = DB \times BC$ i.e. AB is a mean proportional between DB and BC

2 From similarity of ΔΔ DAC and ABC , we get $\frac{DC}{AC} = \frac{AC}{BC}$

$\therefore (AC)^2 = DC \times BC$ i.e. AC is a mean proportional between DC and BC

3 From similarity of ΔΔ DBA and DAC , we get $\frac{DA}{DC} = \frac{DB}{DA}$

$\therefore (DA)^2 = DB \times DC$ i.e. DA is a mean proportional between DB and DC

4 From similarity of ΔΔ DBA and ABC , we get $\frac{AB}{CB} = \frac{AD}{CA}$

$\therefore AD \times CB = AB \times CA$

The previous results are considered as a proof of the Euclidean's theory which we have studied in the preparatory stage.

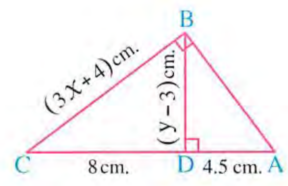
Example 3

In the opposite figure :

ABC is a right-angled triangle at B and $\overline{BD} \perp \overline{AC}$

If AD = 4.5 cm. and DC = 8 cm. ,

find the values of : X and y



Solution

$\because \Delta ABC$ is right-angled at B , $\overline{BD} \perp \overline{AC}$

$\therefore \Delta DBC \sim \Delta BAC$ $\qquad \therefore \frac{BC}{AC} = \frac{DC}{BC}$

$\therefore (BC)^2 = AC \times DC$ $\qquad \therefore (3X + 4)^2 = 12.5 \times 8 = 100$

$\therefore 3X + 4 = 10$ $\qquad \therefore X = 2$

$\because \Delta ABC$ is right-angled at B , $\overline{BD} \perp \overline{AC}$

$\therefore \Delta ABD \sim \Delta BCD$ $\qquad \therefore \frac{DB}{DC} = \frac{DA}{DB}$

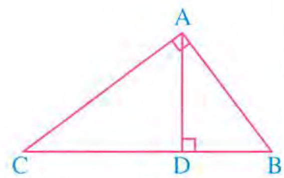
$\therefore (DB)^2 = DC \times DA$ $\qquad \therefore (y - 3)^2 = 8 \times 4.5 = 36$

$\therefore y - 3 = 6$ $\qquad \therefore y = 9$ (The req.)

TRY TO SOLVE

In the opposite figure :

Δ ABC is right-angled at A , $\overline{AD} \perp \overline{BC}$ **Complete :**



1 $\frac{BD}{AD} = \frac{AD}{\cdots\cdots}$

2 $\frac{BD}{AB} = \frac{AD}{\cdots\cdots}$

3 $\frac{AB}{AC} = \frac{AD}{\cdots\cdots}$

4 $\frac{\cdots\cdots}{CB} = \frac{AD}{CA}$

5 $\frac{\cdots\cdots}{AB} = \frac{AB}{\cdots\cdots}$

6 $(DA)^2 = \cdots\cdots\cdots \times \cdots\cdots\cdots$

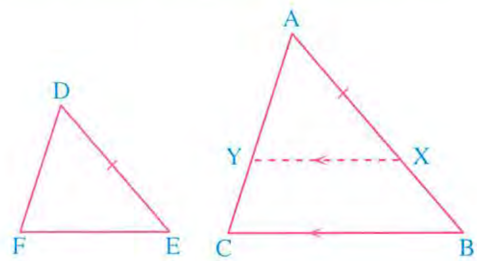
7 $(AC)^2 = \cdots\cdots\cdots \times \cdots\cdots\cdots$

8 $AD = \frac{\cdots\cdots \times CA}{CB}$

Second case

Theorem 1 S.S.S. similarity theorem

If the side lengths of two triangles are in proportion , then the two triangles are similar.



Given In $\Delta\Delta$ ABC , DEF : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

R.T.P. $\Delta ABC \sim \Delta DEF$

Const. Take $X \in \overline{AB}$, where AX = DE

Draw $\overleftrightarrow{XY} \parallel \overline{BC}$ and intersects

$\overline{AC}$ at Y

Proof $\because \overleftrightarrow{XY} \parallel \overline{BC}$ $\therefore \Delta ABC \sim \Delta AXY$ "corollary « 1 »"

$\therefore \frac{AB}{AX} = \frac{BC}{XY} = \frac{CA}{YA}$, $\because$ AX = DE "construction"

$\therefore \frac{AB}{DE} = \frac{BC}{XY} = \frac{CA}{YA}$ (1)

, $\because \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ "given" (2)

From (1) , (2) we deduce that : XY = EF , YA = FD

and $\Delta AXY \equiv \Delta DEF$ "S.S.S. congruency theorem"

$\therefore \Delta DEF \sim \Delta AXY$

, $\because \Delta ABC \sim \Delta AXY$ "proved"

$\therefore \Delta ABC \sim \Delta DEF$ (Q.E.D.)

Remark

For writing the two similar triangles in the same order of their corresponding vertices from the proportionality of their side lengths , we follow the following :

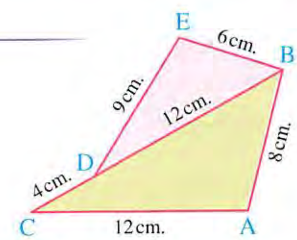
Let the vertices of one of the two triangles be A , B and C and the vertices of the other triangle be D , E and F and we have the proportion : $\frac{AC}{DF} = \frac{AB}{EF} = \frac{BC}{DE}$

We search for the vertices of the triangle which are opposite to the sides $\overline{AC}$, $\overline{AB}$ and $\overline{BC}$ respectively which are B , C and A

and we search for the vertices of the triangle which are opposite to the sides $\overline{DF}$, $\overline{EF}$ and $\overline{DE}$ respectively which are E , D and F , then :

Δ BCA ~ Δ EDF or Δ ABC ~ Δ FED , etc ...

Example 4



In the opposite figure :

Prove that : **1** The two coloured triangles are similar.

2 $\overrightarrow{BD}$ bisects $\angle$ ABE

Solution

$\because \frac{AB}{BE} = \frac{8}{6} = \frac{4}{3}$, $\frac{BC}{BD} = \frac{16}{12} = \frac{4}{3}$, $\frac{AC}{DE} = \frac{12}{9} = \frac{4}{3}$

$\therefore \frac{AB}{BE} = \frac{BC}{BD} = \frac{AC}{DE}$ $\therefore \Delta$ CAB ~ Δ DEB (Q.E.D. 1)

From similarity : m ($\angle$ ABC) = m ($\angle$ EBD)

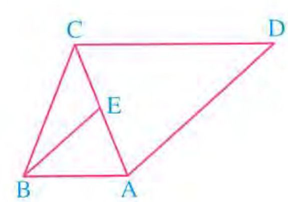
$\therefore \overrightarrow{BD}$ bisects $\angle$ ABE (Q.E.D. 2)

Example 5

ABCD is a quadrilateral , E $\in \overline{AC}$, where $\frac{AC}{AD} = \frac{AE}{BE}$ and $\frac{AB}{AE} = \frac{CD}{AC}$

Prove that : **1** $\overline{CD}$ // $\overline{BA}$ **2** $\overline{AD}$ // $\overline{BE}$

Solution



$\because \frac{AC}{AD} = \frac{AE}{BE}$ $\therefore \frac{AC}{AE} = \frac{AD}{BE}$ (1)

, $\because \frac{AB}{AE} = \frac{CD}{AC}$ $\therefore \frac{AC}{AE} = \frac{CD}{AB}$ (2)

From (1) , (2) we get : $\frac{AC}{AE} = \frac{AD}{BE} = \frac{CD}{AB}$

$\therefore \Delta$ DCA ~ Δ BAE we deduce from the similarity that

m ($\angle$ ACD) = m ($\angle$ EAB) and they are alternative angles. $\therefore \overline{CD}$ // $\overline{BA}$ (Q.E.D. 1)

, m ($\angle$ CAD) = m ($\angle$ AEB) and they are alternative angles. $\therefore \overline{AD}$ // $\overline{BE}$ (Q.E.D. 2)

TRY TO SOLVE

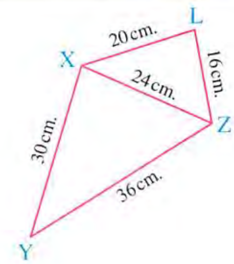
In the opposite figure :

XYZL is a quadrilateral , in which :

XY = 30 cm. , YZ = 36 cm. , ZL = 16 cm.

, LX = 20 cm. and XZ = 24 cm.

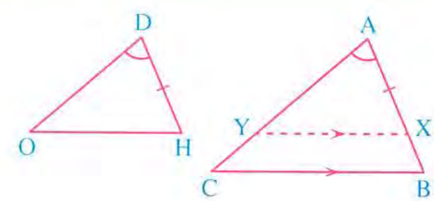
Prove that : Δ XYZ $\sim$ Δ LXZ



Third case

Theorem 2 S.A.S. similarity theorem

If an angle of one triangle is congruent to an angle of another triangle and lengths of the sides including those angles are in proportion , then the triangles are similar.



Given $\angle A \equiv \angle D$ and $\frac{AB}{DH} = \frac{AC}{DO}$

R.T.P. $\Delta ABC \sim \Delta DHO$

Const. Let $X \in \overline{AB}$ such that AX = DH

and draw $\overrightarrow{XY}$ // $\overline{BC}$ and intersects $\overline{AC}$ at Y

Proof

$\because \overline{XY}$ // $\overline{BC}$ $\quad \therefore \Delta ABC \sim \Delta AXY$ "corollary" (1)

$\therefore \frac{AB}{AX} = \frac{AC}{AY}$

, $\because \frac{AB}{DH} = \frac{AC}{DO}$ "given"

, AX = DH "construction"

$\therefore \frac{AB}{AX} = \frac{AC}{DO}$

$\therefore$ AY = DO

$\therefore \Delta AXY \equiv \Delta DHO$ "S.A.S. congruency theorem"

$\therefore \Delta AXY \sim \Delta DHO$ (2)

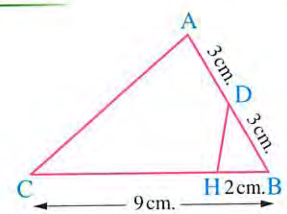
From (1) and (2) we get : $\Delta ABC \sim \Delta DHO$ (Q.E.D.)

Example 6

ABC is a triangle in which : AB = 6 cm. and BC = 9 cm. Let D be the midpoint of $\overline{AB}$ and $H \in \overline{BC}$ such that BH = 2 cm.

Prove that :
1. Δ DBH $\sim$ Δ CBA
2. ADHC is a cyclic quadrilateral.

Solution



In Δ DBH and Δ CBA :

$\because \frac{BH}{BA} = \frac{2}{6} = \frac{1}{3}$, $\frac{BD}{BC} = \frac{3}{9} = \frac{1}{3}$

$\therefore \frac{BH}{BA} = \frac{BD}{BC}$, $\because$ ∠ B is common.

$\therefore$ Δ DBH ~ Δ CBA (Q.E.D. 1)

From the similarity of the two triangles , we get that : m (∠ DHB) = m (∠ A)

, $\because$ ∠ DHB is an exterior angle of the quadrilateral ADHC

$\therefore$ The figure ADHC is a cyclic quadrilateral. (Q.E.D. 2)

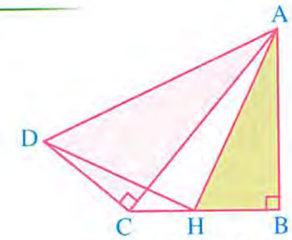
Example 7

ABCD is a quadrilateral in which : m (∠ B) = m (∠ ACD) = 90°

and H $\in \overline{BC}$ such that : $\frac{CD}{CA} = \frac{BH}{BA}$

Prove that : **1** Δ ABH ~ Δ ACD **2** m (∠ AHD) = 90°

Solution



$\because \frac{CD}{CA} = \frac{BH}{BA}$

$\therefore \frac{CD}{BH} = \frac{CA}{BA}$

, $\because$ m (∠ B) = m (∠ ACD)

$\therefore$ Δ ABH ~ Δ ACD (Q.E.D. 1)

and hence m (∠ AHB) = m (∠ ADC)

, $\because$ ∠ AHB is an exterior angle of AHCD

$\therefore$ AHCD is a cyclic quadrilateral.

$\therefore$ m (∠ AHD) = m (∠ ACD) "drawn on $\overline{AD}$ and on the same side of it"

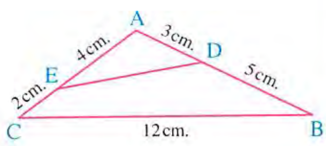
$\therefore$ m (∠ AHD) = 90° (Q.E.D. 2)

TRY TO SOLVE

In the opposite figure :

If AD = 3 cm. , DB = 5 cm. ,

AE = 4 cm. , EC = 2 cm. , BC = 12 cm.



1 **Prove that :** Δ ADE ~ Δ ACB

2 **Find the length of :** $\overline{DE}$

Lesson **3**

The relation between the areas of two similar polygons

- You know that the ratio between the perimeters of two similar polygons equals the ratio between the lengths of any two corresponding sides of them.
- In this lesson you will learn the relation between the areas of two similar polygons.

First — The ratio between the areas of the surfaces of two similar triangles

Theorem 3

The ratio between the areas of the surfaces of two similar triangles equals the square of the ratio between the lengths of any two corresponding sides of the two triangles.

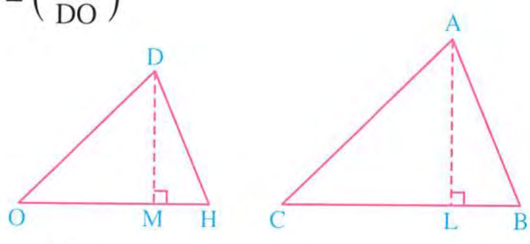
Given $\Delta ABC \sim \Delta DHO$

R.T.P. $\dfrac{\text{The area of } \Delta ABC}{\text{The area of } \Delta DHO} = \left(\dfrac{AB}{DH}\right)^2 = \left(\dfrac{BC}{HO}\right)^2 = \left(\dfrac{AC}{DO}\right)^2$

Const. Draw $\overrightarrow{AL} \perp \overline{BC}$ such that :

$\overrightarrow{AL} \cap \overline{BC} = \{L\}$ and $\overrightarrow{DM} \perp \overline{HO}$

such that $\overrightarrow{DM} \cap \overline{HO} = \{M\}$



Proof $\because \Delta ABC \sim \Delta DHO$

$\therefore m(\angle B) = m(\angle H)$ and $\dfrac{AB}{DH} = \dfrac{BC}{HO} = \dfrac{CA}{OD}$ (1)

In the two right-angled triangles ABL and DHM : $\because m(\angle B) = m(\angle H)$

$\therefore \Delta ABL \sim \Delta DHM \quad \therefore \dfrac{AB}{DH} = \dfrac{AL}{DM}$ (2)

$\because \dfrac{\text{The area of } \Delta ABC}{\text{The area of } \Delta DHO} = \dfrac{\frac{1}{2} BC \times AL}{\frac{1}{2} HO \times DM} = \dfrac{BC}{HO} \times \dfrac{AL}{DM}$ (3)

From (1) , (2) and (3) we get :

$\dfrac{\text{The area of } \Delta ABC}{\text{The area of } \Delta DHO} = \dfrac{BC}{HO} \times \dfrac{BC}{HO} = \left(\dfrac{BC}{HO}\right)^2 = \left(\dfrac{AB}{DH}\right)^2 = \left(\dfrac{CA}{OD}\right)^2$ (Q.E.D.)

Remark 1

From the proof of the previous theorem we can deduce that :

The ratio between areas of two similar triangles equals the square of the ratio between two corresponding heights in them.

Example 1

If the ratio between the areas of two similar triangles is $\frac{9}{16}$, the perimeter of the smaller triangle is 60 cm.

Find : The perimeter of the greater triangle.

Solution

Let the two similar triangles be Δ ABC , Δ XYZ where ΔABC is the smaller

$\therefore \frac{a(\Delta ABC)}{a(\Delta XYZ)} = \left(\frac{AB}{XY}\right)^2 = \frac{9}{16}$ $\quad\therefore \frac{AB}{XY} = \frac{3}{4}$

$\therefore \frac{\text{The perimeter of } \Delta ABC}{\text{The perimeter of } \Delta XYZ} = \frac{AB}{XY} = \frac{3}{4}$ $\quad\therefore \frac{60}{\text{The perimeter of } \Delta XYZ} = \frac{3}{4}$

$\therefore$ The perimeter of $\Delta XYZ = \frac{60 \times 4}{3} = 80$ cm. (The req.)

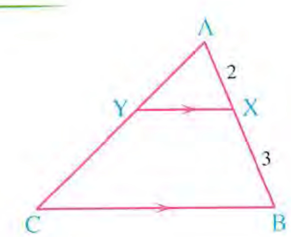
Example 2

ABC is a triangle of area 62.5 cm^2. Draw $\overleftrightarrow{XY}$ // $\overline{BC}$ to intersect $\overline{AB}$ at X and $\overline{AC}$ at Y

If AX : XB = 2 : 3

Find : The area of the figure XBCY

Solution



In Δ ABC : $\because \overline{XY}$ // $\overline{BC}$ $\quad\therefore \Delta AXY \sim \Delta ABC$

$\therefore \frac{a(\Delta AXY)}{a(\Delta ABC)} = \left(\frac{AX}{AB}\right)^2$ $\quad\therefore \frac{a(\Delta AXY)}{62.5} = \left(\frac{2}{5}\right)^2$

$\therefore a(\Delta AXY) = \frac{4}{25} \times 62.5 = 10$ cm^2.

$\therefore$ The area of the figure XBCY = a (Δ ABC) – a (Δ AXY)

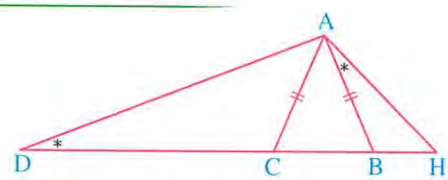
$= 62.5 - 10 = 52.5$ cm^2. (The req.)

Example 3

ABC is a triangle in which : AB = AC , D $\in \overrightarrow{BC}$, D $\notin \overline{BC}$ and H $\in \overrightarrow{CB}$, H $\notin \overline{CB}$ such that m ($\angle$ BAH) = m ($\angle$ D) If the area of Δ ACD equals 4 times the area of Δ ABH

, **then prove that :** DC = 2 AC

Solution



In Δ ABH and Δ DCA :

$\because$ m ($\angle$ BAH) = m ($\angle$ D)

and m ($\angle$ ABH) = m ($\angle$ DCA) "Supplementaries of two equal angles in measure"

$\therefore \Delta ABH \sim \Delta DCA$ $\therefore \frac{a(\Delta ABH)}{a(\Delta DCA)} = \left(\frac{AB}{DC}\right)^2$

$\therefore \frac{1}{4} = \left(\frac{AB}{DC}\right)^2$ $\therefore \frac{1}{2} = \frac{AB}{DC}$ $\therefore DC = 2\,AB$

, $\because AB = AC$ $\therefore DC = 2\,AC$ (Q.E.D.)

Example 4

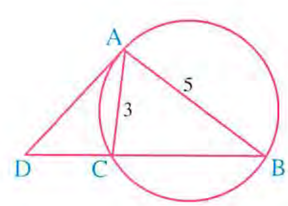
ABC is a triangle inscribed in a circle such that $\frac{AB}{AC} = \frac{5}{3}$

Draw $\overleftrightarrow{AD}$ to be a tangent to the circle at A , to intersect $\overrightarrow{BC}$ at D

Find : The area of Δ ACD : the area of Δ ABC

Solution

In Δ ADC and Δ BDA : $\because \angle$ D is common , m ($\angle$ CAD) = m ($\angle$ B)



$\therefore \Delta ADC \sim \Delta BDA$

$\therefore \frac{\text{The area of } \Delta ADC}{\text{The area of } \Delta BDA} = \left(\frac{AC}{BA}\right)^2 = \frac{9}{25}$

$\therefore \frac{\text{The area of } \Delta ADC}{\text{The area of } \Delta ABC + \text{The area of } \Delta ADC} = \frac{9}{25}$

$\therefore$ 25 (The area of Δ ADC) = 9 (The area of Δ ABC) + 9 (The area of Δ ADC)

$\therefore$ 16 (The area of Δ ADC) = 9 (The area of Δ ABC)

$\therefore \frac{\text{The area of } \Delta ADC}{\text{The area of } \Delta ABC} = \frac{9}{16}$ (The req.)

TRY TO SOLVE

The ratio between the perimeters of two similar triangles is 4 : 5 If the area of the greater one is 150 cm^2 , find the area of the smaller triangle.

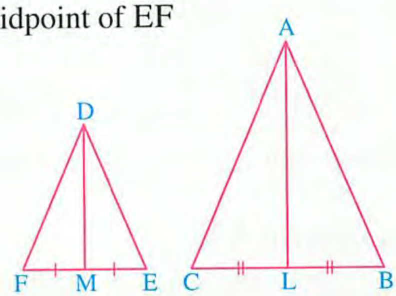
Remark 2

The ratio of the areas of the surfaces of two similar triangles equals the square of the ratio of the lengths of any two corresponding medians of the two triangles.

In the opposite figure :

If $\Delta ABC \sim \Delta DEF$, L is the midpoint of $\overline{BC}$, M is the midpoint of $\overline{EF}$

, then $\frac{a(\Delta ABC)}{a(\Delta DEF)} = \left(\frac{AL}{DM}\right)^2$



Proof :

$\because \Delta ABC \sim \Delta DEF$ $\quad \therefore \frac{AB}{DE} = \frac{BC}{EF}$

, $\because BC = 2\,BL$, $EF = 2\,EM$ $\quad \therefore \frac{AB}{DE} = \frac{2\,BL}{2\,EM}$

$\therefore \frac{AB}{DE} = \frac{BL}{EM}$, $\because \angle B \equiv \angle E$ (Because $\Delta ABC \sim \Delta DEF$)

$\therefore \Delta ABL \sim \Delta DEM$ $\quad \therefore \frac{a(\Delta ABL)}{a(\Delta DEM)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{AL}{DM}\right)^2$ (1)

, $\because \frac{a(\Delta ABC)}{a(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2$ (2)

From (1) , (2) : $\therefore \frac{a(\Delta ABC)}{a(\Delta DEF)} = \left(\frac{AL}{DM}\right)^2$

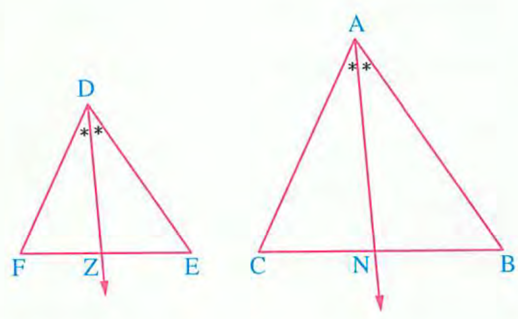
Remark 3

In the opposite figure :

If $\Delta ABC \sim \Delta DEF$, $\overrightarrow{AN}$ bisects $\angle A$ and intersects $\overline{BC}$ at N

, $\overrightarrow{DZ}$ bisects $\angle D$ and intersects $\overline{EF}$ at Z

, then $\frac{a(\Delta ABC)}{a(\Delta DEF)} = \left(\frac{AN}{DZ}\right)^2$



Proof :

$\because \Delta ABC \sim \Delta DEF$

$\therefore m(\angle BAC) = m(\angle EDF)$

$\therefore \frac{1}{2} m(\angle BAC) = \frac{1}{2} m(\angle EDF)$

$\therefore m(\angle BAN) = m(\angle EDZ)$

, $\because m(\angle B) = m(\angle E)$ $\quad \therefore \Delta ABN \sim \Delta DEZ$

$\therefore \frac{a(\Delta ABN)}{a(\Delta DEZ)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{AN}{DZ}\right)^2$ (1)

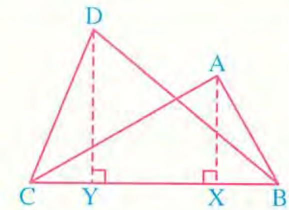
, $\because \frac{a(\Delta ABC)}{a(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2$ (2)

From (1) , (2) : $\therefore \frac{a(\Delta ABC)}{a(\Delta DEF)} = \left(\frac{AN}{DZ}\right)^2$

Remark 4

The ratio of the areas of two triangles having a common base equals the ratio of the two heights of the two triangles.

In the opposite figure :



$\overline{BC}$ is a common base of ΔΔ ABC , DBC

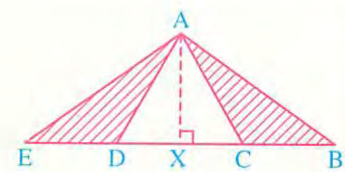
$\therefore \frac{a(\Delta ABC)}{a(\Delta DBC)} = \frac{\frac{1}{2}BC \times AX}{\frac{1}{2}BC \times DY} = \frac{AX}{DY}$

Notice that : It is not necessary that the two triangles are similar.

Remark 5

The ratio of the areas of two triangles having a common height equals the ratio of the lengths of two bases of the two triangles.

In the opposite figure :



AX is a common height for ΔΔ ABC , ADE

$\therefore \frac{a(\Delta ABC)}{a(\Delta ADE)} = \frac{\frac{1}{2}BC \times AX}{\frac{1}{2}DE \times AX} = \frac{BC}{DE}$

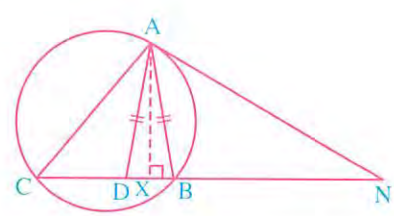
Notice that : It is not necessary that the two triangles are similar.

Example 5

ABC is an inscribed triangle in a circle where AC > AB , D $\in \overline{BC}$, where AD = AB , draw $\overrightarrow{AN}$ a tangent to the circle at A and cuts $\overrightarrow{CB}$ at N

Prove that : BN : DC = $(AN)^2 : (CA)^2$

Solution



$\because \frac{a(\Delta ABN)}{a(\Delta CDA)} = \frac{\frac{1}{2}BN \times AX}{\frac{1}{2}DC \times AX} = \frac{BN}{DC}$ (1)

, ∵ AB = AD ∴ m (∠ ABD) = m (∠ ADB)

∴ m (∠ ABN) = m (∠ ADC)

, ∵ $\overrightarrow{AN}$ is a tangent.

∴ m (∠ BAN) = m (∠ C) (drawn on $\overset{\frown}{AB}$)

∴ Δ ABN ~ Δ CDA ∴ $\frac{a(\Delta ABN)}{a(\Delta CDA)} = \frac{(AN)^2}{(CA)^2}$ (2)

∴ From (1) and (2) : ∴ BN : DC = $(AN)^2 : (CA)^2$ (Q.E.D.)

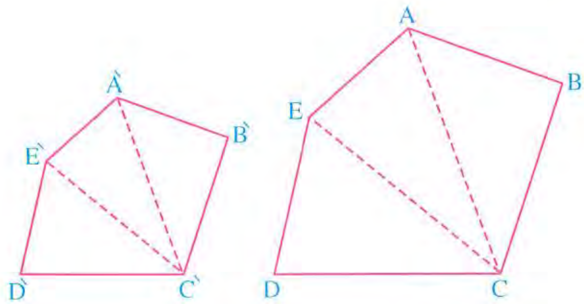
Second The ratio between the areas of the surfaces of two similar polygons

Fact

Any two similar polygons can be divided into the same number of triangles , each is similar to its corresponding one.

In the opposite figure :

If the two polygons ABCDE and $\grave{A}\grave{B}\grave{C}\grave{D}\grave{E}$ are similar and from two corresponding vertices say C and $\grave{C}$ we draw $\overline{CA}$, $\overline{CE}$, $\overline{\grave{C}\grave{A}}$ and $\overline{\grave{C}\grave{E}}$, then each polygon will be divided into three triangles



such that : $\Delta ABC \sim \Delta \grave{A}\grave{B}\grave{C}$, $\Delta ACE \sim \Delta \grave{A}\grave{C}\grave{E}$ and $\Delta ECD \sim \Delta \grave{E}\grave{C}\grave{D}$

Remarks

- The previous fact is correct whatever the number of sides of the two similar polygons (having always the same number of sides)
- If the number of sides of a polygon is n sides , then the number of the triangles that the polygon is divided by drawing the diagonals from one of its vertices = (n – 2) triangles

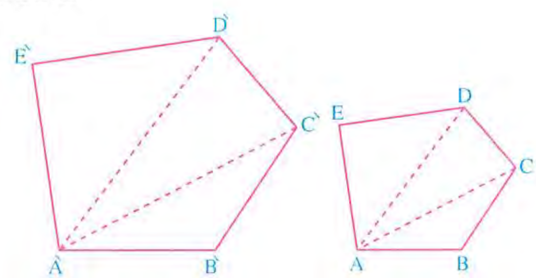
Theorem 4

The ratio between the areas of the surfaces of two similar polygons equals the square of the ratio between the lengths of any two corresponding sides of the polygons.

Given The polygon ABCDE ~ the polygon $\grave{A}\grave{B}\grave{C}\grave{D}\grave{E}$

R.T.P. $\dfrac{a\text{ (the polygon ABCDE)}}{a\text{ (the polygon }\grave{A}\grave{B}\grave{C}\grave{D}\grave{E})} = \left(\dfrac{AB}{\grave{A}\grave{B}}\right)^2$

Const. From A , $\grave{A}$, draw $\overline{AC}$, $\overline{AD}$, $\overline{\grave{A}\grave{C}}$, $\overline{\grave{A}\grave{D}}$



Proof $\because$ The polygon ABCDE ~ The polygon $\grave{A}\grave{B}\grave{C}\grave{D}\grave{E}$

$\therefore$ They are divided into the same number of triangles each is similar to its corresponding one "fact"

$$\therefore \frac{a(\Delta ABC)}{a(\Delta \grave{A}\grave{B}\grave{C})} = \left(\frac{BC}{\grave{B}\grave{C}}\right)^2 , \frac{a(\Delta ACD)}{a(\Delta \grave{A}\grave{C}\grave{D})} = \left(\frac{CD}{\grave{C}\grave{D}}\right)^2 , \frac{a(\Delta ADE)}{a(\Delta \grave{A}\grave{D}\grave{E})} = \left(\frac{DE}{\grave{D}\grave{E}}\right)^2$$

$$, \because \frac{BC}{\grave{B}\grave{C}} = \frac{CD}{\grave{C}\grave{D}} = \frac{DE}{\grave{D}\grave{E}} = \frac{AB}{\grave{A}\grave{B}}$$ "from similar polygons"

$$\therefore \frac{a(\Delta ABC)}{a(\Delta \grave{A}\grave{B}\grave{C})} = \frac{a(\Delta ACD)}{a(\Delta \grave{A}\grave{C}\grave{D})} = \frac{a(\Delta ADE)}{a(\Delta \grave{A}\grave{D}\grave{E})} = \left(\frac{AB}{\grave{A}\grave{B}}\right)^2$$

From proportion properties : $$\frac{a(\Delta ABC) + a(\Delta ACD) + a(\Delta ADE)}{a(\Delta \grave{A}\grave{B}\grave{C}) + a(\Delta \grave{A}\grave{C}\grave{D}) + a(\Delta \grave{A}\grave{D}\grave{E})} = \left(\frac{AB}{\grave{A}\grave{B}}\right)^2$$

$$\therefore \frac{a(\text{the polygon } ABCDE)}{a(\text{the polygon } \grave{A}\grave{B}\grave{C}\grave{D}\grave{E})} = \left(\frac{AB}{\grave{A}\grave{B}}\right)^2$$ (Q.E.D.)

Example 6

The ratio between the perimeters of two similar polygons is 3 : 2

If the sum of their areas is 195 cm^2 , then find the area of each.

Solution

$\because$ The ratio between the perimeters is 3 : 2

$\therefore$ The ratio between the lengths of two corresponding sides is 3 : 2

$\therefore$ The ratio between their areas is 9 : 4

Let the area of the first polygon be $9X$ and the area of the second polygon be $4X$

$\therefore 9X + 4X = 195$ $\qquad$ $\therefore 13X = 195$

$\therefore X = 15$

$\therefore$ The area of the first polygon $= 15 \times 9 = 135$ cm^2

, the area of the second polygon $= 15 \times 4 = 60$ cm^2 (The req.)

Example 7

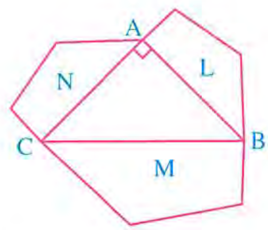
Prove that :

If we construct on the sides of a right-angled triangle , three similar polygons such that the three sides of the triangle correspond to each other , then the area of the polygon constructed on the hypotenuse equals the sum of the areas of the two other polygons.

Solution

$\because$ The polygon L $\sim$ the polygon M

$\therefore \dfrac{\text{The area of L}}{\text{The area of M}} = \left(\dfrac{AB}{BC}\right)^2 = \dfrac{(AB)^2}{(BC)^2}$ (1)



, $\because$ the polygon N $\sim$ the polygon M

$\therefore \dfrac{\text{The area of N}}{\text{The area of M}} = \left(\dfrac{AC}{BC}\right)^2 = \dfrac{(AC)^2}{(BC)^2}$ (2)

Adding (1) and (2) : $\therefore \dfrac{\text{The area of L}}{\text{The area of M}} + \dfrac{\text{the area of N}}{\text{the area of M}} = \dfrac{(AB)^2}{(BC)^2} + \dfrac{(AC)^2}{(BC)^2}$

$\therefore \dfrac{\text{The area of L + the area of N}}{\text{The area of M}} = \dfrac{(AB)^2 + (AC)^2}{(BC)^2} = \dfrac{(BC)^2}{(BC)^2} = 1$ "Pythagoras"

$\therefore$ The area of L + the area of N = the area of M (Q.E.D.)

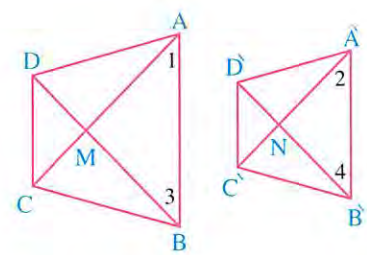
Example 8

ABCD , $\grave{A}\grave{B}\grave{C}\grave{D}$ are two similar polygons , their diagonals intersect at M , N respectively.

Prove that : $\dfrac{a\text{ (the polygon ABCD)}}{a\text{ (the polygon } \grave{A}\grave{B}\grave{C}\grave{D})} = \dfrac{(BM)^2}{(\grave{B}N)^2}$

Solution

$\because$ The two polygons are similar



$\therefore \Delta ABC \sim \Delta \grave{A}\grave{B}\grave{C}$ and we deduce that : m ($\angle$ 1) = m ($\angle$ 2)

, $\Delta ABD \sim \Delta \grave{A}\grave{B}\grave{D}$ and we deduce that : m ($\angle$ 3) = m ($\angle$ 4)

$\therefore \Delta ABM \sim \Delta \grave{A}\grave{B}N$

$\therefore \dfrac{BM}{\grave{B}N} = \dfrac{AB}{\grave{A}\grave{B}}$

$\therefore \dfrac{a\text{ (the polygon ABCD)}}{a\text{ (the polygon } \grave{A}\grave{B}\grave{C}\grave{D})} = \dfrac{(AB)^2}{(\grave{A}\grave{B})^2} = \dfrac{(BM)^2}{(\grave{B}N)^2}$ (Q.E.D.)

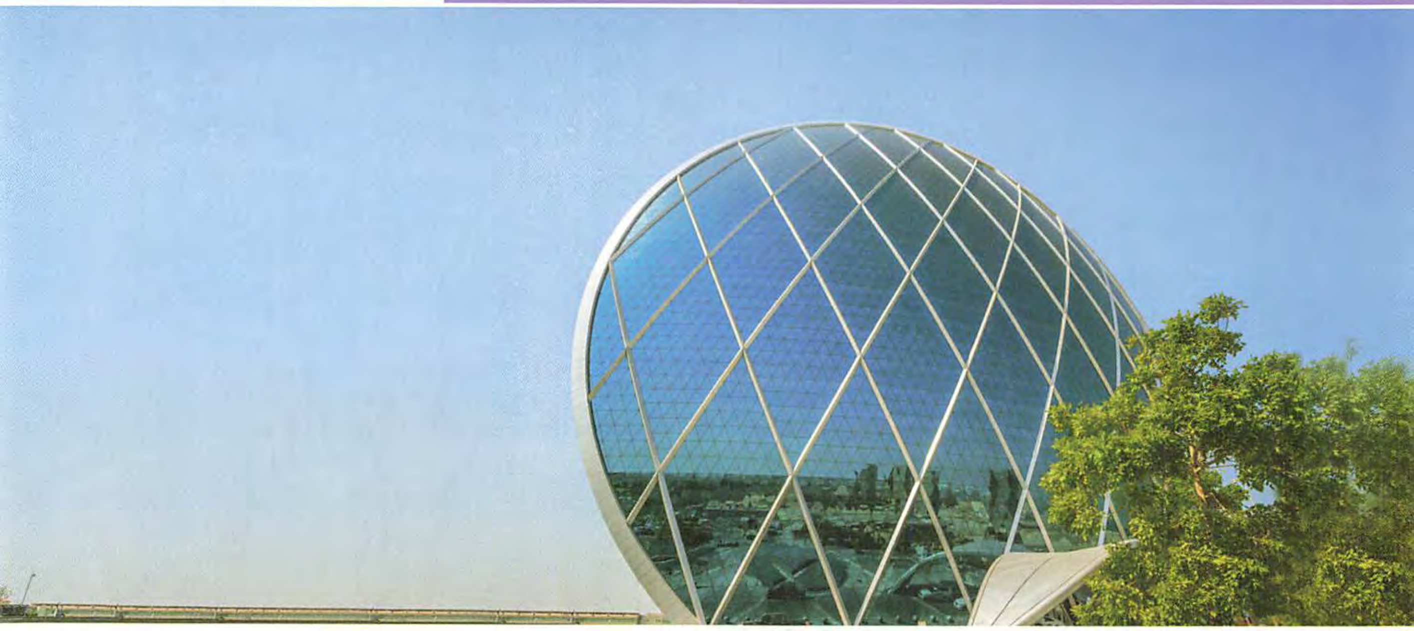
TRY TO SOLVE

ABCD , $\grave{A}\grave{B}\grave{C}\grave{D}$ are two similar polygons , X is the midpoint of $\overline{BC}$, Y is the midpoint of $\overline{\grave{B}\grave{C}}$

Prove that : $\dfrac{a\text{ (the polygon ABCD)}}{a\text{ (the polygon } \grave{A}\grave{B}\grave{C}\grave{D})} = \dfrac{(XD)^2}{(Y\grave{D})^2}$

Lesson **4**

Applications of similarity in the circle



1 In the opposite figure :

$\overline{AB}$, $\overline{CD}$ are two intersecting chords at H

We notice that : Δ HAC $\sim$ Δ HDB

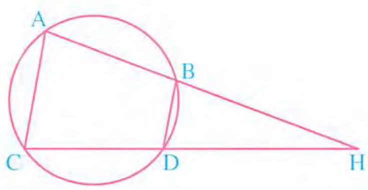
because : m ($\angle$ AHC) = m ($\angle$ DHB) (V.O.A)

, m ($\angle$ A) = m ($\angle$ D) (two inscribed angles subtended by the same arc $\overset{\frown}{CB}$)

From similarity , we deduce that :

$\frac{HA}{HD} = \frac{HC}{HB}$ $\quad\therefore$ HA × HB = HC × HD

2 In the opposite figure :



ABCD is a cyclic quadrilateral , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{H\}$

We notice that : Δ HAC $\sim$ Δ HDB

because : m ($\angle$ HAC) = m ($\angle$ HDB) (properties of cyclic quadrilateral)

, $\angle$ H is a common angle.

From similarity , we deduce that :

$\frac{HA}{HD} = \frac{HC}{HB}$ $\quad\therefore$ HA × HB = HC × HD

Well known problem

If the two lines containing the two chords $\overline{AB}$, $\overline{CD}$ of a circle intersect at the point E , then EA × EB = EC × ED

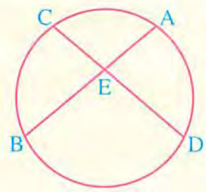


Fig. (1)

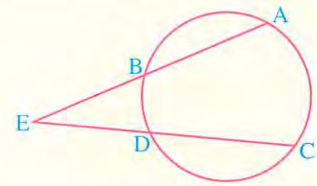
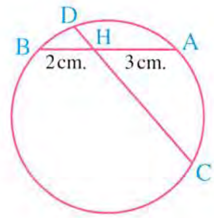


Fig. (2)

Example 1

$\overline{AB}$ and $\overline{CD}$ are two intersecting chords at H in a circle. If AH = 3 cm. , HB = 2 cm. , CD = 5.5 cm. , **calculate the length of each of :** $\overline{CH}$, $\overline{HD}$

Solution



Let CH = x cm.

$\therefore$ HD = $(5.5 - x)$ cm.

, $\because$ $\overline{AB}$, $\overline{CD}$ are two intersecting chords at H

$\therefore$ HA × HB = HC × HD $\qquad$ $\therefore 3 \times 2 = x(5.5 - x)$

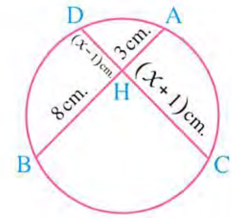
$\therefore 6 = 5.5\,x - x^2$ $\qquad$ $\therefore 2x^2 - 11x + 12 = 0$

$\therefore (2x - 3)(x - 4) = 0$ $\qquad$ $\therefore x = \frac{3}{2}$ or $x = 4$

$\therefore$ CH = 4 cm. , HD = 1.5 cm. (The req.)

TRY TO SOLVE

In the opposite figure :



$\overline{AB} \cap \overline{CD} = \{H\}$

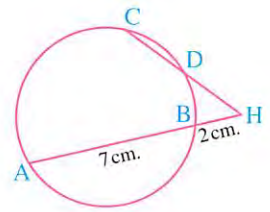
, AH = 3 cm. , HB = 8 cm.

, CH = $(x + 1)$ cm., HD = $(x - 1)$ cm.

Find the value of : x

Example 2

In the opposite figure :



$\overrightarrow{AB} \cap \overrightarrow{CD} = \{H\}$, HB = 2 cm.

, AB = 7 cm. , if $\frac{HD}{HC} = \frac{1}{2}$

Find the length of : $\overline{HC}$

Solution

$\because \frac{HD}{HC} = \frac{1}{2}$ $\therefore HD = k$, $HC = 2k$ where $k \neq 0$

, $\because \overrightarrow{AB} \cap \overrightarrow{CD} = \{H\}$ $\therefore HD \times HC = HB \times HA$

$\therefore k \times 2k = 2 \times 9 = 18$ $\therefore 2k^2 = 18$

$\therefore k^2 = 9$ $\therefore k = 3$ or -3 (refused)

$\therefore HC = 2 \times 3 = 6$ cm. (The req.)

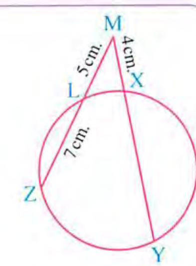
TRY TO SOLVE

In the opposite figure :

$\overrightarrow{YX} \cap \overrightarrow{ZL} = \{M\}$, MX = 4 cm.

, ML = 5 cm. , LZ = 7 cm.

Find the length of : $\overline{XY}$



Remark

In the opposite figure :

$\overline{AB}$ is a tangent to the circle at B

We notice that : $\Delta ABC \sim \Delta ADB$

This is because : m ($\angle$ ABC) = m ($\angle$ D)

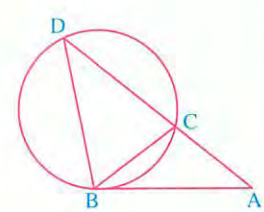
(tangency and inscribed angles subtended by $\overset{\frown}{BC}$)

, $\angle$ A is a common angle

From similarity we deduce that :

$\frac{AB}{AD} = \frac{AC}{AB}$

$\therefore (AB)^2 = AC \times AD$

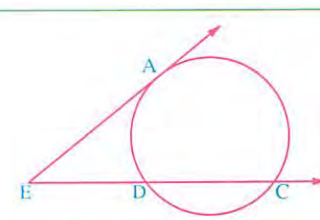


Remember that

AB is a mean proportion of AC , AD

Corollary 1

If E is a point outside the circle , $\overrightarrow{EA}$ is a tangent to the circle at A , $\overrightarrow{EC}$ intersects it at D , C , then $(EA)^2 = ED \times EC$



Example 3

M is a point outside the circle , $\overline{MC}$ is a tangent to the circle at C , $\overrightarrow{MA}$ is a secant intersects it at A and B , where MA > MB If MC = 10 cm. , AB = 15 cm.

Find the length of : $\overline{MB}$

Solution

Let MB = x cm. $\therefore$ MA = (x + 15) cm.

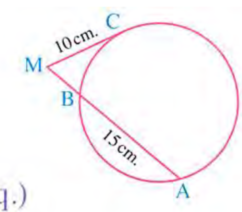
, $\because$ $\overline{MC}$ is a tangent to the circle , $\overrightarrow{MA}$ is a secant to it

$\therefore (MC)^2 = MB \times MA$

$\therefore (10)^2 = x(x + 15)$ $\therefore x^2 + 15x - 100 = 0$

$\therefore (x - 5)(x + 20) = 0$ $\therefore x = 5$

$\therefore$ MB = 5 cm. (The req.)



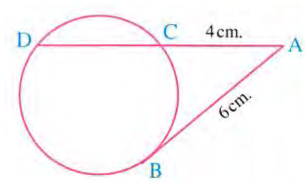
TRY TO SOLVE

In the opposite figure :

$\overline{AD}$ is a secant to the circle at C , D

, $\overline{AB}$ is a tangent to the circle at B

Find the length of : $\overline{CD}$



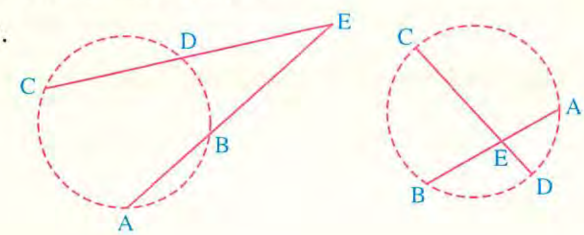
Converse of the well known problem

If the two lines containing the two segments $\overline{AB}$ and $\overline{CD}$ intersect at the point E (A , B , C , D and E are distinct points) and EA × EB = EC × ED , then the points A , B , C and D lie on a circle.

In the opposite figures :

If EA × EB = EC × ED

, then the points A , B , C and D lie on the same circle.

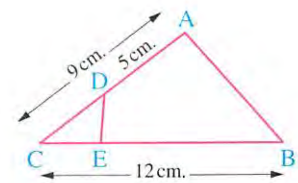


Example 4

ABC is a triangle in which : AC = 9 cm. , BC = 12 cm. Let D $\in \overline{AC}$, where AD = 5 cm.

Let E $\in \overline{BC}$, where $\frac{BE}{EC} = 3$ **Prove that :** The figure ABED is a cyclic quadrilateral.

Solution



$\because$ CD = AC – AD = 9 – 5 = 4 cm. $\therefore$ CD × CA = 4 × 9 = 36

, $\because$ BE = 3 CE $\therefore$ BC = 4 CE

$\therefore$ CE = $\frac{1}{4}$ BC = $\frac{1}{4}$ × 12 = 3 cm. $\therefore$ CE × CB = 3 × 12 = 36

$\therefore$ CD × CA = CE × CB

$\therefore$ The figure ABED is a cyclic quadrilateral. (Q.E.D.)

TRY TO SOLVE

In which of the following figures , do the points A , B , C and D lie on the same circle ?

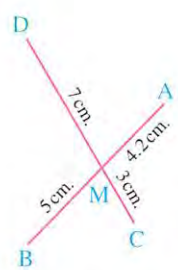


Fig. (1)

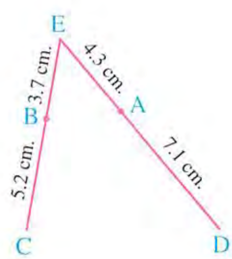


Fig. (2)

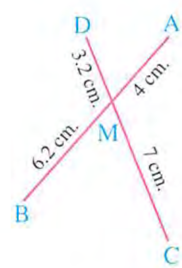


Fig. (3)

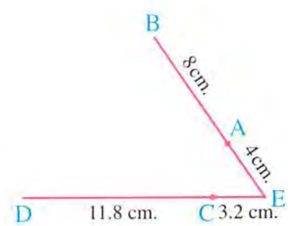
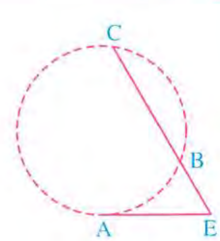


Fig. (4)

Corollary 2

If $(EA)^2$ = EB × EC

, then $\overline{EA}$ is a tangent segment to the circle which passes throught the points A , B and C

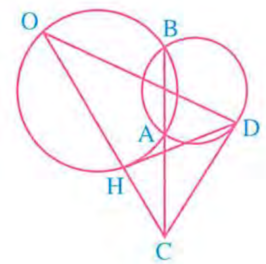


Example 5

Two intersecting circles at A and B , let $C \in \overrightarrow{BA}$ and $C \notin \overline{AB}$, let $\overline{CD}$ be a tangent to one of the two circles at D and $\overrightarrow{CO}$ intersects the other circle at H and O such that CO > CH

Prove that : $\overline{CD}$ is a tangent to the circle passing through D , H and O

Solution



$\because$ $\overline{CB}$ and $\overline{CO}$ intersect one of the two circles

$\therefore$ CA × CB = CH × CO (1)

, $\because$ $\overline{CD}$ is a tangent to the other circle and $\overline{CB}$ intersects it.

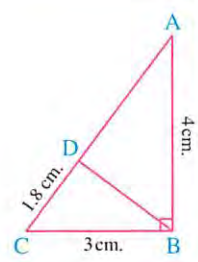
$\therefore$ $(CD)^2$ = CA × CB (2)

From (1) and (2) , we get : $(CD)^2$ = CH × CO

$\therefore$ $\overline{CD}$ is a tangent to the circle passing through D , H and O (Q.E.D.)

TRY TO SOLVE

In the opposite figure :



ABC is a right-angled triangle at B

, AB = 4 cm. , BC = 3 cm. , CD = 1.8 cm.

Prove that :

$\overline{BC}$ is a tangent to the circle passing through the points A , B and D

Unit Four

The triangle proportionality theorems.

Unit Lessons

Lesson 1 Parallel lines and proportional parts.

Lesson 2 Talis' theorem.

Lesson 3 Angle bisector and proportional parts.

Lesson 4 Follow : Angle bisector and proportional parts (Converse of theorem 3).

Lesson 5 Applications of proportionality in the circle.

Learning outcomes

By the end of this unit, the student should be able to :

- Recognize and prove the theorem "If a line is drawn parallel to one side of a triangle and intersects the other two sides , then it divides them into segments whose lengths are proportional" and its corollary and its converse.
- Recognize and prove TALIS' general theorem and its special cases.
- Solve problems and mathematical applications on Talis' general theorem and Talis' special theorem.
- Recognize and prove the theorem "The bisector of the interior or exterior angle of a triangle at any vertex divides the opposite base ..." and its converse.
- Find the length of each of the interior and the exterior bisectors of an angle of a triangle.
- Recognize the fact "The bisectors of angles of a triangle are concurrent".
- Find the power of a point with respect to a circle.
- Deduce the measures of angles resulting from the intersection of the chords and the tangents in a circle.

Preface

Before we study unit 4 (the triangle proportionality theorems)

It is useful and necessary to review the concepts of proportion and some of its properties which will be used in our study in this unit.

- a , b , c , d , e , f , ... are proportional if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$
- a , b , c , d , ... are in continued proportion if $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \ldots$

  and in this case b is called the middle proportion for a and c , where $b^2 = a\,c$

  Also , c is called the middle proportion for b and d where $c^2 = b\,d$
- If $\frac{a}{b} = \frac{c}{d}$, where a , c are called the antecedents and b , d are called the consequents , then :

  1 $a \times d = b \times c$

  2 $\frac{b}{a} = \frac{d}{c}$ (the reciprocal of ratios are equal)

  3 $\frac{a}{c} = \frac{b}{d}$ $\left(\frac{\text{The antecedent of } 1^{\text{st}} \text{ ratio}}{\text{The antecedent of } 2^{\text{nd}} \text{ ratio}} = \frac{\text{The consequent of } 1^{\text{st}} \text{ ratio}}{\text{The consequent of } 2^{\text{nd}} \text{ ratio}}\right)$

  4 $\frac{a+b}{b} = \frac{c+d}{d}$ $\left(\frac{\text{antecedent + consequent}}{\text{consequent}} \text{ of } 1^{\text{st}} \text{ ratio} = \frac{\text{antecedent + consequent}}{\text{consequent}} \text{ of } 2^{\text{nd}} \text{ ratio}\right)$

  5 $\frac{a+b}{a} = \frac{c+d}{c}$ $\left(\frac{\text{antecedent + consequent}}{\text{antecedent}} \text{ of } 1^{\text{st}} \text{ ratio} = \frac{\text{antecedent + consequent}}{\text{antecedent}} \text{ of } 2^{\text{nd}} \text{ ratio}\right)$
- If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$

  , then :

  1 $\frac{a+c+e+\ldots}{b+d+f+\ldots}$ = one of the ratios $\left(\frac{\text{sum of antecedents}}{\text{sum of consequent}} = \text{one of the ratios}\right)$

  2 $\frac{ka+mc+ne}{kb+md+nf}$ = one of the ratios

  , where k , m , n are non zero real numbers

Lesson **1**

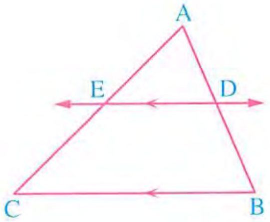
Parallel lines and proportional parts

Theorem 1

If a line is drawn parallel to one side of a triangle and intersects the other two sides , then it divides them into segments whose lengths are proportional.

Given ABC is a triangle , $\overleftrightarrow{DE} \parallel \overline{BC}$

R.T.P. $\frac{AD}{DB} = \frac{AE}{EC}$



Proof $\because \overleftrightarrow{DE} \parallel \overline{BC}$

$\therefore \Delta ABC \sim \Delta ADE$ "similarity postulate"

, then $\frac{AB}{AD} = \frac{AC}{AE}$ (1)

, $\because D \in \overline{AB}$, $E \in \overline{AC}$

$\therefore AB = AD + DB$, $AC = AE + EC$ (2)

From (1) , (2) we get : $\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$

, then : $\frac{AD}{AD} + \frac{DB}{AD} = \frac{AE}{AE} + \frac{EC}{AE}$

$\therefore 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$

$\therefore \frac{DB}{AD} = \frac{EC}{AE}$

From the properties of the proportion , we get : $\frac{AD}{DB} = \frac{AE}{EC}$ (Q.E.D.)

Remark

From the previous figure :

$\because \frac{AD}{DB} = \frac{AE}{EC}$ "Theorem"

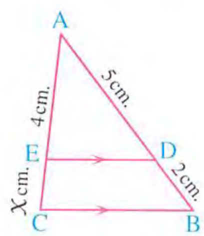
$\therefore \frac{AD + DB}{DB} = \frac{AE + EC}{EC}$ (review the proportion properties)

$\therefore \frac{AB}{DB} = \frac{AC}{EC}$

Example 1

In each of the following figures : $\overline{DE} \mathbin{//} \overline{BC}$ Find the value of x

1



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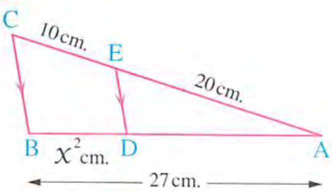

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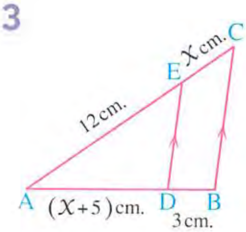


Solution

1 $\because \overline{DE} \mathbin{//} \overline{BC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$

$\therefore \frac{5}{2} = \frac{4}{x}$ $\therefore x = 1.6$

2 $\because \overline{DE} \mathbin{//} \overline{BC}$ $\therefore \frac{AB}{DB} = \frac{AC}{EC}$ $\therefore \frac{27}{x^2} = \frac{30}{10}$

$\therefore x^2 = 9$ $\therefore x = \pm 3$

3 $\because \overline{DE} \mathbin{//} \overline{BC}$ $\therefore \frac{AE}{EC} = \frac{AD}{DB}$ $\therefore \frac{12}{x} = \frac{x+5}{3}$

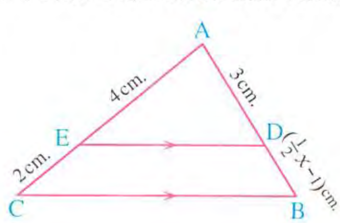
$\therefore x^2 + 5x = 36$ $\therefore x^2 + 5x - 36 = 0$

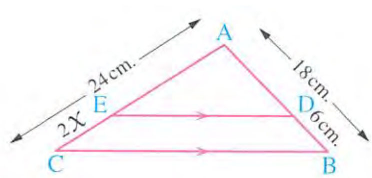
$\therefore (x+9)(x-4) = 0$ $\therefore x = -9$ (refused) or $x = 4$

TRY TO SOLVE

In each of the following figures :

$\overline{DE} \mathbin{//} \overline{BC}$, find the numerical value of x





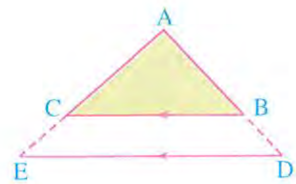
Corollary

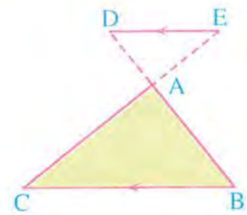
If a straight line is drawn outside the triangle ABC parallel to one side of its sides , say $\overline{BC}$ intersecting $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ at D and E respectively , as shown in the figures , then $\frac{AB}{BD} = \frac{AC}{CE}$

From the properties of the proportion

, we can deduce that :

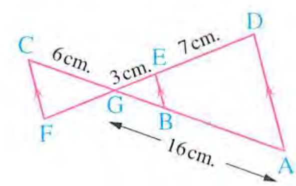
$\frac{AD}{AB} = \frac{AE}{AC}$, $\frac{AD}{BD} = \frac{AE}{CE}$





Example 2

In the opposite figure :



$\overline{AD} // \overline{EB} // \overline{FC}$, $\overline{AC} \cap \overline{DF} = \{G\}$

, DE = 7 cm. , EG = 3 cm.

, GC = 6 cm. , AG = 16 cm.

Find the length of each of : $\overline{GF}$ and $\overline{GB}$

Solution

$\because \overline{AD} // \overline{FC}$ $\quad \therefore \frac{AG}{GC} = \frac{DG}{GF}$

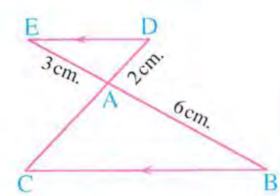
$\therefore \frac{16}{6} = \frac{10}{GF}$ $\quad \therefore GF = \frac{6 \times 10}{16} = 3.75$ cm.

, $\because \overline{BE} // \overline{AD}$ $\quad \therefore \frac{GB}{GA} = \frac{GE}{GD}$

$\therefore \frac{GB}{16} = \frac{3}{10}$ $\quad \therefore GB = \frac{3 \times 16}{10} = 4.8$ cm. (The req.)

TRY TO SOLVE

In the opposite figure :



$\overline{DE} // \overline{BC}$, $\overline{DC} \cap \overline{BE} = \{A\}$

, AE = 3 cm.

, AB = 6 cm.

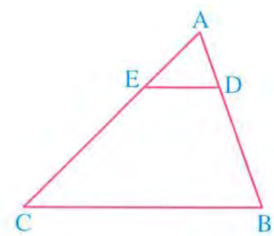
and AD = 2 cm.

Find the length of $\overline{AC}$

Converse of theorem 1

If a straight line intersects two sides of a triangle and divides them into segments whose lengths are proportional , then it is parallel to the third side of the triangle.

In the opposite figure :



ABC is a triangle , $\overleftrightarrow{DE}$ intersects $\overleftrightarrow{AB}$ at D

, $\overleftrightarrow{AC}$ at E and $\frac{AD}{DB} = \frac{AE}{EC}$, then $\frac{AD + DB}{AD} = \frac{AE + EC}{AE}$

$\left(\text{because } \frac{\text{antecedent + consequent}}{\text{antecedent}} = \frac{\text{antecedent + consequent}}{\text{antecedent}}\right)$

$\therefore \frac{AB}{AD} = \frac{AC}{AE}$, $\because \angle A$ is common.

$\therefore \Delta ABC \sim \Delta ADE$

$\therefore \angle B \equiv \angle ADE$ and they are corresponding angles.

$\therefore \overleftrightarrow{DE} \parallel \overleftrightarrow{BC}$

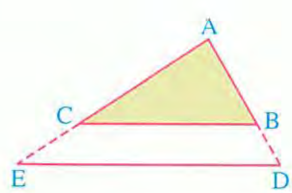
Remark

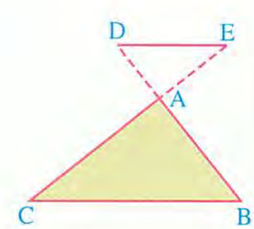
If a straight line (say $\overleftrightarrow{DE}$) is drawn outside the triangle ABC , intersecting $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ at D and E respectively

and if $\frac{AD}{DB} = \frac{AE}{EC}$, then $\overleftrightarrow{DE} \parallel \overline{BC}$

In the opposite figures :

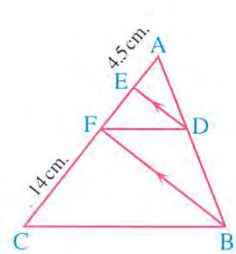
If $\frac{AD}{DB} = \frac{AE}{EC}$, then $\overline{DE} \parallel \overline{BC}$





Example 3

In the opposite figure :



If $\overline{DE} \parallel \overline{BF}$, $AD = \frac{3}{4}DB$, $AE = 4.5$ cm. , $FC = 14$ cm.

Prove that : $\overline{DF} \parallel \overline{BC}$

Solution

$\because AD = \frac{3}{4} DB$ $\qquad \therefore \frac{AD}{DB} = \frac{3}{4}$

$, \because \overline{DE} \parallel \overline{BF}$ $\therefore \frac{AD}{DB} = \frac{AE}{EF}$ $\therefore \frac{3}{4} = \frac{4.5}{EF}$

$\therefore EF = \frac{4 \times 4.5}{3} = 6$ cm. $\therefore AF = 4.5 + 6 = 10.5$ cm $\therefore \frac{AF}{FC} = \frac{10.5}{14} = \frac{3}{4}$

$\therefore \frac{AF}{FC} = \frac{AD}{DB}$ $\therefore \overline{DF} \parallel \overline{BC}$ (Q.E.D.)

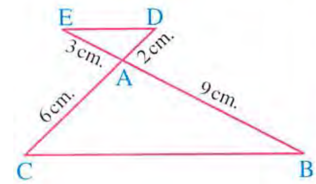
TRY TO SOLVE

In the opposite figure :

$\overline{DC} \cap \overline{BE} = \{A\}$, AD = 2 cm. , AE = 3 cm.

, AB = 9 cm. and AC = 6 cm.

Determine whether $\overline{DE} \parallel \overline{BC}$ and why ?



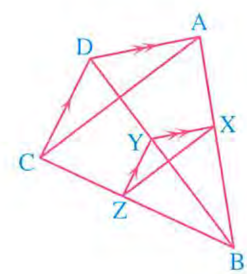
Example 4

In the opposite figure :

ABCD is a quadrilateral , $Y \in \overline{BD}$, $\overline{YX}$ is drawn such that $\overline{YX} \parallel \overline{DA}$ intersecting $\overline{AB}$ at X

, $\overline{YZ}$ is drawn such that $\overline{YZ} \parallel \overline{DC}$ intersecting $\overline{BC}$ at Z

Prove that : $\overline{XZ} \parallel \overline{AC}$



Solution

In Δ ABD : $\because \overline{XY} \parallel \overline{AD}$ $\therefore \frac{BX}{BA} = \frac{BY}{BD}$ (1)

In Δ BCD : $\because \overline{YZ} \parallel \overline{CD}$ $\therefore \frac{BZ}{BC} = \frac{BY}{BD}$ (2)

From (1) , (2) : $\therefore \frac{BX}{BA} = \frac{BZ}{BC}$

$\therefore$ In Δ ABC : $\overline{XZ} \parallel \overline{AC}$ (Q.E.D.)

TRY TO SOLVE

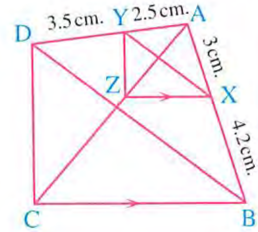
In the opposite figure :

ABCD is a quadrilateral , its diagonals $\overline{AC}$ and $\overline{BD}$ are drawn

, $X \in \overline{AB}$ such that AX = 3 cm. , XB = 4.2 cm. , $Y \in \overline{AD}$ such that AY = 2.5 cm. , YD = 3.5 cm.

, draw $\overrightarrow{XZ} \parallel \overline{BC}$ to intersect $\overline{AC}$ at Z

Prove that : **1** $\overline{XY} \parallel \overline{BD}$ **2** $\overline{YZ} \parallel \overline{CD}$

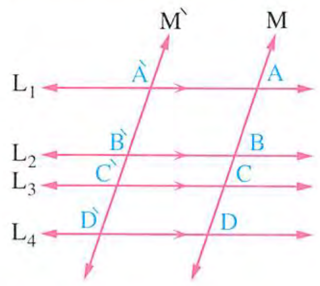


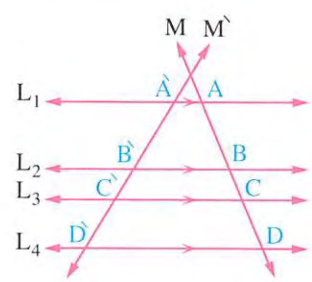
Lesson **2**

Talis' theorem

Theorem 2

Given several coplanar parallel lines and two transversals , then the lengths of the corresponding segments on the transversals are proportional.





In the above two figures :

If $L_1 \parallel L_2 \parallel L_3 \parallel L_4$ and M , M' are two transversals , then $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{AC}{A'C'}$

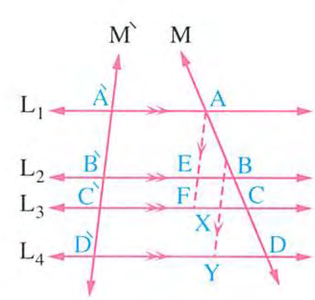
In the following the proof of the theorem

▸ **Given** $L_1 \parallel L_2 \parallel L_3 \parallel L_4$ and M , M' are two transversals to them

▸ **R.T.P.** $AB : BC : CD = A'B' : B'C' : C'D'$

▸ **Const.** Draw $\overrightarrow{AF} \parallel M'$ and intersects L_2 at E ,

L_3 at F , $\overrightarrow{BY} \parallel M'$ and intersects L_3 at X , L_4 at Y



▸ **Proof** $\because \overline{AA'} \parallel \overline{EB'}$, $\overline{AE} \parallel \overline{A'B'}$

$\therefore AEB'A'$ is a parallelogram , then $AE = A'B'$

Similarly : $EF = B'C'$, $BX = B'C'$, $XY = C'D'$

In Δ ACF :

$\because \overline{BE} \mathbin{//} \overline{CF}$ $\quad\quad \therefore \frac{AB}{BC} = \frac{AE}{EF}$

, then $\frac{AB}{BC} = \frac{\grave{A}\grave{B}}{\grave{B}\grave{C}}$, $\frac{AB}{\grave{A}\grave{B}} = \frac{BC}{\grave{B}\grave{C}}$ (exchange the means) (1)

Similarly Δ BDY : $\therefore \frac{BC}{CD} = \frac{\grave{B}\grave{C}}{\grave{C}\grave{D}}$, $\frac{BC}{\grave{B}\grave{C}} = \frac{CD}{\grave{C}\grave{D}}$ (exchange the means) (2)

From (1) , (2) we get :

$$\frac{AB}{\grave{A}\grave{B}} = \frac{BC}{\grave{B}\grave{C}} = \frac{CD}{\grave{C}\grave{D}}$$

$\therefore AB : BC : CD = \grave{A}\grave{B} : \grave{B}\grave{C} : \grave{C}\grave{D}$ (Q.E.D.)

In the previous figure , notice that :

$\frac{AC}{CD} = \frac{\grave{A}\grave{C}}{\grave{C}\grave{D}}$, $\frac{AC}{CB} = \frac{\grave{A}\grave{C}}{\grave{C}\grave{B}}$, $\frac{BD}{DA} = \frac{\grave{B}\grave{D}}{\grave{D}\grave{A}}$

For example :

In the opposite figure :

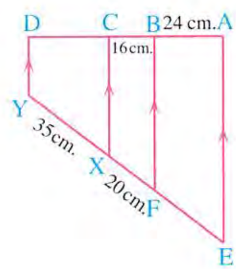
If $\overline{AE} \mathbin{//} \overline{BF} \mathbin{//} \overline{CX} \mathbin{//} \overline{DY}$

such that AB = 24 cm. , BC = 16 cm.

, FX = 20 cm. , XY = 35 cm.

, then $\frac{AB}{EF} = \frac{BC}{FX} = \frac{CD}{XY}$ $\quad$ **i.e.** $\frac{24}{EF} = \frac{16}{20} = \frac{CD}{35}$

, then $EF = \frac{20 \times 24}{16} = 30$ cm. , $CD = \frac{16 \times 35}{20} = 28$ cm.



Example 1

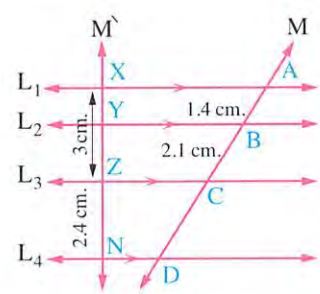
In the opposite figure :

$L_1 \mathbin{//} L_2 \mathbin{//} L_3 \mathbin{//} L_4$ and

M , $\grave{M}$ are two transversals.

Use the lengths shown to

calculate the length of each of $\overline{XY}$ and $\overline{CD}$



Solution

$\because L_1 \mathbin{//} L_2 \mathbin{//} L_3 \mathbin{//} L_4$ and M , $\grave{M}$ are two transversals.

$\therefore \frac{AB}{XY} = \frac{CD}{ZN} = \frac{AC}{XZ}$

$\therefore \frac{1.4}{XY} = \frac{CD}{2.4} = \frac{1.4 + 2.1}{3} = \frac{3.5}{3}$

$\therefore XY = \frac{1.4 \times 3}{3.5} = 1.2$ cm. (First req.)

, $CD = \frac{2.4 \times 3.5}{3} = 2.8$ cm. (Second req.)

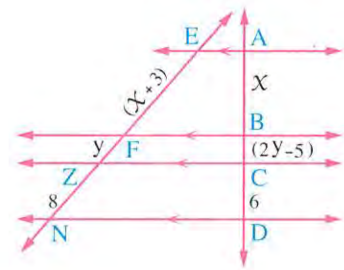
Example 2

In the opposite figure :

If $\overleftrightarrow{AE} \parallel \overleftrightarrow{BF} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DN}$

Find the numerical value of each of x and y

(lengths are measured in centimetres)



Solution

$\because \overleftrightarrow{AE} \parallel \overleftrightarrow{BF} \parallel \overleftrightarrow{CZ} \parallel \overleftrightarrow{DN}$ and $\overleftrightarrow{AB}$, $\overleftrightarrow{EF}$ are two transversals

$\therefore \frac{AB}{EF} = \frac{BC}{FZ} = \frac{CD}{ZN}$ $\qquad \therefore \frac{x}{x+3} = \frac{2y-5}{y} = \frac{6}{8}$

$\therefore 8x = 6(x+3)$ $\qquad \therefore 8x = 6x + 18$ $\qquad \therefore x = 9$

, $\because 6y = 8(2y-5)$ $\qquad \therefore 6y = 16y - 40$ $\qquad \therefore y = 4$ (The req.)

TRY TO SOLVE

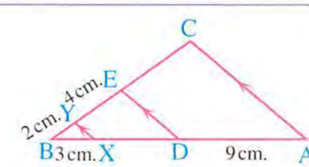
In the opposite figure :

ABC is a triangle ,

$\overline{AC} \parallel \overline{DE} \parallel \overline{XY}$,

AD = 9 cm. , XB = 3 cm. , BY = 2 cm. , EY = 4 cm.

Find : CE and DX

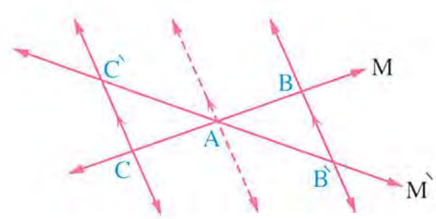


Two special cases

1 If the two lines M and $\grave{M}$ intersect at the point A and $\overleftrightarrow{B\grave{B}} \parallel \overleftrightarrow{C\grave{C}}$

, then $\frac{AB}{AC} = \frac{A\grave{B}}{A\grave{C}}$

and conversely if $\frac{AB}{AC} = \frac{A\grave{B}}{A\grave{C}}$, then $\overleftrightarrow{B\grave{B}} \parallel \overleftrightarrow{C\grave{C}}$



2 Talis' special theorem :

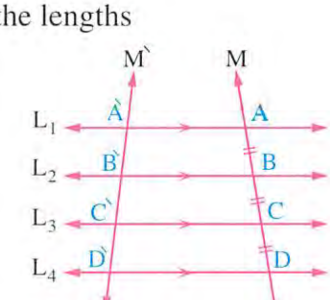
If the lengths of the segments on the transversal are equal , then the lengths of the segments on any other transversal will be also equal.

In the opposite figure :

If $L_1 \parallel L_2 \parallel L_3 \parallel L_4$,

M and $\grave{M}$ are two transversals to them

and if AB = BC = CD , then $\grave{A}\grave{B} = \grave{B}\grave{C} = \grave{C}\grave{D}$

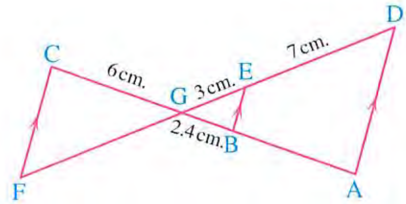


Example 3

In the opposite figure :

$\overline{AD} // \overline{BE} // \overline{FC}$ and $\overline{AC}$, $\overline{DF}$ are two transversals intersecting at G

Use the shown lengths to calculate the length of each of $\overline{GF}$, $\overline{GA}$



Solution

$\because \overline{AD} // \overline{BE} // \overline{FC}$ and $\overleftrightarrow{AC}$, $\overleftrightarrow{DF}$ are two transversals intersecting at G

$\therefore \frac{GF}{GC} = \frac{GE}{GB} = \frac{GD}{GA}$ $\qquad \therefore \frac{GF}{6} = \frac{3}{2.4} = \frac{10}{GA}$

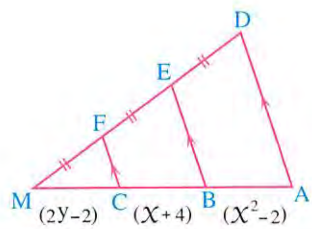
$\therefore GF = \frac{6 \times 3}{2.4} = 7.5$ cm. (First req.) $\qquad$, $GA = \frac{2.4 \times 10}{3} = 8$ cm. (Second req.)

Example 4

In the opposite figure :

$\overline{AD} // \overline{BE} // \overline{CF}$, DE = EF = FM , **find the value of each of x and y**

(lengths are measured in centimetres)



Solution

$\because \overline{AD} // \overline{BE} // \overline{CF}$, DE = EF = FM $\qquad \therefore AB = BC = CM$ $\qquad \therefore x^2 - 2 = x + 4$

$\therefore x^2 - x - 6 = 0$ $\qquad \therefore (x + 2)(x - 3) = 0$ $\qquad \therefore x = -2$ or $x = 3$

$\therefore$ at $x = -2$: $\quad \therefore BC = 2$ cm. $\qquad$, at $x = 3$: $\qquad \therefore BC = 7$ cm.

, $\because BC = CM$ $\qquad \therefore$ at BC = 2 cm. : $\qquad \therefore 2y - 2 = 2 \ \therefore y = 2$

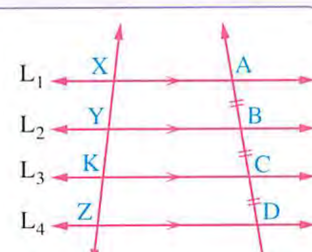
, at BC = 7 cm. : $\therefore 2y - 2 = 7$ $\qquad \therefore y = 4.5$ (The req.)

TRY TO SOLVE

In the opposite figure :

If XK = 6 cm.

Find : The length of $\overline{YK}$



Lesson **3**

Angle bisector and proportional parts

Theorem 3

The bisector of the interior or exterior angle of a triangle at any vertex divides the opposite base of the triangle internally or externally into two parts , the ratio of their lengths is equal to the ratio of the lengths of the other two sides of the triangle.

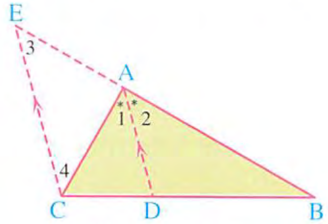


Figure (1)

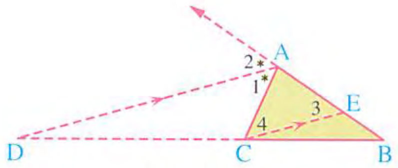


Figure (2)

Given ABC is a triangle , $\overrightarrow{AD}$ bisects $\angle$ BAC internally in figure (1) and externally in figure (2)

R.T.P. $\frac{BD}{DC} = \frac{AB}{AC}$

Const. Draw $\overrightarrow{CE}$ // $\overleftrightarrow{AD}$ and intersects $\overrightarrow{BA}$ at E

Proof $\because$ $\overrightarrow{AD}$ bisects $\angle$ BAC $\therefore \angle 1 \equiv \angle 2$

, $\because$ $\overline{CE}$ // $\overline{AD}$

$\therefore \angle 1 \equiv \angle 4$ (alternate angles) , $\angle 3 \equiv \angle 2$ (corresponding angles)

, $\because \angle 1 \equiv \angle 2$ $\therefore \angle 3 \equiv \angle 4$ $\therefore \overline{AE} \equiv \overline{AC}$ (1)

, $\because$ $\overline{CE}$ // $\overline{AD}$ $\therefore \frac{BD}{DC} = \frac{AB}{AE}$ (2)

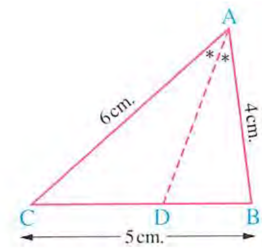
From (1) , (2) : $\therefore \frac{BD}{DC} = \frac{AB}{AC}$ (Q.E.D.)

Example 1

ABC is a triangle in which AB = 4 cm. , BC = 5 cm. , CA = 6 cm. , draw $\overrightarrow{AD}$ to bisect the angle A and intersects $\overline{BC}$ at D

Find the length of each of : $\overline{BD}$, $\overline{DC}$

Solution



$\because \overrightarrow{AD}$ bisects $\angle$ A $\quad \therefore \frac{BD}{DC} = \frac{BA}{AC}$

$\therefore \frac{BD}{DC} = \frac{4}{6} = \frac{2}{3}$ $\quad \therefore \frac{BD}{5 - BD} = \frac{2}{3}$

$\therefore$ 3 BD = 10 – 2 BD $\quad \therefore$ 5 BD = 10

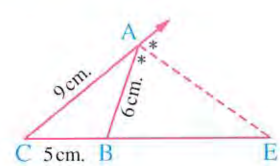
$\therefore$ BD = 2 cm. , DC = 5 – 2 = 3 cm. (The req.)

Example 2

ABC is a triangle in which AB = 6 cm. , BC = 5 cm. , CA = 9 cm. , draw $\overrightarrow{AE}$ to bisect the exterior angle $\angle$ A and intersects $\overleftrightarrow{BC}$ at E

Find the length of each of : $\overline{BE}$, $\overline{EC}$

Solution



$\because$ AB < AC , $\overrightarrow{AE}$ bisects the exterior angle at A

$\therefore E \in \overrightarrow{CB}$, $E \notin \overline{BC}$, $\frac{BE}{EC} = \frac{BA}{AC}$ $\quad \therefore \frac{BE}{EC} = \frac{6}{9} = \frac{2}{3}$

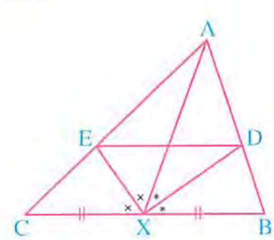
$\therefore \frac{BE}{5 + BE} = \frac{2}{3}$ $\quad \therefore$ 3 BE = 10 + 2 BE

$\therefore$ BE = 10 cm. , EC = 10 + 5 = 15 cm. (The req.)

Example 3

ABC is a triangle , X is the midpoint of $\overline{BC}$, $\overrightarrow{XD}$ bisects $\angle$ AXB and intersects $\overline{AB}$ at D , $\overrightarrow{XE}$ bisects $\angle$ AXC and intersects $\overline{AC}$ at E. **Prove that :** $\overline{DE}$ // $\overline{BC}$

Solution



In Δ AXB : $\because \overrightarrow{XD}$ bisects $\angle$ AXB

$\therefore \frac{AD}{DB} = \frac{AX}{XB}$ (1)

, in Δ AXC : $\because \overrightarrow{XE}$ bisects $\angle$ AXC $\quad \therefore \frac{AE}{EC} = \frac{AX}{XC}$ (2)

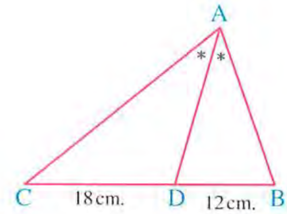
From (1) , (2) and noticing that : XB = XC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ $\quad \therefore$ In Δ ABC : $\overline{DE}$ // $\overline{BC}$ (Q.E.D.)

Example 4

In the opposite figure :

ABC is a triangle , $\overrightarrow{AD}$ bisects $\angle$ A and intersects $\overline{BC}$ at D , where BD = 12 cm. , DC = 18 cm. , if the perimeter of Δ ABC = 80 cm.



Find the length of each of : $\overline{AC}$, $\overline{AB}$

Solution

In Δ ABC : $\because$ $\overrightarrow{AD}$ bisects $\angle$ A $\quad \therefore \frac{AB}{AC} = \frac{BD}{DC} = \frac{12}{18} = \frac{2}{3}$

, $\because$ the perimeter of Δ ABC = 80 cm. , BC = 12 + 18 = 30 cm.

$\therefore$ AB + AC = 80 – 30 = 50 cm.

, $\because \frac{AB}{AC} = \frac{2}{3}$ $\quad \therefore \frac{AB + AC}{AC} = \frac{2+3}{3}$ (from the properties of the proportion)

$\therefore \frac{50}{AC} = \frac{5}{3}$ $\quad \therefore AC = \frac{3 \times 50}{5} = 30$ cm.

$\therefore$ AB = 50 – 30 = 20 cm. (The req.)

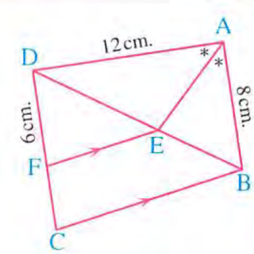
TRY TO SOLVE

In the opposite figure :

ABCD is a quadrilateral in which : AB = 8 cm.

, AD = 12 cm. , $\overrightarrow{AE}$ bisects $\angle$ A and intersects $\overline{BD}$ at E

, $\overrightarrow{EF}$ // $\overline{BC}$ and intersects $\overline{DC}$ at F , if DF = 6 cm. ,



then find the length of : $\overline{DC}$

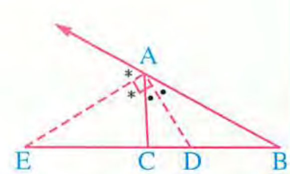
Important Remarks

i.e. The interior and exterior bisectors for any angle in the triangle are perpendicular

1 In the opposite figure :

If $\overrightarrow{AD}$, $\overrightarrow{AE}$ are the bisectors of the angle A and the exterior angle of Δ ABC at A respectively

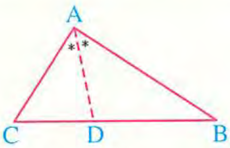
, then $\frac{BD}{DC} = \frac{AB}{AC}$, $\frac{BE}{EC} = \frac{AB}{AC}$ $\quad \therefore \frac{BD}{DC} = \frac{BE}{EC}$



$\therefore$ The base $\overline{BC}$ is divided internally at D , externally at E by the same ratio (AB : AC) and we notice that : the two bisectors $\overrightarrow{AD}$ and $\overrightarrow{AE}$ are perpendicular.

i.e. m ($\angle$ DAE) = 90°

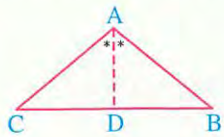
2 **If $\overrightarrow{AD}$ bisects $\angle$ BAC and intersects $\overline{BC}$ at D , then D takes one of the following :**



If AB > AC

, then BD > DC

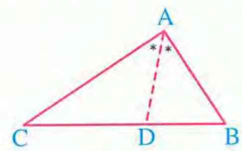
i.e. D is nearer to C than to B



If AB = AC

, then BD = DC

i.e. D is equidistant from each of B and C

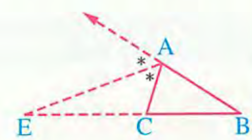


If AB < AC

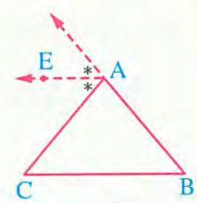
, then BD < DC

i.e. D is nearer to B than to C

3 **If $\overrightarrow{AE}$ bisects the exterior angle of Δ ABC at A , where E $\notin \overline{BC}$, then E takes one of the following cases :**

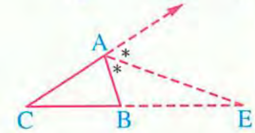


① If AB > AC , then BE > EC i.e. E $\in \overrightarrow{BC}$



② If AB = AC , then $\overrightarrow{AE}$ // $\overline{BC}$

i.e. The exterior bisector of the vertex of isosceles triangle is paralleling to the base.



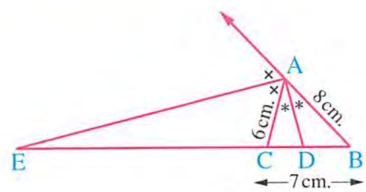
③ If AB < AC , then BE < EC i.e. E $\in \overrightarrow{CB}$

Example 5

ABC is a triangle in which AB = 8 cm. , AC = 6 cm. , BC = 7 cm. , draw $\overrightarrow{AD}$ to bisect $\angle$ A and intersect $\overline{BC}$ at D , draw $\overrightarrow{AE}$ to bisect the exterior angle A and intersect $\overleftrightarrow{BC}$ at E

Find the length of : $\overline{DE}$

Solution



In Δ ABC :

$\because$ $\overrightarrow{AD}$ bisects $\angle$ A , $\overrightarrow{AE}$ bisects the exterior angle A

$\therefore \frac{BD}{DC} = \frac{BE}{CE} = \frac{BA}{AC}$ $\quad \therefore \frac{BD}{DC} = \frac{BE}{CE} = \frac{8}{6} = \frac{4}{3}$ (1)

$\therefore \frac{BD + DC}{DC} = \frac{4 + 3}{3}$ (from the properties of the proportion)

$\therefore \frac{BC}{DC} = \frac{7}{3}$ $\quad \therefore \frac{7}{DC} = \frac{7}{3}$ $\quad \therefore$ DC = 3 cm.

From (1) : $\because \frac{BE}{EC} = \frac{4}{3}$ $\therefore \frac{BE - EC}{CE} = \frac{4-3}{3}$ (from the properties of the proportion)

$\therefore \frac{BC}{CE} = \frac{1}{3}$ $\therefore \frac{7}{CE} = \frac{1}{3}$

$\therefore CE = 21$ cm. $\therefore DE = DC + CE = 3 + 21 = 24$ cm. (The req.)

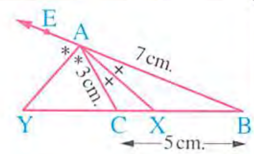
TRY TO SOLVE

In the opposite figure :

$\overrightarrow{AX}$ bisects $\angle BAC$, $\overrightarrow{AY}$ bisects $\angle CAE$

, AB = 7 cm. , AC = 3 cm. , BC = 5 cm.

Find the length of : $\overline{XY}$



Finding the lengths of the interior and the exterior bisectors of an angle of a triangle

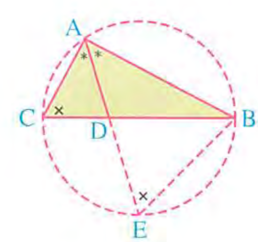
Well known problem

If $\overrightarrow{AD}$ bisects $\angle A$ in ΔABC internally and intersects $\overline{BC}$ at D

, then $AD = \sqrt{AB \times AC - BD \times DC}$

Given ABC is a triangle , $\overrightarrow{AD}$ bisects $\angle BAC$ internally

, $\overrightarrow{AD} \cap \overline{BC} = \{D\}$



R.T.P. $AD = \sqrt{AB \times AC - BD \times DC}$

Const. Draw a circle passing through the vertices of ΔABC

and intersecting $\overrightarrow{AD}$ at E , draw $\overline{BE}$

Proof $\because m(\angle CAD) = m(\angle EAB)$ (given)

, $m(\angle E) = m(\angle C)$ (inscribed angles subtended by $\overset{\frown}{AB}$)

$\therefore \Delta ACD \sim \Delta AEB$, then $\frac{AC}{AE} = \frac{AD}{AB}$

$\therefore AD \times AE = AB \times AC$

$\therefore AD \times (AD + DE) = AB \times AC$

$\therefore (AD)^2 = AB \times AC - AD \times DE$

$\therefore (AD)^2 = AB \times AC - BD \times DC$

$\therefore AD = \sqrt{AB \times AC - BD \times DC}$ (Q.E.D.)

Remember that

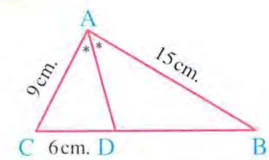
$AD \times DE = BD \times DC$

Example 6

ABC is a triangle in which : AB = 15 cm. , AC = 9 cm. , $\overrightarrow{AD}$ bisects ∠ BAC and intersects $\overline{BC}$ at D , if DC = 6 cm.

Find the length of : $\overline{AD}$

Solution



$\because \overrightarrow{AD}$ bisects ∠ BAC $\quad \therefore \dfrac{BD}{DC} = \dfrac{BA}{CA}$

$\therefore \dfrac{BD}{6} = \dfrac{15}{9} \quad \therefore BD = \dfrac{15 \times 6}{9} = 10$ cm.

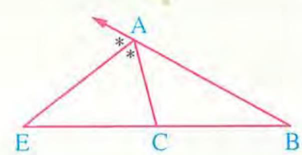
$\therefore AD = \sqrt{AB \times AC - BD \times DC} = \sqrt{15 \times 9 - 10 \times 6} = \sqrt{75} = 5\sqrt{3}$ cm. (The req.)

Remark

In the opposite figure :

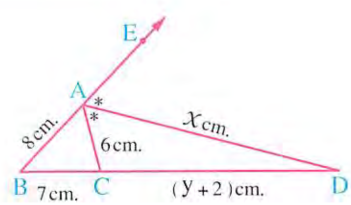
If $\overrightarrow{AE}$ bisects ∠ BAC externally and intersects $\overleftrightarrow{BC}$ at E

, then $AE = \sqrt{BE \times EC - AB \times AC}$



Example 7

In the opposite figure :



ABC is a triangle in which AB = 8 cm.

, BC = 7 cm. , AC = 6 cm. , $\overrightarrow{AD}$ bisects ∠ A externally.

Find the value of each of : x , y

Solution

$\because \overrightarrow{AD}$ bisects ∠ A externally $\quad \therefore \dfrac{BD}{CD} = \dfrac{BA}{AC} = \dfrac{8}{6} = \dfrac{4}{3}$

$\therefore \dfrac{7 + y + 2}{y + 2} = \dfrac{4}{3} \quad \therefore \dfrac{y + 9}{y + 2} = \dfrac{4}{3}$

$\therefore 3y + 27 = 4y + 8 \quad \therefore y = 19$

$\therefore DC = 21$ cm. , $BD = 28$ cm.

, $\because AD = \sqrt{BD \times CD - BA \times AC} = \sqrt{28 \times 21 - 8 \times 6} = \sqrt{540} = 6\sqrt{15}$ cm.

$\therefore x = 6\sqrt{15}$ (The req.)

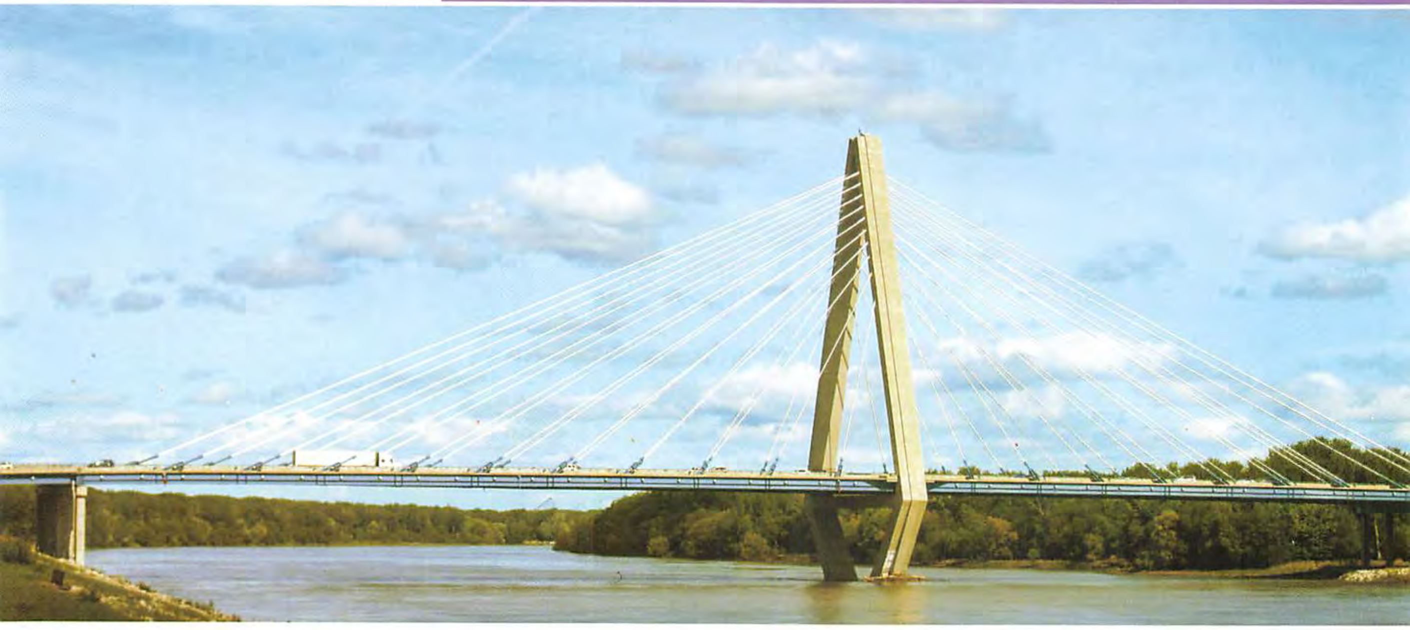
TRY TO SOLVE

ABC is a triangle in which : AB = 27 cm. , AC = 15 cm. , draw $\overrightarrow{AD}$ to bisect ∠ A and intersect $\overline{BC}$ at D , if BD = 18 cm.

Find the length of : $\overline{AD}$

Lesson **4**

Follow : Angle bisector and proportional parts (Converse of theorem 3)



Converse of theorem 3

In the opposite two figures :

- If $D \in \overline{BC}$ (Fig. 1)

  such that : $\frac{BD}{DC} = \frac{BA}{AC}$

  , then $\overrightarrow{AD}$ bisects $\angle$ BAC

- If $D \in \overleftrightarrow{BC}$, $D \notin \overline{BC}$ (Fig. 2)

  such that : $\frac{BD}{DC} = \frac{BA}{AC}$

  , then $\overrightarrow{AD}$ bisects the exterior angle of Δ ABC at A

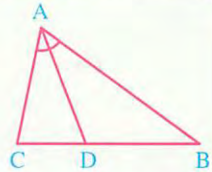


Fig. (1)

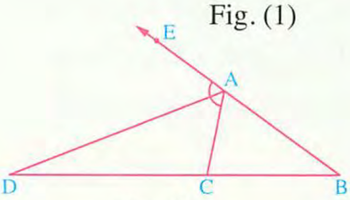


Fig. (2)

Example 1

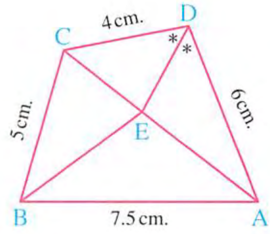
In the opposite figure :

ABCD is a quadrilateral in which AB = 7.5 cm.

, BC = 5 cm. , CD = 4 cm. , AD = 6 cm.

, $\overrightarrow{DE}$ bisects $\angle$ ADC and intersects $\overline{AC}$ at E

Prove that : $\overrightarrow{BE}$ bisects $\angle$ ABC



Solution

In Δ ACD : $\because$ $\overrightarrow{DE}$ bisects $\angle$ ADC

$\therefore \frac{AE}{EC} = \frac{AD}{DC} = \frac{6}{4} = \frac{3}{2}$

, $\because \frac{AB}{BC} = \frac{7.5}{5} = \frac{3}{2}$

$\therefore \frac{AE}{EC} = \frac{AB}{BC}$

$\therefore$ In Δ ABC : $\overrightarrow{BE}$ bisects $\angle$ ABC

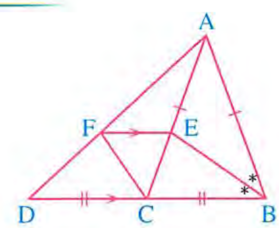
(Q.E.D.)

Example 2

ABC is an isosceles triangle in which AB = AC , D $\in \overrightarrow{BC}$, where BC = CD , draw the bisector of the angle ABC to intersect $\overline{AC}$ at E , draw $\overrightarrow{EF}$ // $\overline{BC}$ and intersects $\overline{AD}$ at F

Prove that : $\overrightarrow{CF}$ bisects $\angle$ ACD

Solution



In Δ ABC : $\because$ $\overrightarrow{BE}$ bisects $\angle$ ABC

$\therefore \frac{AE}{EC} = \frac{AB}{BC}$, but AB = AC , BC = CD (given)

$\therefore \frac{AE}{EC} = \frac{AC}{CD}$ (1)

In Δ ACD :

$\because \overline{EF} \parallel \overline{CD}$ $\therefore \frac{AE}{EC} = \frac{AF}{FD}$ (2)

From (1) , (2) : $\therefore \frac{AF}{FD} = \frac{AC}{CD}$

$\therefore$ In Δ ACD : $\overrightarrow{CF}$ bisects $\angle$ ACD (Q.E.D.)

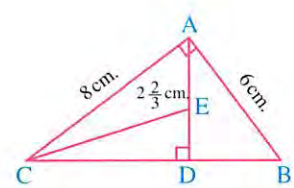
Example 3

In the opposite figure :

ABC is a right-angled triangle at A , $\overline{AD} \perp \overline{BC}$

, AB = 6 cm. , AC = 8 cm. , AE = $2\frac{2}{3}$ cm.

Prove that : $\overrightarrow{CE}$ bisects $\angle$ ACD



Solution

$\because$ Δ ABC is right-angled at A $\therefore (BC)^2 = (AB)^2 + (AC)^2 = 36 + 64 = 100$

$\therefore$ BC = 10 cm.

, $\because \overline{AD} \perp \overline{BC}$ $\therefore \Delta DAC \sim \Delta ABC$

$\therefore \frac{DC}{AC} = \frac{AC}{BC}$ $\therefore \frac{DC}{8} = \frac{8}{10}$ $\therefore$ DC = 6.4 cm.

, $\because \Delta DBA \sim \Delta ABC$ $\therefore \frac{AB}{CB} = \frac{AD}{CA}$

$\therefore \frac{6}{10} = \frac{AD}{8}$ $\therefore$ AD = 4.8 cm. $\therefore DE = 4.8 - 2\frac{2}{3} = 2\frac{2}{15}$ cm.

, $\because \frac{AC}{CD} = \frac{8}{6.4} = \frac{5}{4}$, $\frac{AE}{ED} = \frac{2\frac{2}{3}}{2\frac{2}{15}} = \frac{5}{4}$

$\therefore \frac{AC}{CD} = \frac{AE}{ED}$ $\therefore \overrightarrow{CE}$ bisects $\angle$ ACD (Q.E.D.)

TRY TO SOLVE

ABCD is a quadrilateral in which AB = 20 cm. , AD = 6 cm. , DC = 9 cm. , $E \in \overline{AB}$ such that AE = 8 cm. , draw $\overrightarrow{EX} \parallel \overline{BC}$ to intersect $\overline{AC}$ at X

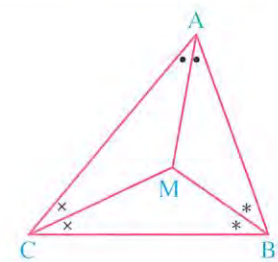
Prove that : $\overrightarrow{DX}$ bisects $\angle$ ADC

Fact

The bisectors of angles of a triangle are concurrent.

In the opposite figure :

$\overrightarrow{AM}$, $\overrightarrow{BM}$ and $\overrightarrow{CM}$ are concurrent at the point M

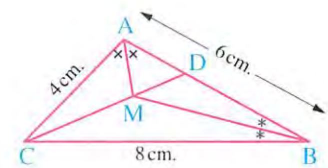


Example 4

In the opposite figure :

ABC is a triangle in which AB = 6 cm. , AC = 4 cm. , BC = 8 cm. , $\overrightarrow{BM}$ bisects $\angle$ ABC , $\overrightarrow{AM}$ bisects $\angle$ BAC

Find the length of : $\overline{AD}$



Solution

$\because$ $\overrightarrow{AM}$ bisects $\angle$ BAC , $\overrightarrow{BM}$ bisects $\angle$ ABC

$\therefore$ M is the point of concurrence of the bisectors of angles of Δ ABC

$\therefore$ $\overrightarrow{CM}$ bisects $\angle$ ACB

$\therefore$ In Δ ABC : $\frac{AD}{DB} = \frac{AC}{CB} = \frac{4}{8} = \frac{1}{2}$

$\therefore \frac{AD}{6 - AD} = \frac{1}{2}$ $\qquad$ $\therefore$ 2 AD = 6 – AD

$\therefore$ 3 AD = 6 $\qquad$ $\therefore$ AD = 2 cm. $\qquad$ (The req.)

TRY TO SOLVE

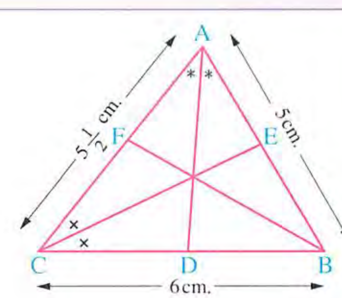
In the opposite figure :

ABC is a triangle in which AB = 5 cm.

, AC = $5\frac{1}{2}$ cm. , BC = 6 cm.

, $\overrightarrow{AD}$ bisects $\angle$ BAC , $\overrightarrow{CE}$ bisects $\angle$ ACB

Find the length of : $\overline{AF}$



Lesson 5 Applications of proportionality in the circle

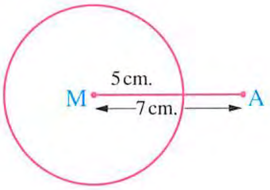
Power of a point with respect to a circle

Definition

Power of the point A with respect to the circle M in which the length of its radius is r , is the real number $P_M(A)$ where $P_M(A) = (AM)^2 - r^2$

For example :

In the opposite figure :



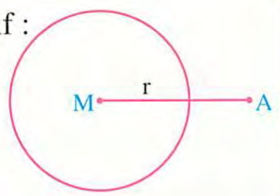
If A is a point outside the circle M whose radius length equals 5 cm.

, where MA = 7 cm.

, then $P_M(A) = 7^2 - 5^2 = 24$

Note 1

We can determine the position of point A with respect to the circle M if :



- $P_M(A) > 0$, then A lies outside the circle.
- $P_M(A) = 0$, then A lies on the circle.
- $P_M(A) < 0$, then A lies inside the circle.

Example 1

If M is a circle of diameter length 12 cm. , A is a point lies on its plane , determine the position of point A with respect to the circle M in each of the following cases , then calculate its distance from the centre of the circle :

1 $P_M(A) = 13$ 2 $P_M(A) = \text{Zero}$ 3 $P_M(A) = -11$

Solution

$\because$ Length of circle diameter = 12 cm. $\therefore r = 6$ cm.

1 $\because P_M(A) = 13 > 0$ $\therefore$ A lies outside the circle

, $\because P_M(A) = (MA)^2 - r^2$ $\therefore 13 = (MA)^2 - 36$

$\therefore MA = 7$ cm.

2 $\because P_M(A) = \text{Zero}$ $\therefore$ A lies on the circle $\therefore MA = 6$ cm.

3 $\because P_M(A) = -11 < 0$ $\therefore$ A lies inside the circle

, $\because P_M(A) = (MA)^2 - r^2$ $\therefore -11 = (MA)^2 - 36$

$\therefore MA = 5$ cm.

TRY TO SOLVE

Determine the position of each of the points A , B and C with respect to the circle M whose radius length is 5 cm. if :

1 $P_M(A) = 11$ 2 $P_M(B) = \text{Zero}$ 3 $P_M(C) = -16$

Then calculate the distance of each point from the circle centre M

Note 2

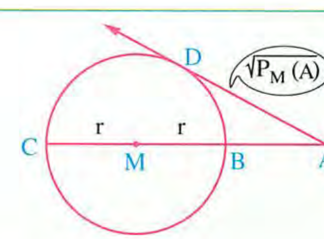
If the point A lies outside the circle M

, then $P_M(A) = (AM)^2 - r^2$

$= (AM - r)(AM + r)$

$= AB \times AC = (AD)^2$

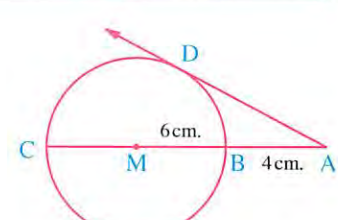
$\therefore$ Length of the tangent drawn from A to circle M $= \sqrt{P_M(A)}$



For example : **In the opposite figure :**

If point A lies outside the circle M whose radius length is 6 cm. , $\overrightarrow{AD}$ is a tangent to the circle at D

If AB = 4 cm. , we can find $P_M(A)$



with one of the following methods :

- Using the definition : $P_M(A) = (AM)^2 - r^2 = (10)^2 - (6)^2 = 64$
- Using the previous note : $P_M(A) = AB \times AC = 4 \times 16 = 64$

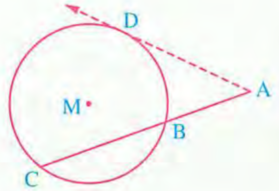
From the previous , we can get : AD where $AD = \sqrt{P_M(A)} = \sqrt{64} = 8$ cm.

Notice that

In the opposite figure :

If point A lies outside the circle , $\overline{AC}$ intersects the circle at B , C

, then $P_M(A) = AB \times AC$



And this can be concluded from the previous note , where :

$P_M(A) = (AD)^2$, where $\overrightarrow{AD}$ is a tangent to the circle M at D

, $\because (AD)^2 = AB \times AC$ $\therefore P_M(A) = AB \times AC$

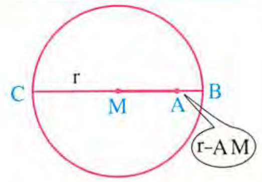
Note 3

If point A lies inside the circle M , then :

$P_M(A) = (AM)^2 - r^2$

$= (AM - r)(AM + r)$

$= -(r - AM)(AM + r) = -AB \times AC$

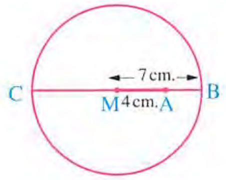


For example : In the opposite figure :

If point A lies inside the circle M whose radius length is 7 cm.

and lies at a distance of 4 cm. from the circle centre

, then $P_M(A) = -AB \times AC = -3 \times 11 = -33$

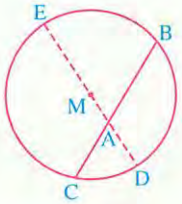


Notice that

In the opposite figure :

If $\overline{BC}$ is a chord in the circle M , $A \in \overline{BC}$

, then $P_M(A) = -AB \times AC$



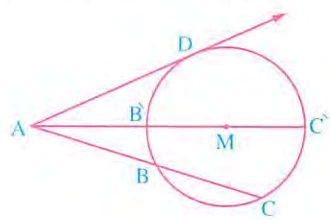
And this could be concluded from the previous note as follows :

$P_M(A) = -AD \times AE$ (where $\overline{DE}$ is a diameter)

, $\because AD \times AE = AB \times AC$ $\therefore P_M(A) = -AB \times AC$

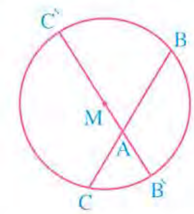
Summary of the previous as follows :

If A lies outside circle M , then :



$P_M(A) = AB \times AC = AB` \times AC` = (AD)^2$

If A lies inside circle M , then :

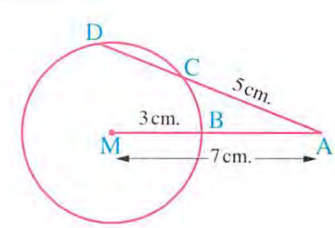


$P_M(A) = -AB \times AC = -AB` \times AC`$

Example 2

A circle of centre M and its radius length is 3 cm. , A is a point at a distance of 7 cm. from its centre , from A a straight line is drawn to intersect the circle at C , D , where $C \in \overline{AD}$, if CA = 5 cm. , calculate the length of the chord $\overline{CD}$

Solution



$\because P_M(A) = (AM)^2 - r^2 = 49 - 9 = 40$

, $\because P_M(A) = AC \times AD$

$\therefore 40 = 5 \times AD$

$\therefore AD = 8$ cm.

$\therefore CD = AD - AC = 8 - 5 = 3$ cm. (The req.)

Example 3

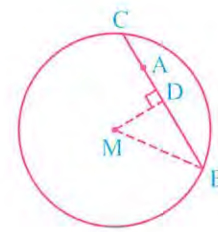
A circle M of radius length 7 cm. , A is a point at a distance of 5 cm. from its centre.

The chord $\overline{BC}$ passes through point A , where AB = 3 AC

Calculate : 1 The length of the chord $\overline{BC}$

2 The distance between $\overline{BC}$ and the centre of the circle.

Solution



$\because P_M(A) = (AM)^2 - r^2 = 25 - 49 = -24$

$, \because P_M(A) = -AB \times AC$ $\quad\therefore -24 = -AB \times AC$

$\therefore 24 = AB \times AC$

$, \because AB = 3\,AC$ $\quad\therefore 24 = 3\,AC \times AC$

$\therefore 8 = (AC)^2$ $\quad\therefore AC = \sqrt{8} = 2\sqrt{2}$ cm.

$, \because AB = 3\,AC$ $\quad\therefore AB = 6\sqrt{2}$ cm.

$\therefore BC = AC + AB = 8\sqrt{2}$ cm. (First req.)

, let the distance between the chord $\overline{BC}$ and the centre of the circle be MD

, where $\overline{MD} \perp \overline{BC}$

$, \because \overline{MD} \perp \overline{BC}$ $\quad\therefore$ D is the midpoint of $\overline{BC}$

$\therefore P_M(D) = (DM)^2 - r^2 = -BD \times DC$ $\quad\therefore (DM)^2 - 49 = -4\sqrt{2} \times 4\sqrt{2}$

$\therefore (DM)^2 = 17$ $\quad\therefore DM = \sqrt{17} \simeq 4.1$ cm. (Second req.)

TRY TO SOLVE

The circle M has radius length 20 cm. , A is a point at a distance 16 cm.

from the centre of the circle , the chord $\overline{BC}$ is drawn where $A \in \overline{BC}$, $AB = 2\,AC$

Calculate : 1 The length of the chord $\overline{BC}$

2 The distance between the chord $\overline{BC}$ and the centre of the circle.

Important Note

The set of points which have the same power with respect to two distinct circles is called the principle axis of the two circles.

If $P_M(A) = P_N(A)$, then A lies on the principle axis of the two circles M and N

For example :

If $P_M(A) = P_N(A)$, $P_M(B) = P_N(B)$

, then $\overleftrightarrow{AB}$ is the principle axis of the two circles M and N

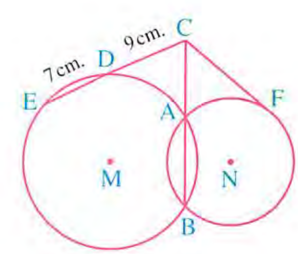
Example 4

Two circles M and N are intersecting at A and B , $C \in \overline{BA}$, $C \notin \overline{BA}$, draw $\overrightarrow{CD}$ to intersect the circle M at D and E , where CD = 9 cm. , DE = 7 cm. , draw $\overrightarrow{CF}$ to touch the circle N at F

1 **Prove that :** C lies on the principle axis of the two circles M and N

2 If AB = 10 cm. , **find the length of each of :** $\overline{AC}$, $\overline{CF}$

Solution



$\because$ A lies on the circle M , A lies on the circle N

$\therefore P_M(A) = P_N(A) = \text{zero}$,

Similarly : $P_M(B) = P_N(B) = \text{zero}$

$\therefore \overleftrightarrow{AB}$ is the principle axis for the two circles M and N

, $\because C \in \overleftrightarrow{AB}$

$\therefore$ C lies on the principle axis of the two circles M and N (First req.)

, $\because P_M(C) = CD \times CE = 9 \times 16 = 144$

, $P_M(C) = CA \times CB$ $\therefore 144 = CA(CA + 10)$

$\therefore 144 = (CA)^2 + 10\,CA$ $\therefore (CA)^2 + 10\,CA - 144 = 0$

$\therefore (CA - 8)(CA + 18) = 0$ $\therefore CA = 8$ cm.

, $\because$ C lies on the principle axis of the two circles M and N

$\therefore P_N(C) = P_M(C)$, $P_N(C) = (CF)^2$

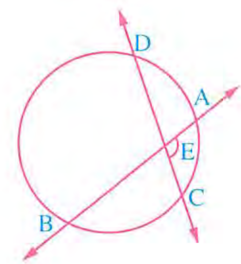
$\therefore (CF)^2 = 144$ $\therefore CF = 12$ cm (Second req.)

Secant , tangent and measures of angles

Remember that

1 The measure of an angle formed by two chords that intersect inside a circle is equal to half the sum of the measures of the intercepted arcs.

In the opposite figure :



$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ are two secants to the circle , where

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$, then

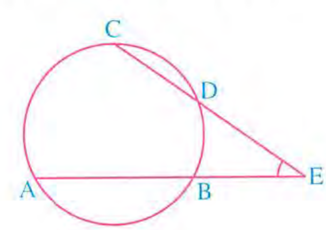
$m(\angle AEC) = \frac{1}{2}[m(\overset{\frown}{AC}) + m(\overset{\frown}{BD})]$

For example If $m(\overset{\frown}{AC}) = 50°$, $m(\overset{\frown}{BD}) = 170°$

$\therefore m(\angle AEC) = \frac{1}{2}(50° + 170°) = 110°$

2 The measure of an angle formed by two secants drawn from a point outside a circle is equal to half the positive difference of the measures of the intercepted arcs.

In the opposite figure :



$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ are two secants to the circle , where

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$, then

$m(\angle E) = \frac{1}{2}[m(\overset{\frown}{AC}) - m(\overset{\frown}{BD})]$

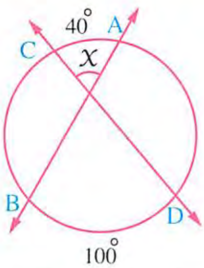
For example If $m(\overset{\frown}{AC}) = 120°$, $m(\overset{\frown}{BD}) = 50°$

$\therefore m(\angle E) = \frac{1}{2}[120° - 50°] = 35°$

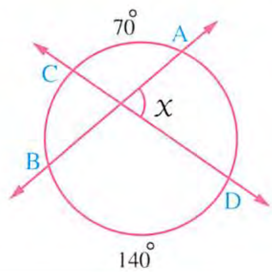
Example 5

In each of the following figures , find the value of x :

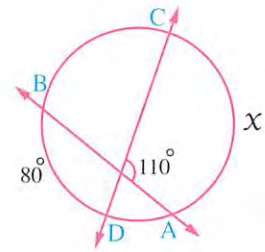
1



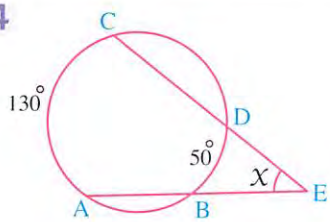
2



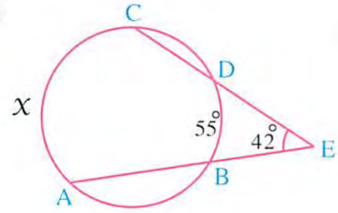
3



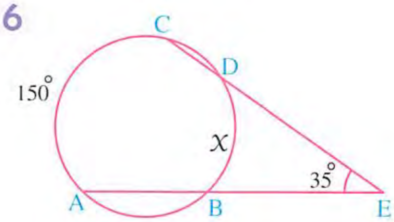
4



5



6



Solution

1 $x = \frac{1}{2}(40° + 100°) = 70°$

2 $\because$ The measure of the circle $= 360°$, $m(\overset{\frown}{AC}) + m(\overset{\frown}{DB}) = 70° + 140° = 210°$

$\therefore m(\overset{\frown}{AD}) + m(\overset{\frown}{BC}) = 360° - 210° = 150°$ $\qquad \therefore x = \frac{1}{2} \times 150° = 75°$

3 $\because \frac{1}{2}(x + 80°) = 110°$ $\qquad \therefore x + 80° = 220°$ $\qquad \therefore x = 140°$

4 $x = \frac{1}{2}(130° - 50°) = 40°$

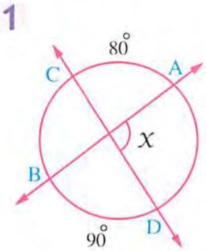
5 $\because \frac{1}{2}(x - 55°) = 42°$ $\qquad \therefore x - 55° = 84°$ $\qquad \therefore x = 139°$

6 $\because \frac{1}{2}(150° - x) = 35°$ $\qquad \therefore 150° - x = 70°$ $\qquad \therefore x = 80°$

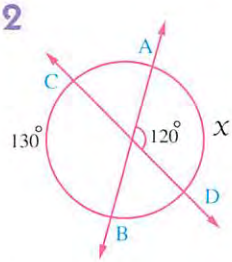
TRY TO SOLVE

Find the value of x in each of the following :

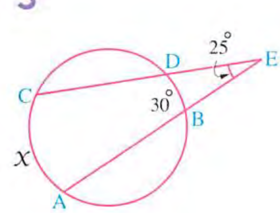
1



2



3



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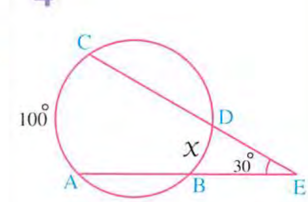
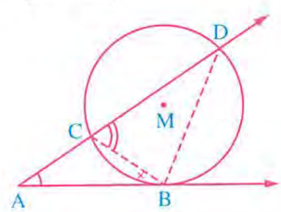


Well known problem

The measure of an angle formed by a secant and a tangent or two tangents drawn from a point outside a circle is equal to half the positive difference of the measures of the intercepted arcs.

First case — Intersection of a secant and a tangent to a circle



Given $\overleftrightarrow{AB}$ is a tangent to the circle M at B , $\overrightarrow{AD} \cap$ the circle M $= \{C , D\}$

R.T.P. $m(\angle A) = \frac{1}{2}[m(\overset{\frown}{BD}) - m(\overset{\frown}{BC})]$

Const. Draw $\overline{BC}$, $\overline{BD}$

Proof $\because \angle BCD$ is an exterior angle of ΔABC

$\therefore m(\angle BCD) = m(\angle A) + m(\angle ABC)$

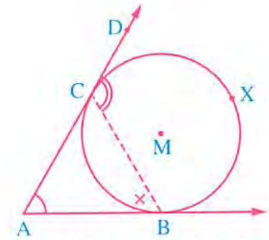
$\therefore m(\angle A) = m(\angle BCD) - m(\angle ABC)$, $\because \angle BCD$ is an inscribed angle.

$\therefore m(\angle BCD) = \frac{1}{2}m(\overset{\frown}{BD})$, $\because \angle ABC$ is a tangency angle.

$\therefore m(\angle ABC) = \frac{1}{2}m(\overset{\frown}{BC})$ $\therefore m(\angle A) = \frac{1}{2}m(\overset{\frown}{BD}) - \frac{1}{2}m(\overset{\frown}{BC})$

$= \frac{1}{2}[m(\overset{\frown}{BD}) - m(\overset{\frown}{BC})]$ (Q.E.D.)

Second case — Intersection of two tangents to a circle



Given $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle M at B and C

R.T.P. $m(\angle A) = \frac{1}{2}[m(\overset{\frown}{BXC}) - m(\overset{\frown}{BC})]$

Const. Draw $\overline{BC}$

Proof $\because \angle BCD$ is an exterior angle of ΔABC

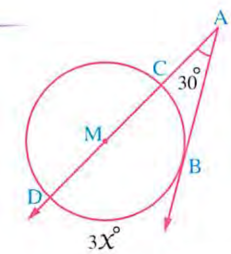
$\therefore m(\angle BCD) = m(\angle A) + m(\angle B)$ $\therefore m(\angle A) = m(\angle BCD) - m(\angle B)$

, $\because \angle BCD$ is a tangency angle. $\therefore m(\angle BCD) = \frac{1}{2}m(\overset{\frown}{BXC})$

, $\because \angle B$ is a tangency angle. $\therefore m(\angle B) = \frac{1}{2}m(\overset{\frown}{BC})$

$\therefore m(\angle A) = \frac{1}{2}m(\overset{\frown}{BXC}) - \frac{1}{2}m(\overset{\frown}{BC})$ $= \frac{1}{2}[m(\overset{\frown}{BXC}) - m(\overset{\frown}{BC})]$ (Q.E.D.)

Example 6



In the opposite figure :

If $\overrightarrow{AB}$ is a tangent to the circle M at B , $m(\angle A) = 30°$

, $\overrightarrow{AM}$ is a secant to the circle at C and D , $m(\overset{\frown}{BD}) = 3X°$

Find the value of : X

Solution

$\because \overrightarrow{AB}$ is a tangent to the circle M , $\overrightarrow{AD}$ is a secant to it.

$\therefore m(\angle A) = \frac{1}{2}[m(\widehat{BD}) - m(\widehat{BC})]$ $\therefore \frac{1}{2}[m(\widehat{BD}) - m(\widehat{BC})] = 30°$

$\therefore m(\widehat{BD}) - m(\widehat{BC}) = 60°$ (1)

, $\because \overline{CD}$ is a diameter in the circle M $\therefore m(\widehat{BD}) + m(\widehat{BC}) = 180°$ (2)

Adding (1) , (2) we get that : $2\,m(\widehat{BD}) = 240°$ $\therefore m(\widehat{BD}) = 120°$

, $\because m(\widehat{BD}) = 3\,X°$ $\therefore 3\,X° = 120°$ $\therefore X = 40°$ (The req.)

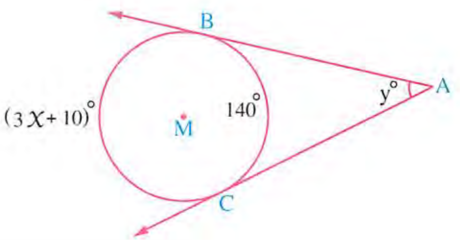
Example 7

In the opposite figure :

If $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle M at B , C respectively , $m(\angle A) = y°$

, $m(\widehat{BC})$ minor $= 140°$, $m(\widehat{BC})$ major $= (3\,X + 10)°$

Find the values of : X and y



Solution

$\because$ The measure of the circle = 360°

$\therefore m(\widehat{BC})$ minor $+ m(\widehat{BC})$ major $= 360°$

$\therefore 140° + (3\,X + 10)° = 360°$ $\therefore 3\,X° + 150° = 360°$

$\therefore 3\,X° = 210°$ $\therefore X = 70°$

$\therefore m(\widehat{BC})$ major $= (3 \times 70° + 10°) = 220°$, $\because \overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to circle M

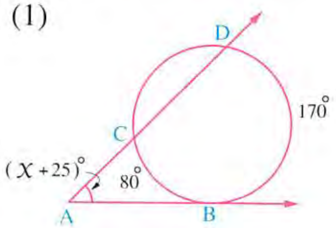
$\therefore m(\angle A) = \frac{1}{2}[m(\widehat{BC})$ major $- m(\widehat{BC})$ minor$]$

$\therefore y° = \frac{1}{2}[220° - 140°] = 40°$ $\therefore y = 40$ (The req.)

TRY TO SOLVE

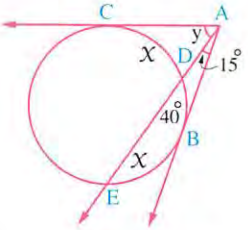
Using the givens in the figure , find the value of the symbol used in measurement :

(1)



$X = \text{..........}°$

(2)

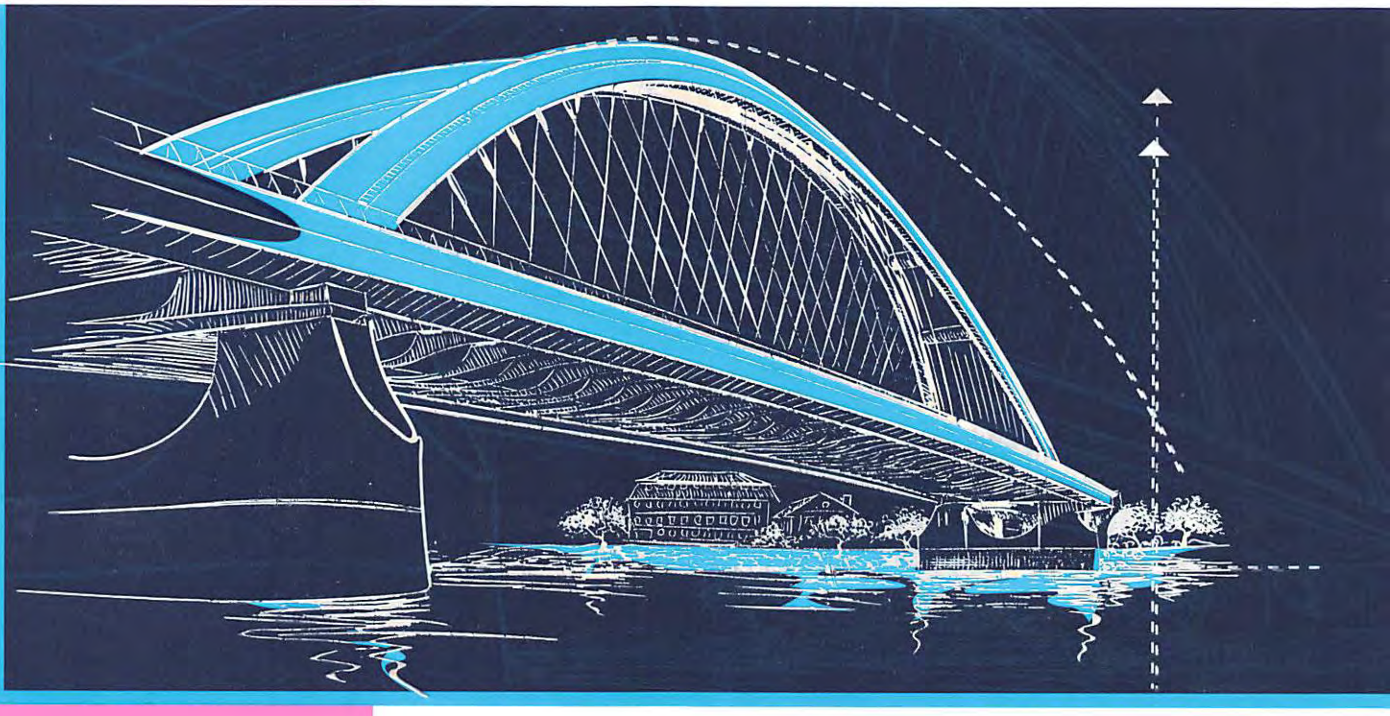


$X = \text{..........}°$, $y = \text{..........}°$

Mathematics

By a group of supervisors

Interactive exams using QR CODE



EXERCISES

FIRST TERM

1 SEC.

2023

EL-MOASSER

CONTENTS

First Algebra and Trigonometry

UNIT 1 Algebra, relations and functions.

UNIT 2 Trigonometry.

Second Geometry

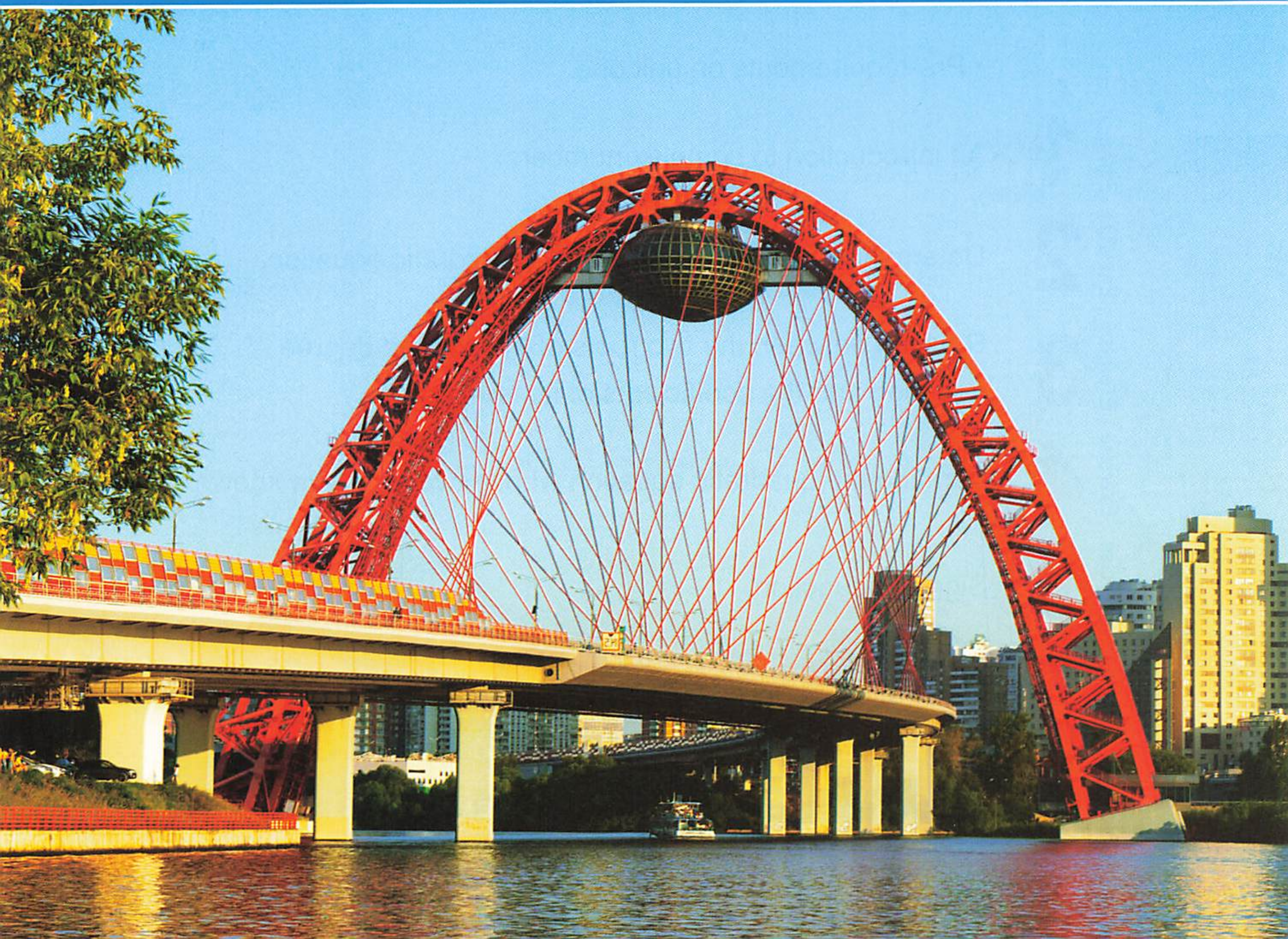
UNIT 3 Similarity.

UNIT 4 The triangle proportionality theorems.

First Algebra and Trigonometry

UNIT 1 Algebra, relations and functions.

UNIT 2 Trigonometry.



Unit One | Algebra, relations and functions.

- Pre-requirements on unit one.

Exercise 1 An introduction to complex numbers.

Exercise 2 Determining the types of roots of a quadratic equation.

Exercise 3 Relation between the two roots of the second degree equation and the coefficients of its terms.

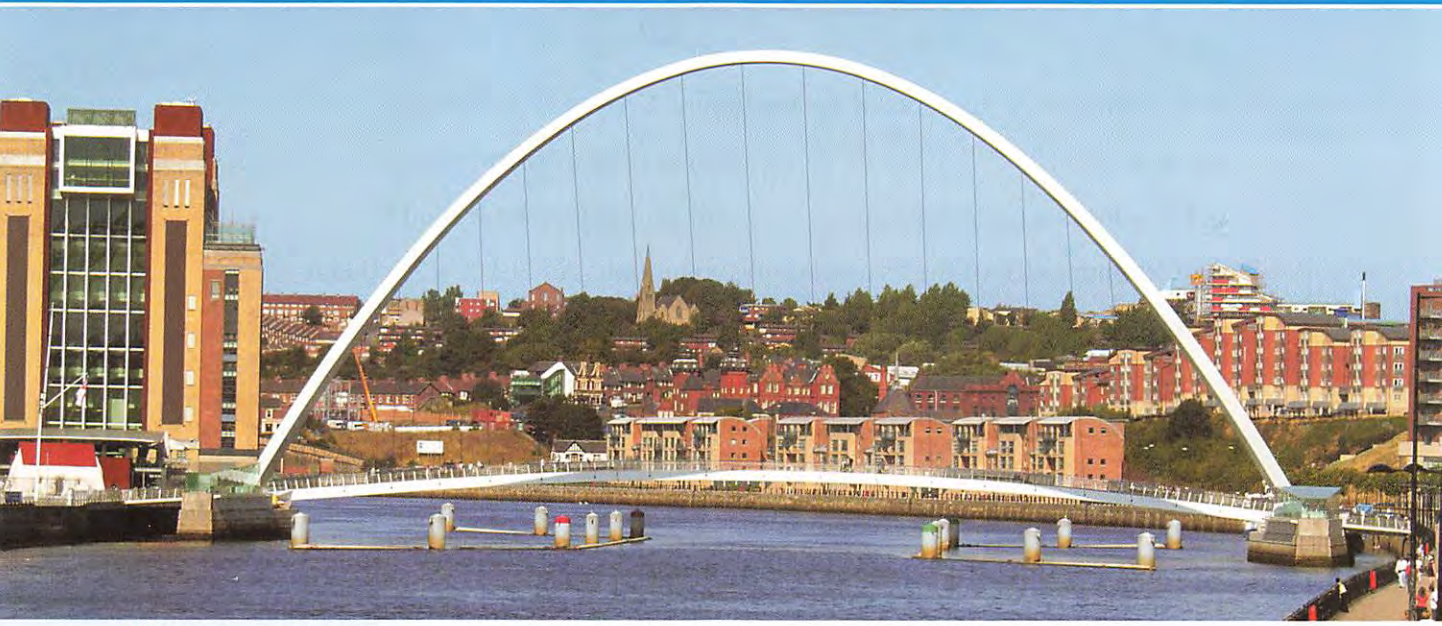
Exercise 4 Forming the quadratic equation whose two roots are known.

Exercise 5 Sign of a function.

Exercise 6 Quadratic inequalities in one variable.

At the end of the unit : Life applications on unit one.

Pre-requirements on unit one



From the school book

First Multiple choice questions

Choose the right answer from those given :

(1) The solution set of the equation : $x^2 - 1 = 0$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) 1 (c) ± 1 (d) $\{1 , -1\}$

(2) The solution set of the equation : $x^2 - 6x + 9 = 0$ in $\mathbb{R}$ is ....

(a) $\{-3\}$ (b) $\{3\}$ (c) $\varnothing$ (d) $\{9\}$

(3) The solution set of the equation : $x^2 - x = 0$ in $\mathbb{R}$ is ....

(a) $\{1 , -1\}$ (b) $\{0\}$ (c) $\{0 , 1\}$ (d) $\varnothing$

(4) The solution set of the equation : $x^2 + 3x = 0$ in $\mathbb{R}^*$ is ....

(a) $\{0 , -3\}$ (b) $\varnothing$ (c) $(0 , 3)$ (d) $\{-3\}$

(5) The number of roots of the equation : $x^2 + 9 = 0$ in $\mathbb{R}$ is ....

(a) 2 (b) 1 (c) 3 (d) zero

(6) The necessary condition which makes the equation $a x^2 + b x + c = 0$ quadratic is ....

(a) $a > 0$ (b) $a < 0$ (c) $a \neq 0$ (d) $a \neq 0 , b \neq 0$

(7) If one of the roots of the equation : $x^2 - 16 = 0$ is 4 , then the other root is ....

(a) -4 (b) 4 (c) 8 (d) zero

(8) If $x = 3$ is a root of the equation : $x^2 + m x = 3$, then m = ...

(a) -1 (b) -2 (c) 2 (d) 1

(9) If $x = -1$ is one of the roots of the equation : $x^2 + k\,x - 6 = 2\,k + 4$, then $k = \cdots\cdots\cdots$

(a) 5 (b) -3 (c) 7 (d) -6

(10) If $x = 4$ is one of the roots of the equation : $x^2 + m\,x = 4$, then $\cdots\cdots\cdots$

(a) $m = -3$ (b) m is an even number

(c) $(1 - m)$ is a perfect square (d) (a) , (c) are both right

(11) The common root of the two quadratic equations : $x^2 - 3\,x + 2 = 0$ and $2\,x^2 - 5\,x + 2 = 0$ is $\cdots\cdots\cdots$

(a) $x = 2$ (b) $x = 1$ (c) $x = -2$ (d) $x = \frac{1}{2}$

(12) If $f(x) = x^2 + b\,x + c$ and $x = 2$ is a root of the equation : $f(x) = 0$, then $f(2) = \cdots\cdots\cdots$

(a) 2 (b) -2 (c) 4 (d) zero

(13) If $(y - 4)^2 = 36$, $y < 0$, then $y + 4 = \cdots\cdots\cdots$

(a) -2 (b) 2 (c) 10 (d) 14

(14) If the curve of the quadratic function f cuts the x-axis at the two points $(2 , 0)$, $(-3 , 0)$, then the solution set of $f(x) = 0$ in $\mathbb{R}$ is $\cdots\cdots\cdots$

(a) $\{2 , 0\}$ (b) $\{-3 , 0\}$ (c) $\{-3 , 2\}$ (d) $\{(2 , -3)\}$

(15) Which of the following statements could be right with respect to the curve of the function $f : f(x) = x\,(a - x)$?

① The curve intersects the x-axis at the two points $(0 , 0)$, $(a , 0)$

② The curve vertex is $\left(\frac{a}{2} , \frac{a^2}{4}\right)$

③ The axis of symmetry of the curve is : $x = a$

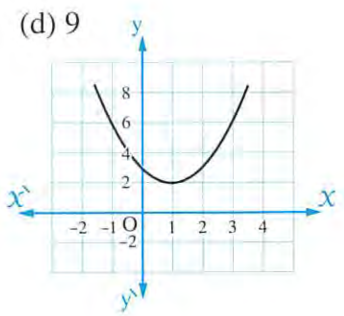
(a) ① , ② only. (b) ① , ③ only.

(c) ② , ③ only. (d) All the previous.

(16) A rectangular piece of land whose dimensions are 6 , 9 metres. It's needed to double its area by increasing each dimension by the same length , then the increased length = $\cdots\cdots\cdots$ m.

(a) 3 (b) 5 (c) 7 (d) 9

(17) If the opposite figure represents the curve of the function f , then the solution set of the equaiton $f(x) = 0$ in $\mathbb{R}$ is $\cdots\cdots\cdots$

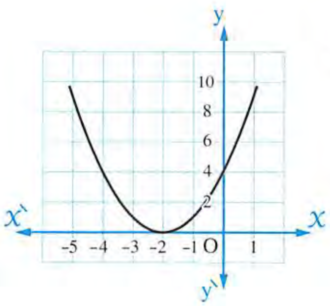


(a) $\{3 , -1\}$ (b) $[2 , 8]$

(c) $\varnothing$ (d) $\{0\}$

(18) In the opposite figure :

The S.S. of the equation $f(\mathcal{X}) = 0$ in $\mathbb{R}$ is ....

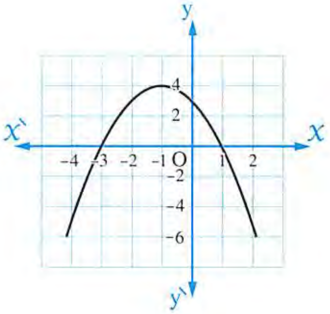


(a) $\{0, -4\}$ (b) $\{(-2, 0)\}$

(c) $\varnothing$ (d) $\{-2\}$

(19) In the opposite figure :

The S.S. of the equation $f(\mathcal{X}) = 0$ in $\mathbb{R}$ is ....

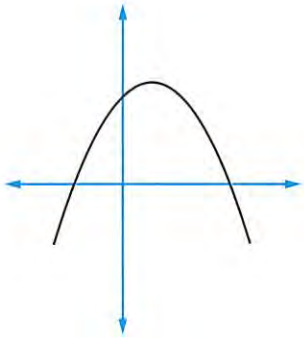


(a) $\{-3, 1\}$ (b) $\{-1, 3\}$

(c) $[-1, 3]$ (d) $[-3, 1]$

(20) The opposite figure represents the curve of the function $f : f(\mathcal{X}) = a\mathcal{X}^2 + b\mathcal{X} + c$

which of the following is true ?



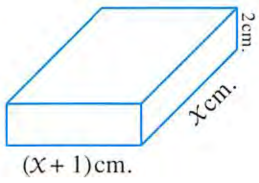
(a) $a > 0, c > 0$ (b) $a > 0, c < 0$

(c) $a < 0, c > 0$ (d) $a < 0, c < 0$

(21) In the opposite figure :

If the volume of the cuboid = 40 cm^3

, then $\mathcal{X}$ = ... cm.



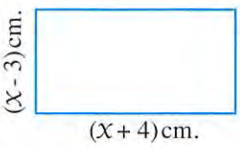
(a) 7 (b) 6

(c) 5 (d) 4

(22) In the opposite figure :

If the area of the rectangle = 78 cm^2

, then the perimeter of the rectangle = ... cm.



(a) 78 (b) 58

(c) 38 (d) 19

Second Essay questions

1 **Find in $\mathbb{R}$ the solution set of each of the following equations by using the general formula approximating the result to the nearest tenth :**

(1) $x^2 - 6x + 1 = 0$

(2) $x^2 + 3x + 5 = 0$

(3) $2x^2 + 3x - 4 = 0$

(4) $3x^2 - 65 = 0$

(5) $x - \frac{5}{x} = 3$

(6) $\frac{3}{x-2} + \frac{2}{x+2} = 2$

---

2 **Find in $\mathbb{R}$ the solution set of each of the following equations algebraically , then check the answer graphically :**

(1) $x^2 - 2x - 4 = 0$ (Hint : draw graphically in the interval $[-2 , 4]$)

(2) $3x - x^2 + 2 = 0$ (Hint : draw graphically in the interval $[-1 , 4]$)

(3) $x^2 + 3 = 0$ (Hint : draw graphically in the interval $[-3 , 3]$)

(4) $-2x^2 - 4x + 1 = 0$

---

3 If the sum of the whole consecutive numbers $(1 + 2 + 3 + ... + n)$ is given by the relation $S = \frac{n}{2}(1 + n)$, how many whole consecutive numbers starting from the number 1 and their sum equals :

(1) 78 (2) 171 (3) 253 (4) 465

---

4 **Find the value of a which makes $x = 2$ is one of the roots of the equation :**

$x^2 - 2ax + 2(a^2 - 6) = 0$ « $1 + \sqrt{5}$ or $1 - \sqrt{5}$ »

---

5 If $f(x) = ax^2 + bx + c$, $f(0) = -3$

, find the value of each of a , b and c if the roots of the equation $f(x) = 0$ are 3 and $-\frac{1}{2}$

« 2 , −5 , −3 »

Exercise **1**

An introduction to complex numbers

Test yourself

From the school book | Remember | Understand | Apply | Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) Which of the following is an imaginary number ?

(a) π (b) $\sqrt{5}$ (c) $\sqrt{-5}$ (d) i^2

(2) $i^{24} =$ ....

(a) -1 (b) i^9 (c) $-i$ (d) 1

(3) The simplest form of the imaginary number i^{45} is ....

(a) i (b) -1 (c) $-i$ (d) 1

(4) $i^{-30} =$ ....

(a) 1 (b) -1 (c) $-i$ (d) i

(5) The simplest form of the experssion $i^{-45} =$ ....

(a) 1 (b) -1 (c) i (d) $-i$

(6) $\frac{1}{i^{199}} =$ ....

(a) 1 (b) $-i$ (c) i (d) -1

(7) $i^{26} + i^{28} =$ ....

(a) i^{54} (b) $-i$ (c) zero (d) 2

(8) $\frac{1}{i^{15}} + i^{21} =$ ....

(a) zero (b) $2i$ (c) $-2i$ (d) $-i$

(9) $5i^7 + 4i^{-1} =$ ....

(a) $9i$ (b) $-9i$ (c) i (d) $-i$

(10) $1 + i + i^2 + i^3 + i^4 =$ ....

(a) $4i + 1$ (b) -1 (c) 1 (d) 5

(11) If $n \in \mathbb{Z}$, then $i^{8n-3} =$ ....

(a) i (b) $-i$ (c) -1 (d) 1

(12) If $n \in \mathbb{Z}$, then $i^{-8n} =$ ....

(a) $\frac{1}{i}$ (b) -1 (c) 1 (d) i

(13) If $n \in \mathbb{Z}$, then $i^{4n+42} =$ ....

(a) 1 (b) -1 (c) $-i$ (d) i

(14) The additive inverse of the complex number $(4 - 7i)$ is ....

(a) $4 + 7i$ (b) $-4 + 7i$ (c) $-4 - 7i$ (d) $4 - 7i$

(15) The conjugate of the number $(3i - 4)$ is ....

(a) $3i + 4$ (b) $-3i - 4$ (c) $-3i + 4$ (d) $3i - 4$

(16) The conjugate of the number $(i - i^2)$ is ....

(a) $1 - i$ (b) $1 + i$ (c) $-i - 1$ (d) $i - 1$

(17) The conjugate of the number (-8) is ....

(a) $8i$ (b) $-8i$ (c) -8 (d) 8

(18) The conjugate of the number $(2 + i)^2$ is ....

(a) $2 + i$ (b) $(2 + i)^{-1}$ (c) $3 + 4i$ (d) $3 - 4i$

(19) $\sqrt{-16} =$ ....

(a) -4 (b) 4 (c) $2i$ (d) $4i$

(20) $\sqrt{2} \times \sqrt{-8} =$ ....

(a) i (b) $-2i$ (c) $4i$ (d) $-4i$

(21) $\sqrt{-18} \times \sqrt{-12} =$ ....

(a) $6\sqrt{6}\,i$ (b) $6\sqrt{6}$ (c) $-6\sqrt{6}$ (d) $-6\sqrt{6}\,i$

(22) $(-4i)(-6i) =$ ....

(a) $-10i$ (b) $24i$ (c) $-24i$ (d) -24

(23) $3i(-2i) =$ ....

(a) $6i$ (b) 6 (c) -6 (d) $-6i$

(24) 📖 $(-2\,i)^3\,(-3\,i)^2 = \cdots\cdots$

(a) $-72\,i$ (b) $72\,i$ (c) 72 (d) -72

(25) 📖 $(3+2\,i)+(2-5\,i) = \cdots\cdots$

(a) $5+2\,i$ (b) $5-3\,i$ (c) $3-5\,i$ (d) $5+3\,i$

(26) If $(2+5\,i)-(4-2\,i) = x+y\,i$, then $x+y = \cdots\cdots$

(a) 9 (b) -1 (c) 1 (d) 5

(27) $(12-5\,i^{17})-\left(7-\sqrt{-81}\right) = \cdots\cdots$

(a) $5-4\,i$ (b) $-5+4\,i$ (c) $5+4\,i$ (d) $-5-4\,i$

(28) $2-(1-2\,i)+(4-5\,i)-(1-3\,i) = \cdots\cdots$

(a) $4\,i$ (b) $-5\,i$ (c) $7\,i$ (d) 4

(29) $(4-3\,i)(4+3\,i) = \cdots\cdots$

(a) $25\,i$ (b) 14 (c) $14\,i$ (d) 25

(30) If $(1+i^4)(1-i^7) = x+y\,i$, then $x+y = \cdots\cdots$

(a) 4 (b) 3 (c) 2 (d) 1

(31) If x , y are real numbers and $x+y\,i = i^{43}+3\sqrt{-4}$, then $x+y = \cdots\cdots$

(a) 3 (b) 5 (c) $3+2\,i$ (d) $5\,i$

(32) If $x+y\,i = (3+2\,i)+(2-i)$, then $(x , y) = \cdots\cdots$

(a) $(1 , 5)$ (b) $(-5 , 1)$ (c) $(1 , -5)$ (d) $(5 , 1)$

(33) If $x+y\,i = (2-3\,i)^2$, then $x+y = \cdots\cdots$

(a) $-5-12\,i$ (b) -17 (c) 17 (d) 60

(34) If $x+y\,i = \frac{1}{i}$ where $x , y \in \mathbb{R}$, then $x+y = \cdots\cdots$

(a) zero (b) 1 (c) -1 (d) 2

(35) If $12+3\,a\,i = 4\,b-27\,i$, then $a+b = \cdots\cdots$

(a) -9 (b) 12 (c) -6 (d) 6

(36) If $3\,x-2\,y\,i = (5-2\,i)^2$, then $y-x = \cdots\cdots$

(a) 17 (b) -3 (c) 3 (d) $21-20\,i$

(37) The solution set of the equation : $x^2+4=0$ in the set of complex numbers is $\cdots\cdots$

(a) $\{2\}$ (b) $\{-2\}$ (c) $\varnothing$ (d) $\{2\,i , -2\,i\}$

(38) The solution set of the equation : $9X^2 + 4 = 0$ in the set of complex numbers is ....

(a) $\left\{\frac{-2}{3}\right\}$ (b) $\left\{\frac{-2}{3}, \frac{2}{3}\right\}$ (c) $\left\{\frac{2}{3}\right\}$ (d) $\left\{\frac{-2}{3}i, \frac{2}{3}i\right\}$

(39) If $X - 2i = 3 + yi$, then the conjugate of the number $X + yi$ is ....

(a) $3 - 2i$ (b) $3 + 2i$ (c) $-3 - 2i$ (d) $-3 + 2i$

(40) If $X^2 - 2X + 2 = 0$, then $X =$ ....

(a) $2 \pm 2i$ (b) $2 \pm i$ (c) $1 \pm i$ (d) $1 \pm 2i$

(41) The multiplicative inverse of the number $\frac{1}{2i+1}$ is ....

(a) $2i - 1$ (b) $-2i + 1$ (c) $2i + 1$ (d) $-2i - 1$

(42) If Z_1 is the conjugate of the number Z_2 , then $Z_1 Z_2 + (Z_1 + Z_2) =$ ....

(a) a real number. (b) an imaginary.

(c) complex , not real. (d) undetermined.

(43) All of the following are imaginary numbers except ....

(a) $\sqrt{-18}$ (b) i^{19} (c) $(2 + 2i)^4$ (d) $(1 + i)^6$

(44) All the following are not real numbers except ....

(a) $(1 + i)^4$ (b) $\sqrt{-8}$ (c) i^3 (d) $\sqrt{-\pi^2}$

(45) $3 + 3i + 3i^2 + 3i^3 =$ ....

(a) zero (b) 3 (c) 12 (d) 12 i

(46) $3 \times 3i \times 3i^2 \times 3i^3 =$ ....

(a) 81 (b) – 81 (c) 81 i (d) – 81 i

(47) $\sqrt{-9} \times \sqrt{\frac{-1}{9}} =$ ....

(a) i (b) – i (c) – 1 (d) 1

Second Essay questions

1 **Find the result of each of the following in the simplest form :**

(1) $(2 + \sqrt{-9})(3 - 4i)$

(2) $(2 - 5i)^2$

(3) $(3 - 2i)^2 + (3 + 2i)$

(4) $(1 + i)^4$

(5) $\left(1 + \sqrt{-1}\right)^4 - \left(1 - \sqrt{-1}\right)^4$

(6) $(1 - i)^{10}$

(7) $(1 + 2i^2)(2 + 3i^5 + 4i^6)$

2 **Put each of the following in the form (a + b i) where a and b are real numbers :**

(1) $\frac{4-5i}{7i}$ (2) $\frac{26}{3-2i}$ (3) $\frac{2-3i}{3+i}$

(4) $\frac{3+4i}{5-2i}$ (5) $\frac{(3+2i)(2-i)}{3+i}$ (6) $\frac{(3+i)(3-i)}{3-4i}$

(7) $\frac{1}{(1+2i)^2}$ (8) $\frac{1+i+2i^2+2i^3}{1-5i+3i^2-3i^3}$ (9) $\frac{2\sqrt{3}+\sqrt{-8}}{\sqrt{3}-\sqrt{-18}}$

3 **Solve each of the following equations in the set of complex numbers :**

(1) $3x^2+12=0$ (2) $4x^2+100=75$

(3) $x^2-4x+5=0$ (4) $2x^2+6x+5=0$

4 **Find the values of x and y that satisfy each of the following equations :**

(1) $(2x-3)+(3y+1)i=7+10i$ (2) $(2x-y)+(x-2y)i=5+i$

(3) $3x+xi-2y+yi=5$ (4) $x^2-y^2+(x+y)i=4i$

(5) $\frac{10}{2+i}=x+yi$ (6) $\frac{6-4i}{1-i}=x+yi$

(7) $\frac{(2+i)(2-i)}{3+4i}=x+yi$

5 If $x=\frac{13}{5-i}$, $y=\frac{3+2i}{1+i}$ **, prove that :** x and y are two conjugate numbers.

6 If $a+bi=\frac{2+i}{2-i}$ **, prove that :** $a^2+b^2=1$

Discover the error

7 **Find the simplest form of the expression :** $(2+3i)^2(2-3i)$

Ahmed's answer

$(2+3i)(2+3i)(2-3i)$
$=(2+3i)(4-9i^2)$
$=(2+3i)(4+9)$
$=13(2+3i)$
$=26+39i$

Karim's answer

$(2+3i)^2(2-3i)$
$=(4+9i^2)(2-3i)$
$=(4-9)(2-3i)$
$=-5(2-3i)$
$=-10+15i$

Which of the two answers is correct ? Why ?

Third Higher skills

1 Choose the correct answer from those given :

(1) If L , M are the roots of a quadratic equations : $x^2 + 1 = 0$, then $L^{2018} + M^{2018} =$ ....

(a) $-2i$ (b) $2i$ (c) -2 (d) 2018

(2) $(1 + i)^{2020} =$ ....

(a) $(1 - i)^{2020}$ (b) 2^{1010} (c) $2^{1010}\,i$ (d) i^{2020}

(3) If $\left(\frac{1-i}{1+i}\right)^{100} = x + y\,i$, then $(x , y) =$ ....

(a) (0 , 1) (b) (– 1 , 0) (c) (0 , – 1) (d) (1 , 0)

(4) The conjugate of the number $(2 + i)^{-1}$ is ....

(a) $2 + i$ (b) $2 - i$ (c) $\frac{2-i}{5}$ (d) $\frac{2+i}{5}$

(5) Which of the following considering factorization of the expression : $x^2 + 4$?

(a) $(x - 2)(x + 2)$ (b) $(x + 2)^2$

(c) $(x - 2i)^2$ (d) $(x - 2i)(x + 2i)$

(6) To find the real value of each of x , y , it is sufficient to have ....

(a) $(x + 2) + 4yi = 3 - 4i$ only. (b) $(2x + y) + 5i = 7 + 5i$ only.

(c) (a) , (b) together. (d) nothing of the previous.

(7) The smallest positive integer (n) which makes

$\left(\frac{1+i}{1-i}\right)^n = 1$ is ....

(a) 2 (b) 4

(c) 8 (d) 12

(8) If a , b , c , d are four positive consecutive integers , then $i^a + i^b + i^c + i^d =$ ....

(a) zero (b) – 1 (c) 1 (d) i

(9) $i + i^2 + i^3 + i^4 + \cdots + i^{100} =$ ....

(a) i (b) –1 (c) zero (d) $i^{1+2+3+\cdots}$

(10) $(1 + i)(1 + i^2)(1 + i^3)(1 + i^4) \ldots (1 + i^{100}) =$ ....

(a) 2 (b) 1 (c) zero (d) Nothing of the previous.

(11) If $i^m = i^n$, then which of the following is always correct ?

① $m = n$

② $(m + n)$ is an even number

③ $(n - m)$ is multiple of 4

(a) ① only. (b) ① , ③ only.

(c) ② , ③ only. (d) All the previous.

(12) If $a < b < 0 < c$ where a , b , c are real numbers and $\sqrt{b(c-a)} + \sqrt{ab} = 2 + 3i$, then $bc = \cdots\cdots\cdots$

(a) 3 (b) – 3 (c) 2 (d) – 5

(13) Which of the following is true ?

(a) $2 + 3i < 3 + 4i$ (b) $3 - 4i < 2 - 3i$

(c) $1 + i > -1 - i$ (d) Nothing of the previous.

---

2 If $7i = (x + 3i)(y - i) - 9$, find the values of the two real numbers x and y which satisfy the previous equation.

---

3 If $x = \frac{2+i}{2-i}$, $y = \frac{2+3i}{2+i}$ and $2x - y = a + bi$, **prove that :** $9a^2 + b^2 = 1$

Exercise 2

Determining the types of roots of a quadratic equation

Test yourself

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) The two roots of the equation : $x^2 - 5x + 11 = 0$ are ....

(a) two complex and non real roots. (b) two rational roots.

(c) two different real roots. (d) two equal real roots.

(2) The two roots of the equation : $x^2 - 11x + 10 = 0$ are ....

(a) two complex and non real roots. (b) two different real roots.

(c) two equal real roots. (d) Two conjugate complex numbers.

(3) The two roots of the equation : $49x^2 - 14x + 1 = 0$ are ....

(a) two different real roots. (b) two equal real roots.

(c) two complex and non real roots. (d) two non conjugate complex numbers.

(4) The two roots of the equation : $6x^2 = 19x - 15$ are ....

(a) two non real roots. (b) two equal real roots.

(c) two different rational numbers. (d) two conjugate imaginary numbers.

(5) The two roots of the equation : $x(x - 2) = 5$ are ....

(a) two complex and non real roots. (b) two equal real roots.

(c) two different real roots. (d) 2 and zero.

(6) The two roots of the equation : $X + \frac{9}{X} = 6$ are ....

(a) two equal real roots. (b) two complex and non real roots.

(c) two different real roots. (d) two equal imaginary numbers.

(7) Number of values of real X which satisfy the equation : $2X^2 - 7X = 5$ is ....

(a) zero (b) 1 (c) 2 (d) 3

(8) The discriminant of the equation : $(X + 2)^2 + 5 = 0$ is ....

(a) perfect square. (b) more than zero.

(c) negative number. (d) irrational number.

(9) In the quadratic equation : $b X^2 + a X = c$ the discriminant is ....

(a) $b^2 - 4ac$ (b) $a^2 + 4bc$ (c) $b^2 + 4ac$ (d) $c^2 - 4ab$

(10) The quadratic equation : $a^2 X^2 + 2abX + b^2 = 0$ where $a, b \in \mathbb{R}$ ....

(a) has two different real roots. (b) has two equal real roots.

(c) hasn't any real roots.

(d) Can't determine the type of its two roots because we don't know the value of a and b

(11) The two roots of the equation : $c X^2 + a X + b = 0$ are two complex and non real roots if ....

(a) $b^2 - 4ac < 0$ (b) $a^2 - 4bc < 0$

(c) $c^2 - 4ab < 0$ (d) $b^2 - 4ac > 0$

(12) If the two roots of the equation : $a X^2 + b = 0$ are two different real roots , then ....

(a) $ab > 0$ (b) $a = 0$ (c) $a > 0 , b > 0$ (d) $ab < 0$

(13) If $a X^2 + b X + c = 0$ and $ac < 0$, then the two roots of the equation are ....

(a) equal real. (b) different real.

(c) conjugate complex. (d) rational.

(14) If $a X^2 + b X + c = 0$ is a quadratic equation , then which of the following inequalities does satisfy that the equation has two real roots ?

(a) $b^2 + 4ac \geq 0$ (b) $b^2 - 4ac < 0$

(c) $b^2 \geq 5ac$ (d) $b^2 - 4ac \leq 0$

(15) If $a X^2 + b X + c = 0$ where a , b , c are rational numbers and $b^2 - 4ac = 25$, then the two roots of the equation are ....

(a) equal real. (b) complex and non real.

(c) conjugate complex. (d) different rational.

(16) If the two roots of the equation : $X^2 - k X + 25 = 0$ are equal real roots , then k = ...

(a) 10 (b) – 10 (c) ± 10 (d) – 5

(17) If the two roots of the quadratic equation : $k X^2 - 2 k X + 3 = 0$ are equal real roots , then k = ...

(a) zero or 3 (b) ± 1 (c) zero only. (d) 3 only.

(18) If the two roots of the equation : $3 X^2 - 6 X + k = 0$ are equal real roots , then k = ...

(a) 2 (b) 3 (c) 6 (d) 9

(19) If the discriminant of the quedratic equation : $2 X^2 + 5 X + 4 k = 0$ equal zero , then k ....

(a) ± 14 (b) zero (c) $\pm \frac{25}{32}$ (d) $\frac{25}{32}$

(20) If the roots of the equation : $X^2 + 3 X - m = 0$ are different real roots , then one of the values of m which satisfy the equation : is m = ...

(a) – 2 (b) – 3 (c) – 4 (d) – 5

(21) If the two roots of the equation : $X^2 - 4 X + k = 0$ are real , then k ∈ ....

(a) $[4 , \infty[$ (b) $]-\infty , 4[$ (c) $]4 , \infty[$ (d) $]-\infty , 4]$

(22) If the roots of the equation : $X^2 + 4 X + k = 0$ are different real , then ....

(a) k = 0 (b) k < 4 (c) k ≤ 0 (d) k ≤ 4

(23) If the roots of the equation : $k X^2 - 8 X + 16 = 0$ are two complex and non real , then ....

(a) k > 2 (b) k < 2 (c) $k \in]1 , 10[$ (d) k > 1

(24) In the equation : $75 X^2 + 7 k X + 3 = 0$ if k ≥ 5 , then the two roots of the equation ....

(a) equal real. (b) complex and non real.

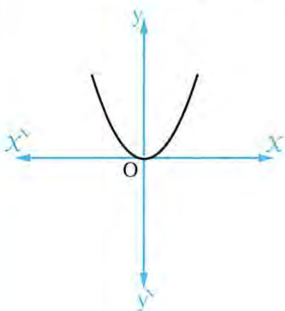
(c) different rational. (d) different real.

(25) If the graph of the quadratic function $f : f(X)$ does not intersect the X-axis , then which of the following can be the rule of the function ?

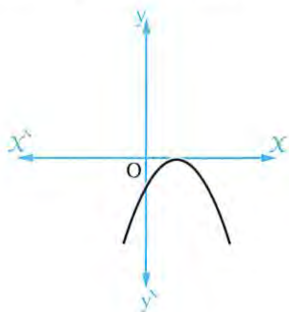
(a) $2X^2 + 3X - 5$ (b) $-X^2 + 5X + 1$

(c) $4X^2 - 20X + 25$ (d) $3X^2 - X + 2$

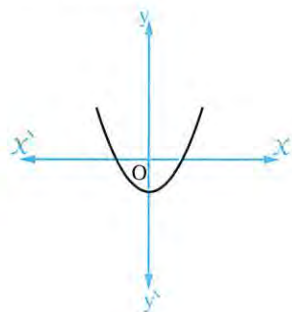
(26) In the quadratic equation $f(X) = 0$, if the discriminant is negative , then which of the following graphs is the graph of the function $f(X)$?



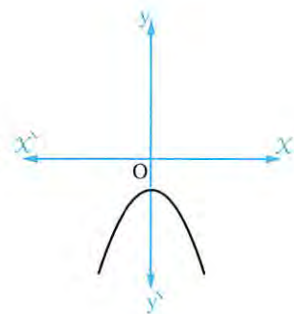
(a)



(b)

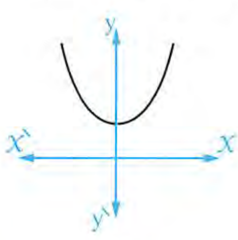


(c)

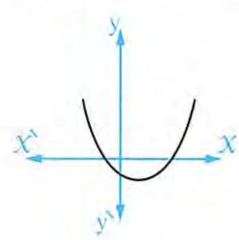


(d)

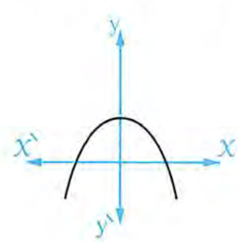
(27) Each of the following figures represents the curve of the function f :
$f(X) = a X^2 + b X + c$ which of these figures does have $b^2 - 4ac = 0$



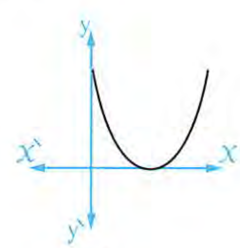
(a)



(b)



(c)



(d)

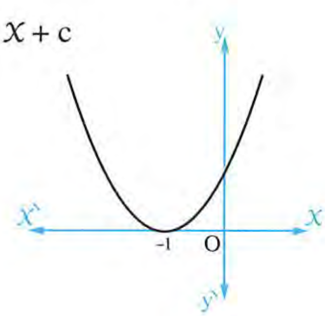
(28) If the curve of the quadratic equation $f : f(X) = X^2 - 2(m - 2)X + m^2 - 8$ touches the X-axis , then m ....

(a) 2 (b) 3 (c) 4 (d) 5

(29) The given figure represents the function $f : f(X) = a X^2 + b X + c$
, then $(b^2 - 4ac) \times f(3) =$ ....

(a) 3 (b) -1

(c) -3 (d) zero



(30) The curve of the quadratic function $f : f(X) = -aX^2 + bX + c$ is drawn on the cartesian coordinate and the vertex of the curve is $(3, 1)$, the curve intersects the X-axis twice where a, b, c are constants which of the following could be a value of c

(a) -8 (b) 2 (c) 3 (d) 7

(31) The roots of the equation : $X^2 = k - 2$ has distinct imaginary roots, then ....

(a) $k > 2$ (b) $k < 2$ (c) $k \geq 2$ (d) $k \leq 2$

(32) If the roots of the equation : $X^2 + kX + k^2 = 0$ are complex and not real, then $k \in$ ....

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R}$ (c) $]0, \infty[$ (d) $]-\infty, 0[$

(33) Which of the following equations does have two complex non real roots ?

(a) $-5X^2 + 9X - 2 = 0$ (b) $-5X^2 + 9X + 2 = 0$

(c) $-5X^2 + 2X - 9 = 0$ (d) $-5X^2 + 2X + 9 = 0$

(34) For the equation : $X^2 - 3X + k = 0$ two unequal roots if $k \neq$ ....

(a) 9 (b) 3 (c) $\frac{9}{4}$ (d) -3

(35) The equation : $X^2 - (2m - 1)X + m^2 = 0$ has no real roots if $m \in$ ....

(a) $]\frac{1}{4}, \infty[$ (b) $]-\infty, \frac{1}{4}[$ (c) $]4, \infty[$ (d) $]-\infty, 4[$

(36) The roots of the equation : $X^2 + k = 0$, where $k > 0$ are ....

(a) conjugate complex and not real. (b) distinct real.

(c) equal and real. (d) rational.

(37) The equation : $(X - 3)^2 + (X - 4)^2 = 0$ has ....

(a) two unequal real roots. (b) two equal real roots.

(c) two rational roots. (d) two non real complex roots.

(38) The two roots of the equation : $(a^2 + 1)X^2 - 2a^3X + a^4 = 0$ where $a \in \mathbb{R} - \{0\}$ are ....

(a) distinct and real. (b) complex and not real.

(c) equal and real. (d) distinct rational.

(39) If a and b are real numbers, $a \neq b$, then the roots of the equation : $(a - b)X^2 - 5(a + b)X - 2(a - b) = 0$ are ....

(a) real equal. (b) complex not real.

(c) unequal real. (d) nothing of the previous.

(40) The number of real distinct roots of the equation : $x(x-a)=a^2$ in $\mathbb{R}$ where $a \in \mathbb{R}-\{0\}$ equals ....

(a) 1 (b) 2 (c) 3 (d) zero

(41) a , b , c are rational numbers , then the equation : $a x^2 + b x + c = 0$ has rational roots if $b^2 - 4ac =$ ....

(a) positive real number. (b) negative real number.

(c) perfect square real number. (d) zero.

(42) To calculate the value of k in the equation : $x^2 + 6x + 2k + 1 = 0$ it is sufficient to knowthat ....

(a) its roots are equal only. (b) k < zero only.

(c) both (a) and (b) (d) nothing of the previous.

(43) If the two roots of the equation : $a x^2 + b x + c = 0$ are ℓ , ℓ where $\ell \in \mathbb{R}$ then ....

(a) $a = c$ (b) $c = \ell$ (c) $b = 0$ (d) $\frac{b^2}{4ac} = 1$

Second Essay questions

1 Determine the type of the two roots of each of the following equations :

(1) $x^2 - 2x + 5 = 0$

(2) $x^2 - 10x + 25 = 0$

(3) $-x^2 + 5x - 30 = 0$

(4) $(x-11) - x(x-6) = 0$

(5) $x - \frac{2}{x-1} = 4$

(6) $\frac{x}{x+1} + \frac{x}{x-1} = 3$

(7) $(x-1)(x-7) = 2(x-3)(x-4)$

---

2 Prove that : The two roots of the equation : $2x^2 - 3x + 2 = 0$ are complex and not real , then use the general formula to find those two roots.

---

3 If the two roots of each of the following quadratic equations are equal , then find the value of k :

(1) $x^2 - 3x + 2 + \frac{1}{k} = 0$ « 4 »

(2) $x^2 + (2k+3)x + k^2 = 0$ « $-\frac{3}{4}$ »

(3) $x^2 + 2(k-1)x + (2k+1) = 0$, then find the two roots. « 0 , 1 , 1 or 4 , – 3 , – 3 »

(4) $x^2 - 2kx + 7k - 6x + 9 = 0$, then find the two roots. « 0 , 3 , 3 or 1 , 4 , 4 »

4 **Find the values of the real number m that make the equation :**

$(m-1)\,x^2 - 2\,m\,x + m = 0$ has no real roots. « $m \in\,]-\infty , 0[$ »

5 **Without solving any of the following equations , show which of them has two rational roots and which of them doesn't have rational roots , then check your answer by solving the equation :**

(1) $2\,x^2 - 3\,x - 2 = 0$

(2) $x^2 + \sqrt{5}\,x - 5 = 0$

(3) $2\,(x+3) + x\,(x-1) = 9$

6 **If a and b are rational numbers , prove that the two roots of the equation :**

$a\,x^2 + b\,x + b - a = 0$ are rational.

7 **If L and M are two rational numbers , then prove that the two roots of the equation :**

$L\,x^2 + (L-M)\,x - M = 0$ are rational numbers.

8 **Prove that the two roots of the equation :**

$x^2 + k\,x + k = 1$ are always rational where $k \in \mathbb{Q}$

9 **If a and b are two rational numbers , prove that the two roots of the equation :**

$x^2 - 2\,a^3\,x + a^6 - b^6 = 0$ are rational numbers.

10 **Find the interval to which a belongs that makes the two roots of the equation :**

$(a+2)\,x^2 + (2\,a+3)\,x + a - 1 = 0$ real numbers. « $a \in \left[-\frac{17}{8} , \infty\right[$ »

11 **Prove that for all the real values of a except zero the equation :**

$(a^2+1)\,x^2 - 2\,a^3\,x + a^4 = 0$ has no real roots.

12 **Prove that for all real values of a and b , the roots of the equation :**

$(x-a)\,(x-b) = 5$ are real.

13 **Prove that for all real values of a except (a = 2) the equation :**

$(a-1)\,x^2 - a\,x + 1 = 0$ has two real and different roots.

Third Higher skills

1 Choose the correct answer from those given :

(1) The two roots of the equation $x^2 - 2\sqrt{5}\,x + 1 = 0$ are ....

(a) real and rational. (b) not real.

(c) real and equal. (d) real and irrational.

(2) If $a x^2 + b x + c = 0$, $a \in \mathbb{R}^*$, $b \in \mathbb{R}$, $c \in \mathbb{R}$ and $(b^2 - 4ac)$ is non-positive , then the two roots of the equation are ....

(a) equal. (b) not real.

(c) complex and conjugate to each other. (d) real and different.

(3) If a , b , c are real numbers , $a + b + c = 0$, $a \neq c$, then the two roots of the equation $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are ....

(a) real and equal. (b) real different and rational.

(c) real different and irrational. (d) not real.

(4) In which of the following quadratic equations the roots are conjugate complex ?

(a) $x^2 - 4x - 5 = 0$ (b) $\sqrt{3}\,x^2 + \sqrt{5}\,x - 1 = 0$

(c) $x^2 - 3\sqrt{2}\,x + 4 = 0$ (d) $3x^2 - \sqrt{7}\,x + 5 = 0$

(5) If the roots of the equation $x^2 - 2\sqrt{2}\,x + a = 0$ are conjugate complex , then $a \in$ ....

(a) $[-2 , 2]$ (b) $]-\infty , 2]$

(c) $]2 , \infty[$ (d) $[2 , \infty[$

---

2 If a , b and c are real numbers , then prove that the two roots of the equation :

$x^2 + 2ax + a^2 = b^2 + c^2$ are real.

---

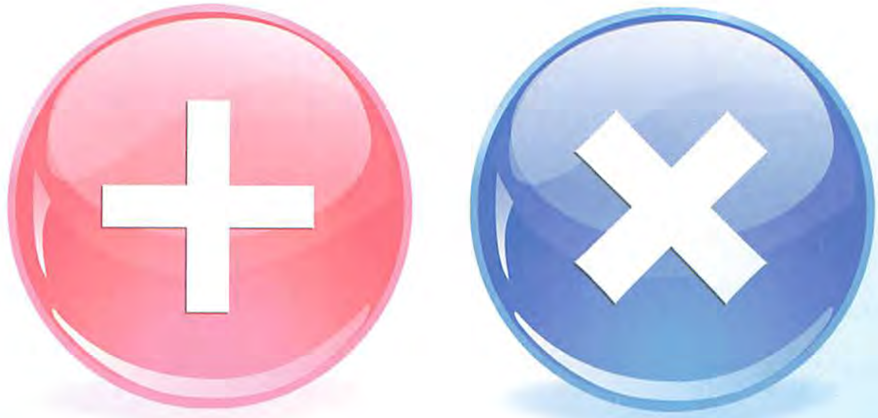
3 Prove that the two roots of the equation :

$\frac{1}{x + a} = \frac{1}{x} + \frac{1}{a}$ are always not real if $a \in \mathbb{R}^*$, $x \notin \{0 , -a\}$

Exercise **3**

Relation between the two roots of the second degree equation and the coefficients of its terms

Test yourself



From the school book · Remember · Understand · Apply · Higher Order Thinking Skills

First — Multiple choice questions

Choose the correct answer from those given :

(1) The sum of the two roots of the equation : $x^2 + 3x - 10 = 0$ is ....

(a) 10 (b) -10 (c) 3 (d) -3

(2) The sum of the two roots of the equation : $4x^2 + 4x - 35 = 0$ is ....

(a) -1 (b) -4 (c) 1 (d) $\frac{-35}{4}$

(3) The sum of the two roots of the equation : $5x^2 - 3 = 0$ is ....

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) zero (d) $\frac{5}{3}$

(4) The product of the two roots of the equation : $x^2 - 5x + 6 = 0$ is ....

(a) -6 (b) 5 (c) -5 (d) 6

(5) The product of the two roots of the equation : $2x^2 - 7x - 6 = 0$ equal ....

(a) -6 (b) $\frac{7}{2}$ (c) 3 (d) -3

(6) The product of the two roots of the equation : $3 + 2x - \frac{1}{4}x^2 = 0$ equals ....

(a) $\frac{-2}{3}$ (b) 12 (c) -12 (d) $\frac{3}{4}$

(7) The product of the two roots of the equation : $b x^2 + c x + a = 0$ equals ....

(a) $\frac{-c}{a}$ (b) $\frac{a}{b}$ (c) $\frac{-c}{b}$ (d) $\frac{a}{c}$

(8) The product of the two roots of the equation : $3x^2 - 4 = 0$ multiplying by the sum of the two roots of the equation $x^2 - 3x = 0$ is ....

(a) 12 (b) -3 (c) -4 (d) 3

(9) If the product of the two roots of the equation : $(k - 2) x^2 - 6x + 12 = 0$ is 3 , then $k = \cdots\cdots$

(a) zero (b) 4 (c) 6 (d) 38

(10) If M , (5 – M) are the two roots of the equation : $x^2 - kx + 6 = 0$, then $k = \cdots\cdots$

(a) – 5 (b) 5 (c) 6 (d) – 8

(11) In the quadratic equation : $a x^2 - b x + c = 0$, if the sum of the two roots equal the product of them , then $b = \cdots\cdots$

(a) – a (b) a (c) – c (d) c

(12) If $x = -1$ is one of the two roots of the equation : $x^2 - kx - 6 = 0$, then the sum of the two roots $= \cdots\cdots$

(a) – 5 (b) 6 (c) – 6 (d) 5

(13) If (2 + i) is one of the roots of the equation : $x^2 - 4x + c = 0$, then $c = \cdots\cdots$

(a) 16 (b) – 16 (c) – 5 (d) 5

(14) If L , M are the two roots of the equation : $x^2 - (k + 2) x - 3 = 0$ and $L + M = 0$, then $k = \cdots\cdots$

(a) – 2 (b) – 3 (c) 2 (d) 3

(15) If M , $\frac{2}{M}$ are the roots of the equation : $a x^2 + b x + 12 = 0$, then $a = \cdots\cdots$

(a) 3 (b) 5 (c) 6 (d) 9

(16) If (L + 1) , (M + 1) are the two roots of the equation : $x^2 - 3x + 2 = 0$ and $L < M$, then $L = \cdots\cdots$

(a) zero (b) 1 (c) 2 (d) 3

(17) If L , M are the two roots of the equation : $x^2 + x + 1 = 0$, then $L + M + LM = \cdots\cdots$

(a) zero (b) 1 (c) – 1 (d) 2

(18) If L , M are the two roots of the equation : $x^2 - 21x + 4 = 0$, then : $\sqrt{L} + \sqrt{M} = \cdots\cdots$

(a) 25 (b) 5 (c) – 5 (d) ± 5

(19) If the two roots of the equation : $x^2 + bx + c = 0$ are L and L , then $b^2 + 4c = \cdots\cdots$

(a) 0 (b) $4L^2$ (c) 8 L (d) $8L^2$

(20) The product of the roots of the equations : $a x^2 + b x + c = 0$, $b x^2 + c x + a = 0$ and $c x^2 + a x + b = 0$ equal $\cdots\cdots$

(a) a b c (b) – 1 (c) 1 (d) zero

(21) If L , L^2 are the two roots of the equation : $2x^2 + bx + 54 = 0$, then $b = \cdots\cdots$

(a) – 12 (b) – 24 (c) 6 (d) 9

(22) If one of the roots of the equation : $X^2 - 5X + n = 0$ more than the other root by 1 , then n = ...

(a) 2 (b) 2 or 3 (c) 6 (d) 8

(23) If one of the roots of the equation : $X^2 - 3X + c = 0$ is twice the other root , then c = ...

(a) – 4 (b) – 2 (c) 2 (d) 4

(24) If one of the two roots of the equation : $X^2 + kX - 98 = 0$ is twice the additive inverse of the other root , then k = ...

(a) ± 14 (b) ± 7 (c) ± 8 (d) 49

(25) If one of the roots of the equation : $3X^2 + (a + 3)X + 7 = 0$ is the additive inverse of the other root , then a = ...

(a) – 3 (b) 3 (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

(26) If one of the two roots of the equation : $X^2 - (b - 3)X + 5 = 0$ is the additive inverse of the other root , then b = ...

(a) – 5 (b) – 3 (c) 3 (d) 5

(27) If one of the two roots of the equation : $X^2 - (b^2 - 2b + 1)X - 9 = 0$ is additive inverse of the other , then b = ...

(a) zero (b) 3 (c) 1 (d) – 1

(28) If one of the roots of the equation : $(2X + k)^2 - 12X = 0$ is the additive inverse of the other root , then k = ...

(a) 3 (b) 2 (c) $\frac{1}{2}$ (d) 12

(29) If one of the two roots of the equation : $aX^2 - 3X + 2 = 0$ is the multiplicative inverse of the other , then a = ...

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 2 (d) 3

(30) If one of the two roots of the equation : $2kX^2 + 7X + 1 + k^2 = 0$ is the multiplicative inverse of the other root , then k = ...

(a) 1 (b) ± 1 (c) – 1 (d) 2

(31) If one of the two roots of the equation : $2kX^2 + 3X + k^2 + 2k - 1 = 0$ is the multiplicative inverse of the other root , then k = ...

(a) ± 1 (b) – 1 (c) 2 (d) – 2

(32) If one of the two roots of the equation : $(k - 3)X^2 - 5X + 2k = 8$ is the multiplicative inverse of the other root , then the value of k = ...

(a) 5 (b) 3 (c) – 5 (d) – 3

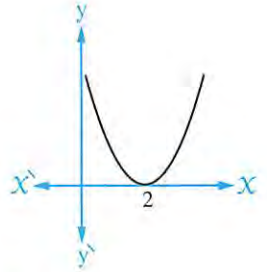
(33) If one of the roots of the equation : $3x^2 - (k + 2)x + k^2 + 2k = 0$ is the multiplicative inverse of the other , then k = ...

(a) – 3 or 1 (b) – 3 or – 1 (c) 3 or – 1 (d) 3 or 1

(34) The opposite figure represents the curve of the function f :

$f(x) = a x^2 + b x + c$

, then b + c = ...

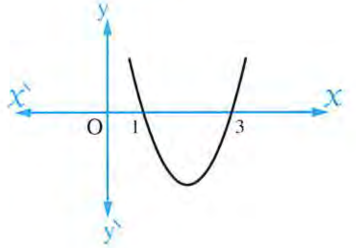


(a) zero (b) 2

(c) 4 (d) 8

(35) The opposite figure represents the curve of the function $f : f(x) = x^2 + k x + n$

, then k + n = ...

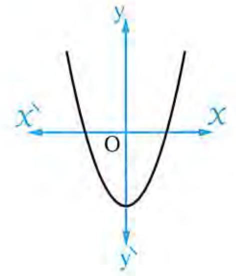


(a) 1 (b) – 1

(c) 7 (d) – 7

(36) The opposite figure represents the curve of the function $f : f(x) = a x^2 + b x + c$

, then which of the following is true ?

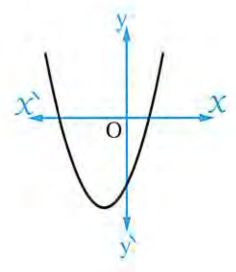


(a) $a > 0$, $c > 0$ (b) $a > 0$, $c < 0$

(c) $a < 0$, $b > 0$ (d) $a < 0$, $c < 0$

(37) The opposite figure represents the curve of the quadratic function $f : f(x) = a x^2 + b x + c$

, then ....



(a) $a c > 0$

(b) $a c < 0$

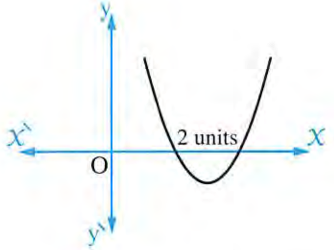
(c) $a c = 0$

(d) a c is an imaginary number.

(38) The opposite figure represents the curve of the function f :

$f(x) = x^2 - 8x + k + 1$

, then k = ...



(a) – 14 (b) 14

(c) 8 (d) – 8

(39) If $X = -3$ is one of the two roots of the equation : $2X^2 + kX - 3 = 0$, then the other root equals ....

(a) 2 (b) $\frac{-3}{2}$ (c) $\frac{1}{2}$ (d) 4

(40) If $X = 3$ is one of the two roots of the equation : $2X^2 - 5X + k = 0$, then the other root equals ....

(a) 3 (b) $-\frac{1}{2}$ (c) $\frac{-5}{2}$ (d) -3

(41) If $X = 2$, $X = -3$ are the two roots of the equation : $2X^2 + aX + b = 0$, then $a + b =$ ....

(a) -6 (b) -1 (c) -10 (d) 12

(42) If one of the roots of the equation : $aX^2 + bX + c = 0$ is one , then the other root equals ....

(a) $\frac{a}{c}$ (b) $\frac{c}{a}$ (c) $\frac{-b}{a}$ (d) $\frac{-a}{b}$

(43) If the roots of the equation : $aX^2 + bX + c = 0$ are h , 1 then ....

(a) $a = h$ (b) $b = ah + 1$ (c) $h + 1 = \frac{-b}{a}$ (d) $h + 1 = \frac{b}{a}$

(44) The roots sum of the equation : $(X - a)(X - b) = c$ is ....

(a) $a + b$ (b) $-(a + b)$ (c) $a + b + c$ (d) $a + b - c$

(45) The products of the two roots of the equation : $\frac{X}{a} + \frac{b}{X} = c$ is ....

(a) $\frac{c}{a}$ (b) ac (c) ab (d) bc

(46) If the sum of the two roots of the equation : $2X^2 + bX - 5 = 0$ is $\frac{-3}{2}$, then $b =$ ....

(a) $\frac{3}{2}$ (b) $\frac{-3}{2}$ (c) 3 (d) -3

(47) If the product of the two roots of the equation : $3X^2 + 8X + c = 0$ equals $\frac{4}{3}$, then $c =$ ....

(a) 4 (b) -4 (c) $\frac{4}{3}$ (d) $\frac{-4}{3}$

(48) If $2 - i$ is one of the roots of the equation : $X^2 + bX + c = 0$, $b , c \in \mathbb{R}$, then $(b , c) =$ ....

(a) $(4 , 5)$ (b) $(-4 , 5)$ (c) $(4 , -5)$ (d) $(-4 , -5)$

(49) If the two roots of the equation : $a X^2 + b X + c = 0$ are $(m - n - 1)$, $(n - m + 2)$, then ....

(a) $\frac{c}{a} = 1$ (b) $\frac{b}{a} = 1$ (c) $\frac{c}{a} = -1$ (d) $\frac{b}{a} = -1$

(50) If one of the two roots of the equation : $(a - b) X^2 + (b - c) X + (c - a) = 0$ is additive inverse of the other , then $\frac{c - a}{a - b} =$ ....

(a) 1 (b) -1 (c) zero (d) 2

Second Essay questions

1 **Without solving the equation , find the sum and the product of the two roots of each of the following equations :**

(1) $3 X^2 = 23 X - 30$

(2) $(4 X + 1)(X + 6) = (X - 2)(3 X - 4)$

(3) $\frac{X}{2} + \frac{1}{X} = \frac{3}{2}$

(4) $\frac{3 X + 2}{X + 2} = \frac{X + 1}{X - 1}$

(5) $(a - 1) X^2 + X - a^2 X - 1 + a = 0$

(6) $(a + b) X^2 + (a^2 - b^2) X + a^2 + 2 ab + b^2 = 0$

---

2 If the product of the two roots of the equation : $3 X^2 + 10 X - c = 0$ is $\frac{-8}{3}$, find the value of c , then solve the equation in the set of complex numbers. « $c = 8$, $X = \frac{2}{3}$ or $X = -4$ »

---

3 If the sum of the two roots of the equation : $2 X^2 + b X - 5 = 0$ is $\frac{-3}{2}$, find the value of b , then solve the equation in the set of complex numbers. « $b = 3$, $X = \frac{-5}{2}$ or $X = 1$ »

---

4 **Find the other root of the equation , then find the value of a in each of the following where $a \in \mathbb{R}$:**

(1) If $X = -1$ is one of the two roots of the equation : $X^2 - 2 X + a = 0$ « 3 , -3 »

(2) If $X = \frac{1}{2}$ is one of the two roots of the equation : $2 X^2 - a X + 3 = 0$ « 3 , 7 »

(3) If $(1 + i)$ is one of the two roots of the equation : $X^2 - 2 X + a = 0$ « $1 - i$, 2 »

(4) If $(2 + i)$ is one of the two roots of the equation : $X^2 + a X + 5 = 0$ « $2 - i$, -4 »

---

5 **Find the values of a , b in each of the following equations , if :**

(1) 2 , 5 are the two roots of the equation : $X^2 + a X + b = 0$ « $a = -7$, $b = 10$ »

(2) -3 , 7 are the two roots of the equation : $a X^2 - b X - 21 = 0$ « $a = 1$, $b = 4$ »

(3) -1 , $\frac{3}{2}$ are the two roots of the equation : $a X^2 - X + b = 0$ « $a = 2$, $b = -3$ »

(4) $\sqrt{3}\, i$, $-\sqrt{3}\, i$ are the two roots of the equation : $X^2 + a X + b = 0$ « $a = 0$, $b = 3$ »

6 **Find the value of k in each of the following which makes :**

(1) One of the roots of the equation : $x^2 + (k - 1)x - 3 = 0$ is the additive inverse of the other roots. « 1 »

(2) One of the roots of the equation : $(k - 2)x^2 + (k - 3)x - 4 = 0$ is the multiplicative inverse of the other root. « – 2 »

(3) One of the roots of the equation : $4kx^2 + 7x + k^2 + 4 = 0$ is the multiplicative inverse of the other. « 2 »

(4) One of the roots of the equation : $2x^2 + k^2 = 5x + 2$ is the multiplicative inverse of the other root. « ± 2 »

---

7 Find the value of a which makes one of the two roots of the equation : $x^2 - ax + 21 = 0$ exceeds double the other root by one. « – 9.5 or 10 »

---

8 **In the equation $(a - 2)x^2 + (a - 3)x - 4 = 0$, find the value of a if :**

(1) The sum of its roots equals 3

(2) The product of its roots equals – 4 « $\frac{9}{4}$, 3 »

---

9 **In the equation $(k - 4)x^2 - (3 - k)x - 3 = 0$, find the value of k if :**

(1) The sum of its two roots equals 5

(2) The product of its two roots equals – 3

(3) One of its two roots equals the additive inverse of the other root.

(4) One of its two roots equals the multiplicative inverse of the other root. « $\frac{23}{6}$, 5 , 3 , 1 »

---

10 Find the value of k which makes one of the two roots of the equation :

$2x^2 - (k - 1)x + (k^2 + 2k - 3) = 0$ double the other root. « – 3.5 or 1 »

---

11 Find the value of a which makes one of the two roots of the equation :

$x^2 - ax + 2a - 4 = 0$ four times the other root. « 10 or $2\frac{1}{2}$ »

---

12 If the sum of the two roots of the equation : $(a - 2)x^2 - ax + b^2 = 0$ equals 3 and the product of the roots is 5 , find the value of each of a , b « 3 , $\pm\sqrt{5}$ »

13 Find the value of c which makes one of the two roots of the equation : $x^2 - 6x + c = 0$ equals the square of the other root. « – 27 or 8 »

14 If one of the two roots of the equation : $8x^2 - 30x + c = 0$ equals the square of the other root , find the value of c « 27 or – 125 »

15 Find the value of a which makes one of the two roots of the equation : $4x^2 - ax - 3 = 0$ exceeds the additive inverse of the other root by 1 « 4 »

16 Find the value of a which makes one of the two roots of the equation : $2x^2 - ax + 3 = 0$ exceeds the multiplicative inverse of the other root by 1 « 7 »

17 Find the value of c , if one of the two roots of the equation : $x^2 - 10x + c = 0$ is less by 2 than the square of the other root. « – 56 or 21 »

18 If the ratio between the two roots of the equation : $ax^2 + bx + c = 0$ as the ratio 2 : 3 , **prove that :** $25\,ac = 6\,b^2$

19 If the two roots of the equation : $8x^2 - bx + 3 = 0$ are positive and the ratio between them is 2 : 3 , find the value of b « 10 »

20 If the sum of the two roots of the equation : $(a + 1)x^2 + (3a - 1)x + a^2 + 1 = 0$ equals the product of its roots , find the value of a « 0 or – 3 »

21 **Find the satisfying condition such that one of the two roots of the equation $ax^2 + bx + c = 0$:**

(1) Is double the other root.

(2) Exceeds the other root by 3 « $9\,ac = 2\,b^2$, $4\,ac = b^2 - 9\,a^2$ »

22 **Find the value of a which makes the sum of the two roots of the equation :**

$x^2 - (a + 4)x + 3a^2 = 0$ equals the product of the two roots of the equation :

$2x^2 - 7ax + a^2 = 0$ « 4 or – 2 »

Discover the error

23 If the product of the two roots of the equation : $x^2 + 4x + k = 2$ is 12 , find the value of k

Mona's answer

$\because$ Product of the two roots = 12

$\therefore \frac{k}{1} = 12$

$\therefore k = 12$

Noura's answer

$\because x^2 + 4x + k = 2$

$\therefore x^2 + 4x + k - 2 = 0$

$\because$ Product of the two roots = 12

$\therefore \frac{k-2}{1} = 12 \quad \therefore k - 2 = 12 \quad \therefore k = 14$

Which answer is correct ? Why ?

Third Higher skills

1 **Choose the correct answer from those given :**

(1) If (2 i) is one root of the equation : $x^2 + a x + b = 0$

where coefficients of its terms are real numbers , then all of the following are true except ....

(a) the other root is (– 2 i)   (b) sum of the two roots = zero

(c) product of the two roots = – 4   (d) discriminant of the equation < 0

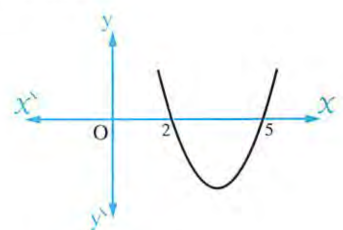
(2) To evaluate the real values of b , c in the equation : $x^2 + b x + c = 0$, it is sufficient to have ....

(a) real roots sum = 6 only.   (b) one of the roots = (3 + i) only.

(c) (a) , (b) together.   (d) nothing of the previous.

(3) If the opposite figure represents the curve of the function

$f : f(x) = a x^2 + b x + c$, then $\frac{b+c}{a} =$ ....



(a) 3   (b) 5

(c) 7   (d) 10

(4) If x_1 , x_2 are the roots of the equation : $a x^2 + b x + c = 0$ and $x_1 < 0 < x_2$, $|x_1| > |x_2|$, which of the following statements could be true ?

(a) $a < 0$ (b) $b c > 0$ (c) $b c < 0$ (d) $x_1 + x_2 > 0$

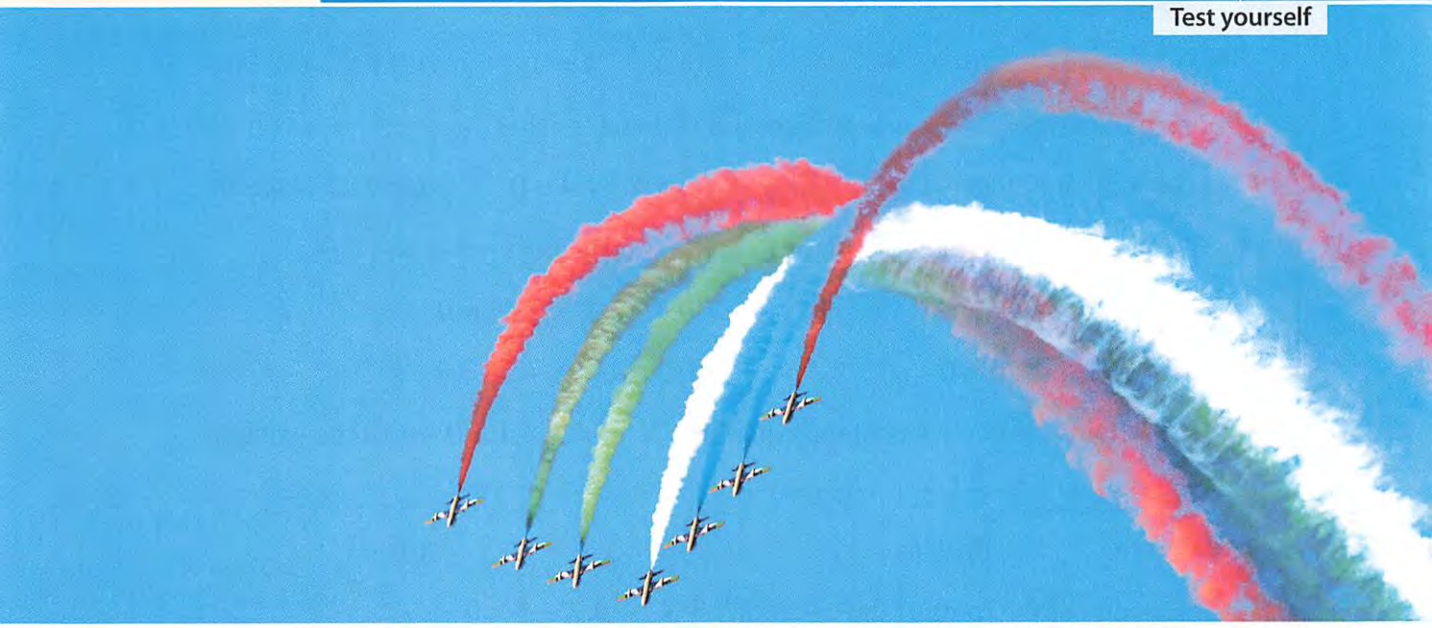
2 **Find the value of a which makes the two roots of the equation :**

$3 x^2 - (2 a - 1) x + (a - 4) = 0$ are different in sign.   « $a \in]-\infty , 4[$ »

Exercise **4**

Forming the quadratic equation whose two roots are known

Test yourself



From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) The quadratic equation whose roots sum equals – 1 and their product equals – 3 is ....

(a) $x^2 - x - 3 = 0$ (b) $x^2 + x + 3 = 0$

(c) $x^2 - x + 3 = 0$ (d) $x^2 + x - 3 = 0$

(2) The quadratic equation whose roots are 3 , – 5 is ....

(a) $x^2 + 2x - 15 = 0$ (b) $x^2 - 2x - 15 = 0$

(c) $x^2 - 2x + 15 = 0$ (d) $x^2 + 2x + 15 = 0$

(3) The quadratic equation whose roots are – 2 , 3 is ....

(a) $(x + 2)(x + 3) = 0$ (b) $x^2 - 4x + 6 = 0$

(c) $x^2 - x = 6$ (d) $4x^2 - 2x + 3 = 0$

(4) The quadratic equation whose roots are 8 , 8 is ....

(a) $2x = 16$ (b) $(x + 8)^2 = 0$

(c) $x^2 + 16x - 64 = 0$ (d) $x^2 - 16x + 64 = 0$

(5) If the two roots of a quadratic equation are – 9 and zero , then this equation is ....

(a) $x + 9 = 0$ (b) $(x - 9)(x) = 0$ (c) $x^2 + 9x = 0$ (d) $x^2 + 9x + 9 = 0$

(6) The quadratic equation whose roots are i and – i is ....

(a) $x^2 - 1 = 0$ (b) $(x + 1)^2 = 0$ (c) $x^2 + 1 = 0$ (d) $(x - 1)^2 = 0$

(7) The quadratic equation whose roots are $-2i$ and $2i$ is ....

(a) $x^2 = 4i$ (b) $x^2 + 4 = 0$ (c) $x^2 - 4 = 0$ (d) $i x^2 + 4 = 0$

(8) The quadratic equation whose roots are $\frac{3}{2}$ i and $\frac{3}{2}$ i^3 is ....

(a) $4x^2 - 9 = 0$ (b) $4x^2 + 9 = 0$ (c) $4x^2 - 4 = 0$ (d) $9x^2 + 4 = 0$

(9) The quadratic equation whose roots are $(1 - 5i)$ and $(1 + 5i)$ is ....

(a) $x^2 - 2x + 26 = 0$ (b) $x^2 + 2x - 26 = 0$

(c) $x^2 - 2x - 26 = 0$ (d) $x^2 + 2x + 26 = 0$

(10) If L , M are the two roots of the equation : $x^2 - 4x + 1 = 0$, then the value of expression : $L^2 - 4L + 1 =$ ....

(a) zero (b) -4 (c) 1 (d) -1

(11) If L is one of the roots of the equation : $3x^2 + 4x - 5 = 0$, then $3L^2 + 4L + 5 =$ ....

(a) zero (b) 10 (c) -5 (d) 5

(12) If L is one of the roots of the equation : $x^2 + 4x + 7 = 0$, then $(L + 2)^2 =$ ....

(a) -11 (b) 11 (c) 3 (d) -3

(13) If L , M are the two roots of the equation : $x^2 - 7x + 3 = 0$, then the value of the expression : $L^2 M + L M^2 =$ ....

(a) 7 (b) 3 (c) 10 (d) 21

(14) If L , M are the two roots of the equation : $x^2 - 7x + 3 = 0$, then $L^2 + M^2 =$ ....

(a) 7 (b) 43 (c) 58 (d) 79

(15) If L , M are the two roots of the equation : $x^2 - 8x + c = 0$ and $L^2 + M^2 = 40$, then c = ...

(a) 8 (b) 10 (c) 12 (d) 14

(16) If L , M are the two roots of the equation : $x^2 - 7x + 9 = 0$ where $L > M$, then $L^3 - M^3 =$ ....

(a) 31 (b) 63 (c) $40\sqrt{13}$ (d) $9\sqrt{7}$

(17) If L , M are the two roots of the equation : $x^2 - 5x + 7 = 0$, then $L(M + 1) + M =$ ....

(a) 2 (b) -2 (c) 12 (d) 7

(18) If L , M are the two roots of the equation : $3x^2 - 8x + 2 = 0$, then $\frac{1}{L} + \frac{1}{M} = \cdots\cdots$

(a) $\frac{4}{3}$ (b) 4 (c) $\frac{-4}{3}$ (d) $\frac{2}{3}$

(19) If L , M are the two roots of the equation : $x^2 - 7x + 3 = 0$, then the equation whose two roots are (L + M) and L M is ....

(a) $x^2 - 10x + 21 = 0$ (b) $x^2 + 10x + 21 = 0$

(c) $x^2 - 21x + 10 = 0$ (d) $x^2 - 21x - 10 = 0$

(20) If L , M are the two roots of the equation : $x^2 - 5x + 3 = 0$, then the equation whose two roots are 2 L , 2 M is ....

(a) $2x^2 - 10x + 6 = 0$ (b) $x^2 - 10x + 12 = 0$

(c) $2x^2 - 10x - 6 = 0$ (d) $x^2 + 10x + 12 = 0$

(21) If L , M are the two roots of the equation : $2x^2 - 3x - 6 = 0$, then the equation whose two roots are $\frac{L}{4}$ and $\frac{M}{4}$ is ....

(a) $x^2 - 3x - 6 = 0$ (b) $4x^2 - 6x - 3 = 0$

(c) $16x^2 + 6x - 3 = 0$ (d) $16x^2 - 6x - 3 = 0$

(22) If L , M are the two roots of the equation : $x^2 - 5x + 7 = 0$, then the equation whose two roots are L^2 and M^2 is ....

(a) $x^2 + 11x + 49 = 0$ (b) $x^2 - 11x + 49 = 0$

(c) $x^2 - 49x + 11 = 0$ (d) $x^2 + 11x - 49 = 0$

(23) If L , M are the two roots of the equation : $x^2 + 5x + 6 = 0$, then the equation whose two roots are (L – M) and (M – L) is ....

(a) $x^2 + x + 1 = 0$ (b) $x^2 + 1 = 0$

(c) $x^2 - x + 1 = 0$ (d) $x^2 - 1 = 0$

(24) The quadratic equation in which each of its two roots more than the two roots of the equation : $x^2 - 3x + 2 = 0$ by 2 is ....

(a) $x^2 - 3x + 2 = 0$ (b) $x^2 + 7x + 12 = 0$

(c) $x^2 - 7x + 12 = 0$ (d) $x^2 - 7x - 12 = 0$

(25) If $\frac{2}{L}$, $\frac{2}{M}$ are the roots of the equation : $4x^2 + 3x = 2$, then the equation whose two roots are L and M is ....

(a) $3x^2 - 8x + 3 = 0$ (b) $x^2 - 3x + 8 = 0$

(c) $x^2 - 3x - 8 = 0$ (d) $3x^2 + 8x - 3 = 0$

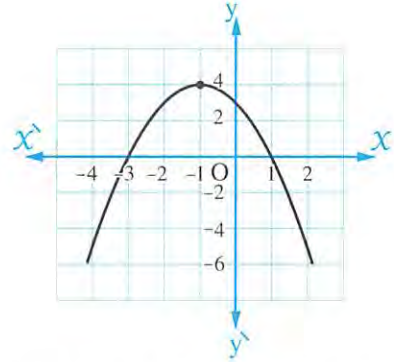
(26) If L , L^2 are the roots of the equation : $2X^2 + bX + 54 = 0$, then $-3L^2 - b = $ ....

(a) -12 (b) -3 (c) -51 (d) ± 3

(27) If L , M are the roots of the equation : $2X^2 + 3X - 1 = 0$, then $4L^2 + 6L = $ ....

(a) 0 (b) 1 (c) 2 (d) 3

(28) If the opposite figure represents a graph of a quadratic function in one variable , then the rule of the function can be written as ....



(a) $f(X) = -X^2 - 2X + 3$

(b) $f(X) = -X^2 + 2X - 3$

(c) $f(X) = X^2 + 2X + 3$

(d) $f(X) = -X^2 + 2X - 3$

(29) The quadratic equation whose terms coefficients are real numbers and one of its roots is $(3 - i)$ is ....

(a) $X^2 - 6X - 10 = 0$ (b) $2X^2 + 6X + 10 = 0$

(c) $X^2 - 6X + 10 = 0$ (d) $X^2 + 6X + 10 = 0$

(30) The quadratic equation whose roots are : $2 - \sqrt{3}$, $2 + \sqrt{3}$ is ....

(a) $X^2 + 2X + 3 = 0$ (b) $X^2 - 4X + 1 = 0$

(c) $X^2 - 4X + 7 = 0$ (d) $X^2 + 4X + 1 = 0$

(31) If L, M are the roots of the equation : $X^2 + 4X + 5 = 0$, then the equation whose roots are $(4L + 5)$ and $(4M + 5)$ is ....

(a) $X^2 + 16X + 25 = 0$ (b) $X^2 + 6X + 25 = 0$

(c) $X^2 - 16X + 25 = 0$ (d) $X^2 - 6X + 25 = 0$

(32) If L , M are the roots of the equation : $X^2 + bX + c = 0$, then the equation whose roots $\frac{1}{L}$, $\frac{1}{M}$ is ....

(a) $X^2 + bX + c = 0$ (b) $X^2 + cX + b = 0$

(c) $cX^2 + bX + 1 = 0$ (d) $cX^2 + X + b = 0$

(33) If L + 1 , M + 1 are roots of the equation : $X^2 + 4X + 2 = 0$, then the quadratic equation whose roots are L , M is ....

(a) $X^2 + 5X + 3 = 0$ (b) $X^2 + 5X + 5 = 0$

(c) $X^2 + 4X + 3 = 0$ (d) $X^2 + 6X + 7 = 0$

(34) The absolute value of the difference between the two roots of the equation :

$x^2 - 4x + 2 = 0$ equals ....

(a) 2 (b) $\sqrt{2}$ (c) 8 (d) $\sqrt{8}$

(35) If L , M are roots of the equation : $x^2 - 4x + 2 = 0$, then the equation whose roots $L^2 - 4L + 7$, $2M^2 - 8M + 9$ is ....

(a) $x^2 - 10x + 25 = 0$ (b) $x^2 - 25 = 0$

(c) $x^2 + 25 = 0$ (d) $x^2 - 7x - 9 = 0$

(36) If L , M are roots of the equation : $x^2 - 4x + 5 = 0$, then the equation whose roots L^2 , $4M - 5$ is ....

(a) $x^2 - 5x + 4 = 0$ (b) $5x^2 - 4x + 1 = 0$

(c) $x^2 - 6x + 25 = 0$ (d) $x^2 + 5x + 4 = 0$

Second Essay questions

1 **Form the quadratic equation whose two roots are :**

(1) -2 , 4 (2) 7 , 7 (3) -7 , 0

(4) $\frac{2}{3}$, $\frac{3}{2}$ (5) $\frac{3}{5}$, $-2\frac{1}{5}$ (6) $5\sqrt{3}$, $-2\sqrt{3}$

(7) $7 + 2\sqrt{5}$, $7 - 2\sqrt{5}$ (8) $-5i$, $5i$ (9) $1 - 3i$, $1 + 3i$

(10) $3 - 2\sqrt{2}\,i$, $3 + 2\sqrt{2}\,i$ (11) $\frac{3}{i}$, $\frac{3+3i}{1-i}$ (12) $\frac{-2+2i}{1+i}$, $\frac{-2-4i}{2-i}$

(13) $a - b$, $a + b$ (14) $\frac{a^2 - b^2}{a - b}$, $\frac{a^3 - b^3}{a^2 + ab + b^2}$

---

2 If L and M are the two roots of the equation : $x^2 - 7x + 5 = 0$,

then find the numerical value of each of the following expressions :

(1) $L^2 M + M^2 L$ (2) $\frac{1}{M} + \frac{1}{L}$

(3) $(L - 2)(M - 2)$ (4) $\left(L + \frac{1}{M}\right)\left(M + \frac{1}{L}\right)$

« 35 , $\frac{7}{5}$, -5 , $7\frac{1}{5}$ »

---

3 If L and M are the two roots of the equation : $x^2 - 4x + 2 = 0$, where $L > M$

, find the numerical value of each of the following expressions :

(1) $L^2 + M^2$ (2) $L - M$ (3) $L^3 + M^3$

(4) $L^2 - 4L + 7$ (5) $2M^2 - 8M + 15$

« 12 , $2\sqrt{2}$, 40 , 5 , 11 »

4 If L and M are the two roots of the equation : $x^2 - 3x - 5 = 0$, then find the equation whose roots are : L – 4 and M – 4 «$x^2 + 5x - 1 = 0$»

---

5 If L and M are the two roots of the equation : $2x^2 - 5x - 7 = 0$, then find the equation whose roots are : 1 – L and 1 – M «$2x^2 + x - 10 = 0$»

---

6 If L and M are the two roots of the equation : $x^2 - 3x - 4 = 0$, then find the equation whose roots are : $\frac{1}{L}$ and $\frac{1}{M}$ «$4x^2 + 3x - 1 = 0$»

---

7 If L and M are the roots of the equation : $2x^2 - 5x + 1 = 0$, then find the equation whose roots are : $2L^2$ and $2M^2$ «$2x^2 - 21x + 2 = 0$»

---

8 Find the quadratic equation in which each of the two roots exceeds one of the two roots of the equation : $x^2 - 7x - 9 = 0$ «$x^2 - 9x - 1 = 0$»

---

9 Form the quadratic equation in which each of its two roots equals half of its corresponding root of the equation : $4x^2 - 12x + 7 = 0$ «$16x^2 - 24x + 7 = 0$»

---

10 Find the quadratic equation in which each of its two roots equals the square of the corresponding root of the equation : $x^2 + 3x - 5 = 0$ «$x^2 - 19x + 25 = 0$»

---

11 If L and M are the two roots of the equation : $2x^2 - 3x - 1 = 0$, then form the quadratic equations whose two roots are : $\frac{L}{M}$, $\frac{M}{L}$ «$2x^2 + 13x + 2 = 0$»

---

12 If L and M are the two roots of the equation : $x^2 - 2x - 4 = 0$, find the equation whose roots are : $\frac{1}{L^2}$ and $\frac{1}{M^2}$ «$16x^2 - 12x + 1 = 0$»

---

13 If L and M are the two roots of the equation : $3x^2 - 5x + 2 = 0$, form the equation whose roots are : $\frac{L^2}{M}$ and $\frac{M^2}{L}$ «$18x^2 - 35x + 12 = 0$»

---

14 If L and M are the two roots of the equation : $10x^2 + 12x - 1 = 0$, form the equation whose roots are : $2L + \frac{1}{M}$, $2M + \frac{1}{L}$ «$5x^2 - 48x - 32 = 0$»

---

15 If L and M are the two roots of the equation : $x^2 - 3x - 5 = 0$, find the equation whose roots are : $L^2 M$ and $M^2 L$ «$x^2 + 15x - 125 = 0$»

---

16 If L and M are the two roots of the equation : $x^2 - 3x + 5 = 0$, find the equation whose roots are : 6 , $L^2 + M^2$ «$x^2 - 5x - 6 = 0$»

17 If L and M are the two roots of the equation : $X^2 - 3X - 1 = 0$, where $L > M$, form the equation whose roots are : $3L - 2M$, $2L - 3M$ « $X^2 - 5\sqrt{13}X + 79 = 0$ »

18 If $L + 2$ and $M + 2$ are the two roots of the equation : $X^2 - 11X + 3 = 0$, find the equation whose roots are : L , M « $X^2 - 7X - 15 = 0$ »

19 If $L + 3$ and $M + 3$ are the two roots of the equation : $X^2 - 5X + 11 = 0$, form the equation whose roots are : $L^2 M$ and $M^2 L$ « $X^2 + 5X + 125 = 0$ »

20 If $\frac{1}{L}$, $\frac{1}{M}$ are the two roots of the equation : $X^2 - 3X + 1 = 0$, form the equation whose roots are : $LM - 7$, $L + M + 3$ « $X^2 - 36 = 0$ »

21 If L and M are the two roots of the equation : $X^2 - 2X - 5 = 0$, form the equation whose roots are : $L^2 + M$, $M^2 + L$ « $X^2 - 16X + 58 = 0$ »

22 If $\frac{3}{L}$ and $\frac{3}{M}$ are the two roots of the equation : $X^2 - 12X + 9 = 0$, form the equation whose roots are : $\frac{1}{L^3}$, $\frac{1}{M^3}$ « $X^2 - 52X + 1 = 0$ »

23 If the difference between the two roots of the equation : $6X^2 - 7X + 1 = c$ is $\frac{11}{6}$, find the value of c « 4 »

24 If the difference between the two roots of the equation : $3X^2 - 2X + c = 0$ equals the difference between the two roots of the equation : $2X^2 - cX + 3 = 0$

, **prove that :** $9c^2 + 48c - 232 = 0$

25 If the difference between the two roots of the equation : $X^2 + kX + 2k = 0$ equals twice the product of the two roots of the equation : $X^2 + 3X + k = 0$, then find the value of k

« 0 or $-\frac{8}{3}$ »

26 If L and M are the two roots of the equation : $4X^2 - 6X + a = 0$ and $L^2 + M^2 = 7LM$, find the value of a « 1 »

27 If L and M are the two roots of the equation : $X^2 - 8X + c = 0$ and $L^2 + M^2 = 40$, find the numerical value of c , then form the equation whose roots are : $L^2 M + M^2 L$, LM

« $c = 12$, $X^2 - 108X + 1152 = 0$ »

28 If L and M are the two roots of the equation : $X^2 - 4X - 5 = 0$, where $L > M$, then form the equation whose roots are : $L - 7$, $2M^2 + 1$ « $X^2 - X - 6 = 0$ »

Discover the error

29 If L + 1 and M + 1 are the roots of the equation : $x^2 + 5x + 3 = 0$, then find the quadratic equation whose roots are : L and M

Yousef's answer

$\because (L + 1) + (M + 1) = -5$

$\therefore L + M + 2 = -5$

$\therefore L + M = -7$

, $\because (L + 1)(M + 1) = 3$

$\therefore LM + (L + M) + 1 = 3$

$\therefore LM - 7 + 1 = 3$

$\therefore LM = 9$

$\therefore$ The equation is : $x^2 + 7x + 9 = 0$

Amira's answer

$\because L + M = -5$

, $LM = 3$

$\therefore (L + 1) + (M + 1)$

$= L + M + 2 = -5 + 2 = -3$

, $\because (L + 1)(M + 1)$

$= LM + (L + M) + 1$

$= 3 - 3 + 1 = 1$

$\therefore$ The equation is : $x^2 + 3x + 1 = 0$

Which of the two answers is correct ? Why ?

Third Higher skills

1 **Choose the correct answer from those given :**

(1) The quadratic equation whose roots are the dimensions of a rectangle of area 15 cm^2 and its perimeter 26 cm. is ....

(a) $x^2 - 26x + 15 = 0$ (b) $x^2 + 26x - 15 = 0$

(c) $x^2 - 13x - 15 = 0$ (d) $x^2 - 13x + 15 = 0$

(2) If $a^2 + 3a + 1 = 0$, $b^2 + 3b + 1 = 0$ where a , b are real different numbers , then $\frac{a}{b} + \frac{b}{a} =$ ....

(a) 2 (b) 7 (c) -5 (d) 11

(3) If L , M are the roots of the quadratic equation : $(x - a)(x - b) = k$, then the quadratic equation whose roots are a and b is ....

(a) $(x - L)(x - M) = 0$ (b) $(x - L)(x - M) + k = 0$

(c) $(x - L)(x - M) = k$ (d) $x^2 - (L + M)x + k = 0$

(4) To form the quadratic equation whose roots 4 L , 4 M where L , M are real numbers it is sufficient to have ....

(a) L + M = 5 only. (b) $(L + M + 4)^2 + (LM - 3)^2 =$ zero only.

(c) (a) , (b) together. (d) nothing of the previous.

(5) Omar and Khaled are trying to solve a quadratic equation Omar miswrite the absolute term of the equation and he got the roots of the equation 3 , 4 , while Khaled miswrite the coefficient of X in the equation so he got the roots of the equation 2 , 3 then the right roots of the equation are ....

(a) 2 , 4 (b) – 2 , – 4 (c) 1 , 6 (d) – 1 , – 6

(6) If the roots of the quadratic equation : $X^2 + b\,X + c = 0$ are two consecutive odd numbers , then $b^2 - 4\,c =$ ....

(a) – 1 (b) 2 (c) 3 (d) 4

(7) If the roots of the quadratic equation : $X^2 - b\,X + c = 0$ are two different integers and b , c are prime numbers which of the following statements could be right ?

① The difference between the equation roots is odd.

② $b^2 - c$ is a prime number ③ $b + c$ is a prime number

(a) ① only (b) ① , ③ only. (c) ② , ③ only. (d) All the previous.

(8) If the curve of the function f where $f(X) = a\,X^2 + b\,X + c$ intersects X-axis at $X = L$, $X = M$ where $|L - M| > 1$, then ....

(a) $f(L+1) > f(L) > f(L-1)$ (b) $f(L-1) > f(L) > f(L+1)$

(c) $f(L) > f(L+1) > f(L-1)$ (d) $f(L+1) \times f(L-1) < 0$

(9) If L , M are the roots of the equation : $X^2 - (\tan\theta)\,X - 1 = 0$ and $L^2 + M^2 = 3$ where $0° < \theta < 90°$, then $\theta =$ ....

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

2 If L and M are the two roots of the equation : $a\,X^2 + 2\,b\,X + c = 0$, $a \neq 0$, $L > M$ and $L - M = 2$, **prove that :**

(1) $b^2 = a\,(a + c)$ (2) $L = 1 - \frac{b}{a}$

3 If the difference between the two roots of the equation : $a\,X^2 + b\,X + c = 0$, where $a \neq 0$ equals twice the sum of their multiplicative inverses

, **prove that :** $c^2\,(b^2 - 4\,ac) = 4\,a^2\,b^2$

Exercise 5

Sign of a function

Test yourself

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) The function $f : f(X) = -4$ is negative in the interval ....

(a) $]-\infty , 4[$ only. (b) $]-4 , 4[$ only.

(c) $]-\infty , \infty[$ (d) $]-2 , 2[$ only.

(2) The function $f : f(X) = 5X - 3$ is positive at ....

(a) $X > \frac{3}{5}$ (b) $X < \frac{3}{5}$ (c) $X > \frac{1}{3}$ (d) $X < \frac{-5}{3}$

(3) If $f(X) = 2X - 4$, then f is negative at $X \in$ ....

(a) $[2 , \infty[$ (b) $]-\infty , 2[$ (c) $]2 , \infty[$ (d) $]-\infty , 2]$

(4) The sign of the function $f : f(X) = 6 - 2X$ is non positive at ....

(a) $X > 3$ (b) $X \leq 3$ (c) $X < 3$ (d) $X \geq 3$

(5) The function $f : f(X) = 3 - \frac{1}{2}X$ is non negative at $X \in$ ....

(a) $]-\infty , 6]$ (b) $]-\infty , 6[$ (c) $[6 , \infty[$ (d) $]6 , \infty[$

(6) If the function $f : f(X) = X + 2$ where $X \in]-4 , 3[$
, then $f(X)$ is positive at $X \in$ ....

(a) $]-\infty , -2[$ (b) $]-2 , \infty[$ (c) $]-4 , -2[$ (d) $]-2 , 3[$

(7) If the function $f : f(X) = X + 3 , X \in]-5 , 6[$
, then $f(X)$ is negative at $X \in$ ....

(a) $]-5 , -3[$ (b) $]-\infty , -3[$ (c) $]-3 , \infty[$ (d) $]-3 , 6[$

(8) The function $f : f(X) = c$ has a sign ………. always.

(a) positive (b) negative

(c) like the sign of X (d) like the sign of c

(9) The sign of the function $f : f(X) = aX + b$ on $\mathbb{R}$ is the same as the sign of b if ……….

(a) $a = b$ (b) $a = 0$ (c) $a > 0$ (d) $a < 0$

(10) The function $f : f(X) = aX^2 + bX + c$ has one sign on $\mathbb{R}$ if ……….

(a) $b^2 - 4ac > 0$ (b) $b^2 - 4ac < 0$

(c) $b^2 - 4ac = 0$ (d) $b^2 - 4ac \geq 0$

(11) If $f(X) = 3X$, then the sign of the function f is negative in the interval ……….

(a) $]-\infty , 3[$ (b) $]3 , \infty[$ (c) $]-\infty , 0[$ (d) $]-3 , \infty[$

(12) The function $f : f(X) = X^2 - 9$ is negative at $X \in$ ……….

(a) $\mathbb{R} - [-3 , 3]$ (b) $]-3 , 3[$ (c) $]-\infty , -9[$ (d) $]-\infty , -3[$

(13) The function $f : f(X) = X^2 + 1$ is positive at $X \in$ ……….

(a) $]0 , \infty[$ only. (b) $]1 , \infty[$ only. (c) $]-\infty , 1[$ only. (d) $\mathbb{R}$

(14) The function $f : f(X) = X^2 - 6X + 9$ is positive in the interval ……….

(a) $]0 , \infty[$ (b) $]-\infty , 3]$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R} - \{0\}$

(15) The interval in which the function $f : f(X) = X^2 - 5X + 6$ is positive is ……….

(a) $[2 , 3]$ (b) $\mathbb{R} - \{2 , 3\}$ (c) $\mathbb{R} - [2 , 3]$ (d) $\mathbb{R} -]2 , 3[$

(16) If $f(X)$ is positive at $X \in]-2 , 5[$, then $f(X) =$ ……….

(a) $X^2 - 3X - 10$ (b) $10 - 3X - X^2$

(c) $X^2 + 3X - 10$ (d) $10 + 3X - X^2$

(17) If $f(X) = X^2 + bX + c$ is negative at $X \in]2 , 3[$, then the product of the two roots of the equation : $X^2 + bX + c = 0$ equal ……….

(a) -6 (b) 6 (c) b (d) $-c$

(18) The sign of the two function $f : f(X) = (X - 1)(X + 2)$ and $g : g(X) = -X^2 + 9$ are both positive at $X \in$ ……….

(a) $]1 , 3[\cup]-3 , -2[$ (b) $]-2 , 0[$

(c) $]3 , \infty[\cup]-\infty , -3[$ (d) $]-3 , 3[$

(19) The sign of the two functions f and g where $f(X) = X - 2$, $g(X) = 4 - X^2$ are both negative in the interval ……….

(a) $]2 , \infty[$ (b) $]-\infty , -2[$ (c) $]-2 , 2[$ (d) $]-\infty , -2]$

(20) If the function $f : f(X) = aX^2 + bX + c$ and $a < 0$ and the two roots of $f(X) = 0$ are $2 , -5$, then the function f is positive in ……….

(a) $\{-5 , 2\}$ (b) $\mathbb{R} -]-5 , 2[$ (c) $]-5 , 2[$ (d) $]-\infty , -5[$

(21) When investigate the sign of the function f its sufficient that you know ....

(a) the curve of the function f is parallel to X-axis only.

(b) the curve of the function f lies completely below X-axis only.

(c) (a) and (b) together.

(d) nothing of the previous.

(22) If $f(X) = aX + b$ and $X = L$ is a root of the equation $f(X) = 0$, then $f(L+1) \times f(L-1) \in$ ....

(a) $\mathbb{R}^+$ (b) $\mathbb{R}^-$ (c) $[-1, 1]$ (d) $[-5, 5]$

(23) Which of the following functions is positive for all values of $X \in \mathbb{R}$?

(a) $f : f(X) = X^2 + 4$

(b) $f : f(X) = 3$

(c) $f : f(X) = (X - 1)^2 + 9$

(d) All the previous.

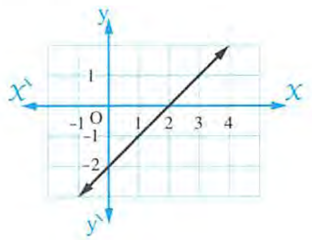
(24) The function $f : f(X) = 12 + 4X - X^2$ is not negative in the interval ....

(a) $]-2, 6[$ (b) $[-2, 6]$ (c) $\mathbb{R} -]-2, 6[$ (d) $]-\infty, \infty[$

(25) The function $f : f(X) = -(X - 1)(X + 2)$ is positive in the interval ....

(a) $]1, 2[$ (b) $[-1, 2]$ (c) $]-2, 1[$ (d) $]-\infty, \infty[$

(26) The opposite figure represents a first degree function of X



First : The function is positive in the interval ....

(a) $[2, \infty[$

(b) $]1, \infty[$

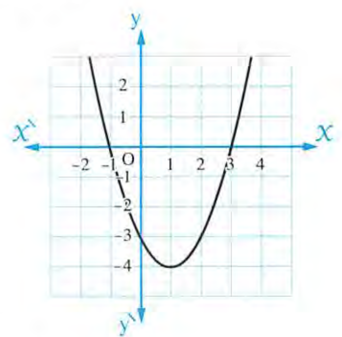
(c) $]-\infty, 2[$

(d) $]2, \infty[$

Second : The function is negative in the interval ....

(a) $]-\infty, 2]$ (b) $]-2, 2]$ (c) $]-\infty, 2[$ (d) $]2, \infty[$

(27) The opposite figure represents a second degree function f of X



First : $f(X) = 0$ at $X \in$ ....

(a) $\mathbb{R}$

(b) $\mathbb{N}$

(c) $[-1, 3]$

(d) $\{3, -1\}$

Second : $f(X) > 0$ at $X \in$ ....

(a) $]-1, 3[$

(b) $[-1, 3]$

(c) $\mathbb{R} - [-1, 3]$

(d) $\mathbb{R}$

Third : $f(X) < 0$ at $X \in$ ....

(a) $]-1, 3[$ (b) $[-1, 3]$ (c) $\mathbb{R} - [-1, 3]$ (d) $\mathbb{R}$

(28) If $f(X) = (X - a)^2$, then $f(a+1) \times f(a-1) \in$ ....

(a) $\mathbb{R}^-$ (b) $\mathbb{R}^+$ (c) $[-1, 1]$ (d) $]-1, 1[$

(29) If the roots of the equation : $f(X) = 0$ are L , M where f is a quadratic function , L > M , then $f(L+1) \times f(M-1) \in$ ....

(a) $]0, \infty[$ (b) $]-\infty, 0[$ (c) $[-1, 1]$ (d) $\{0\}$

(30) If L is a root of the function : $f(X) = 0$ where $f(X) = aX + b$, then $f(L+1) \times f(L+3) \in$ ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\mathbb{R}^-$ (d) $[1, 3]$

(31) If the curve of the function f , where f is a linear function , intersects the X-axis at $(3, 0)$ which of the following statements is always true ?

(a) $f(2) > f(3)$ (b) $f(4) < f(3)$

(c) $f(2) \times f(4) > f(3)$ (d) $f(2) \times f(4) < f(3)$

(32) The sign of function $f : f(X) = (X-3)^2$ is non-negative on ....

(a) $\{3\}$ only. (b) $]3, \infty[$ only. (c) $\mathbb{R}$ (d) $\varnothing$

(33) If $f(X) = aX^2 + bX + c$, $a > 0$ and the roots of the equation $f(X) = 0$ are -2 , 1 , then the function f is non-positive at $X \in$ ....

(a) $\{-2, 1\}$ (b) $]-2, 1[$ (c) $[-2, 1]$ (d) $\mathbb{R} - [-2, 1]$

(34) The function $f : f(X) = a^2 X^2 + c$ where $a \neq 0$, $c > 0$ has a sign .... alawys.

(a) negative (b) positive (c) like the sign of X (d) like the sign of a

(35) The function $f : f(X) = X^2 - 6X + 9$ is negative on ....

(a) $\{3\}$ (b) $\mathbb{R} - \{3\}$ (c) $]3, \infty[$ (d) $\varnothing$

(36) All functions defined by the following rules are positive on $\mathbb{R}$ except ....

(a) $f(X) = 3$ (b) $f(X) = X + 3$

(c) $f(X) = X^2 - 3X + 3$ (d) $f(X) = X^2 + X + 3$

(37) If the minimum value of a quadratic function $y = f(X)$ is 3 , then the function is negative at $X \in$ ....

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\{3\}$ (d) $]3, \infty[$

Second Essay questions

1 **Determine the sign of the functions which are defined by the following rules , then represent your answer on the number line :**

(1) $f(X) = (X-2)(X+3)$

(2) $f(X) = (2X-3)^2$

(3) $f(X) = 2X^2 + 5X - 7$

(4) $f(X) = X^2 - 4X + 3$

(5) $f(X) = X^2 - 8X + 16$

(6) $f(X) = 2X^2 - 3X + 5$

(7) $f(X) = 4X - 7 - X^2$

(8) $f(X) = 9 - 4X^2$

(9) $f(X) = 2X^2$

2 Draw the curve of the function $f : f(X) = 2X^2 - 8$ in $[-2, 2]$

From the graph, determine the sign of f in $\mathbb{R}$

3 Draw the curve of the function $f : f(X) = 2X^2 - 3X + 4$ in $\left[-1, 2\frac{1}{2}\right]$

From the graph, determine the sign of f in $\mathbb{R}$

4 Draw the curve of the function $f : f(X) = -X^2 + 8X - 15$ in $[1, 7]$ From the graph, determine the sign of f in $\mathbb{R}$ and the solution of the equation $f(X) = 0$ « $\{3, 5\}$ »

5 Draw the curve of the function $f : f(X) = X^2 - 9$ in the interval $[-3, 4]$

From the graph, determine the sign of f in that interval.

6 Draw the curve of the function $f : f(X) = -X^2 + 2X + 4$ in $[-3, 5]$

From the graph, determine the sign of f in that interval.

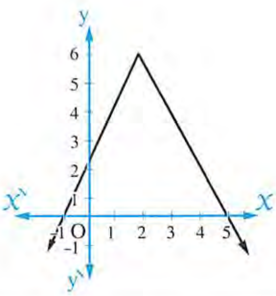
7 **Investigate the sign of each of the following functions :**

(1) $f : [-1, 6] \longrightarrow \mathbb{R}$ where $f(X) = 3 - X$

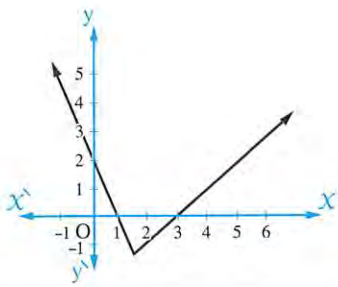
(2) $f : [-2, 8] \longrightarrow \mathbb{R}$ where $f(X) = X^2 - 5X - 6$

8 **Determine the sign of the functions represented by the following figures :**

(1)



(2)



9 Determine the sign of each of the two functions : $f : f(X) = X - 3$, $g : g(X) = X^2 - 5X - 6$ and when the two functions are positive together.

10 If $f_1(X) = X - 3$, $f_2(X) = 5 + 4X - X^2$, determine the sign of each of f_1, f_2 on the number line and determine the intervals at which the two functions are negative together.

11 If $f(X) = X^2 - 5X + 6$ and $g(X) = 2X^2 - 5X - 18$, state the two functions f, g when they are positive together or negative together.

12 Prove that for all the values of $k \in \mathbb{R}$ the two roots of the equation :

$2X^2 - kX + k - 3 = 0$ are real and different.

Discover the error

13 If $f(X) = X + 1$, $g(X) = 1 - X^2$

, determine the interval at which the two functions are positive together.

Yousef's answer

$X = -1$ makes $f(X) = 0$
$f(X)$ is positive in the interval $]-1, \infty[$
, $X = \pm 1$, makes $g(X) = 0$, $g(X)$ is positive in the interval $]-1, 1[$, thus the two functions are positive together in the interval
$]-1, \infty[\cup]-1, 1[=]-1, \infty[$

Amira's answer

$X = -1$ makes $f(X) = 0$
$f(X)$ is positive in the interval $]-1, \infty[$
, $X = \pm 1$, it makes $g(X) = 0$
$g(X)$ is positive in the interval $]-1, 1[$
thus the two functions are positive together in the interval
$]-1, \infty[\cap]-1, 1[=]-1, 1[$

Which of the two answers is correct ? Represent each of the two functions graphically and check the correct answer.

Third Higher skills

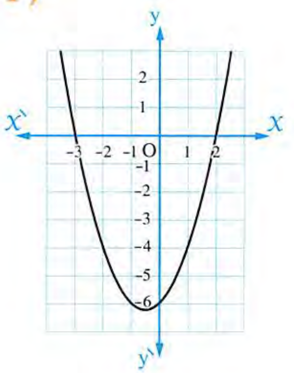
1 **Study the sign of each of the following two functions :**

(1) $f : f(X) = -2X^2 - 2\sqrt{2}X - 1$

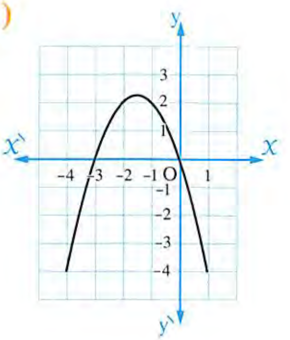
(2) $f : f(X) = X + (X + 1)(2X + 3) - 4(X + 1) + 1$

2 **Each of the following figures shows the graphical representation of a second degree function in one variable. Study the sign of each function in $\mathbb{R}$, then find the rule of each function :**

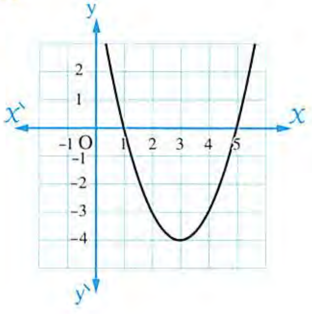
(1)



(2)



(3)



Exercise **6**

Quadratic inequalities in one variable

Test yourself

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) The solution set of the inequality : $(X-2)(X-5)<0$ in $\mathbb{R}$ is ....

(a) $\{2,5\}$ (b) $]2,5[$ (c) $[2,5]$ (d) $\mathbb{R}-[2,5]$

(2) The solution set of the inequality : $X^2+3X-4\geq 0$ in $\mathbb{R}$ is ....

(a) $\{-4,1\}$ (b) $[-4,1]$ (c) $\mathbb{R}-]-4,1[$ (d) $\mathbb{R}-[-4,1]$

(3) The solution set of the inequality : $7+X^2-4X<0$ in $\mathbb{R}$ is ....

(a) $]-4,7[$ (b) $\mathbb{R}-[-4,7]$ (c) $\mathbb{R}$ (d) $\varnothing$

(4) The solution set of the inequality : $2X+X^2+5>0$ in $\mathbb{R}$ is ....

(a) $\mathbb{R}-[-2,3]$ (b) $[-2,3]$ (c) $\mathbb{R}$ (d) $\varnothing$

(5) The solution set of the inequality : $X^2+9>6X$ in $\mathbb{R}$ is ....

(a) $]-3,3[$ (b) $\mathbb{R}$ (c) $\mathbb{R}-[-3,3]$ (d) $\mathbb{R}-\{3\}$

(6) The solution set of the inequality : $4X-X^2-4<0$ in $\mathbb{R}$ is ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\mathbb{R}^-$ (d) $\mathbb{R}-\{2\}$

(7) The S.S. of the inequality $(X-1)^2\leq 0$ in $\mathbb{R}$ is ....

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\{1\}$ (d) $\mathbb{R}-\{1\}$

(8) The solution set of the inequality : $-X(X+2)\geq 0$ in $\mathbb{R}$ is ....

(a) $\{0,-2\}$ (b) $[-2,0]$ (c) $]-2,0[$ (d) $[-2,2]$

(9) The solution set of the inequality : $X(X-1)>0$ in $\mathbb{R}$ is ....

(a) $\{0,1\}$ (b) $]0,1[$ (c) $[0,1]$ (d) $\mathbb{R}-[0,1]$

(10) The solution set of the inequality : $X(X-2)<0$ is ....

(a) $\{0,2\}$ (b) $]-2,2[$ (c) $]0,2[$ (d) $]1,2[$

(11) The solution set of the inequality : $X^2<3X$ is ....

(a) $\mathbb{R}-[0,3]$ (b) $[0,3]$ (c) $]0,3[$ (d) $\mathbb{R}-]0,3[$

(12) The solution set of the inequality : $X^2+49<0$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $[-7,7]$ (d) $\mathbb{R}-[-7,7]$

(13) The solution set of the inequality : $X^2+1\le 0$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $[-1,1]$ (d) $\mathbb{R}-]-1,1[$

(14) The solution set of the inequality : $X^2+9>0$ in $\mathbb{R}$ is ....

(a) $\varnothing$ (b) $\mathbb{R}$ (c) $]-3,3[$ (d) $\mathbb{R}-[-3,3]$

(15) If $f(X)=X^2-6X+9$, then the solution set of the inequality : $f(X)\le 0$ in $\mathbb{R}$ is ....

(a) $\mathbb{R}$ (b) $\{3\}$ (c) $\mathbb{R}-]-3,3[$ (d) $[-3,3]$

(16) The solution set of the inequality : $X^2\le 9$ in $\mathbb{R}^+$ is ....

(a) $[-3,3]$ (b) $\mathbb{R}-]-3,3[$ (c) $]0,3]$ (d) $\varnothing$

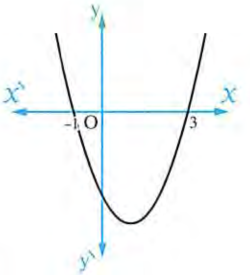
(17) The solution set of the inequality : $X^2>16$ in the interval $[-4,4]$ is ....

(a) $[-4,4]$ (b) $\mathbb{R}-[-4,4]$ (c) $\varnothing$ (d) $\{-4,4\}$

(18) Which of the following answers does not belong to the solution set of the inequality $3X-5\ge 4X-3$?

(a) -1 (b) -2 (c) -3 (d) -5

(19) If the opposite figure represents the function $f:f(X)=X^2-2X-3$, then the solution set of the inequality $X^2-2X-3\ge 0$ in $\mathbb{R}$ is ....



(a) $]-1,3[$ (b) $]-\infty,2[$

(c) $]3,\infty[$ (d) $]-\infty,-1]\cup[3,\infty[$

(20) If the solution set in $\mathbb{R}$ of the inequality : $aX^2+bX+c>0$ is $\mathbb{R}$, then ....

(a) $a,b,c\in\mathbb{R}^+$ (b) a , c have the same sign

(c) $4ac>b^2$ (d) $\sqrt{b^2-4ac}\in\mathbb{R}$

(21) If the solution set of the inequality : $a\,x^2 + b\,x + c > 0$ is $\mathbb{R} - \{d\}$, then which of the following is wrong ?

(a) $b^2 = 4\,a\,c$ (b) $a \in \mathbb{R}^+$ (c) $a\,d^2 + b\,d + c > 0$ (d) $d^2 = \frac{c}{a}$

(22) If the solution set of the inequality : $a\,x^2 + b\,x + c < 0$ is $\mathbb{R} - [L , M]$, then which of the following is wrong ?

(a) The S.S. of the equation $a\,x^2 + b\,x + c = 0$ in $\mathbb{R}$ is $\{L , M\}$

(b) $L + M = \frac{-b}{a}$

(c) $b^2 > 4\,a\,c$

(d) The S.S. of the inequality $a\,x^2 + b\,x + c > 0$ is $[L , M]$

(23) The solution set of the inequality : $(x + 5)(x - 1) \geq (x + 5)$ is ....

(a) $[1 , \infty[$ (b) $[-5 , 2]$ (c) $\mathbb{R} -]-5 , 2[$ (d) $\mathbb{R} -]-5 , 1[$

(24) $]-2 , 4[$ is the solution set of the inequality : ....

(a) $x^2 - 8 > 2\,x$ (b) $x^2 - 2\,x \leq 8$ (c) $8 + 2\,x > x^2$ (d) $x^2 - 2\,x \geq 8$

(25) The number of integers belong to the solution set of the inequality $(2\,x + 1)(x - 2) < 0$ is ....

(a) zero (b) 1 (c) 2 (d) 3

(26) If $5 \leq x \leq 8$, then ....

(a) $(x - 5)(x - 8) \geq 0$ (b) $(x - 5)(x - 8) > 0$

(c) $(x - 5)(x - 8) \leq 0$ (d) $(x - 5)(x - 8) < 0$

(27) If $a , b \in \mathbb{R}^+$, $a < b$, then ....

(a) $\frac{1}{a} > \frac{1}{b}$ (b) $\frac{1}{a} < \frac{1}{b}$

(c) $a^2 > b^2$ (d) nothing of the previous.

(28) The values of x satisfy both : $x^2 - 2\,x - 3 < 0$, $x - 2 < 0$ are ....

(a) $]-1 , 3[$ (b) $]-1 , 2[$ (c) $]2 , 3[$ (d) $[-1 , 3]$

Second Essay questions

1 Find in $\mathbb{R}$ the solution set of each of the following inequalities :

(1) $x^2 + 2\,x - 8 > 0$ (2) $x^2 - 5\,x - 6 < 0$ (3) $x^2 - x - 2 \leq 0$

(4) $4 - 3\,x - x^2 \geq 0$ (5) $5\,x - x^2 - 6 < 0$ (6) $x^2 - 1 \leq 0$

(7) $4 - x^2 < 0$ (8) $x^2 - 4\,x + 4 \geq 0$ (9) $6\,x - x^2 - 9 < 0$

(10) $x^2 - 8\,x + 16 < 0$ (11) $-x^2 - 10\,x - 25 \geq 0$ (12) $2\,x - x^2 < 0$

2 **Find in $\mathbb{R}$ the solution set of each of the following inequalities :**

(1) $x^2 + 5x < -4$

(2) $5x^2 + 12x \geq 44$

(3) $3x^2 \leq 11x + 4$

(4) $x^2 \geq 6x - 9$

(5) $3 - 2x \geq x^2$

(6) $7x + 15 \leq 2x^2$

(7) $x^2 + 5 \leq 1$

(8) $-x^2 - 7 < 2$

(9) $(x - 2)^2 \geq 9$

(10) $(x - 2)^2 \leq -5$

(11) $x(x + 2) - 3 \leq 0$

(12) $(x + 2)^2 + (x + 1)(x - 4) < 0$

(13) $(x + 3)^2 < 10 - 3(x + 3)$

(14) $5 - 2x \leq x^2$

3 Determine the sign of the function $f : f(x) = x^2 - 5x + 6$ and from that find in $\mathbb{R}$ the solution set of the inequality : $f(x) < 0$

4 Determine the sign of the function $f : f(x) = 2x^2 + 7x - 15$ and from that find in $\mathbb{R}$ the solution set of the inequality : $2x^2 + 7x \leq 15$

5 Determine the sign of the function $f : f(x) = x^2 + 4$
, then find in $\mathbb{R}$ the solution set of the inequality : $f(x) \leq$ zero

6 **Draw the graph of the function $f : f(x) = -x^2 + 2x + 3$ in the interval $[-2, 4]$, from the graph find in $\mathbb{R}$:**

(1) The solution set of the equality $f(x) = 0$

(2) The solution set of the inequality $f(x) \leq 0$

(3) The solution set of the inequality $f(x) > 0$

Discover the error

7 **Find in $\mathbb{R}$ the solution set of the inequality :** $(x + 1)^2 < 4(2x - 1)^2$

Yousef's answer

$\because (x + 1)^2 < 4(2x - 1)^2$

$\because x + 1 < 2(2x - 1)$ by taking the square root to both sides

$\therefore -4x + x + 2 + 1 < 0$

$\therefore -3x + 3 < 0$

$\therefore$ The equation related to the inequality is : $-3x + 3 = 0$

Nour's answer

$\because (x + 1)^2 < 4(2x - 1)^2$

$\therefore x^2 + 2x + 1 < 16x^2 - 16x + 4$

$\therefore 15x^2 - 18x + 3 > 0$

$\therefore$ The equation related to the inequality is $3(5x - 1)(x - 1) = 0$

$\therefore$ The solution set $= \left\{1, \frac{1}{5}\right\}$

$\therefore$ The S.S. is $\{1\}$

• By investigating the sign of f where

$f(X) = -3X + 3$

X	$-\infty$		1		∞
$f(X)$		$+ + + + + + +$	0	$- - - - - - - -$	

$\therefore$ The solution set $=]1, \infty[$

• By investigating the sign of f where

$f(X) = 15X^2 - 18X + 3$

X	$-\infty$		$\frac{1}{5}$		1		∞
$f(X)$		$+ + + +$	0	$- - - - - - -$	0	$+ + + +$	

$\therefore$ The solution set $= \mathbb{R} - [\frac{1}{5}, 1]$

Which of the two answers is correct ?

8 **Find in $\mathbb{R}$ the solution set of the inequality : $X^2 - 2X + 1 \geq 0$**

Basem's answer

$\because$ The related equation to the inequality is

$X^2 - 2X + 1 = 0 \quad \therefore (X - 1)^2 = 0$

$\therefore$ The S.S. $= \{1\}$

• Investigating the sign of the function f where $f(X) = X^2 - 2X + 1$

X	$-\infty$		1		∞
$f(X)$		$+ + + + + + + +$	0	$+ + + + + + + +$	

$\therefore$ The solution set $= \mathbb{R} - \{1\}$

Eslam's answer

$\because$ The related equation to the inequality is

$X^2 - 2X + 1 = 0 \quad \therefore (X - 1)^2 = 0$

$\therefore$ The S.S. $= \{1\}$

• Investigating the sign of the function f :

$f(X) = X^2 - 2X + 1$

X	$-\infty$		1		∞
$f(X)$		$+ + + + + + + +$	0	$+ + + + + + + +$	

$\therefore$ The solution set $= \mathbb{R}$

Which of the two answers is correct ? Why ?

Third Higher skills

1 **Choose the correct answer from those given :**

(1) If $f(X) = X^2 - 7X + 12$, $X \in \mathbb{R}$, then all the following are true except ....

(a) solution set of the equation $f(X) = 0$ is $\{3, 4\}$

(b) solution set of the inequality $f(X) > 0$ is $\mathbb{R} - [3, 4]$

(c) solution set of the inequality $f(X) < 0$ is $]3, 4[$

(d) $f(X)$ is positive in the interval $\mathbb{R} -]3, 4[$

(2) The sum of integers belong to the solution set of the inequality $(X - 2)(3X - 1) \leq 0$ ....

(a) -1 (b) 1 (c) 2 (d) 3

(3) The solution set of the inequality $(X + 3)^2 < 4(X + 1)^2$ in $\mathbb{R}$ is ....

(a) $]\frac{-5}{3}, 1[$ (b) $\mathbb{R} -]\frac{-5}{3}, 1[$ (c) $[\frac{-5}{2}, 1]$ (d) $\mathbb{R} - [\frac{-5}{3}, 1]$

(4) If L , M are the roots of the equation : $a x^2 + b x + c = 0$ where $a > 0$, $L < M$, then the solution set of the inequality $a x^2 + b x + c < 0$ in $\mathbb{R}$ is ....

(a) $]-\infty , L[$ (b) $]L , M[$ (c) $]M , \infty[$ (d) $\mathbb{R} - [L , M]$

(5) If the discriminant of the equation : $a x^2 + b x + c = 0$ is negative , then the solution set of the inequality $a x^2 + b x + c < 0$ where $a < 0$ in $\mathbb{R}$ is ....

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{R}^+$ (d) $\mathbb{R}^-$

(6) If L , M are the two roots of the equation : $2 x^2 + (k - 2) x - 5 = 0$ and $-1 < L < M$, then ....

(a) $-1 < k < 0$ (b) $k > 6$ (c) $k < -1$ (d) $-1 < k < 6$

(7) If each one of the two roots of a quadratic equation : $x^2 - 2 k x + k^2 + k - 5 = 0$ is less than 5 , then $k \in$ ....

(a) $[4 , 5]$ (b) $[4 , \infty[$ (c) $]-\infty , 4[$ (d) $\mathbb{R} - [4 , 5]$

(8) If the two roots of the quadratic equation : $x^2 - k x + 1 = 0$ are not real , then ....

(a) $k \in \mathbb{Z}^-$ (b) $-2 < k < 2$ (c) $k > 2$ (d) $k < -2$

(9) If the solution set of the inequality : $x^2 - 4 \le x + k$ is $[-2 , 3]$, then $k =$ ....

(a) -6 (b) 1 (c) 2 (d) 10

(10) If the solution set of the inequality : $x^2 - 10 < b x$ is $]-2 , 5[$, then $b =$ ....

(a) -10 (b) -2 (c) 3 (d) 5

(11) If one of the roots of the equation : $x^2 - b x + 3 = 0$ belongs to the interval $]1 , 2[$, then $b \in$ ....

(a) $]1 , 2[$ (b) $]-\infty , 3[$ (c) $]3\frac{1}{2} , 4[$ (d) $\mathbb{R} -]3\frac{1}{2} , 4[$

(12) If S_1 is the solution set of the inequality : $x^2 - x - 2 \le 0$ and S_2 is the solution set of the inequality : $x^2 + x - 2 \le 0$, then $S_1 \cap S_2 =$ ....

(a) $\varnothing$ (b) $[-2 , 2]$ (c) $[-1 , 1]$ (d) $\mathbb{R} -]-1 , 1[$

(13) If L , M are the roots of the equation : $a x^2 + a x + a + 2 = 0$ and $2 \in]L , M[$, then $a \in$ ....

(a) $[1 , 2]$ (b) $\mathbb{R}^+$ (c) $]\frac{-2}{7} , 0[$ (d) $]\frac{2}{L} , \frac{2}{M}[$

(14) If the two roots of the quadratic equation : $4 x^2 - 2 x + m = 0$ belong to the interval $]-1 , 1[$, then ....

(a) $0 \le m < 2$ (b) $-6 < m < \frac{1}{8}$ (c) $-2 < m \le \frac{1}{4}$ (d) $-6 < m < -2$

2 **Find the S.S. of the inequality :** $10 > x^2 + 2 x - 5 \ge 3$ in $\mathbb{R}$

Life Applications on Unit One

From the school book

1 A missile is launched vertically upwards with speed $u = 24.5$ m./sec. Calculate the time "t" in seconds elapsed such that the missile reaches a height $S = 29.4$ m. , given that the relation between the height "S" and the time "t" is as follows : $S = u\,t - 4.9\,t^2$

« 2 sec. or 3 sec. »

---

2 A diver starts jumping from a platform of height 10 metres above water surface. If the height of the diver above water surface "S" metres is determined by the relation :
$S = -4.9\,t^2 + 3.5\,t + 10$, where "t" is the time in seconds.
After how many seconds the diver will reach the water surface ?

« $\frac{5}{7}$ sec. »

---

3 The dimensions of a rectangular piece of land are 6 and 9 metres , it is required to double its area by increasing each of its dimensions with the same magnitude. Find the additional magnitude.

« 3 metres »

---

4 A golf player strikes the ball to a certain place , the following relation represents the height "y" in feet :
$y = -16\,t^2 + 80\,t + 20$ where "t" is the time by sec.

(1) After how many seconds it will reach the ground surface ?

(2) Does the ball reach a height 130 feet ?

« 5.24 sec. »

---

5 Population of Egypt in 2013 is estimated by the relation : $Z = n^2 + 1.2\,n + 91$, where (n) is the number of years and (Z) is the population in millions :

(1) What is the population in 2013 ?

(2) Estimate the population in 2023

(3) Estimate the number of years at which the population will be 334 million.

« 91 million , 203 million , 15 years i.e. in 2028 »

6 Find the total electric current intensity passing through two resistances connected in parallel in a closed circuit , if the current intensity in the first resistance is $(4 - 2\ i)$ ampere and the second resistance is $\left(\frac{6 + 3\ i}{2 + i}\right)$ ampere (given that the total current intensity equals the sum of the two current intensities which passes through the two resistances).

« $(7 - 2\ i)$ ampere »

7 If the electric current intensity passing in two resistances connected on parallel in a closed circuit equals $6 + 4\ i$ ampere , and the current intensity passing in one of them equals $\frac{17}{4 - i}$, then find the current intensity passing in the other resistance.

« $(2 + 3\ i)$ ampere »

8 The production of a gold mine from 1990 to 2010 estimated in determined ounce was determined by the function $f : f(n) = 12\ n^2 - 96\ n + 480$ where 'n' is the number of years and $f(n)$ is the production of gold.

(1) Investigate the sign of the production function f

(2) Find the production of the gold mine (in thousand ounce) in each of the two years 1990 – 2005

(3) In which year , the production of the gold was 2016 thousand ounce ?

« 480 thousands ounces , 1740 thousands ounces , 2006 »

Unit Two | Trigonometry.

Exercise **7** Directed angle.

Exercise **8** Systems of measuring angle (Degree measure - radian measure).

Exercise **9** Trigonometric functions.

Exercise **10** Related angles.

Exercise **11** Graphing trigonometric functions.

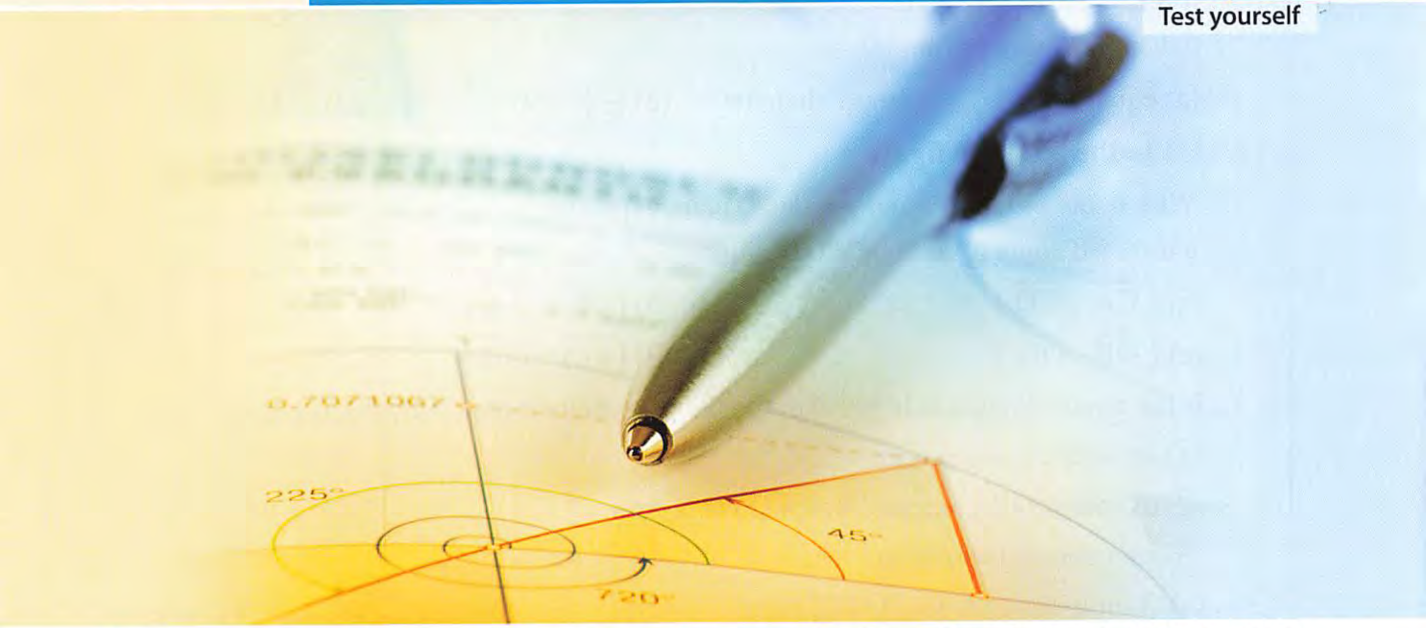
Exercise **12** Finding the measure of an angle given the value of one of its trigonometric ratios.

At the end of the unit : Life applications on unit two.

Exercise 7

Directed angle

Test yourself



From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

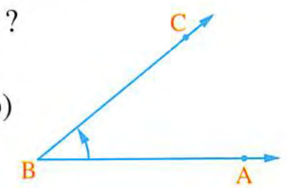
(1) The ordered pair ($\overrightarrow{OB}$, $\overrightarrow{OC}$) represents the directed angle ....

(a) ∠ OBC (b) ∠ BOC (c) ∠ BCO (d) ∠ OCB

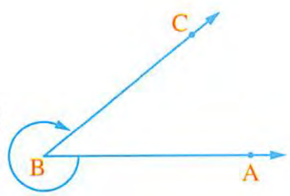
(2) Which of the angles is not the directed ∠ ABC ?

(a) ($\overrightarrow{BA}$, $\overrightarrow{BC}$)

(b)



(c)



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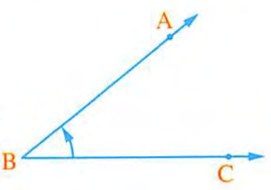


(3) If θ is the smallest positive measure of a directed angle , then its negative measure is ....

(a) – θ (b) θ – 180° (c) θ – 360° (d) 360°– θ

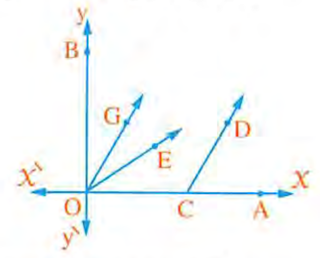
(4) If θ_1 is the positive measure of a directed angle and θ_2 is the negative measure of the same directed angle , then $\theta_1 - \theta_2$ = ...°

(a) zero (b) ± 360 (c) 360 (d) – 360

(5) If θ is the directed angle , then the sum of its positive and negative measure ....°
(where θ is not zero angle)

(a) equal 360° (b) more than 360° (c) $\in]-360°, 360°[$ (d) $\in]0, 360°[$

(6) **In the opposite figure :**



Which one of the following ordered pairs expresses a directed angle in its standard position ?

(a) $(\overrightarrow{CA}, \overrightarrow{CD})$ (b) $(\overrightarrow{OE}, \overrightarrow{OA})$

(c) $(\overrightarrow{OB}, \overrightarrow{OG})$ (d) $(\overrightarrow{OA}, \overrightarrow{OB})$

(7) If the directed angle is in standard position , which of the following is correct ?

① its vertex is the origin.

② its initial side coincides the positive X-axis.

③ its measure is positive.

(a) ① only (b) ① , ② only (c) ① , ③ only (d) All the previous.

(8) It is said that the directed angles in the standard positions are equivalent if they have the same ....

(a) initial side. (b) terminal side.

(c) vertex. (d) rotation direction.

(9) If θ is the directed angle measure in standard position , $n \in \mathbb{Z}$, then the angles whose measures $(\theta \pm n \times 360°)$ are called ....

(a) equivalent. (b) quadrantal. (c) supplementary. (d) adjacent.

(10) If A and B are the measures of two equivalent angles , then – A and – B are ....

(a) supplementary. (b) equivalent. (c) complementary. (d) of sum – 360°

(11) The quadrantal angle measure is multiple of ....

(a) 360° (b) 180° (c) 90° (d) 60°

(12) The angle whose measure is 60° in the standard position is equivalent to the angle of measure ....

(a) 120° (b) 240° (c) 300° (d) 420°

(13) The angle of measure 585° is equivalent to the angle in the standard position of measure ....

(a) 45° (b) 135° (c) 225° (d) 315°

(14) The angle whose measure is 950° is equivalent to the angle in the standard position of measure ....

(a) 130° (b) – 130° (c) 235° (d) – 230°

(15) All the following angles are equivalent to 75° in the standard position except ....

(a) – 285° (b) – 645° (c) 285° (d) 435°

(16) The quadrant in which the angle of measure 1670° lies is the ....

(a) first. (b) second. (c) third. (d) fourth.

(17) The angle whose measure is (– 135°) lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(18) The angle whose measure is (– 850°) lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(19) All the following are measures of angles lying in the second quadrant except ....

(a) – 240° (b) 100° (c) – 120° (d) 860°

(20) The angle of measure $45° + (4n + 1) \times 90°$ lies in the .... quadrant ($n \in \mathbb{Z}$)

(a) first (b) second (c) third (d) fourth

(21) If the terminal side of angle of measure 60° in standard position rotates two and quarter revolutions anticlockwise then the terminal side represents the angle of measure ....

(a) 60° (b) 120° (c) 150° (d) 240°

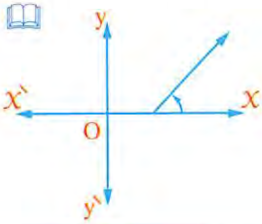
(22) If the terminal side of an angle of measure 30° in standard position rotates three and half revolutions clockwise , then the terminal side will be in the .... quadrant.

(a) first (b) second (c) third (d) fourth

Second Essay questions

1 **Which of the following directed angles is in its standard position ? Explain your answer.**

(1)



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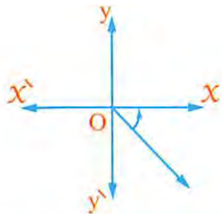

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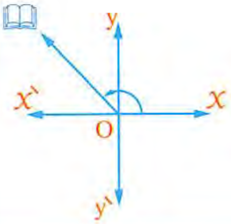

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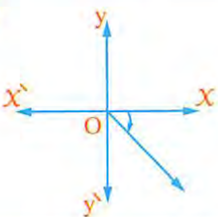

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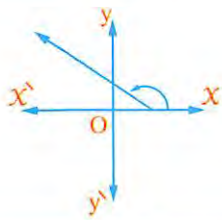

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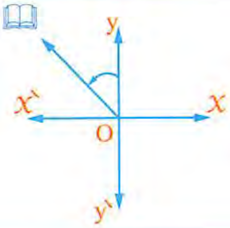

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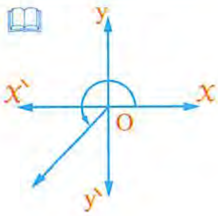

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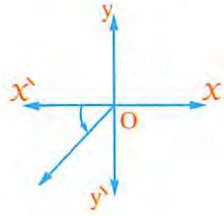

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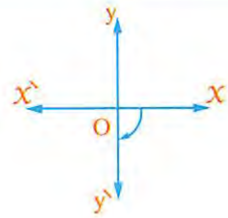
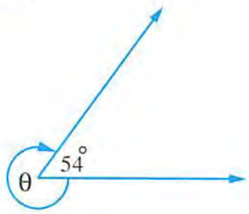


2 Find the measure of the directed angle θ in each of the following :

(1)



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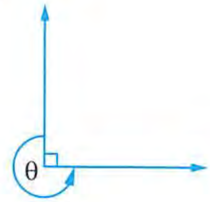

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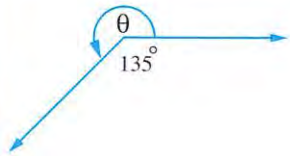

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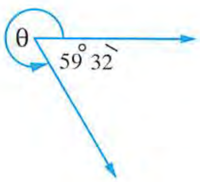

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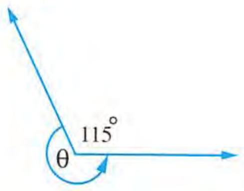

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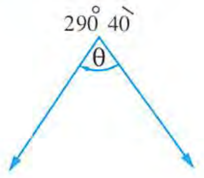


3 Show by drawing , each of the following angles in the standard position :

(1) 32° (2) 140° (3) – 80° (4) – 110° (5) – 315°

4 Determine the quadrant in which each of the following angles lies :

(1) 24° (2) 215° (3) –50° (4) –210°

(5) 150° 14′ (6) 89° 59′ (7) –180° (8) 269° 59′ 60″

5 Determine the smallest positive measure for each of the angles whose measures are as follows , then determine the quadrant in which each angle lies :

(1) –56° (2) 600° (3) –215° (4) 940°

(5) 415° (6) –870° (7) 1120° 15′ (8) –590° 18′

6 Determine one of the negative measures for each of the angles of the following measures :

(1) 83° (2) 136° (3) 90°

(4) 264° (5) 964° (6) 1070°

7 Find two angles , one of them with positive measure and the other with negative measure having common terminal side for each of the following angles :

(1) 40° (2) 150° (3) – 125° (4) – 240° (5) – 180°

Discover the error

8 **Write the positive measure of the smallest angle and another angle with negative measure sharing with the terminal side for the angle whose measure is (– 135°) :**

Karim's answer

The smallest angle with positive measure $= -135° + 180° = 45°$

An angle with negative measure $= -135° - 180° = -315°$

Ziad's answer

The smallest angle with positive measure $= -135° + 360° = 225°$

An angle with negative measure $= -135° - 360° = -495°$

Which of the two answers is correct ?

Third Higher skills

Choose the correct answer from those given :

(1) If A , B are two measures of equivalent angles , then which of the following represents the measures of equivalent angles , where $C \in \mathbb{Z}$?

(a) (A + C) , (B + C) (b) (A – C) , (B – C)

(c) (CA) , (CB) (d) All the previous.

(2) If A , – A are measures of two equivalent angles , then one of the values of A is ....

(a) 150° (b) 90° (c) 180° (d) 270°

(3) If $(3\,X - 5)°$ is the smallest positive measure , $(3\,y - 5)°$ is the greatest negative measure of equivalent angles , then $X - y =$ ....

(a) 360° (b) 180° (c) 120° (d) 90°

(4) If $(\theta + 20)°$, $(20 - 8\,\theta)°$ are the positive and negative measures of a directed angle respectively , then the smallest positive value of θ is ....

(a) 20° (b) 10° (c) 30° (d) 40°

(5) If the terminal side of an angle in standard position passes through the point (– 1 , 0) , then its terminal side lies in ....

(a) first quadrant. (b) second quadrant.

(c) third quadrant. (d) otherwise.

Exercise **8**

Systems of measuring angle (Degree measure - Radian measure)

Test yourself

From the school book Remember Understand Apply Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) The angle of measure $\frac{25\pi}{9}$ lies in the ·········· quadrant.

(a) first (b) second (c) third (d) fourth

(2) The angle of measure $\frac{31\pi}{6}$ lies in the ·········· quadrant.

(a) first (b) second (c) third (d) fourth

(3) The angle of measure $\frac{9\pi}{4}$ lies in the ·········· quadrant.

(a) first (b) second (c) third (d) fourth

(4) The angle of measure $\frac{-\pi}{4}$ lies in the ·········· quadrant.

(a) first (b) second (c) third (d) fourth

(5) The angle of measure $\frac{-9\pi}{4}$ lies in the ·········· quadrant.

(a) first (b) second (c) third (d) fourth

(6) If the degree measure of an angle is $43^\circ\ 12'$, then its radian measure is ··········

(a) 0.24^{rad} (b) 0.24π (c) 0.28^{rad} (d) 0.28π

(7) The degree measure of the angle of measure $\frac{8\pi}{3}$ is ··········

(a) 540° (b) 820° (c) 150° (d) 480°

(8) The sum of the measures of the angles of the quadrilateral in radian equals ....

(a) 2π (b) π (c) $\frac{3\pi}{2}$ (d) 3π

(9) If the sum of measures of the interior angles of a regular polygon equals 180° (n – 2) where n is the number of its sides , then the measure of the interior angle in radian of a regular pentagon equals ....

(a) $\frac{\pi}{3}$ (b) $\frac{7\pi}{2}$ (c) $\frac{3\pi}{5}$ (d) $\frac{2\pi}{3}$

(10) In a circle of diameter length 12 cm. , the length of the arc subtended by a central angle of measure 60° equals .... cm.

(a) 5π (b) 4π (c) 3π (d) 2π

(11) The length of the arc subtended by a inscribed angle of measure 67.5° in a circle of radius length 8 cm. equal .... cm.

(a) 3π (b) 6π (c) 1080 (d) 12π

(12) The measure of the central angle in a circle of radius length 15 cm. and opposite to an arc of length 5π cm. equals ....

(a) 30° (b) 60° (c) 90° (d) 180°

(13) The measure of the central angle in a circle of radius length 12 cm. and opposite to an arc of length 2π cm. equal ....

(a) 2π (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

(14) The meausre of the central angle subtended by an arc of length equal the diameter length of the circle approximately to the nearest degree equal ....

(a) 113° (b) 115° (c) 120° (d) 180°

(15) If the measure of one of the angles of a triangle is 75° and the measure of another angle is $\frac{\pi}{3}$, then the radian measure of the third angle equals ....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{12}$

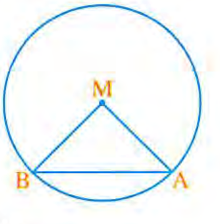
(16) The string length of a simple pendulum is 14 cm. swings in an angle of measure $\frac{1}{10}\pi$, then its arc length $\simeq$ .... cm.

(a) 4.6 (b) 4.4 (c) 4.2 (d) 4.8

(17) ABCD is a cyclic quadrilateral , m ($\angle$ A) = 60° , then m ($\angle$ C) = ...

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

(18) **In the opposite figure :**



To find the length of $\overset{\frown}{AB}$ it is sufficient to get ....

(a) Δ AMB is an equilateral triangle of perimeter 30 cm. only.

(b) the circle circumference = 10π cm only.

(c) (a) , (b) together. (d) nothing of the previous.

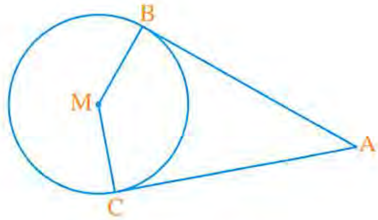
(19) The radian measure of a regular heptagon exterior angle equals ....

(a) $\frac{1}{7}\pi$ (b) $\frac{2}{7}\pi$ (c) $\frac{3}{7}\pi$ (d) $\frac{4}{7}\pi$

(20) In the opposite figure :

If $\overline{AB}$, $\overline{AC}$ are two tangents to the circle M and m ($\angle$ A) $= \frac{5}{12}\pi$ and the circle circumference = 96 cm.

, then the smaller arc length $\overset{\frown}{BC}$ = ...



(a) 20 (b) $\frac{28}{\pi}$ (c) 28 (d) 20π

(21) The angle whose measure $30° + 180° (2n + 1)$ where $n \in \mathbb{Z}$, its radian measure is equivalent to ....

(a) $\frac{\pi}{6}$ (b) π (c) $\frac{7}{6}\pi$ (d) $\frac{5}{3}\pi$

(22) If the length of an arc in a circle equals $\frac{3}{8}$ of its circumference , then the measure of the central angle subtending this arc in degrees equals ....

(a) 30° (b) $67° 30'$

(c) 135° (d) 43° approximately.

(23) In the circle whose radius length is the unit length , then measure of any central angle in it in radian is ....

(a) $\frac{1}{4}$ its arc length. (b) $\frac{1}{2}$ its arc length.

(c) the length of the arc. (d) double its arc length.

(24) The radian measure and the degree measure of the central angle that subtends an arc of length 3 cm. in a cricle of area 16π cm^2. = ...

(a) (1^{rad} , 180°) (b) (1.5^{rad} , 86°)

(c) (1.75^{rad} , 90°) (d) (0.75^{rad} , $42° 58'$)

(25) The angle of measure 1^{rad} is called .... angle.

(a) quadrantal (b) obtuse (c) central (d) radian

Second Essay questions

1 **Find in terms of π the radian measure of each of the angles whose degree measures are as follows :**

(1) 135°	(2) 90°	(3) 300°	(4) −235°
(5) −210°	(6) $112° 30'$	(7) 390°	(8) 780°

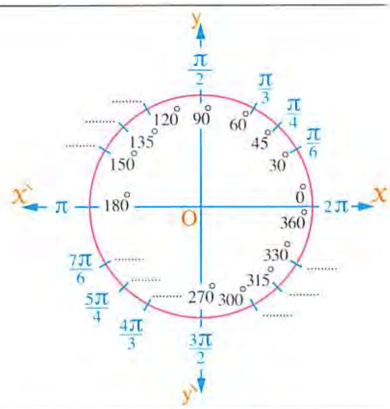
2 **Find the radian measure of each of the angles whose degree measures are as follows approximating the result to three decimal places :**

(1) 58°	(2) 📖 56.6°	(3) $37^\circ\ 15'$
(4) $115^\circ\ 38'\ 6''$	(5) $257^\circ\ 54''$	(6) 📖 $160^\circ\ 50'\ 48''$

3 **Find the degree measure (in degrees , minutes and seconds) of each of the angles whose radian measures are as follows :**

(1) $\frac{11\pi}{15}$	(2) 📖 0.72π	(3) 📖 0.49^{rad}
(4) -1.67^{rad}	(5) 📖 2.27^{rad}	(6) 📖 $-3\frac{1}{2}^{rad}$

4 📖 The opposite figure represents the measures of some special angles , some of them is written in radian outside the circle , and the other is written in degrees inside the circle. Write the corresponding measure of each angle in the opposite figure.



5 **Determine the degree measure and the radian measure for the central angle that subtends an arc of length (ℓ) in a circle of radius (r) in each of the following cases :**

(1) $\ell = 12$ cm. , $r = 10$ cm.	(2) $\ell = 14$ cm. , $r = 7$ cm.
(3) $\ell = 2\pi$ cm. , $r = 6$ cm.	(4) $\ell = 15.72$ cm. , $r = 9.17$ cm.

6 **Find the length of the radius of the circle in which a central angle (θ) is drawn subtending an arc of length (ℓ) in each of the following cases :**

(1) $\theta = \frac{9\pi}{8}$, $\ell = 22.5$ cm.	(2) $\theta = 0.767^{rad}$, $\ell = 38.35$ cm.
(3) $\theta = 139^\circ$, $\ell = 24.325$ cm.	(4) $\theta = 78^\circ\ 36'\ 26''$, $\ell = 43.92$ cm.

7 **Find to the nearest one decimal place of a centimetre the length of an arc in a circle of radius length (r) subtending a central angle of measure (θ) in each of the following cases :**

(1) $r = 12.5$ cm. , $\theta = 1.6^{rad}$	(2) $r = 20$ cm. , $\theta = 2.43^{rad}$
(3) $r = 7.5$ cm. , $\theta = 67^\circ\ 40'$	(4) $r = 15$ cm. , $\theta = 104^\circ\ 58'\ 6''$

8 Find the circumference of a circle which has an arc of length 12 cm. subtended by an inscribed angle of measure 45° « 48 cm. »

9 Find in radian and degrees the measure of a central angle subtending an arc of length three times the length of the radius of its circle. « 3^{rad} , $171^\circ\ 53'\ 14''$ »

---

10 If the measure of a central angle in a circle equals 105° and it is subtending an arc of length $\frac{7\pi}{3}$ cm. , find the length of the diameter of the circle. « 8 cm. »

---

11 In a triangle , the measure of one of its angles is 60° , and the measure of another angle is $\frac{\pi}{4}$ Find the radian measure and the degree measure of the third angle. « $\frac{5}{12}\pi$, 75° »

---

12 In a quadrilateral , the measure of one of its angles is $\left(\frac{11}{6}\right)^{rad}$, the measure of another angle is $\left(2\frac{4}{9}\right)^{rad}$ and the measure of a third angle is 45°

Find the degree measure and the radian measure of the fourth angle $\left(\pi \approx \frac{22}{7}\right)$ « 70° , $\left(\frac{11}{9}\right)^{rad}$ »

---

13 Two angles , the sum of their measures equals 70° , and the difference between them equals $\frac{\pi}{5}$, find the measure of each angle in degrees and in radian. « 53° , 17° , $\frac{53}{180}\pi$, $\frac{17}{180}\pi$ »

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14 Two supplementary angles , the difference between their measures is $\frac{\pi}{3}$ Find the measures of the two angles in radian and in degrees. « $\frac{2\pi}{3}$, $\frac{\pi}{3}$, 120° , 60° »

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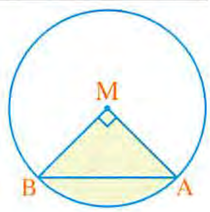
15 **In the opposite figure :**

If the area of the right-angled triangle MAB at M equals 32 cm^2 , find the perimeter of the shaded area to the nearest hundredth.



« 28.57 cm. »

---

16 $\overline{XY}$ is a diameter in circle M its length is 18 cm., the chord $\overline{YZ}$ is drawn such that $m(\angle XYZ) = 10^\circ$. Determine the length of the minor arc $\overset{\frown}{XZ}$ approximating the result to the nearest two decimal places. « 3.14 cm. »

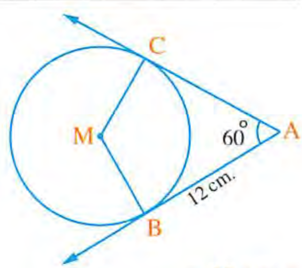
---

17 **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle M ,

$m(\angle CAB) = 60^\circ$, AB = 12 cm.

Find to the nearest integer the length of the greater arc $\overset{\frown}{BC}$



« 29 cm. »

18 ABC is a right-angled triangle at C drawn inside a circle , if AB = 24 cm. , BC = 12 cm. , find the lengths of the three arcs into which the circle is divided by the vertices of this triangle approximating the result to the nearest one decimel place.

« 12.6 cm. , 25.1 cm. , 37.7 cm. »

19 A circle of radius length 7.5 cm. passing through the vertices of the triangle ABC , if m ($\angle$ BAC) = 60° , m ($\angle$ ABC) = 54° , find the lengths of the three arcs into which the circle is divided by the vertices of this triangle. « 15.7 cm. , 14.1 cm. , 17.3 cm. »

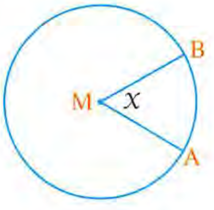
Third Higher skills

1 **Choose the correct answer from those given :**

(1) If an arc opposite to central angle of measure 72° was cut from a circle whose radius length 14 cm. and bent to form a circle , then the radius length of the resulted circle = ... cm.

(a) 1.4 (b) 2.8 (c) 5.6 (d) 7

(2) **In the opposite figure :**



Circle whose centre M , the radius length 10 cm. , if the length of $\widehat{AB} \in]5 , 6[$, then the value of X could be ....

(a) 90° (b) 60° (c) 28° (d) 34°

(3) If the ratio between measures of angles of a quadrilateral is 5 : 4 : 9 : 6 , then the measure of the smallest angle = ...^{rad}

(a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$ (c) $\frac{5\pi}{12}$ (d) $\frac{3\pi}{4}$

(4) The positive measure of an angle that formed between the hour hand and the minute hand at exactly half past two equals ....^{rad}

(a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{12}$ (c) $\frac{7\pi}{12}$ (d) $\frac{3\pi}{4}$

(5) If the arc length opposite to central angle of measure 60° in a circle equals the arc length opposite to central angle of measure 80° in another circle , then the ratio between the two radii of the two circles is ....

(a) $\frac{5}{4}$ (b) $\frac{4}{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{9}{16}$

(6) A cylinder rotates 45 revolutions per minute around its axis , then the measure of the angle at which a point on the lateral surface rotates in one second equals ....

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

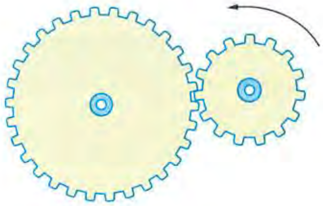
(7) (The measure of the circle)^{rad} > n where n is a positive integer , then the greatest value for n is ....

(a) 3 (b) 5 (c) 6 (d) 8

(8) The distance covered by the tip of the minute hand whose length 8 cm. from 6 am till quarter past three pm equals .... cm.

(a) 592π (b) 148π (c) $\frac{37}{2}\pi$ (d) $\frac{37}{4}\pi$

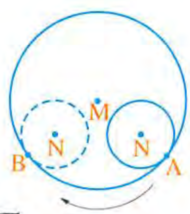
(9) In the opposite figure :



When the greater gear revolves one revolution then the smaller gear revolves 3 revolutions. If the smaller gear revolves one revolution in the direction of the arrow shown on the figure , then the measure of the central angle of revolving the greater gear is ....^{rad}

(a) $-\frac{\pi}{2}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) 2π

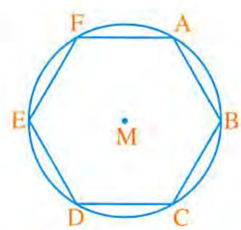
(10) In the opposite figure :



Two circles M and N , their radii length are 21 cm. , 7 cm. respectively. If a circle N rotated a complete revolution from a point A to point B , then m (∠ AMB) = ...

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{2\pi}{5}$ (d) π

(11) In the opposite figure :



ABCDEF is a regular hexagon of side length 4 cm. inscribed in a circle M , then the length of $\overset{\frown}{AB}$ = ... cm.

(a) π (b) $\frac{4}{3}\pi$ (c) 2π (d) $\frac{5}{3}\pi$

---

2 A straight line makes an angle of radian measure $\frac{\pi}{3}$ with the positive direction of the x-axis in the standard position in the unit circle. Find the equation of the straight line.

« $y = \sqrt{3}\,x$ »

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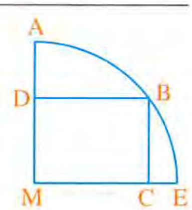
3 **In the opposite figure :**



A quarter circle , BCMD is a rectangle which is drawn inside it , where CD = 10 cm.

Find the length of arc : $\overset{\frown}{ABE}$

« 5π cm. »

Exercise 9

Trigonometric functions

Test yourself

From the school book · Remember · Understand · Apply · Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) If θ is the measure of an angle in the standard position , its terminal side intersects the unit circle at the point $\left(\frac{\sqrt{3}}{2} , \frac{1}{2}\right)$, then sin θ = ...

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

(2) If the terminal side of the angle whose measure θ drawn in the standard position intersect the unit circle at the point B $\left(\frac{-3}{5} , \frac{4}{5}\right)$, then cot θ = ...

(a) $\frac{5}{4}$ (b) $\frac{-5}{3}$ (c) $\frac{-4}{3}$ (d) – 0.75

(3) If θ is a directed angle in the standard position its terminal side intersect the unit circle at $\left(\frac{-5}{13} , \frac{12}{13}\right)$, then cos θ – sin θ = ...

(a) $\frac{17}{13}$ (b) $\frac{7}{13}$ (c) $\frac{-7}{13}$ (d) $\frac{-17}{13}$

(4) A directed angle in the standard position its terminal side passes through the point (3 , 4) , then its initial side intersect the unit circle at the point ....

(a) (3 , 0) (b) (1 , 0) (c) (0.6 , 0.8) (d) $\left(\frac{4}{3} , \frac{5}{3}\right)$

(5) If $\tan \theta = \frac{1}{2}$ where θ is an acute angle in standard position , then its terminal side intersects the unit circle at the point ....

(a) (2 , 1) (b) (1 , 2) (c) $\left(\frac{1}{\sqrt{5}} , \frac{2}{\sqrt{5}}\right)$ (d) $\left(\frac{2}{\sqrt{5}} , \frac{1}{\sqrt{5}}\right)$

(6) If $\sin \theta = \frac{1}{\sqrt{2}}$, where θ is the measure of a positive acute angle , then the measure of angle θ = ...

(a) 30° (b) 60° (c) 45° (d) 90°

(7) If $\sin \theta = -1$, $\cos \theta = 0$, then the measure of angle θ = ...

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

(8) If $\csc \theta = 2$, where θ is a positive acute angle , then the measure of angle θ = ...

(a) 15° (b) 30° (c) 45° (d) 60°

(9) If $\tan \theta = 1$, where θ is a positive acute angle , then the measure of angle θ = ...

(a) 60° (b) 30° (c) 45° (d) 90°

(10) If $\cos \theta = \frac{1}{2}$, $\sin \theta = \frac{\sqrt{3}}{2}$, then the measure of angle θ = ...

(a) $\frac{\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{5\pi}{3}$ (d) $\frac{11\pi}{6}$

(11) If $\cos \theta = \frac{1}{2}$, $\sin \theta = \frac{-\sqrt{3}}{2}$, then $\tan \theta$ = ...

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{-1}{\sqrt{3}}$ (d) $-\sqrt{3}$

(12) If the terminal side of a directed angle in the standard position intersect the unit circle at the point $\left(\frac{1}{2} , \frac{\sqrt{3}}{2}\right)$, then the measure of this angle = ...

(a) 150° (b) 30° (c) 60° (d) 210°

(13) If $\cos \theta = \frac{\sqrt{3}}{2}$, where θ is a positive acute angle , then $\sin \theta$ = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

(14) If $\cos \theta > 0$, $\sin \theta < 0$, then θ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(15) If $\sin \theta = \frac{-1}{2}$, $\sec \theta = \frac{-2}{\sqrt{3}}$, then θ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(16) If $\sin\theta = \frac{-1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$, then the angle whose measure θ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(17) If θ is measure of an angle lies in the third quadrant , which of the following is always true ?

(a) $\sin\theta \cos\theta < 0$ (b) $\sec\theta \csc\theta < 0$ (c) $\tan\theta \cot\theta < 0$ (d) $\sin\theta \tan\theta < 0$

(18) $2\sin 45° =$ ....

(a) $\sin 90°$ (b) $\frac{\sqrt{2}}{2}$ (c) $\sqrt{2}$ (d) 2

(19) $\cot^2 30° - \sec^2 60° + \csc^2 45° =$ ....

(a) 1 (b) 0 (c) -1 (d) 2

(20) $\sin\left(-\frac{12}{5}\pi\right) =$ ....

(a) $\sin\frac{12}{5}\pi$ (b) $\sin 72°$ (c) $\sin 288°$ (d) $\sin\frac{1}{5}\pi$

(21) $\sin 0° + \cos 0° + \tan 0° =$ ....

(a) 0 (b) 1 (c) 2 (d) 3

(22) $\cos^2\frac{\pi}{4} - \sin^2\frac{\pi}{4} =$ ....

(a) $\cos^2\pi$ (b) $\sin^2\frac{\pi}{2}$ (c) $\cos\pi$ (d) $\cos\frac{\pi}{2}$

(23) $\cos\frac{\pi}{2}\cos 0 + \sin\frac{3\pi}{2}\sin\frac{\pi}{2} =$ ....

(a) zero (b) 1 (c) -1 (d) 2

(24) $\sin 0° + \sin 90° + \sin 180° + \sin 270° =$ ....

(a) 4 (b) 2 (c) 3 (d) zero

(25) $\cot^2 30° + 2\sin^2 45° + \cos^2 90° =$ ....

(a) zero (b) 3 (c) 4 (d) 2

(26) $2\sin 45° \cos 45° \cot 45° =$ ....

(a) $\cos 60°$ (b) $2\cos 30°$ (c) $2\sin\frac{\pi}{6}$ (d) $\tan\pi$

(27) $\sin 30° + \cos 60° - \cot 45° =$ ....

(a) 2 (b) zero (c) $\sqrt{3}-\sqrt{2}$ (d) 1

(28) $\frac{\tan^2 60° - \tan^2 45°}{\sec^2 30° - \csc^2 45°} =$ ....

(a) zero (b) 3 (c) -2 (d) -3

(29) If ABCD is a square , then $\sin^2(\angle ACD) + \sin^2(\angle ABD) + \tan(\angle ADB) =$ ....

(a) $\frac{3}{2}$ (b) 3 (c) 2 (d) $1+\sqrt{2}$

(30) ABC is an isosceles triangle in which m ($\angle$ A) = 120°

, then $\sin B + \cos^2 C =$ ....

(a) $1 + \sqrt{3}$ (b) $1 \frac{1}{2}$ (c) $1 \frac{2}{3}$ (d) $1 \frac{1}{4}$

(31) If ABC is a right-angled triangle at B , m ($\angle$ A) = 2 m ($\angle$ C)

, then sec A + csc C = ...

(a) 2 (b) 4 (c) 6 (d) 8

(32) If $\theta \in \left]0 , \frac{\pi}{2}\right[$, $\cos \theta = \frac{3}{5}$, then $\csc \theta \sin \theta - \tan \theta \csc \theta =$ ....

(a) zero (b) 1 (c) $\frac{-3}{2}$ (d) $\frac{-2}{3}$

(33) If $\sin \theta = \frac{-24}{25}$, $\theta \in \left]\frac{3\pi}{2} , 2\pi\right[$, then $\frac{\sin \theta + \cos \theta}{\sin \theta} =$ ....

(a) $\frac{17}{24}$ (b) $\frac{-17}{24}$ (c) $\frac{24}{17}$ (d) $\frac{-24}{17}$

(34) If $x \in [0° , 90°]$ and $\cos x = \frac{\sin 60°}{\sin 90°} - \frac{\sin 0°}{\sin 45°}$, then $x =$ ....

(a) 30° (b) 60° (c) 0° (d) 90°

(35) If $\theta \in \left]\frac{\pi}{2} , \pi\right[$, $\sin \theta = \frac{12}{13}$, then $\sqrt{\csc \theta \sin \theta - \tan \theta \cot \theta + \cos^2 \theta} =$ ....

(a) zero (b) $\frac{5}{13}$ (c) $\frac{4}{3}$ (d) $\frac{15}{26}$

(36) If the terminal side of an angle in standard position intersects the unit circle of point A which lies in the fourth quadrant where the x-coordinate of A equals $\frac{5}{13}$

, then A = ...

(a) $\left(\frac{5}{13} , \frac{-12}{13}\right)$ (b) $\left(\frac{5}{13} , \frac{1}{13}\right)$ (c) $\left(\frac{5}{13} , \frac{12}{13}\right)$ (d) $\left(\frac{5}{13} , \frac{-8}{13}\right)$

(37) If θ is a measure of an angle in standard position and its terminal side intersects the unit circle at the point $\left(\frac{1}{2} , y\right)$ where $y > 0$, then $\sin \theta =$ ....

(a) $\frac{1}{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

(38) If the terminal side of a directed angle in the standard position intersect the unit circle at $(-x , x)$ where $x < 0$, then the sine of this angle = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{-1}{\sqrt{2}}$

(39) The terminal side of angle of measure 30° in its standard position intersects the circle whose centre is the origin and its radius length is 6 cm. at the point ....

(a) (3 , 6) (b) $\left(\frac{\sqrt{3}}{2} , \frac{1}{2}\right)$ (c) $\left(3 , 3\sqrt{3}\right)$ (d) $\left(3\sqrt{3} , 3\right)$

(40) The sine of a directed angle θ in the standard position its terminal side intersect the unit circle at the point (1 , 0) equal the cosine of a directed angle X in the standard position and its terminal side intersect the unit circle at the point ....

(a) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ (b) (–1 , 0) (c) (0 , – 1) (d) $\left(X , \frac{-1}{\sqrt{2}}\right)$

(41) sine of the quadrantal angle ....

(a) equal zero. (b) $\in]-1 , 1[$

(c) $\in \{0 , 1 , -1\}$ (d) more than or equal zero.

(42) All the following trigonometric ratios are for the same angle θ and lies in the third quadrant except ....

(a) $\sin \theta = \frac{-3}{\sqrt{10}}$ (b) $\sec \theta = -\sqrt{10}$

(c) $\cot \theta = \frac{1}{3}$ (d) $\csc \theta = 3$

(43) If $\sin X + \cos y = 2$, X , $y \in [0° , 360°[$, then $X + y =$ ....

(a) 2 (b) 1 (c) 90° (d) 180°

(44) If $\theta = \frac{\pi}{4}(8 n + 2)$, $n \in \mathbb{Z}$, then $\cos \theta =$ ....

(a) 1 (b) – 1 (c) zero (d) $\frac{1}{\sqrt{2}}$

(45) If the equation of a straight line : $y = \frac{3}{4} X + 1$ and it makes with the positive direction of the X-axis an angle of measure θ , then $\sin \theta =$ ....

(a) $\frac{3}{4}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{4}{3}$

(46) If Δ ABC is right-angled triangle at A , $\overline{AD} \perp \overline{BC}$, AD = 6 cm. , and $\cot B + \cot C = \frac{5}{2}$ then BC = ... cm.

(a) 5 (b) 10 (c) 3.6 (d) 15

(47) If θ is the measure of a directed angle in its standared position where its terminal side interescts the unit circle in the point B (X , y) where $X > 0$ and $\tan \theta = \frac{-3}{4}$, then $X + y =$ ....

(a) $-\frac{1}{5}$ (b) $\frac{1}{5}$ (c) zero (d) 1

(48) The sign of the function $f : f(X) = \sec X$ is ....

(a) positive in $]0 , \frac{\pi}{2}[$, positive in $]\frac{3\pi}{2} , 2\pi[$

(b) negative in $]0 , \frac{\pi}{2}[$, negative in $]\frac{3\pi}{2} , 2\pi[$

(c) negative in $]0 , \frac{\pi}{2}[$, positive in $]\frac{3\pi}{2} , 2\pi[$

(d) positive in $]0 , \frac{\pi}{2}[$, negative in $]\frac{3\pi}{2} , 2\pi[$

Second Essay questions

1 **Determine the signs of the following trigonometric ratios :**

(1) $\cos 350°$
(2) $\tan 100°$
(3) $\sec 265°$
(4) $\sin \frac{5\pi}{4}$
(5) $\csc \frac{3\pi}{7}$
(6) $\cot \frac{3\pi}{4}$
(7) $\tan 410°$
(8) $\csc 1200°$
(9) $\cos (-165°)$
(10) $\cot \frac{32\pi}{3}$
(11) $\cot \left(\frac{-3\pi}{4}\right)$
(12) $\sec \left(\frac{-25\pi}{6}\right)$

2 **Find all trigonometric functions of the angle whose measure is θ drawn in the standard position , its terminal side intersects the unit circle at the point :**

(1) $\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$
(2) $\left(-\frac{3}{5}, -\frac{4}{5}\right)$
(3) $(0, -1)$

3 **If θ is the measure of a directed angle in the standard position and B is the intersection point of its terminal side with the unit circle , then find all trigonometric functions of the angle θ in each of the following cases :**

(1) $B (0.6 , y) , y > 0$
(2) $B (x , -0.6) , x > 0$
(3) $B \left(-\frac{\sqrt{3}}{2}, y\right)$, where $90° < \theta < 180°$
(4) $B \left(x , \frac{\sqrt{5}}{3}\right) , x < 0$
(5) $B (-1 , y)$
(6) $B (-x , x) , x > 0$
(7) $B (-x , -x) , x > 0$
(8) $B (9a , 12a)$ where $180° < \theta < 270°$
(9) $B \left(\frac{3}{2} a , -2a\right)$, where $\frac{3\pi}{2} < \theta < 2\pi$

4 **Find the value of each of :**

(1) $\tan 0° + \tan 45° + \tan 180°$
(2) $\sin 180° \cos 45° - \cos 180° \sin 45°$
(3) $\sec \frac{\pi}{6} \tan \frac{\pi}{3} - \cot \frac{\pi}{3} \cos \frac{\pi}{6}$
(4) $\frac{4 \sin^2 30° - 3 \tan 45° \cos 0°}{2 \cos 60° + 2 \sin 45° \cos 45°}$
(5) $3 \sin 30° \sin^2 60° - \cos 0° \sec 60° + \sin 270° \cos^2 45°$

5 **Prove each of the following equalities :**

(1) $2 \sin^2 90° = -2 \cos 180°$
(2) $3 \cos 30° \tan 60° - 2 \sec 45° \csc 45° = \frac{1}{2}$
(3) $3 \cot^2 45° - 2 \sin 60° \cos 30° = \frac{3}{2} \sin^2 90°$

(4) $\sec 30° \tan 60° + \csc^2 60 - \tan^2 45 = \frac{7}{3}$

(5) $\sin 60° \cos 30° - \cos 60° \sin 30° = \sin^2 \frac{\pi}{4}$

(6) $3 \tan^2 30° + \frac{4}{3} \cos^2 30° - \frac{1}{4} \cot^2 45° \csc^2 30° = 1$

(7) $2 \cos^2 \frac{\pi}{3} + 3 \sin^2 \frac{\pi}{4} + 4 \tan^2 \frac{\pi}{3} - 4 \sin \frac{\pi}{2} = 10$

(8) $\frac{\tan 60° - \tan 30°}{1 + \tan 60° \tan 30°} = \cot 60°$

(9) $\frac{\sin 30° \cos 45° + \sin 45° \cos 30°}{\sin 45° \cos 60° + \cos 45° \sin 60°} = \sin 90°$

6 **Find the value of X if :**

(1) $X \sin^2 \frac{\pi}{4} \cos \pi = \tan^2 \frac{\pi}{3} \sin \frac{3\pi}{2}$ « 6 »

(2) $X \sin \frac{\pi}{4} \cos \frac{\pi}{4} \cot \frac{\pi}{6} = \tan^2 \frac{\pi}{4} - \cos^2 \frac{\pi}{3}$ « $\frac{\sqrt{3}}{2}$ »

7 **If $X \in [0° , 90°]$, then find the value of X which satisfies each of the following equations :**

(1) $\cos X = \frac{\sin 60°}{\sin 90°} - \frac{\sin 0°}{\sin 45°}$ « 30° »

(2) $\sin X = \sin 30° \cos 60° + \cos 30° \sin 60°$ « 90° »

8 **Find all trigonometric ratios for the angle AOB whose measure is θ in each of the following cases :**

(1) $\theta \in]0 , \frac{\pi}{2}[$, $\cos \theta = 0.6$

(2) $\theta \in]\frac{\pi}{2} , \pi[$, $\sin \theta = \frac{12}{13}$

(3) $\theta \in]\frac{\pi}{2} , \pi[$, $\tan \theta = -\frac{3}{4}$

(4) $\theta \in]\pi , \frac{3\pi}{2}[$, $\csc \theta = -\frac{25}{7}$

(5) $\theta \in]\frac{3\pi}{2} , 2\pi[$, $\sec \theta = 2$

9 If the terminal side of the angle θ in the standard position intersects the unit circle at the point (2 a , 3 a) , where $0 < \theta < \frac{\pi}{2}$, find the value of a **, then find the value of :**

$\sec^2 \theta - \tan^2 \theta$ « $\frac{1}{\sqrt{13}}$, 1 »

10 **If $\theta \in]\frac{3\pi}{2} , 2\pi[$, $\sin \theta = -\frac{24}{25}$, then find :**

(1) $\frac{\cot \theta - \csc \theta}{\tan \theta - \sec \theta}$

(2) $\cos \theta - \csc \theta \tan \theta$ « $\frac{-3}{28}$, $\frac{-576}{175}$ »

Discover the error

11 **The teacher asks the students to find the value of :** $2 \sin 45°$

Karim's answer

$2 \sin 45° = \sin 2 \times 45°$
$= \sin 90° = 1$

Ahmed's answer

$2 \sin 45° = 2 \times \frac{1}{\sqrt{2}}$
$= \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

Which of the two answers is correct ? Why ?

Third Higher skills

Choose the correct answer from those given :

(1) In the unit circle whose centre is (O) if the length of $\overset{\frown}{BC} = \frac{1}{3}\pi$, then $\sec (\angle BOC) =$ ....

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{-1}{2}$ (d) 2

(2) If A is the greatest acute angle measure in a triangle whose side lengths are 5 , 12 , 13 cm. , then $\cot A =$ ....

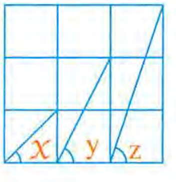
(a) $\frac{12}{13}$ (b) $\frac{5}{13}$ (c) $\frac{5}{12}$ (d) $\frac{12}{5}$

(3) If the side lengths of right-angled triangle ABC are $x - 7$, x , $x + 1$ and $\overline{BC}$ is the smallest side , then $\sec A =$ ....

(a) $\frac{5}{13}$ (b) $\frac{12}{13}$ (c) $\frac{13}{12}$ (d) $\frac{5}{4}$

(4) In the opposite figure :

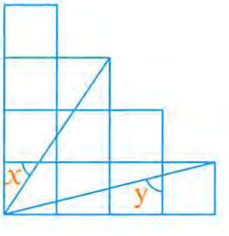
All squares are identical , then $\cot x + \cot y + \cot z =$ ....



(a) 6 (b) $\frac{11}{6}$ (c) $\frac{6}{11}$ (d) $\sqrt{5} + 3$

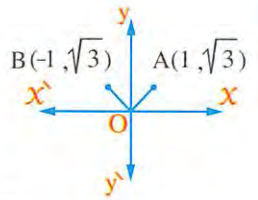
(5) In the opposite figure :

All squares are identical , then $\tan x + \cot y =$ ....



(a) $\frac{11}{12}$ (b) $\frac{7}{4}$ (c) $\frac{5}{3}$ (d) $\frac{14}{3}$

(6) In the opposite figure :

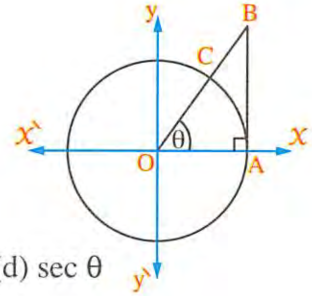


If A$\left(1 , \sqrt{3}\right)$, B$\left(-1 , \sqrt{3}\right)$

, then cot (∠ AOB) = ...

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

(7) In the opposite figure :



O is the centre of the unit circle ,

$\overline{AB}$ is a tangent segment , then :

First : OB = ...

(a) sin θ (b) cos θ (c) csc θ (d) sec θ

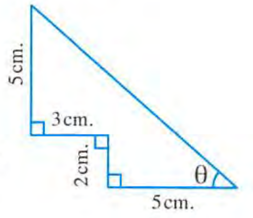
Second : BC = ...

(a) cot θ (b) (sec θ) – 1 (c) (csc θ) – 1 (d) cos θ

Third : The area of triangle ABO = ...

(a) $\frac{1}{2}$ cos θ (b) $\frac{1}{2}$ tan θ (c) $\frac{1}{2}$ sin θ (d) $\frac{1}{2}$ sin θ cos θ

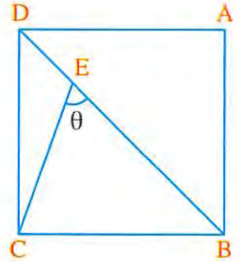
(8) In the opposite figure :



cot θ = ...

(a) $\frac{2}{5}$ (b) $\frac{7}{8}$ (c) $\frac{3}{2}$ (d) $\frac{8}{7}$

(9) In the opposite figure :

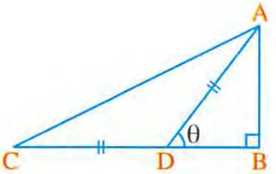


If ABCD is a square and $\frac{DE}{EB} = \frac{2}{5}$

, then tan θ = ...

(a) $\frac{7}{3}$ (b) $\frac{3}{7}$ (c) $\frac{2}{7}$ (d) $\frac{7}{2}$

(10) In the opposite figure :

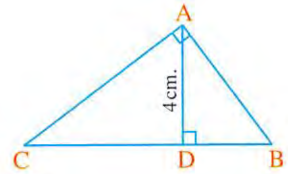


If D ∈ $\overline{BC}$ and AD = DC

, tan θ = $\frac{4}{3}$, then cot $\frac{\theta}{2}$ = ...

(a) $\frac{3}{4}$ (b) 2 (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(11) In the opposite figure :

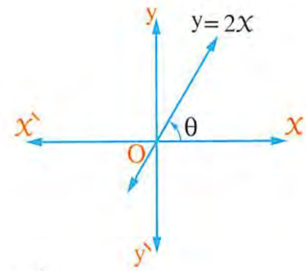


If $\tan B + \tan C = \frac{5}{2}$

, then BC = ... cm.

(a) 6 (b) 8 (c) 10 (d) 14

(12) In the opposite figure :



If θ is the measure of the included angle between the straight line $y = 2X$ and the positive direction of X-axis

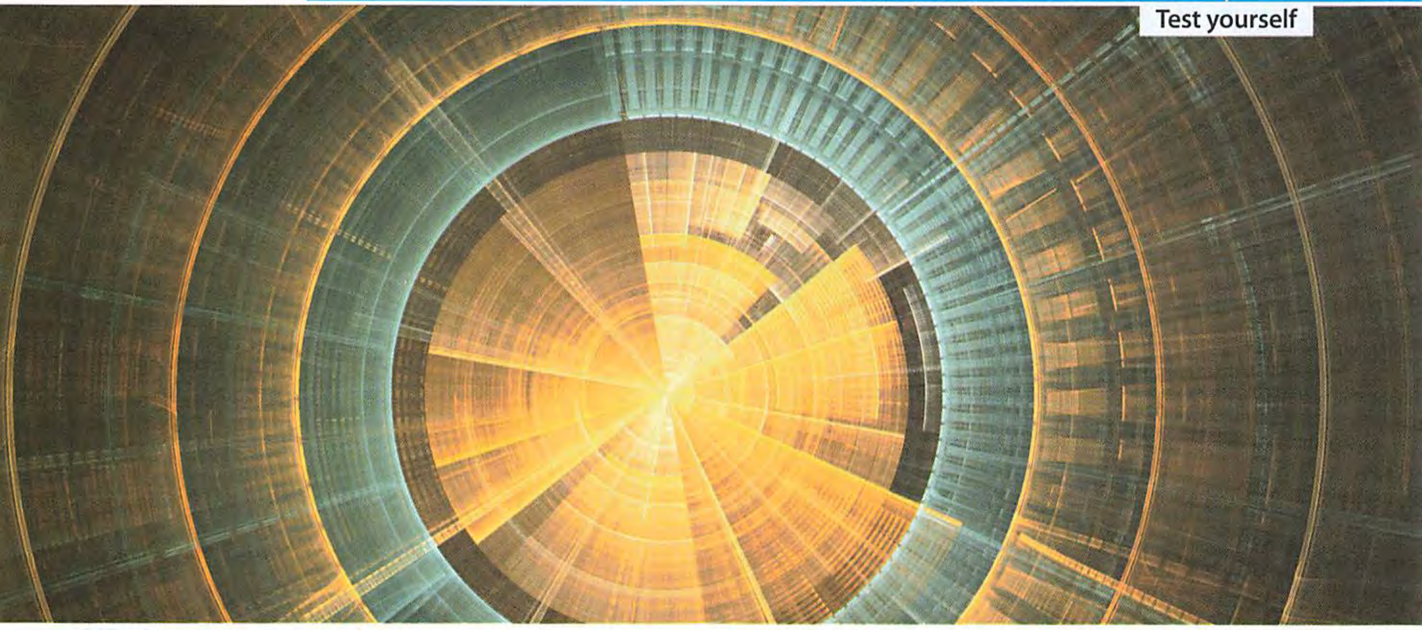
, then $\sin \theta =$ ....

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{1}{3}$

Exercise 10

Related angles

Test yourself



From the school book | Remember | Understand | Apply | Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) $\tan 42^\circ = \cdots\cdots\cdots$

(a) $\cot 42^\circ$ (b) $\tan 48^\circ$ (c) $\cot 48^\circ$ (d) $\csc 48^\circ$

(2) $\cot (90^\circ + \theta) = \cdots\cdots\cdots$

(a) $\tan (90^\circ - \theta)$ (b) $-\tan \theta$ (c) $\tan (90^\circ + \theta)$ (d) $\tan (270^\circ + \theta)$

(3) $\frac{\sec 105^\circ}{\csc 15^\circ} = \cdots\cdots\cdots$

(a) $\frac{\sin 105^\circ}{\cos 15^\circ}$ (b) $\tan 135^\circ$ (c) $\cot 15^\circ$ (d) $\cos 90^\circ$

(4) $\tan (180^\circ - \theta) = \cdots\cdots\cdots$

(a) $\tan \theta$ (b) $-\tan \theta$ (c) $\cot \theta$ (d) $-\cot \theta$

(5) $\sec (90^\circ + \theta) = \cdots\cdots\cdots$

(a) $\csc (180^\circ - \theta)$ (b) $\csc (180^\circ + \theta)$ (c) $\csc (270^\circ - \theta)$ (d) $\csc (270^\circ + \theta)$

(6) $\cos (270^\circ - \theta) = \cdots\cdots\cdots$

(a) $\sin \theta$ (b) $\cos \theta$ (c) $-\sin \theta$ (d) $-\cos \theta$

(7) If $\sin \theta = \frac{3}{5}$, then $\cos (270^\circ - \theta) = \cdots\cdots\cdots$

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{-4}{5}$

(8) $\cos(90° - \theta) \times \csc\theta = \cdots\cdots$

(a) zero (b) 1 (c) -1 (d) $\frac{-4}{5}$

(9) If $\frac{\sin 50°}{\cos 40°} + \frac{\sin 70°}{\sin 110°} = k$, then $k = \cdots\cdots$

(a) 1 (b) 2 (c) 3 (d) zero

(10) The simplest form of the expression : $\tan(90° - \theta) + \tan(90° + \theta)$ is $\cdots\cdots$

(a) $2\cot\theta$ (b) $2\tan\theta$ (c) zero (d) $\tan\theta + \cot\theta$

(11) $\tan(45° + x) = \cdots\cdots$

(a) $\tan x$ (b) $-\tan x$ (c) $\tan(45° - x)$ (d) $\cot(45° - x)$

(12) $\frac{\sin(30° + x)}{\cos(60° - x)} = \cdots\cdots$

(a) 1 (b) -1 (c) zero (d) $\tan x$

(13) $\frac{\tan(45° + x)}{\cot(45° - x)} = \cdots\cdots$

(a) -1 (b) 1 (c) $\tan(90° + x)$ (d) $\cot(90° + x)$

(14) $\sin(90° - \theta)\sec(360° - \theta) - \cos(270° + \theta)\csc(180° + \theta) = \cdots\cdots$

(a) -2 (b) -1 (c) 1 (d) 2

(15) If $A + B = 90°$, $\tan A = \frac{1}{3}$, then $\tan B = \cdots\cdots$

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) 3

(16) If $x + y = \frac{\pi}{2}$, then $\frac{\sin x - \sin y}{\cos x - \cos y} = \cdots\cdots$

(a) -1 (b) zero (c) 1 (d) 2

(17) $\cos\theta + \cos(180° - \theta) = \cdots\cdots$

(a) zero (b) 1 (c) $2\cos\theta$ (d) $\cos\theta$

(18) $\sin\theta + \cos(270° + \theta) = \cdots\cdots$

(a) zero (b) 1 (c) $2\sin\theta$ (d) $\sin\theta\cos\theta$

(19) The simplest form of the expression :

$\sin(180° - \theta) + \cos(-60°) + \cos(90° + \theta) + \sin(-150°) = \cdots\cdots$

(a) zero (b) 1 (c) -1 (d) $2\sin\theta$

(20) If $\cos\theta = -\sin 2\theta$, θ is the smallest positive measure , then $\theta = \cdots\cdots°$

(a) 60 (b) 150 (c) 90 (d) 330

(21) If $\sqrt{3}\csc\theta = -2$ where θ is the smallest positive angle, then $= \theta \cdots\cdots$

(a) 60° (b) 120° (c) 300° (d) 240°

(22) If $\cos\theta = \frac{-1}{2}$, θ is measure of the smallest positive angle , then $\theta =$ ....

(a) $60°$ (b) $120°$ (c) $240°$ (d) $300°$

(23) If $\cos(270° - \theta) = \frac{1}{2}$ where θ is the measure of the smallest positive angle , then $\theta =$ ....

(a) $30°$ (b) $150°$ (c) $210°$ (d) $330°$

(24) If $\cos(90° + \theta) = \frac{\sqrt{3}}{2}$ where θ is the smallest positive angle , then $\theta =$ ....

(a) $150°$ (b) $240°$ (c) $210°$ (d) $330°$

(25) If $\tan\theta = \tan(90 - \theta)$ where θ is an acute angle , then $\theta =$ ....°

(a) 15 (b) 30 (c) 45 (d) 60

(26) If $\cos(990° - \theta) = \frac{1}{2}$ where θ is measure of the smallest positive angle , then $\theta =$ ....

(a) $30°$ (b) $150°$ (c) $210°$ (d) $330°$

(27) If $2\cos\theta + \sqrt{3} = 0$ where $180° < \theta < 270°$, then $\theta =$ ....

(a) $150°$ (b) $240°$ (c) $210°$ (d) $300°$

(28) If $5\sin X = 3$, then $\sec(270° + X) =$ ....

(a) $\frac{5}{3}$ (b) $\frac{-5}{4}$ (c) $\frac{-5}{3}$ (d) $\frac{5}{4}$

(29) If $\sin\theta = -\frac{1}{2}$, $\tan\theta > 0$, then $\theta =$ ....

(a) $30°$ (b) $150°$ (c) $210°$ (d) $330°$

(30) If $\tan\theta = \frac{-5}{12}$, $\cos\theta < 0$, then $\csc\theta =$ ....

(a) $\frac{5}{13}$ (b) $\frac{-5}{13}$ (c) $\frac{13}{5}$ (d) $\frac{-13}{5}$

(31) If $2\sin(90° - \theta) = 1$, where $0 < \theta < \frac{\pi}{2}$, then $\theta =$ ....

(a) $90°$ (b) $60°$ (c) $30°$ (d) $45°$

(32) If $5\cos(90° - \theta) = 4$, $0° < \theta < 90°$, then $\sin\theta =$ ....

(a) $\frac{5}{4}$ (b) $\frac{-3}{5}$ (c) $\frac{4}{5}$ (d) $\frac{3}{5}$

(33) If $\sin\theta = -0.8$ where $180° < \theta < 270°$, then $3\cot(270 - \theta) =$ ....

(a) -3 (b) 3 (c) -4 (d) 4

(34) If $24\tan\theta + 7 = 0$, $90° < \theta < 270°$, then $\sec(1080° + \theta) =$ ....

(a) $\frac{24}{7}$ (b) $\frac{-24}{7}$ (c) $\frac{25}{24}$ (d) $\frac{-25}{24}$

(35) If $13 \sin (90° - \theta) = 5$, then $\cos \theta =$ ....

(a) $\frac{12}{13}$ (b) $\frac{-12}{13}$ (c) $\frac{5}{13}$ (d) $\frac{-5}{13}$

(36) If $\cot (90° + \theta) + 1 = 0$ where $0° < \theta < 90°$, then $\cos 4\theta =$ ....

(a) $\frac{1}{2}$ (b) 1 (c) zero (d) -1

(37) If $\cos (90° + \theta) + \sin (90° - 2\theta) = 0$, where $\theta \in \left]0 , \frac{\pi}{4}\right[$, then $\sin 2\theta =$ ....

(a) $\frac{1}{2}$ (b) 1 (c) zero (d) $\frac{\sqrt{3}}{2}$

(38) If $\cot (90° + \theta) + \tan (90° - 2\theta) = 0$, where $\theta \in \left]0 , \frac{\pi}{4}\right[$, then $\tan 2\theta =$ ....

(a) $\frac{1}{\sqrt{3}}$ (b) 1 (c) zero (d) $\sqrt{3}$

(39) If $\tan B = \frac{3}{4}$ where $\pi < B < \frac{3\pi}{2}$, then $\cos (360° - B) - \cos (90° - B) =$ ....

(a) $\frac{-7}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{-4}{5}$ (d) $\frac{-1}{5}$

(40) If $13 \sin \theta - 5 = 0$, where $\theta \in \left]\frac{\pi}{2} , \pi\right[$, then the value of $\sin (270° - \theta) \times \sec (90 + \theta)$ $=$ ....

(a) $\frac{-12}{5}$ (b) $\frac{12}{5}$ (c) $\frac{5}{12}$ (d) $\frac{-5}{12}$

(41) If the terminal side of an angle whose measure is θ in standard position intersects the unit circle at the point $\left(\frac{-\sqrt{3}}{2} , y\right)$ where $y \in \mathbb{R}^+$, then $\theta =$ ....

(a) 30° (b) 150° (c) 210° (d) 330°

(42) If $\left(x , \frac{1}{2}\right)$ is the intersection point of the terminal side of a directed angle in the standard position with the unit circle where $90° < \theta < 180°$
, then $\sin (90° - \theta) \tan \theta =$ ....

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{1}{3}$ (d) -3

(43) If θ is the measure of an angle in standard position and its terminal side intersects the unit circle at $(x , -x)$ where $x > 0$, then $\theta =$ ....°

(a) 45 (b) 135 (c) 225 (d) 315

(44) If the terminal side of an angle whose measure is θ in its standard position intersects the unit circle at the point $\left(\frac{-3}{5} , \frac{4}{5}\right)$, then $\csc \left(\frac{3\pi}{2} - \theta\right) =$ ....

(a) $\frac{5}{3}$ (b) $\frac{-5}{3}$ (c) $\frac{5}{4}$ (d) $\frac{-5}{3}$

(45) If the terminal side of the directed angle $(90° - \theta)$ in the standard position intersect the unit circle at the point $\left(\frac{-4}{5}, \frac{3}{5}\right)$, then $\sin \theta = \cdots\cdots\cdots$

(a) $\frac{-4}{5}$ (b) $\frac{4}{5}$ (c) $\frac{-3}{5}$ (d) $\frac{3}{5}$

(46) If $\sin \propto = \cos \beta$, then $\csc (\propto + \beta) = \cdots\cdots\cdots$

(a) 1 (b) -1 (c) $\frac{1}{\sqrt{3}}$ (d) undefined.

(47) If $\sin \propto = \cos \beta$, then $\cot (\propto + \beta) = \cdots\cdots\cdots$

(a) 1 (b) -1 (c) zero (d) undefined.

(48) If $\sin \theta = \cos 2\theta$, $\theta \in]0, \frac{\pi}{2}[$, then $\sin 3\theta = \cdots\cdots\cdots$

(a) $\frac{1}{2}$ (b) 1 (c) zero (d) $\frac{\sqrt{3}}{2}$

(49) If $\sin 2\theta = \cos 4\theta$ where θ is a positive acute angle , then $\tan (90° - 3\theta) = \cdots\cdots\cdots$

(a) -1 (b) $\frac{1}{\sqrt{3}}$ (c) 1 (d) $\sqrt{3}$

(50) If $\tan \theta = \cot 2\theta$, $0° < \theta < 90°$, then $\sin \theta + \cos 2\theta = \cdots\cdots\cdots$

(a) 1 (b) -1 (c) 2 (d) $\frac{1}{4}$

(51) If $\sin (\theta + 13°) = \cos (\theta + 17°)$ where θ is a positive acute angle , then $\tan \theta = \cdots\cdots\cdots$

(a) $\sqrt{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$

(52) If $\cos \frac{20 + \theta}{2} = \sin \frac{40 + \theta}{2}$, $0° < \theta < 90°$, then $\theta = \cdots\cdots\cdots$

(a) 20° (b) 30° (c) 45° (d) 60°

(53) The general solution of the equation $\tan 2\theta = \cot \theta$ is $\cdots\cdots\cdots$

(a) $\frac{\pi}{2} + \pi n$ (b) $\frac{\pi}{6} + \frac{\pi}{3} n$ (c) $\frac{\pi}{6} + 2\pi n$ (d) $\frac{\pi}{6} + \pi n$

(54) For every $n \in \mathbb{Z}$, the general solution of the equation : $\tan 2\theta = \cot 4\theta$ is $\cdots\cdots\cdots$

(a) $15° + 360° n$ (b) $90° + 180° n$ (c) $15° + 30° n$ (d) $30° + 180° n$

(55) For every $n \in \mathbb{Z}$, the general solution of the equation : $\csc \theta = \sec (30° + \theta)$ is $\cdots\cdots\cdots$

(a) $60° + 180° n$ (b) $30° + 360° n$ (c) $60° + 360° n$ (d) $30° + 180° n$

(56) If ABCD is a cyclic quadrilateral and $\sin A = \frac{3}{5}$, then $\sin C = \cdots\cdots\cdots$

(a) $\frac{3}{5}$ (b) $-\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $-\frac{4}{5}$

(57) If XYZL is a cyclic quadrilateral , $\cos X = \frac{1}{2}$ then $\sin (270° - Z) =$ ....

(a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(58) In a right-angled triangle and one of its angles is $X°$, if $\sin X = \frac{4}{5}$, then $\cos (90 - X°) =$ ....

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{-4}{5}$ (d) $\frac{4}{5}$

(59) If Δ ABC is an obtuse-angled triangle at A , $\sin A = \frac{4}{5}$, then $\sin (2A + B + C) =$ ....

(a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{-4}{5}$ (d) $\frac{4}{5}$

(60) ABC is a right-angled triangle at B , if $\cos A = \frac{1}{2}$, then the value of $\sin (A + B + 2C) =$ ....

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) zero

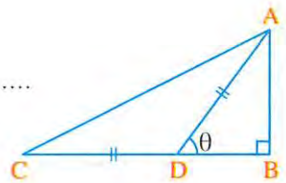
(61) If XYZ is an acute-angled triangle and $\tan Z = \sqrt{3}$, then $\sin (X + y + 2z) =$ ....

(a) $-\sqrt{3}$ (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{-\sqrt{3}}{2}$

(62) If ABC is an acute-angled triangle , then $\cos A + \cos (B + C) =$ ....

(a) -1 (b) zero (c) 1 (d) $\frac{1}{2}$

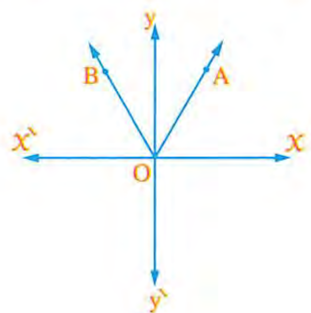
(63) In the opposite figure :



If $D \in \overline{BC}$, AD = DC , $\sin \theta = \frac{4}{5}$, then $\cot \left(270° - \frac{\theta}{2}\right) =$ ....

(a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{2}{3}$

(64) In the opposite figure :

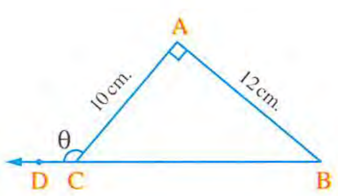


If $A = (2 , 2\sqrt{3})$, $B = (-2 , 2\sqrt{3})$, then $\cot \left(180° - m(\angle AOB)\right) =$ ....

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{-1}{\sqrt{3}}$ (d) $\sqrt{3}$

(65) In the opposite figure :

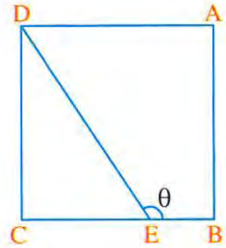


$D \in \overrightarrow{BC}$, AC = 10 cm. , AB = 12 cm. , then $\cot \theta =$ ....

(a) $\frac{6}{5}$ (b) $-\frac{6}{5}$

(c) $\frac{5}{6}$ (d) $-\frac{5}{6}$

(66) In the opposite figure :

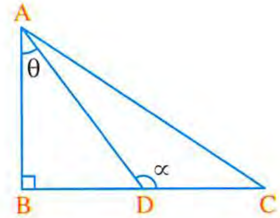


ABCD is a square , CE = 2 BE , then tan θ = ...

(a) $-\frac{3}{2}$ (b) $-\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(67) In the opposite figure :



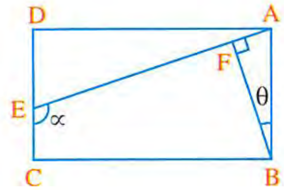
Δ ABC is a right-angled triangle at B , tan θ = $\frac{3}{4}$,

then cos α = ...

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$

(c) $-\frac{4}{5}$ (d) $-\frac{3}{5}$

(68) In the opposite figure :



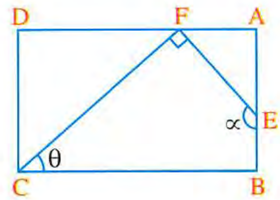
ABCD is a rectangle , tan θ = $\frac{1}{3}$, $\overline{BF} \perp \overline{AE}$,

then cot α = ...

(a) $\frac{1}{3}$ (b) $\frac{3}{4}$

(c) $-\frac{1}{3}$ (d) $\frac{2}{3}$

(69) In the opposite figure :



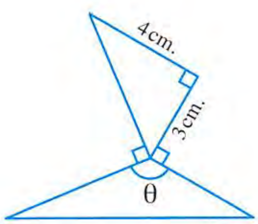
ABCD is a rectangle , cos θ = $\frac{3}{4}$, $\overline{EF} \perp \overline{FC}$,

then cos α = ...

(a) $\frac{3}{5}$ (b) $-\frac{4}{5}$

(c) $-\frac{3}{4}$ (d) $\frac{3}{4}$

(70) In the opposite figure :

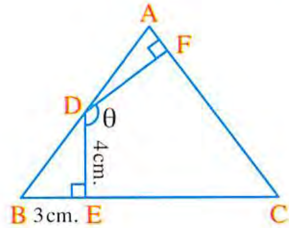


cos θ = ...

(a) $\frac{3}{5}$ (b) $-\frac{3}{5}$

(c) $-\frac{4}{3}$ (d) $-\frac{4}{5}$

(71) In the opposite figure :



ABC is an isosceles triangle in which

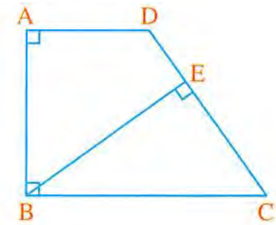
AB = AC , D ∈ $\overline{AB}$, $\overline{DE} \perp \overline{BC}$, $\overline{DF} \perp \overline{AC}$

, m (∠ EDF) = θ , DE = 4 cm. , BE = 3 cm.

, then cos θ = ...

(a) $\frac{3}{5}$ (b) $-\frac{3}{5}$ (c) $-\frac{4}{5}$ (d) $\frac{4}{5}$

(72) In the opposite figure :



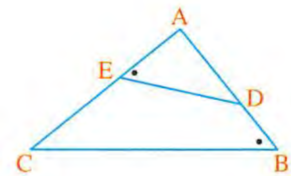
If 3 BE = 4 CE

, then tan (∠ ADC) = ...

(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$

(c) $\frac{3}{4}$ (d) $-\frac{3}{4}$

(73) In the opposite figure :



m (∠ AED) = m (∠ B)

, then cos C + cos (∠ BDE) = ...

(a) 1 (b) – 1 (c) π (d) zero

Second Essay questions

1 Find the value of each of the following :

(1) sin 150°	(2) sec 210°	(3) tan 240°	(4) cos (– 150°)
(5) tan 225°	(6) csc $\frac{11\pi}{6}$	(7) cot 780°	(8) cos (– 900°)
(9) sin $\left(\frac{-4\pi}{3}\right)$	(10) sec $\left(\frac{-2\pi}{3}\right)$	(11) sec (– 480°)	(12) sin $\left(\frac{-7\pi}{4}\right)$

2 Find the value of each of the following :

(1) cos 120° + tan 225° + csc 330° + cos 420° «– 1»

(2) sin 390° cos (– 60°) + cos 30° sin 120° «1»

(3) sin 150° cos (– 300°) + cos (930°) cot 240° «$-\frac{1}{4}$»

(4) $\tan \frac{2\pi}{3} \sec \frac{11\pi}{3} + \cot \frac{11\pi}{6} \csc \frac{19\pi}{6} + \tan \frac{25\pi}{6} \csc \left(\frac{-19\pi}{3}\right)$ «$-\frac{2}{3}$»

3 Prove each of the following equalities :

(1) cos (– 300°) sin 420° – cos 750° cos 660° = zero

(2) sin 600° cos (– 30°) + sin 150° cos (– 240°) = – 1

(3) sin 480° cos (– 60°) + cos 300° sin (– 120°) = zero

(4) sin 150° tan 225° + cos 315° sec (– 120°) + sin (– 135°) csc 210° = $\frac{1}{2}$

4 **If the terminal side of an angle of measure θ in its standard position intersects the unit circle at the point $\left(-\frac{3}{5}, \frac{4}{5}\right)$, find :**

(1) $\sin(180° + \theta)$
(2) $\cos\left(\frac{\pi}{2} - \theta\right)$
(3) $\tan(360° - \theta)$
(4) $\csc\left(\frac{3\pi}{2} - \theta\right)$
(5) $\sec(\theta + \pi)$
(6) $\sin(\theta - \pi)$

5 **If the directed angle of measure θ in the standard position , its terminal side passes by the point $\left(\frac{\sqrt{5}}{3}, \frac{2}{3}\right)$, find the following trigonometric functions :**

(1) $\sin(270° + \theta)$
(2) $\sec(270° + \theta)$
(3) $\csc\left(\theta + \frac{\pi}{2}\right)$
(4) $\tan\left(\frac{\pi}{2} - \theta\right)$
(5) $\cot(\theta - 180°)$
(6) $\sec(-\theta)$

6 **If θ is the measure of a positive acute angle in the standard position and its terminal side intersects the unit circle at the point B $\left(x, \frac{3}{5}\right)$, find the value of :**

$\sin(90° - \theta) + \tan(90° - \theta)\cos(90° + \theta)$ « zero »

7 **If $\sin\theta = \frac{3}{5}$ where $90° < \theta < 180°$, find the value of :**

(1) $\cos(180° - \theta)$
(2) $\tan(180° + \theta)$
(3) $\csc(-\theta)$
(4) $\cot(360° - \theta)$
(5) $\sin(90° - \theta)$
(6) $\sin(270° - \theta)$

8 **If $\cos\theta = \frac{-3}{5}$ where $180° < \theta < 270°$, find the value of each of :**

(1) $\csc(180° + \theta)$
(2) $\sec(-\theta)$
(3) $\tan(360° - \theta)$
(4) $\cot(\theta - 90°)$
(5) $\sec(90° + \theta)$
(6) $\tan(270° - \theta)$

9 **Find one of the values of θ , where $0° < \theta < 90°$, which satisfies each of the following :**

(1) $\sin(3\theta + 15°) = \cos(2\theta - 5°)$ « 16° »
(2) $\sec(\theta + 25°) = \csc(\theta + 15°)$ « 25° »
(3) $\tan(\theta + 20°) = \cot(3\theta + 30°)$ « 10° »
(4) $\cos\left(\frac{\theta + 20°}{2}\right) = \sin\left(\frac{\theta + 40°}{2}\right)$ « 60° »
(5) $\tan(\theta + 18°\ 24') = \cot(\theta + 52°\ 10')$ « 9° 43' »

10 **Find the general solution for each of the following equations :**

(1) $\sin 2\theta = \cos\theta$
(2) $\cos 5\theta = \sin\theta$

11 **Find the values of θ in the following cases where $\theta \in \left]0, \frac{\pi}{2}\right]$:**

(1) $\csc(\theta + 15°) = \sec 42°$

(2) $\sin(\theta + 30°) = \cos\theta$

(3) $\sin\theta - \cos\theta = 0$

(4) $\csc\left(\theta - \frac{\pi}{6}\right) = \sec\theta$

(5) $\tan(\theta + 27°) = \cot 2\theta$

(6) $\tan(\theta + 10°) = \cot(4\theta - 10°)$

(7) $\sec(2\theta + 35°) = \csc(3\theta - 10°)$

(8) $\sec\theta = \csc(3\theta - 90°)$

(9) $\sin(4\theta + 48°) = \cos(\theta - 33°)$

(10) $\csc 8\theta = \sec 2\theta$

12 **Find all values of θ , where $\theta \in \left]0, \frac{\pi}{2}\right[$ which satisfies each of the following equations :**

(1) $\tan\theta - 1 = 0$

(2) $2\cos\theta - 1 = 0$

(3) $2\cos\left(\frac{\pi}{2} - \theta\right) = 1$

(4) $2\sin\left(\frac{\pi}{2} - \theta\right) = \sqrt{3}$

13 **Find the S.S. of each of the following equations knowing that $\theta \in]0, 2\pi[$:**

(1) $2\cos\theta + 1 = 0$

(2) $\sec\theta - \sqrt{2} = 0$

(3) $2\sin\theta - \sqrt{3} = 0$

(4) $\cos\theta + 1 = 0$

(5) $2\sin\theta + \sqrt{3} = 0$

(6) $\tan\theta + 1 = 0$

(7) $\sqrt{3}\csc\theta = -2$

(8) $\sin^2\theta = \frac{1}{4}$

14 If $\cos\left(\frac{3\pi}{2} - \theta\right) = \frac{\sqrt{3}}{2}$, $\sin\left(\frac{\pi}{2} + \theta\right) = \frac{1}{2}$

, find the measure of the smallest positive angle θ « 300° »

15 If $\sin(2\theta + 15°) = \cos(\theta + 30°)$, where $0° < \theta < 90°$

, **find the value of :** $\csc^2 2\theta + \cot^2 3\theta + \sec^2 4\theta$ « 9 »

16 If $\frac{\sin(3\theta - 25°)}{\cos(2\theta - 35°)} = 1$, find the value of θ , where $\theta \in \left]0, \frac{\pi}{4}\right[$

, **then find the value of :** $\frac{\sin 18°}{\cos 72°} + \sin(180° - \theta)$ « 30° , $1\frac{1}{2}$ »

17 If $\frac{\tan\theta}{\cot 2\theta} = 1$ where $0° < \theta < 90°$, **find the value of θ , then find the value of :**

$\sin(180° - 3\theta)\cos(360° - 2\theta) + \tan 2\theta \cot(\theta - 180°)$ « 30° , $3\frac{1}{2}$ »

18 If $\tan(\theta - 15°) = \cot(2\theta + 15°)$ where $\theta \in \left]0, \frac{\pi}{2}\right[$

, **find the value of θ , then prove that :** $\frac{1 + \sin(270° + 2\theta)}{1 + \sin(90° + 2\theta)} = \frac{1}{3}$ « 30° »

19 If $\cos\theta = \frac{3}{5}$ where $270° < \theta < 360°$,

find the value of : $\sin(180° - \theta) + \tan(90° - \theta) - \tan(270° - \theta)$ « $-\frac{4}{5}$ »

20 If $13\cos\theta = 12$ where $90° < \theta < 360°$,

find the value of : $13\sin(180° - \theta) - 10\sin^2 45° \tan^2 60° + 50\sin 150°$ « 5 »

21 If $15\tan\theta + 8 = 0$, $90° < \theta < 180°$, find the values of the trigonometric functions of the angle θ , **then find the value of each of :** $2\sin\theta\cos\theta$, $\sec(1080° + \theta)$ « $-\frac{240}{289}$, $\frac{-17}{15}$ »

22 If $\sin\theta = \frac{\sqrt{2}}{2}$, where $\theta \in]0 , \frac{\pi}{2}[$, find the value of θ , then :

(1) **Find the value of :** $\frac{1 - 2\cot(270° - \theta)}{1 + \cos^2(270° + \theta)}$

(2) **Prove that :** $\cos 2\theta = \frac{1 - \tan^2(270° - \theta)}{\csc^2(90° + \theta)}$ « 45° , $\frac{-2}{3}$ »

23 If B $(-5k , -12k)$ is the point of intersection of the terminal side of the directed angle of measure θ in its standard position with the unit circle , $180° < \theta < 270°$

, **find the value of :** $\csc(90° - \theta)\sin(90° + \theta) + 12\tan(270° + \theta)$ « -4 »

24 If $13\sin\theta - 5 = 0$ where $\theta \in]\frac{\pi}{2} , \pi[$,

find the value of each of : $\csc(270° + \theta)$, $\cos(\theta - 270°)$, $\tan(270° + \theta)$,

then prove that : $\sin(270° - \theta) \times \sec(270° + \theta) \times \cot(270° + \theta) = \sin 90°$

25 If $\cos^2\alpha = \frac{9}{25}$, where $90° < \alpha < 180°$, **find the value of :** $25\sin\alpha - 4\cot\alpha$ « 23 »

26 If $\tan\alpha = \frac{3}{4}$ where α is the smallest positive angle , $\tan\beta = \frac{5}{12}$ where $180° < \beta < 270°$

, find the trigonometric functions for each of the two angles α , β ,

then find the value of : $\sin\alpha\cos\beta - \cos\alpha\sin\beta$ « $-\frac{16}{65}$ »

27 If $\sin\alpha = \frac{3}{5}$ where $\alpha \in]\frac{\pi}{2} , \pi[$, $13\cos\beta - 5 = 0$ where $\beta \in]\frac{3\pi}{2} , 2\pi[$,

find the value of : $\cos\alpha\cos\beta + \sin\alpha\sin\beta$ « $-\frac{56}{65}$ »

28 If $25\sin\alpha + 24 = 0$ where $180° < \alpha < 270°$, $5\tan\beta + 12 = 0$

where β is the greatest positive angle , $\beta \in]0° , 360°[$,

find the value of :

(1) $\sin(180° + \alpha) + \cos(180° - \beta)$

(2) $\csc(180° + \alpha)\cot(90° - \beta) - \sec(360° + \alpha)\tan(360° - \beta)$

(3) $\csc(90° + \alpha)\cot(270° + \beta)\tan(270° - \alpha)\csc(270° + \beta)$ «$\frac{187}{325}$, $\frac{85}{14}$, $6\frac{1}{2}$»

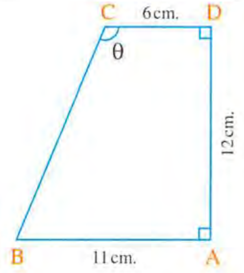
29 If the terminal side of the angle whose measure is $(90° - \theta)$ intersects the unit circle at the point $\left(\frac{5}{13}, y\right)$, find the trigonometric functions for the angle θ where $\theta \in \left]0 , \frac{\pi}{2}\right[$

30 **In the opposite figure :**

ABCD is a trapezium , $m(\angle A) = m(\angle D) = 90°$

, CD = 6 cm. , AD = 12 cm. , AB = 11 cm.

Find : $\sin\theta$

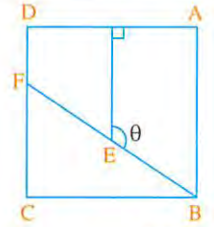


«$\frac{12}{13}$»

31 **In the opposite figure :**

ABCD is a square , 2 DF = FC

Find : $\csc\theta$



«$\frac{\sqrt{13}}{3}$»

Discover the error

32 In one of the mathematical competitions , the teacher asked Karim and Ziad to find the value of $\sin\left(\theta - \frac{\pi}{2}\right)$, then who of them has a correct answer ? Explain your answer.

Karim's answer	Ziad's answer
$\sin\left(\theta - \frac{\pi}{2}\right) = \sin\left(2\pi + \theta - \frac{\pi}{2}\right)$ $= \sin\left(\frac{3}{2}\pi + \theta\right)$ $= -\cos\theta$	$\sin\left(\theta - \frac{\pi}{2}\right) = \sin\left[-\left(\frac{\pi}{2} - \theta\right)\right]$ $= -\sin\left(\frac{\pi}{2} - \theta\right)$ $= -(-\cos\theta) = \cos\theta$

Third Higher skills

1 **Choose the correct answer from those given :**

(1) $\cos 45° \times \cos 46° \times \cos 47° \times \cdots \times \cos 135° = \dots\dots$

(a) zero (b) -1 (c) 1 (d) $\frac{\sqrt{3}}{2}$

(2) $\sin 75° \times \cos 12° \times \sec 15° \times \csc 78° = \cdots\cdots$

(a) $1+\sqrt{2}$ (b) $\sqrt{3}-1$ (c) 2 (d) 1

(3) The points A , B , C are placed on the coordinate system where

A (0 , 0) , B (4 , 1) , C (0 , – 2) , then $\sin (\angle BAC) = \cdots\cdots$

(a) $\frac{3}{4}$ (b) $\frac{-3}{4}$ (c) $\frac{4}{\sqrt{17}}$ (d) $\frac{-4}{\sqrt{17}}$

(4) $\frac{\sec 1° \times \sec 2° \times \cdots \times \sec 88° \times \sec 89°}{\csc 1° \times \csc 2° \times \cdots \times \csc 88° \times \csc 89°} = \cdots\cdots$

(a) zero (b) – 1 (c) 1 (d) 90

(5) $\frac{\sin (60\pi + \theta) + \cos (90\pi + \theta)}{\cos\left(\frac{5\pi}{2} + \theta\right) - \sin\left(\frac{9\pi}{2} + \theta\right)} = \cdots\cdots$

(a) 2 (b) 1 (c) zero (d) – 1

(6) If $7X = \frac{\pi}{2}$, then $\frac{\sin 3X}{\cos 4X} + \frac{\tan 2X}{\cot 5X} = \cdots\cdots$

(a) – 2 (b) – 1 (c) 1 (d) 2

(7) If $X + y = 30°$, then :

First: $\tan (X + 2y) \tan (2X + y) = \cdots\cdots$

(a) – 1 (b) 1 (c) $\sin (X - y)$ (d) $\cos (X - y)$

Second: $\sin (3X + 2y) + \sin (9X + 8y) = \cdots\cdots$

(a) zero (b) 1 (c) $\cos X$ (d) $\cos y$

(8) If $f(X) = \sin 2X$, then $f(\theta) + f\left(\theta + \frac{\pi}{2}\right) + f(\theta + \pi) + f\left(\theta + \frac{3\pi}{2}\right) + \cdots$

$+ f(\theta + 99\pi) + f\left(\theta + \frac{199}{2}\pi\right) = \cdots\cdots$

(a) 1 (b) zero (c) 99 (d) 100

(9) If $\cos^2 \theta = 1$, then $\theta = \cdots\cdots$ where $n \in \mathbb{Z}$

(a) $n\pi$ (b) $\frac{n}{2}\pi$ (c) $2n\pi$ (d) $(2n+1)\pi$

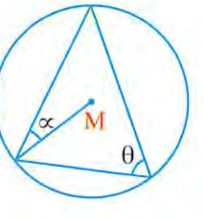
(10) The number of solutions of the equation : $\tan X = -\sqrt{3}$ where $0 \le X \le 15\pi$ is $\cdots\cdots$

(a) 2 (b) 4 (c) 15 (d) 30

(11) In the opposite figure :

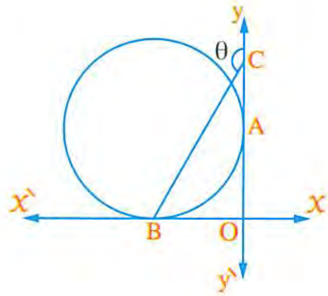
M is the centre of the circle

, then $\tan \theta = \cdots\cdots$



(a) $\tan \alpha$ (b) $\cot \alpha$ (c) $\cos \alpha$ (d) $\sin \alpha$

(12) In the opposite figure :



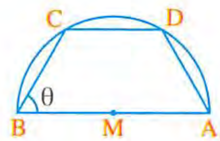
If A (0 , 3) , C (0 , 4)

, then cos θ = ...

(a) $\frac{-4}{5}$ (b) $\frac{3}{4}$

(c) $\frac{-3}{5}$ (d) $\frac{-3}{4}$

(13) In the opposite figure :

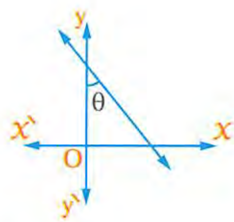


$\overline{AB}$ is a diameter of the semi-circle M

and 13 sin θ = 12 , then cos (∠ ADC) = ...

(a) $\frac{-12}{13}$ (b) $\frac{-5}{13}$ (c) $\frac{5}{13}$ (d) $\frac{12}{13}$

(14) In the opposite figure :



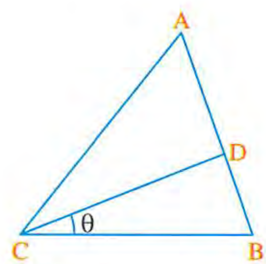
If the equation of the straight line is $y = \frac{-3}{4}x + 5$

, θ is an acute angle between

the straight line and y-axis , then ....

(a) $\cos\theta = \frac{3}{4}$ (b) $\sin\theta = \frac{4}{3}$ (c) $\tan\theta = \frac{4}{3}$ (d) $\sin\theta = \frac{3}{5}$

(15) In the opposite figure :



ABC is an equilateral triangle

, D ∈ $\overline{AB}$ such that : 2 AD = 3 BD

, then tan θ = ...

(a) $\frac{2}{3}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{\sqrt{3}}{5}$ (d) $\frac{2}{5}$

2 Find the value of each of :

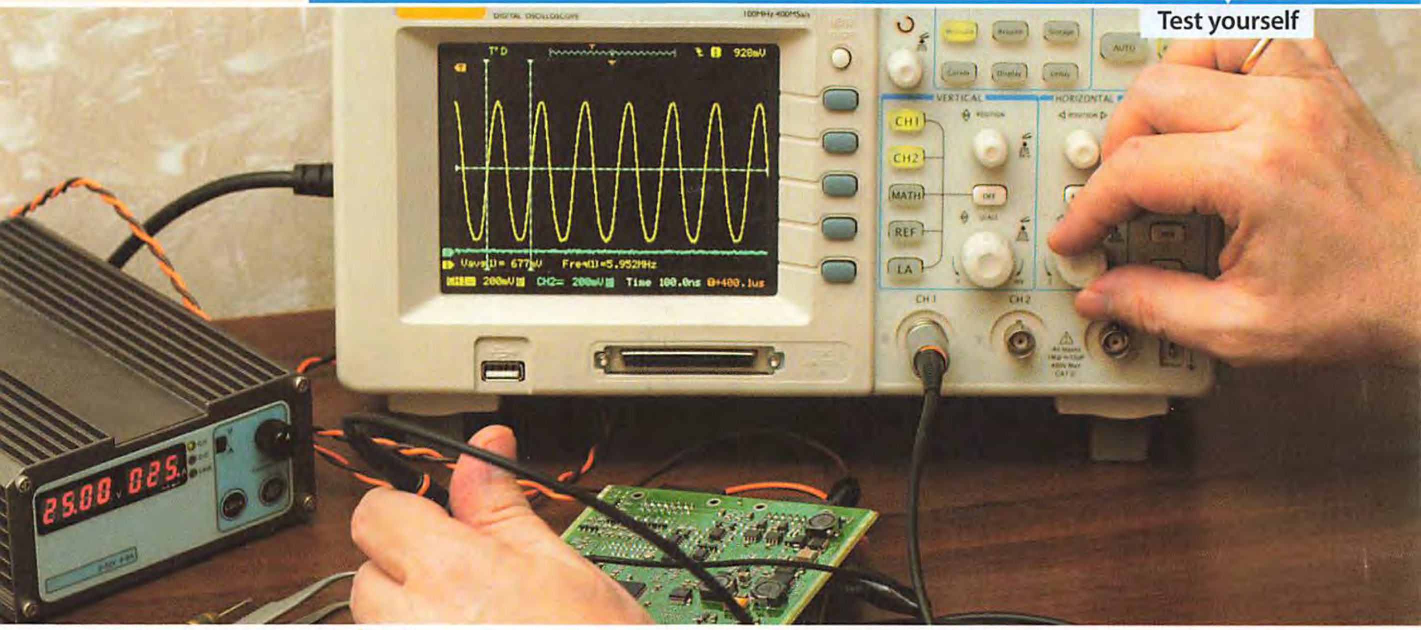
(1) cos 20° + cos 40° + cos 60° + ⋯ + cos 160° + cos 180° « – 1 »

(2) sin 1° + sin 2° + sin 3° + ⋯ + sin 358° + sin 359° « zero »

Exercise 11

Graphing trigonometric functions

Test yourself



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First Multiple choice questions

Choose the correct answer from those given :

(1) The range of the function $f : f(\theta) = \sin \theta$ is ....

(a) $\{-1 , 1\}$ (b) $[-1 , 1]$ (c) $]-1 , 1[$ (d) $]-\infty , \infty[$

(2) If $f(\theta) = \cos 5\theta$, then the range of the function is ....

(a) $\{-5 , 5\}$ (b) $[-1 , 1]$ (c) $]-5 , 5[$ (d) $[-5 , 5]$

(3) The range of the function $f : f(\theta) = 4 \sin 2\theta$ where $\theta \in [0 , 2\pi]$ equal ....

(a) $[-4 , 4]$ (b) $]-4 , 4[$ (c) $[-2 , 2]$ (d) $]-2 , 2[$

(4) If $f(\theta) = \sin \theta$, $\theta \in [0 , \pi[$, then the range of f is ....

(a) $[-1 , 1]$ (b) $[0 , 1]$ (c) $[-1 , 0]$ (d) $\mathbb{R}$

(5) The range of the function $f : f(X) = \frac{\cos X}{5}$ where $X \in \mathbb{R}$ is ....

(a) $\left[-\frac{1}{5} , \frac{1}{5}\right]$ (b) $[-1 , 1]$ (c) $[-5 , 5]$ (d) $\left[0 , \frac{2}{5}\right]$

(6) If the range of the function $f : f(\theta) = 2 a \sin \theta$ is $[-6 , 6]$, then a = ...

(a) 3 (b) -3 (c) 6 (d) a and b together.

(7) The minimum value of the function h : $h(\theta) = 5 \cos 7\theta$ is ....

(a) 5 (b) zero (c) -5 (d) -7

(8) The minimum value of the function $f : f(\theta) = 1 + \sin 3\theta$ is ....

(a) -3 (b) -2 (c) zero (d) -4

(9) The minimum value of the function $f : f(X) = 2 \cos X - 1$ is ....

(a) -3 (b) -2 (c) zero (d) -1

(10) The maximum value of the function g : g (θ) = 4 sin θ is ....

(a) 4 (b) 1 (c) zero (d) ∞

(11) The function $f : f(X) = 3 + \sin(X)$ reaches its maximum value at $X =$ ....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{7\pi}{6}$

(12) If $f(\theta) = 4 \sin 3\theta$, then the sum of the maximum value and the minimum value of the function $f(\theta) =$ ....

(a) 8 (b) 6 (c) 2 (d) zero

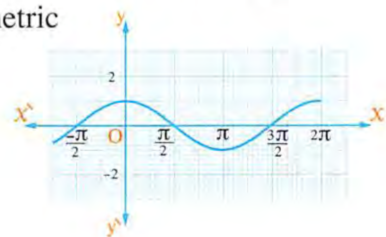
(13) The function $f : f(\theta) = 2 \sin 4\theta$ is a periodic function and its period equals ....

(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

(14) If f is a periodic function and its period equals $\frac{\pi}{2}$, then $f(X)$ could be ....

(a) $4 \sin X$ (b) $\sin 4X$ (c) $\frac{1}{4} \sin X$ (d) $\sin \frac{1}{4} X$

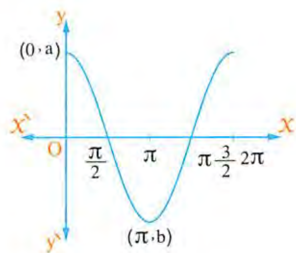
(15) The opposite figure represents the curve of the trigonometric function $y = f(X)$ then the rule of the function is ....



(a) $y = \sin\theta$ (b) $y = \cos\theta$

(c) $y = 2\cos\theta$ (d) $y = 2\sin\theta$

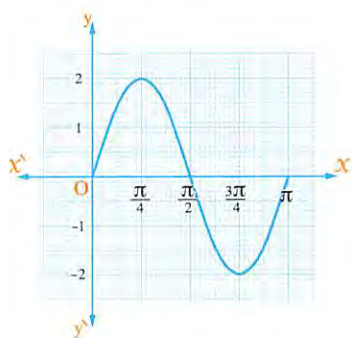
(16) If the opposite figure represents the curve of the function $f : f(X) = \cos X$, then a + b = ...



(a) 1 (b) zero

(c) π (d) 2π

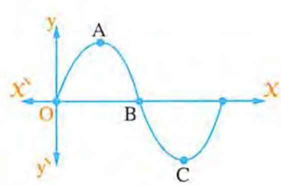
(17) The opposite figure represents one cycle of the trigonometric function $y = f(X)$, then the rule of the function is ....



(a) $y = 2\sin X$ (b) $y = \sin 2X$

(c) $y = 2\sin 2X$ (d) $y = \sin X$

(18) If the opposite figure represents the curve of the function $f : f(X) = 2\sin\frac{1}{3}X$, then the coordinates of the point C ....



(a) $\left(\frac{3}{2}\pi , -1\right)$ (b) $(9\pi , -2)$

(c) $\left(\frac{2}{9}\pi , -2\right)$ (d) $\left(\frac{9}{2}\pi , -2\right)$

(19) Number of times of intersections between the curve $y = \sin X$ with the X-axis on the interval $[0, 2\pi]$ equals ....

(a) 1 (b) 2 (c) 3 (d) 4

Second Essay questions

1 **Find the maximum and minimum values, then write the range of each of the following functions :**

(1) $y = \frac{1}{2} \sin \theta$ (2) $y = \frac{1}{3} \sin 2\theta$ (3) $y = 2 \sin 3\theta$

2 **Represent graphically each of the following functions and from the graph determine the minimum and maximum values of the function and write the range :**

(1) $y = 4 \cos \theta$ where $\theta \in [0, 2\pi]$

(2) $y = 4 \sin \theta$ where $\theta \in [0, 2\pi]$

(3) $y = 2 \cos \theta$ where $\theta \in [-2\pi, 2\pi]$

(4) $y = 3 \sin \theta$ where $\theta \in [-2\pi, 2\pi]$

3 **Represent graphically each of the following functions, and from the graph determine the minimum and maximum values of the function, and write the range :**

(1) $y = \cos 3\theta$ where $0° \le \theta \le 120°$

(2) $y = 5 \sin 2\theta$ where $0° \le \theta \le 180°$

4 Use the graph calculator or graphing program on your computer to graph each of the functions : $y = 4 \cos \theta$, $y = 3 \sin \theta$ **, then find from the graph :**

(1) The range of the function.

(2) The maximum and minimum values of the function.

Third Higher skills

Choose the correct answer from those given :

(1) If $\frac{2 - \sin X}{3} = m$, then ....

(a) $\frac{1}{3} \le m \le 1$ (b) $\frac{2}{3} \le m \le 3$ (c) $1 \le m \le 3$ (d) $2 \le m \le 4$

(2) The function $y = \sin\left(\frac{\pi}{4} + X\right)$ has maximum value at $X =$ ....

(a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$ (c) $\frac{\pi}{4}$ (d) zero

(3) The function $f : f(X) = \sin(bX)$ is a periodic function its period $\frac{2\pi}{3}$, then $b =$ ....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 3 (d) 6

(4) If the two points $(X_1, \cos X_1), (X_2, \cos X_2)$ lie on the curve of the function $f : f(X) = \cos X$, then the greatest value of the expression $(\cos X_1 - \cos X_2) = \cdots\cdots$

(a) 1 (b) 2 (c) zero (d) 180°

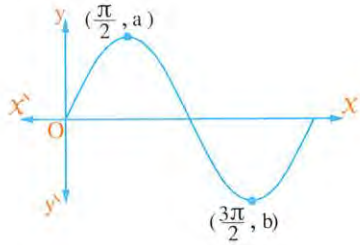
(5) If the function $f : f(X) = a \cos b X$ where $a > 0$ is a periodic function and its period $\frac{\pi}{2}$ and its range $[-1, 1]$, then $\frac{a}{b} = \cdots\cdots$

(a) $\frac{1}{2}$ (b) $\frac{-1}{4}$ (c) $\frac{-1}{2}$ (d) $\frac{1}{4}$

(6) If $f(X) = a \cos b X$ where $a > 0$, $b > 0$ is a periodic function and its period π and its range $[-3, 3]$, the $a + b = \cdots\cdots$

(a) 4 (b) 7 (c) 6 (d) 5

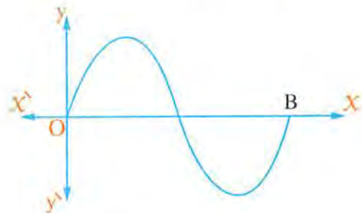
(7) The opposite figure represents the curve $y = \sin X$, then $|a| + |b| = \cdots\cdots$



(a) 1 (b) 2

(c) π (d) 2π

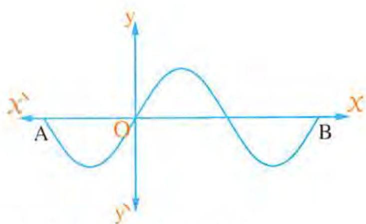
(8) The opposite figure represents the curve $y = 3 \sin \frac{1}{2} X$, then the X-coordinate of B equals $\cdots\cdots$



(a) $\frac{\pi}{2}$ (b) π (c) 2π (d) 4π

(9) **In the opposite figure :**

If $y = \sin X$, then $B - A = \cdots\cdots$



(a) π (b) 2π

(c) 3π (d) 4π

(10) The number of intersections of the curve $y = \sin 3 X$ with X-axis in the interval $[0, 2\pi]$ equals $\cdots\cdots$

(a) 2 (b) 3 (c) 4 (d) 7

(11) If the number of times that the function $f : f(X) = \sin a X$ intersect X-axis is 9 times in the interval $[0, 2\pi]$, then $a = \cdots\cdots$

(a) 3 (b) 6 (c) 9 (d) 4

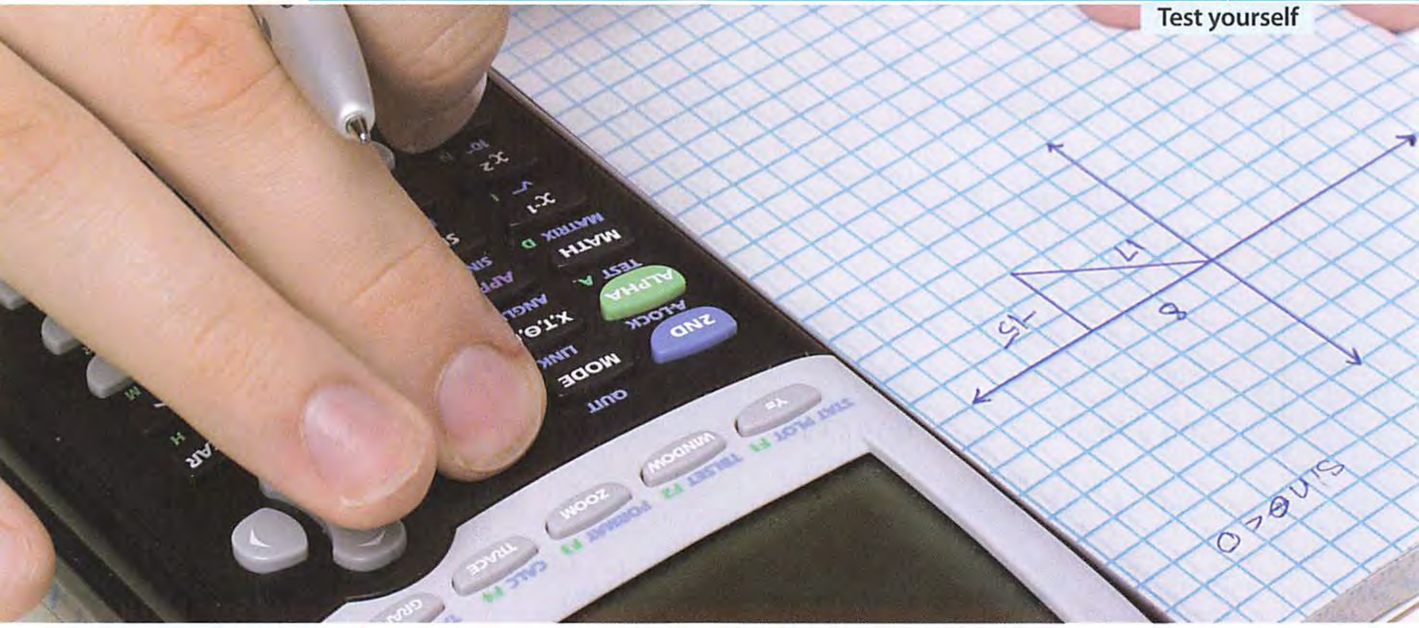
(12) Number of times that the function $f : f(X) = \sin 2 X + 1$ reaches to its maximum value on the interval $[0, 2\pi[$ is $\cdots\cdots$

(a) 1 (b) 2 (c) 3 (d) 4

Exercise **12**

Finding the measure of an angle given the value of one of its trigonometric ratios

Test yourself



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First Multiple choice questions

Choose the correct answer from those given :

(1) If $\theta = \sin^{-1} \frac{\sqrt{3}}{2}$, then $\theta =$ ....

(a) 60° (b) 120° (c) 240° (d) 300°

(2) If $\csc \theta = -2$, $270^\circ < \theta < 360^\circ$, then $\theta =$ ....

(a) 30° (b) 300° (c) 330° (d) 150°

(3) If $\tan \theta = \frac{-1}{\sqrt{3}}$, $90^\circ < \theta < 180^\circ$, then $\theta =$ ....

(a) 30° (b) 120° (c) 150° (d) 210°

(4) If $\tan \theta = 2.1$ and $90^\circ \le \theta \le 360^\circ$, then $\theta \simeq$ ....

(a) 64.5° (b) 115.5° (c) 244.5° (d) 295.5°

(5) If $\tan \theta = 1.8$ and $90^\circ \le \theta \le 360^\circ$, then $\theta \simeq$ ....

(a) $60^\circ\ 57'$ (b) $119^\circ\ 3'$ (c) $240^\circ\ 57'$ (d) $299^\circ\ 3'$

(6) If $5 \cot (90^\circ + \theta) = 12$, where $90^\circ < \theta < 180^\circ$, then $\cos (90^\circ + \theta) =$ ....

(a) $\frac{-12}{13}$ (b) $\frac{12}{13}$ (c) $\frac{5}{13}$ (d) $\frac{-5}{13}$

(7) If $y = \sin (90^\circ - \theta)$, then $\theta =$ ....

(a) $\sin^{-1} y$ (b) $\cos^{-1} y$ (c) $\sin^{-1} \theta$ (d) $\cos^{-1} \theta$

(8) If $\csc \theta = -\sqrt{2}$, then each of the following could be a value of θ except ....

(a) $45°$ (b) $-45°$ (c) $-135°$ (d) $225°$

(9) If $90° < \theta < 180°$, $\tan \theta = -2.4$, then $\sec (90° - \theta) =$ ....

(a) $\frac{-5}{13}$ (b) $\frac{-13}{5}$ (c) $\frac{12}{13}$ (d) $\frac{13}{12}$

(10) $\sin^{-1} 0.7 \simeq$ ....

(a) $44° 25' 37''$ (b) $135° 34' 23''$ (c) $224° 25' 37''$ (d) $315° 34' 23''$

(11) $\sin^{-1} (-0.6) \simeq$ ....

(a) $-36.87°$ (b) $143.13°$ (c) $216.87°$ (d) $323.13°$

(12) $\sin^{-1} \left(\frac{\sqrt{3}}{2}\right) + \cos^{-1} \left(\frac{1}{2}\right) =$ ....

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

(13) If $\sin \theta = \frac{1}{2}$, where θ is measure of the smallest positive angle , then $\theta =$ ....

(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$

(14) If $\cos \theta = 0.436$, where θ is the measure of the smallest positive angle , then $\theta \simeq$ ....

(a) $64° 9'$ (b) $115° 51'$ (c) $244° 9'$ (d) $295° 51'$

(15) If $\sin \theta = \frac{-1}{2}$ where θ is the measure of the smallest positive angle , then $\theta =$ ....

(a) $-30°$ (b) $30°$ (c) $210°$ (d) $150°$

(16) If the terminal side of a directed angle θ in the standard position intersect the unit circle at $\left(\frac{-\sqrt{3}}{2} , y\right)$ where $y \in \mathbb{Z}^+$, then $\theta =$ ....

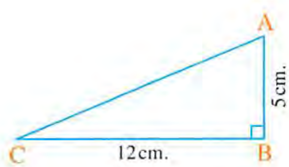
(a) $30°$ (b) $150°$ (c) $210°$ (d) $330°$

(17) If the terminal side of an angle of measure θ in standard position intersects the unit circle at the point $\left(-\frac{1}{\sqrt{2}} , \frac{1}{\sqrt{2}}\right)$, then $\theta =$ ....

(a) $45°$ (b) $135°$ (c) $225°$ (d) $315°$

(18) **In the opposite figure :**

$m (\angle ACB) =$ ....



(a) $\tan^{-1} \left(\frac{12}{5}\right)$ (b) $\sin^{-1} \left(\frac{12}{13}\right)$

(c) $\csc^{-1} \left(\frac{12}{13}\right)$ (d) $\cos^{-1} \left(\frac{12}{13}\right)$

(19) $\cos\left(\frac{1}{2}\right)^{\circ} \times \cos^{-1}\left(\frac{1}{2}\right) \simeq$ ....

(a) 1 (b) $\frac{1}{4}$ (c) 60° (d) $\cos \frac{1}{4}$

Second Essay questions

1 **Find in degrees the measure of the smallest positive angle θ satisfying :**

(1) $\sin \theta = 0.6$
(2) $\cos \theta = 0.7865$
(3) $\tan \theta = 2.4577$
(4) $\tan \theta = -0.8227$
(5) $\sin \theta = -0.4652$
(6) $\cos \theta = -0.5206$
(7) $\cot \theta = 3.6218$
(8) $\cot \theta = -1.4612$
(9) $\sec \theta = 1.0478$
(10) $\csc \theta = -2.5466$
(11) $\sec \theta = -3.57$
(12) $\csc \theta = 2.9811$

2 **If $0^\circ < \theta < 360^\circ$, find θ which satisfies each of the following :**

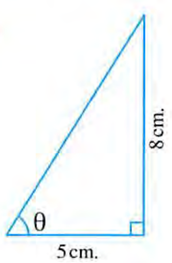
(1) $\sin \theta = 0.86603$
(2) $\cos \theta = -0.4752$
(3) $\csc \theta = -1.2576$
(4) $\tan \theta = 1.5417$
(5) $\cos \theta = -0.642$
(6) $\sec \theta = 2.0515$
(7) $\csc \theta = -1.8715$
(8) $\cot \theta = -2.7012$
(9) $\tan \theta = -2.1456$

3 **If the terminal side of angle θ in the standard position intersects the unit circle at point B , then find m (∠ θ) where $0^\circ < \theta < 360^\circ$ when :**

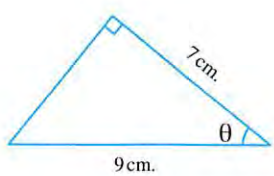
(1) $B\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
(2) $B\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
(3) $B\left(\frac{6}{10}, -\frac{8}{10}\right)$

4 **Find the degree measure of the angle θ in each of the following figures :**

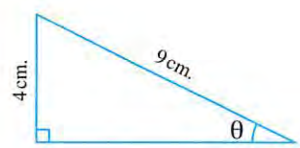
(1)



(2)



(3)



5 **If $\sin \theta = \frac{1}{3}$ and $90^\circ \leq \theta \leq 180^\circ$:**

(1) Calculate the measure of the angle θ to the nearest second.

(2) Find the value of each of the following : $\cos \theta$, $\tan \theta$, $\sec \theta$

6 ABC is a triangle in which $\cos A = -0.5807$, $\tan B = 0.4578$

Find to the nearest minute m (∠ C) « 29° 54' »

7 If $0^\circ < \theta < 360^\circ$ **, find the values of θ in degrees and minutes which satisfy :**

$\tan \theta = \sin 23^\circ\, 48' + \cos 84^\circ\, 32'$ « $26^\circ\, 31'$ or $206^\circ\, 31'$ »

---

8 If $0^\circ < \theta < 360^\circ$ **, find the values of θ in degrees and minutes which satisfy :**

$\cos \theta = \sin 70^\circ - 2 \cos 80^\circ \tan 75^\circ$ « $110^\circ\, 53'$ or $249^\circ\, 7'$ »

---

9 If $\tan \theta = \frac{4}{3}$ where θ is the measure of the greatest positive angle $\theta \in]0 , 2\pi[$

Find the value of α to the nearest minute if :

$\sin \alpha = \sin 150^\circ \sin (-\theta) + \frac{1}{5} \csc (180^\circ + \theta) \tan 225^\circ$ « $40^\circ\, 32'$ or $139^\circ\, 28'$ »

---

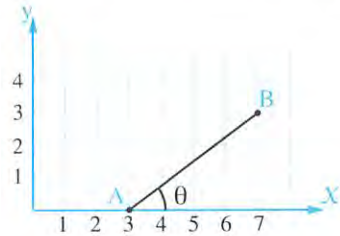
10 If $\sin \alpha = \frac{3}{5}$ where $90^\circ < \alpha < 180^\circ$ **, find θ from the equation :**

$\frac{-5}{4} \cos (360^\circ - \alpha) + \cot (270^\circ - \theta) = 2$ where $0^\circ < \theta < 360^\circ$ « 45° or 225° »

---

11 The opposite figure represents a line segment joining between the two points A (3 , 0) , B (7 , 3)

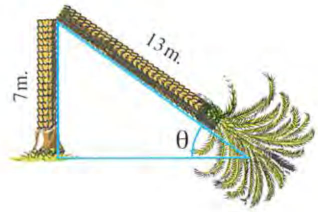
Find the measure of the angle θ included between $\overline{AB}$ and the X-axis.



« $36^\circ\, 52'\, 12''$ »

Discover the error

12 A palm of length 20 metres was broken due to the wind as in the opposite figure , if the length of the vertical part equals 7 metres , and the inclined part is of length 13 metres and θ is the angle which the inclined part makes with the horizontal , find in degrees the measure of θ



Karim's answer

$\because \csc \theta = \frac{13}{7}$

$\therefore \theta = \csc^{-1} \frac{13}{7}$

$\therefore m (\angle \theta) \simeq 32^\circ\, 34'\, 44''$

Omar's answer

$\because \sec \theta = \frac{13}{7}$

$\therefore \theta = \sec^{-1} \frac{13}{7}$

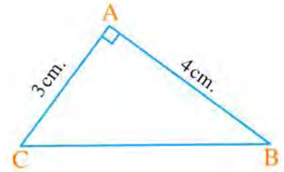
$\therefore m (\angle \theta) \simeq 57^\circ\, 25'\, 16''$

Which answer is right ? Why ?

Third Higher skills

Choose the correct answer from those given :

(1) In the opposite figure :



m (∠ ABC) = ...

(a) $\sin^{-1} \frac{3}{4}$ (b) $\sin^{-1} \frac{4}{3}$

(c) $\tan^{-1} \frac{3}{4}$ (d) $\cot^{-1} \frac{3}{4}$

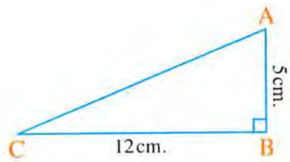
(2) $\sin\left(\cos^{-1} \frac{\sqrt{3}}{2}\right) =$ ....

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) 30° (d) 60°

(3) $\csc\left(\cos^{-1} \text{zero}\right) =$ ....

(a) 1 (b) – 1 (c) $\frac{\pi}{2}$ (d) zero

(4) In the opposite figure :

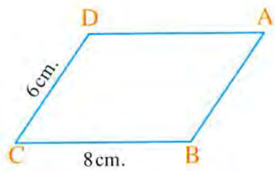


$\sin\left(\tan^{-1} \frac{5}{12}\right) =$ ....

(a) $\frac{5}{12}$ (b) $\frac{5}{13}$

(c) $\frac{12}{13}$ (d) 13

(5) In the opposite figure :



ABCD is a parallelogram , its area = 40 cm^2

, then m (∠ A) ≈ ....

(a) 37° (b) 56° (c) 53° (d) 34°

(6) $\tan^{-1} \frac{1}{\sqrt{3}} + \cot^{-1} \sqrt{3} =$ ....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) $\frac{\pi}{6}$

(7) $\cos^{-1} x + \sin^{-1} x =$ ....

(a) zero (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

Life Applications on Unit Two

From the school book

1 One of the gymansts spins on the play device by an angle of measure 200°. Draw this angle in the standard position , then find its measure in radian. « 3.49^{rad} »

---

2 What is the distance covered by a point on the end of the minute hand in 10 minutes, if the hand length is 6 cm. ? « 2π cm. »

---

3 A satellite revolves around the Earth in a circular path way a full revolution every 6 hours , if the radius length of its path from the center of the Earth is 9000 km. Find its speed in kilometre per hour. « 9424.78 km./hr. »

---

4 A satellite spins around the Earth in a circular path a complete revolution every 3 hours. If the radius length of the Earth approximately equals 6400 km. and the distance between the satellite and the surface of the Earth equals 3600 km. , find the distance which the satellite covers during one hour approximating the result to the nearest km.

« 20944 km. »

---

5 A sundial is used to determine the time during the day through the shadow length falling on a graduated surface to show the clock and its parts. If the shadow rotates on the disk by the rate 15° every hour.

(1) Find the radian measure of the angle which the shadow rotates from it after 4 hours.

(2) After how many hours does the shadow rotate by an angle of radian measure $\frac{2\pi}{3}$?

(3) The radius of a sundial is 24 cm. In terms of π , find the arc length which the rotation of the shadow makes on the edge of the disk after 10 hours.

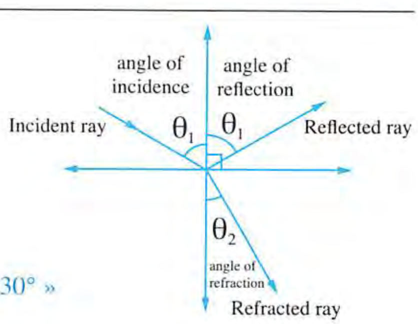
« 1.05^{rad}, 8 hours , 20π cm. »

---

6 When the sun rays fall on a translucent surface , they are reflected with the same angle of incidence but some rays are refracted when they pass through this surface as shown in the opposite figure.

If $\sin \theta_1 = k \sin \theta_2$ and $k = \sqrt{3}$, $\theta_1 = 60°$,

find the measure of angle θ_2 « 30° »



7 When Karim uses his labtop , the measure of the angle of inclination of his labtop on the horizontal is 132° as shown in the opposite figure.

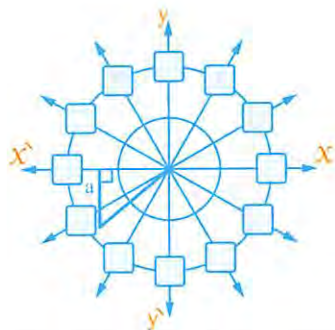
(1) Draw the figure on the coordinate plane such that the angle of measure 132° is in the standard position , then find its related angle.

(2) Write a trigonometric function you can use to find the value of a , then find the value of a to the nearest centimetre. « 17 cm. »

---

8 The spinning wheel is commonly spreading out in the amusement parks. It contains a number of boxes rotating in a circular arc of radius length 12 m.

If the measure of the common angle with the terminal side in the standard position is $\frac{5\pi}{4}$



(1) Draw the angle of measure $\frac{5\pi}{4}$ in the standard position.

(2) Write a trigonometric function you can use to find the value of a , then find the value of a in metre to the nearest hundredth. « 8.49 m. »

---

9 It is possible for the ships entering the port , if the level of water is high as a result of the movement of the ebb and tide , where the depth of water is at least 10 metres.

The movement of the ebb and tide in that day is given by the relation ,

$S = 6 \sin (15\, n)^\circ + 10$ where n is the time elapsed after the mid-night in hour according to 24 hours system.

(1) How many times did the depth of water completely reach 10 metres in the port ?

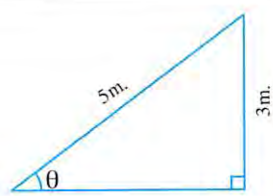
(2) Draw a graph representation to show how the depth of water vary with the movement of the ebb and tide during the day.

(3) How many hours during the day at which the ship be able to enter the port ?

---

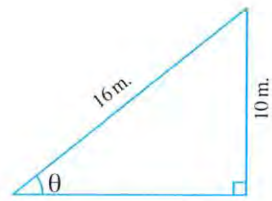
10 A ladder of length 5 metres rests on a wall.

If the height of the ladder from the ground is 3 metres , find in radian the measure of the angle of inclination of the ladder to the horizontal.



« 0.644^{rad} »

11 There is a skiing game in the theme parks.
If the height of one of these games is 10 metres, and its length is 16 metres as in the opposite figure, write a trigonometric function you can use to find the value of the angle θ, then find the value of the angle in degrees to the nearest thousands.



« 38.682° »

---

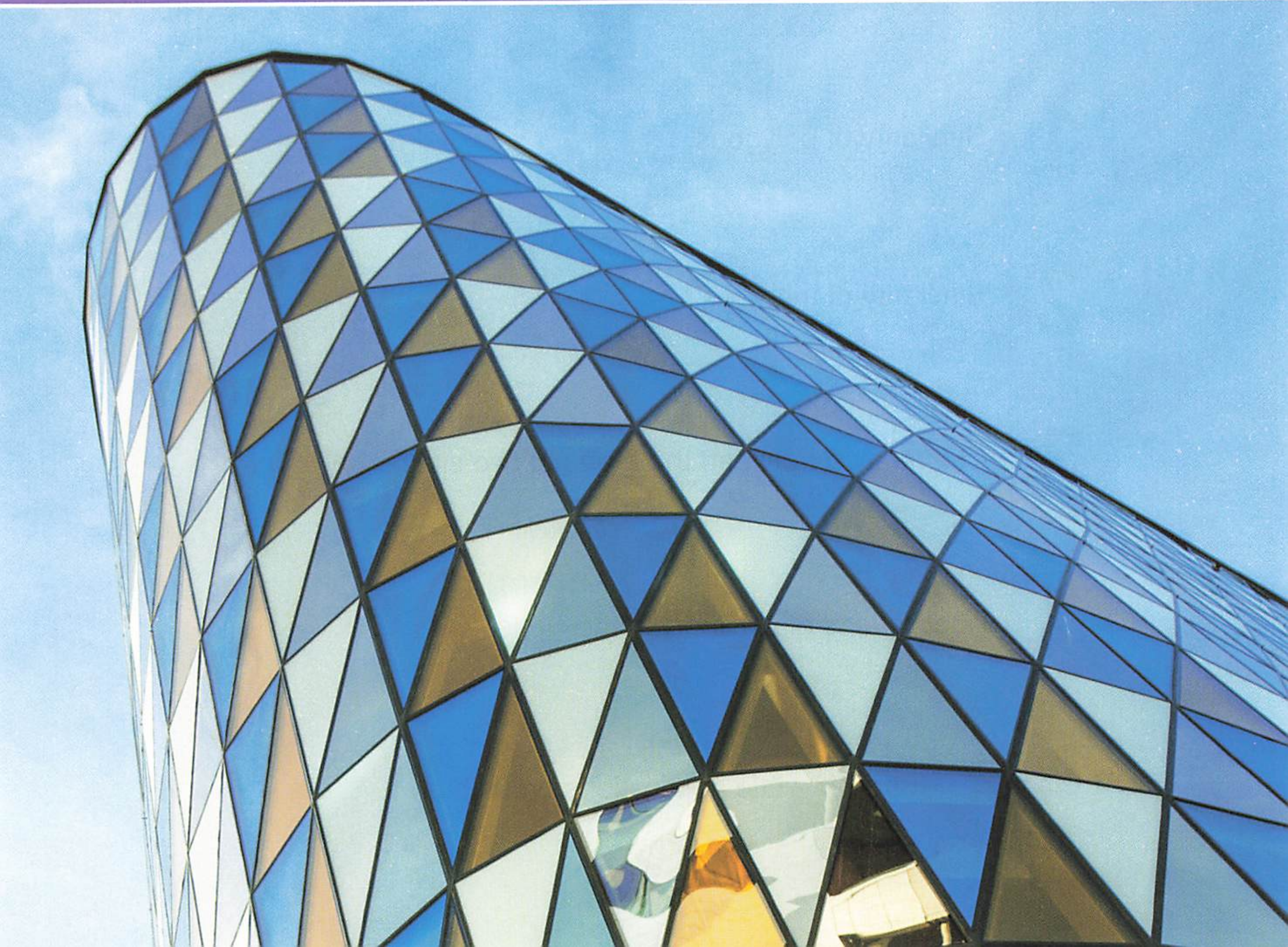
12 Karim descends by his car down a ramp of length 65 m. and its height is 8 m. If the ramp makes an angle θ with the horizontal, find m (∠ θ) in degree measure.

« $7^\circ\ 4'\ 11''$ »

Second **Geometry**

UNIT 3 Similarity.

UNIT 4 The triangle proportionality theorems.



Unit Three | Similarity

Exercise **1** Similarity of polygons.

Exercise **2** Similarity of triangles.

Exercise **3** The relation between the areas of two similar polygons.

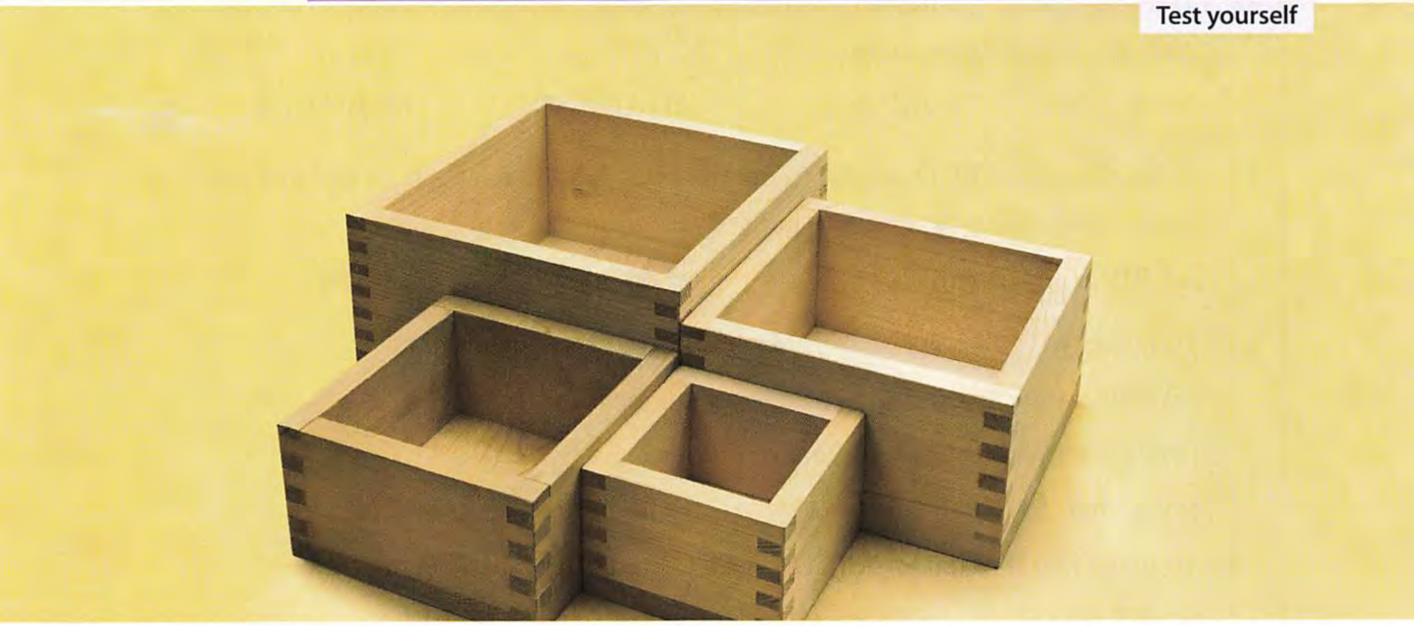
Exercise **4** Applications of similarity in the circle.

At the end of the unit : Life applications on unit three.

Exercise 1

Similarity of polygons

Test yourself



From the school book | Remember | Understand | Apply | Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) If K is the scale factor of similarity of polygon M_1 to polygon M_2 and $0 < K < 1$, then the polygon M_1 is .... to polygon M_2

(a) congruent to (b) enlargement (c) minimization (d) of double area

(2) If k is the scale factor of similarity of polygon M_1 to polygon M_2 and the polygon M_1 is minimization to polygon M_2 , then K may be equal ....

(a) 1 (b) $\frac{3}{5}$ (c) $\frac{3}{2}$ (d) zero

(3) If K_1 is the scale factor of similarity of polygon M_1 to polygon M_2 and K_2 is the scale factor of similarity of polygon M_2 to polygon M_3 , then the scale factor of similarity of polygon M_1 to polygon M_3 is ....

(a) $K_1 + K_2$ (b) K_1K_2 (c) $\frac{K_1}{K_2}$ (d) $\frac{K_2}{K_1}$

(4) The two similar polygons are congruent if the scale factor K satisfies ....

(a) $K = \frac{1}{2}$ (b) $K = 1$ (c) $K > 1$ (d) $0 < K < 1$

(5) If $\Delta ABC \sim \Delta DEF$, $BC = 3\ EF$, then the scale factor of similarity of the two triangles = ...

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 3

(6) The scale factor of similarity between the square ABCD and the square XYZL equals each of the following except ....

(a) AC : XZ (b) AB : YZ (c) $(AB)^2 : (XY)^2$ (d) BC : YZ

(7) If the rhombus ABCD similar to the rhombus XYZL , m (∠ A) = 60° and the scale factor of similarity = $\frac{1}{2}$, then m (∠ Z) = ...

(a) 30° (b) 120° (c) 60° (d) 150°

(8) To make two polygons M_1 and M_2 similar , it is sufficient to have ....

(a) their corresponding angles are equal in measures only.

(b) their corresponding sides are in proportion only.

(c) (a) and (b) together. (d) nothing of the previous.

(9) To make two rhombuses ABCD , XYZL similar it is sufficient to have ....

(a) m (∠ A) = 60° , m (∠ Y) = 120° only.

(b) the perimeter of rhombus ABCD = 2 the perimeter of the rhombus XYZL only.

(c) (a) and (b) together. (d) nothing of the previous.

(10) Which of the following statements is not true ?

(a) each two squares are similar.

(b) each two equilateral triangles are similar.

(c) each two rhombuses are similar.

(d) each two regular polygons with the same number of sides are similar.

(11) The true statement from the following is ....

(a) all the isosceles triangles are similar.

(b) all the right angled triangles are similar.

(c) all the squares uses are similar. (d) all the regular polygons are similar.

(12) Which of the following statements is true ?

(a) all the regular polygons are similar.

(b) all the squares are congruent.

(c) all the equilateral triangles are similar.

(d) all the rhombuses are similar.

(13) If M_1 , M_2 are two similar polygons and the lengths of two corresponding sides are 20 cm. , 16 cm respectively , then the perimeter of polygon M_1 : the perimeter of M_2 = ...

(a) 25 : 16 (b) 41 : 9 (c) 9 : 41 (d) 5 : 4

(14) Two similar polygons , the ratio between their perimeters equal 4 : 9 , then the ratio between the lengths of two corresponding sides is ....

(a) 4 : 9 (b) 2 : 3 (c) 16 : 81 (d) 9 : 4

(15) Two similar polygons , the ratio between the lengths of two corresponding sides is 3 : 4 , if the perimeter of the smaller is 15 cm. , then the perimeter of the bigger is .... cm.

(a) 20 (b) $\frac{80}{3}$ (c) 27 (d) $\frac{95}{4}$

(16) If polygon ABCD ~ polygon XYZL and AB = 32 cm. , BC = 40 cm. , XY = 3 m – 1 , YZ = 3 m + 1 , then m = ...

(a) 3 (b) 2 (c) 1 (d) 4

(17) Two similar rectangles , the dimensions of the first are 12 cm. , 8 cm. and the perimeter of the second equals 60 cm. , then the length of the second rectangle = ... cm.

(a) 12 (b) 18 (c) 24 (d) 16

(18) Two similar rectangles , the dimensions of the first are 4 cm. , 10 cm. and the perimeter of the second rectangle = 140 cm. , then the area of the second rectangle = ... cm^2.

(a) 100 (b) 200 (c) 500 (d) 1000

(19) If Δ ABC ~ Δ DEF , AB = 3 cm. , DE = 6 cm. , EF = 8 cm. , then BC = ... cm.

(a) 4 (b) 3 (c) 2 (d) 15

(20) The perimeter of one triangle of two similar triangles is 74 cm. and the side lengths of the second are 4.5 cm. , 6 cm. , 8 cm. , then the length of the greatest side in the first triangle equals .... cm.

(a) 4 (b) 64 (c) 32 (d) 16

(21) If polygon ABCD ~ polygon XYZL , then $\frac{AB}{BC}$ = ...

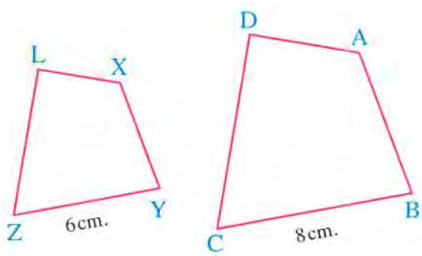
(a) $\frac{YZ}{XL}$ (b) $\frac{AD}{XL}$ (c) $\frac{XL}{AD}$ (d) $\frac{XY}{YZ}$

(22) In the opposite figure :

If the polygon ABCD ~ the polygon XYZL and the perimeter of polygon ABCD = 48 cm. , then the perimeter of polygon XYZL = ... cm.

(a) 48 (b) 36

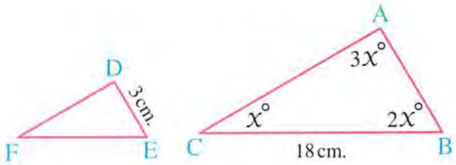
(c) 64 (d) 32



(23) In the opposite figure :

If Δ ABC $\sim \Delta$ DEF ,

then the length of $\overline{FE}$ = ... cm.



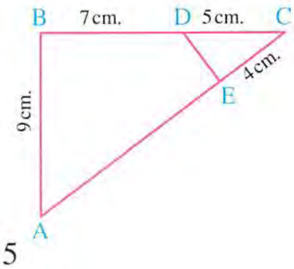
(a) 3 (b) 4 (c) 6 (d) 8

(24) In the opposite figure :

If Δ CBA $\sim \Delta$ CED

using the lengths shown on the figure ,

then ED + EA = ... cm.

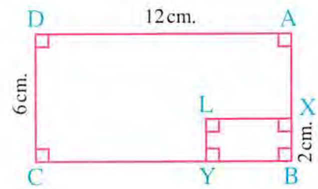


(a) 12 (b) 13 (c) 14 (d) 15

(25) In the opposite figure :

Rectangle ABCD $\sim$ rectangle XBYL ,

then the length of $\overline{YC}$ = ... cm.

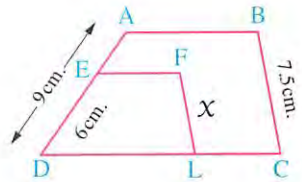


(a) 6 (b) 8 (c) 10 (d) 11

(26) In the opposite figure :

Polygon ABCD ~ polygon EFLD

then x = ... cm.



(a) 5 (b) 3

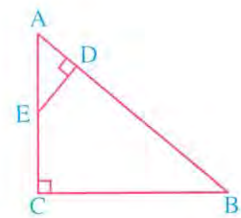
(c) 7.5 (d) 6

(27) In the opposite figure :

If Δ ABC $\sim \Delta$ AED ,

m ($\angle$ B) = 3 x + 10° , m ($\angle$ AED) = x + 30° ,

then m ($\angle$ A) = ...



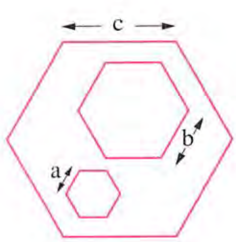
(a) 50° (b) 40° (c) 30° (d) 60°

(28) The opposite figure shows three regular hexagons , the ratio between their sides lengths is as follows

a : b = 1 : 2 , b : c = 3 : 8

if the length of the side of the greatest hexagon = 32 cm.

, then the perimeter of the smallest hexagon = ... cm.

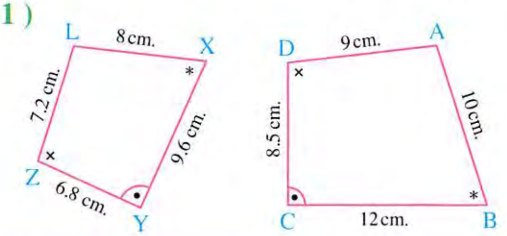


(a) 12 (b) 6 (c) 36 (d) 48

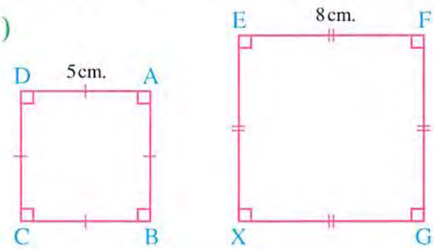
Second Essay questions

1 **Show which of the following pairs of polygons are similar. Write the similar polygons in the order of their corresponding vertices and determine the similarity ratio :**

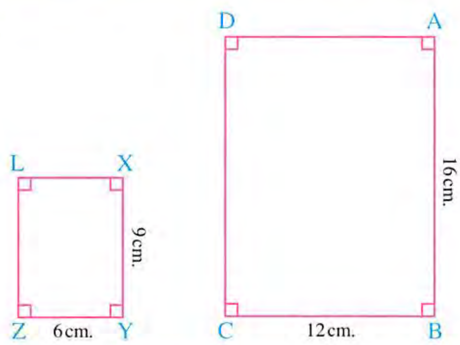
(1)



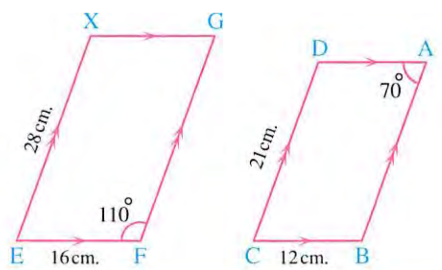
(2)



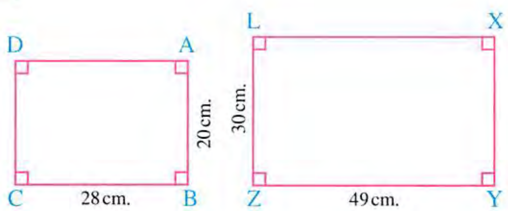
(3)



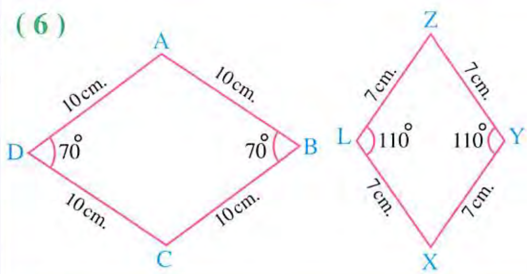
(4)



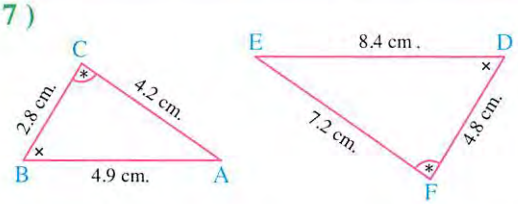
(5)



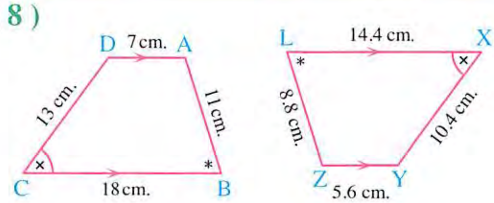
(6)



(7)



(8)



2 **In the opposite figure :**

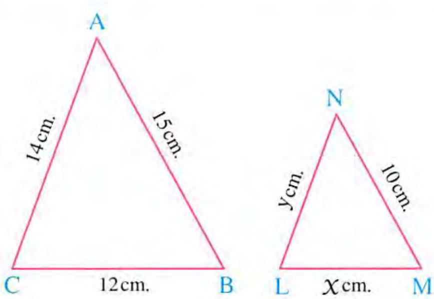
$\Delta ABC \sim \Delta NML$

The lengths of sides are shown on the figures.

Find :

(1) The scale factor of similarity of triangle ABC to triangle NML

(2) The values of x and y



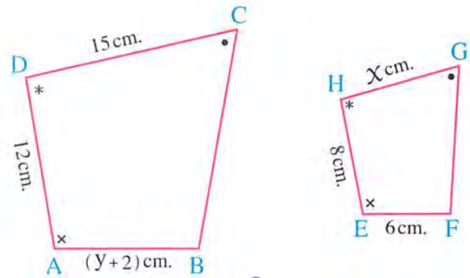
« $\frac{3}{2}$, 8 cm. , $9\frac{1}{3}$ cm. »

3 **In the opposite figure :**

Polygon ABCD ~ polygon EFGH

(1) **Find :** The scale factor of similarity of polygon ABCD to polygon EFGH

(2) **Find the values of :** X and y



« $\frac{3}{2}$, 10 cm. , 7 cm. »

---

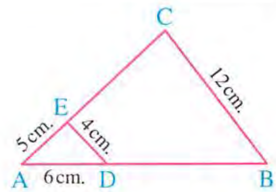
4 **In the opposite figure :**

Δ ADE ~ Δ ABC

Prove that : $\overline{DE}$ // $\overline{BC}$,

and from the lengths shown on the figure ,

find the length of each of : $\overline{BD}$ and $\overline{CE}$



« 12 cm. , 10 cm. »

---

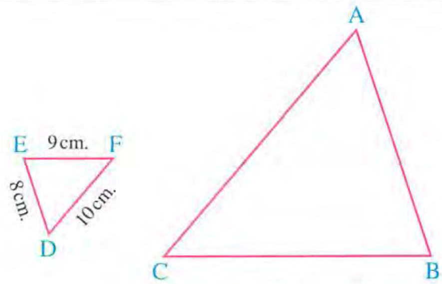
5 **In the opposite figure :**

Δ ABC ~ Δ DEF

, DE = 8 cm. , EF = 9 cm. , FD = 10 cm.

If the perimeter of Δ ABC = 81 cm.

, find the side lengths of : Δ ABC



« 24 cm. , 27 cm. , 30 cm. »

---

6 Two similar rectangles , the dimensions of the first are 8 cm. and 12 cm. , and the perimeter of the second is 200 cm. Find the length of the second rectangle and its area.

« 60 cm. , 2400 cm^2. »

---

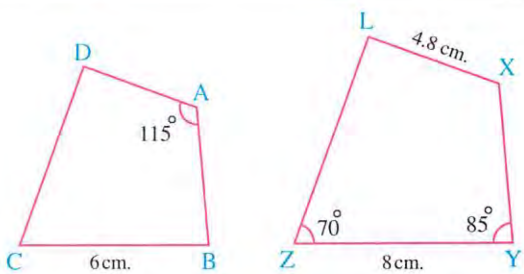
7 **In the opposite figure :**

Polygon ABCD ~ polygon XYZL

(1) **Calculate :** m (∠ XLZ) , length of $\overline{AD}$

(2) If the perimeter of the polygon ABCD = 19.5 cm.

Find : The perimeter of the polygon XYZL



« 90° , 3.6 cm. , 26 cm. »

---

8 **If polygon ABCD ~ polygon XYZL , complete :**

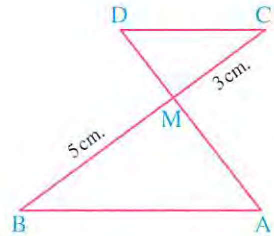
(1) $\frac{AB}{BC} = \frac{\cdots\cdots}{YZ}$

(2) AB × ZL = XY × ....

(3) $\frac{BC + YZ}{YZ} = \frac{\cdots\cdots + LX}{LX}$

(4) $\frac{\text{perimeter of polygon } \cdots\cdots}{\text{perimeter of polygon } \cdots\cdots} = \frac{XY}{AB}$

9 **In the opposite figure :**



Δ MAB $\sim$ Δ MDC

Prove that : $\overline{AB}$ // $\overline{CD}$

and if MC = 3 cm. , MB = 5 cm. , AD = 6 cm.

Find : The length of $\overline{AM}$ « $3\frac{3}{4}$ cm. »

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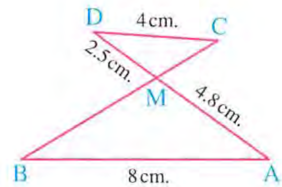
10 **In the opposite figure :**



Δ MAB $\sim$ Δ MCD

Prove that : The figure ABDC is a cyclic quadrilateral.

And if AB = 8 cm. , CD = 4 cm. , MA = 4.8 cm.

, MD = 2.5 cm.

Find : The length of $\overline{BC}$ « 7.4 cm. »

---

11 **Triangle ABC has : AB = 5 cm. , BC = 6 cm. , AC = 9 cm. Find the lengths of the sides of a similar triangle if :**

(1) The scale factor of similarity = 2.5

(2) The scale factor of similarity = 0.6

---

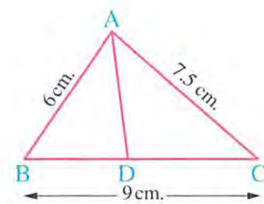
12 **The dimensions of a rectangle are 10 cm. and 6 cm. Find the perimeter and the area of another rectangle similar to it if :**

(1) The scale factor equals 3

(2) The scale factor equals 0.4

---

13 **In the opposite figure :**



Δ ABC $\sim$ Δ DBA

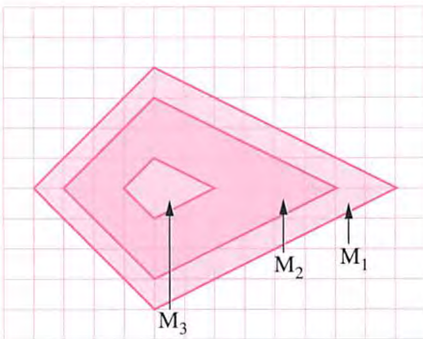
Prove that : $\overline{AB}$ is a tangent to the circle passing through the vertices of Δ ADC and that AB is a mean proportional between BD and BC and if AB = 6 cm. , AC = 7.5 cm.

Find : The length of each of $\overline{AD}$, $\overline{CD}$ « 5 cm. , 5 cm. »

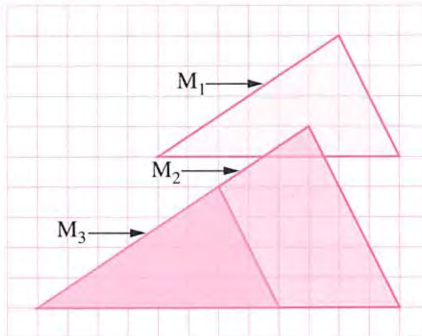
14 **In each of the following figures :** Polygon $M_1 \sim$ polygon $M_2 \sim$ polygon M_3

Find the scale factor of similarity of each of polygon M_1 and polygon M_2 with respect to polygon M_3

(1)

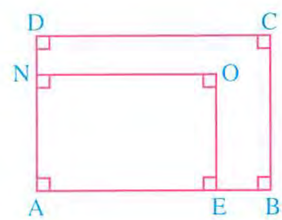


(2)



Third Higher skills

In the opposite figure :



Rectangle ABCD ~ rectangle AEON

Prove that :

Perimeter of rectangle ABCD : perimeter of rectangle AEON

= (AB – AD) : (AE – AN)

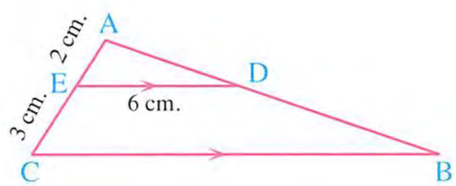
Exercise 2 Similarity of triangles

Test yourself

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :



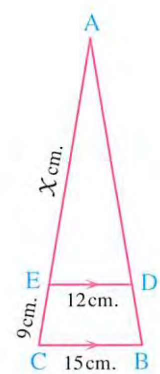
(1) In the opposite figure :

If $\overline{ED} // \overline{BC}$, AE = 2 cm.

, EC = 3 cm. , ED = 6 cm.

, then BC = ... cm.

(a) 9 (b) 15 (c) 12 (d) 10

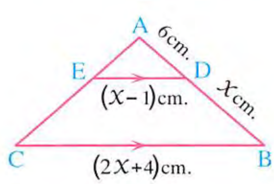


(2) In the opposite figure :

x = ... cm.

(a) 12 (b) 24

(c) 36 (d) 48



(3) In the opposite figure :

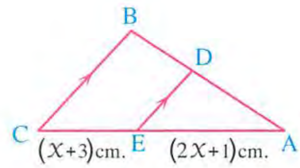
If $\overline{DE} // \overline{BC}$, then x = ...

(a) 10 (b) 30

(c) 3 (d) 24

(4) In the opposite figure :

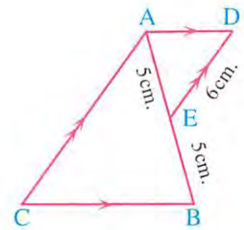
If AD : AB = 3 : 5 , $\overline{DE}$ // $\overline{BC}$, then X = ... cm.



(a) 5 (b) 3
(c) 4 (d) 7

(5) In the opposite figure :

AC = ... cm.

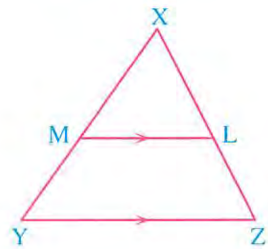


(a) 6 (b) 9
(c) 12 (d) 15

(6) In the opposite figure :

If $\overline{LM}$ // $\overline{YZ}$, $\frac{LM}{YZ} = \frac{4}{7}$

, then $\frac{YM}{MX}$ = ...

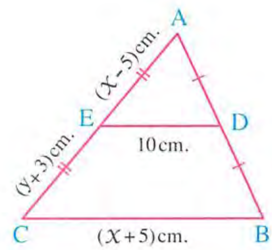


(a) $\frac{11}{4}$ (b) $\frac{3}{4}$
(c) $\frac{4}{3}$ (d) $\frac{4}{11}$

(7) In the opposite figure :

D , E are midpoints of $\overline{AB}$, $\overline{AC}$

, then the length of X + y = ... cm.



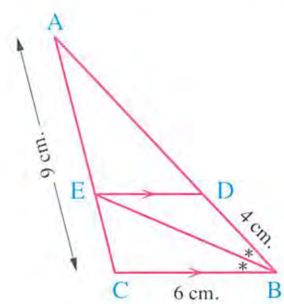
(a) 15 (b) 7
(c) 22 (d) 11

(8) In the opposite figure :

If AC = 9 cm. , BD = 4 cm.

, BC = 6 cm. ,

then the perimeter of Δ ADE = ... cm.

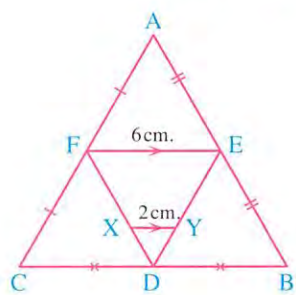


(a) 18 (b) 16
(c) 14 (d) 12

(9) In the opposite figure :

If the perimeter of Δ DXY = 8 cm.

, then the perimeter of Δ ABC = ... cm.



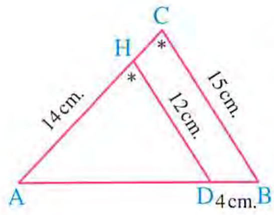
(a) 18 (b) 24
(c) 36 (d) 48

(10) In the opposite figure :

If m (∠ AHD) = m (∠ C) , AH = 14 cm. , HD = 12 cm.

, CB = 15 cm. , DB = 4 cm.

, then AC + AD + AB = ... cm.



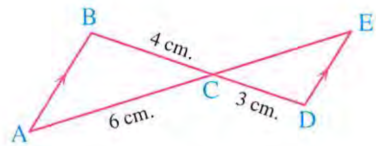
(a) 62.5 (b) 48 (c) 56 (d) 53.5

(11) In the opposite figure :

If $\overline{AB}$ // $\overline{DE}$, CD = 3 cm.

, AC = 6 cm. , BC = 4 cm.

, then CE = ... cm.



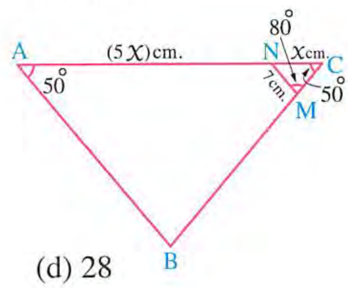
(a) 5.4 (b) 4.5 (c) 8 (d) 2.5

(12) In the opposite figure :

If CN = x cm. , NA = (5 x) cm. , MN = 7 cm.

, m (∠ C) = m (∠ A) = 50° , m (∠ CMN) = 80°

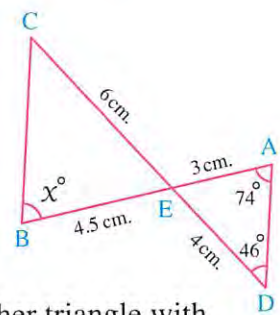
, then AB = ... cm.



(a) 21 (b) 35 (c) 42 (d) 28

(13) In the opposite figure :

x = ...°



(a) 60 (b) 46

(c) 74 (d) 30

(14) Two angles of a triangle with measures 50° , 70° similar to another triangle with angles of measures 50° and ....°

(a) 60 (b) 80 (c) 55 (d) 40

(15) If two triangles , the first has two angles of measures 50° and 60° , the second has two angles of measures 60° and 70° , then the two triangles are ....

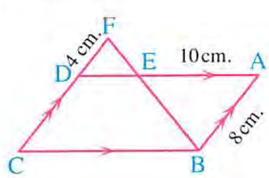
(a) congruent and not similar. (b) similar and not necessary congruent.

(c) congruent and similar. (d) not congruent and not similar.

(16) In the opposite figure :

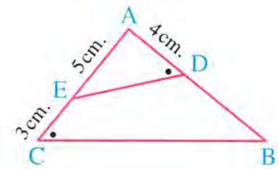
ABCD is a parallelogram , F ∈ $\overrightarrow{CD}$

, then BC = ... cm.



(a) 5 (b) 15 (c) 10 (d) 8

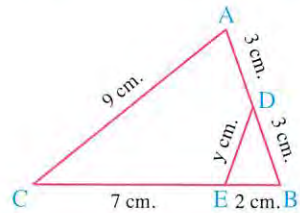
(17) In the opposite figure :



BD = ... cm.

(a) 5 (b) 6 (c) 4 (d) 7

(18) In the opposite figure :

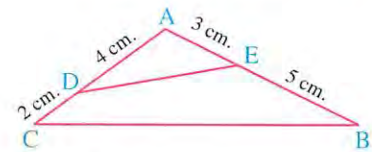


y = ... cm.

(a) 2 (b) 4.5

(c) 3.5 (d) 3

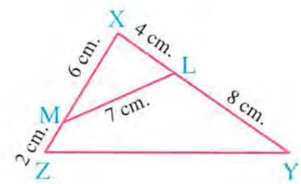
(19) In the opposite figure :



The ratio between the perimeters of the two triangles ADE , ABC is ....

(a) 2 : 1 (b) 3 : 5 (c) 1 : 2 (d) 1 : 4

(20) In the opposite figure :



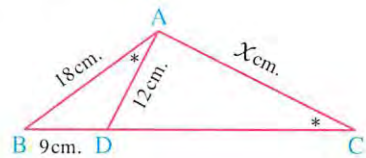
If L $\in \overline{XY}$ where XL = 4 cm. , YL = 8 cm.

, M $\in \overline{XZ}$ where XM = 6 cm. , ZM = 2 cm.

, LM = 7 cm. , then the length of $\overline{YZ}$ = ... cm.

(a) 21 (b) 28 (c) 14 (d) 3

(21) In the opposite figure :

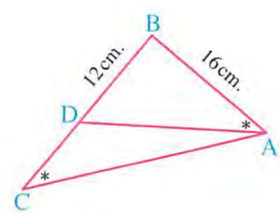


If m (∠ DAB) = m (∠ C)

, then x = ...

(a) 6 (b) 18 (c) 21 (d) 24

(22) In the opposite figure :



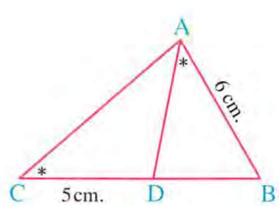
m (∠ BAD) = m (∠ C) , AB = 16 cm.

BD = 12 cm. , then DC = ... cm.

(a) 16 (b) 12

(c) $9\frac{1}{3}$ (d) $23\frac{1}{3}$

(23) In the opposite figure :

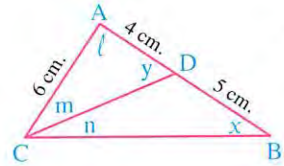


If m (∠ BAD) = m (∠ C)

, then BD = ... cm.

(a) 3 (b) 4 (c) 5 (d) 6

(24) In the opposite figure :

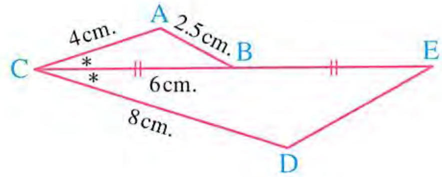


$\mathcal{X}$ = ...

(a) m (b) n

(c) y (d) ℓ

(25) In the opposite figure :



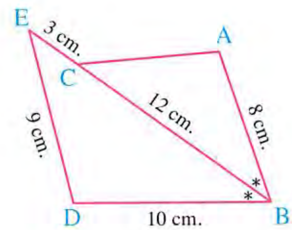
If B is the midpoint of $\overline{CE}$

, then DE = ... cm.

(a) 4 (b) 5

(c) 6 (d) 7

(26) In the opposite figure :

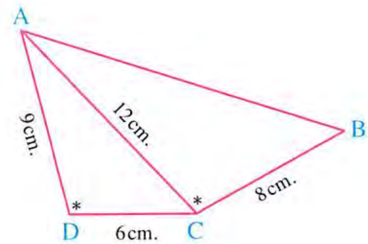


AC = ... cm.

(a) 6.2 (b) 6

(c) 7.2 (d) 7

(27) In the opposite figure :



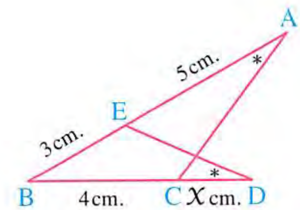
If m (∠ ADC) = m (∠ ACB)

, then AB = ... cm.

(a) 12 (b) 16

(c) 18 (d) 20

(28) In the opposite figure :



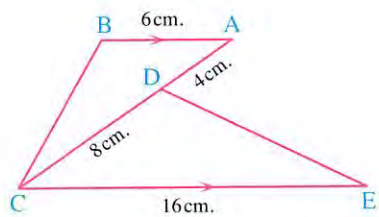
If m (∠ A) = m (∠ D)

, then $\mathcal{X}$ = ...

(a) 5 (b) 4

(c) 3 (d) 2

(29) In the opposite figure :



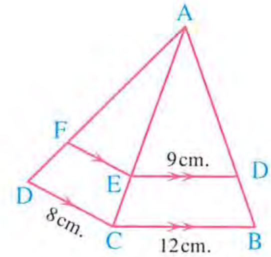
If $\overline{AB}$ // $\overline{EC}$

, then $\frac{ED}{BC}$ = ...

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

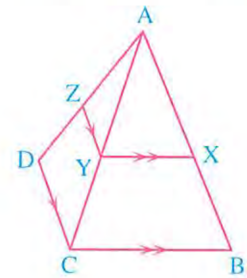
(30) In the opposite figure :



EF = ... cm.

(a) 3 (b) 6
(c) 9 (d) 12

(31) In the opposite figure :



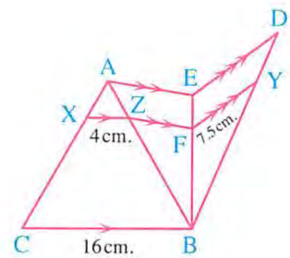
If $\overline{XY} \parallel \overline{BC}$, $\overline{YZ} \parallel \overline{CD}$

and XY = CD , YZ = 2 cm. , BC = 6 cm.

, then the length of $\overline{XY}$ = ... cm.

(a) $2\sqrt{2}$ (b) $3\sqrt{2}$
(c) $2\sqrt{3}$ (d) 4

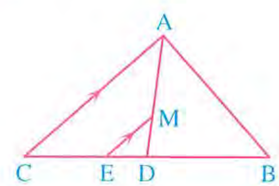
(32) In the opposite figure :



DE = ... cm.

(a) 8 (b) 10
(c) 12 (d) 15

(33) In the opposite figure :



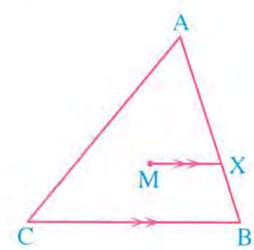
If M is the point of intersection of the medians of Δ ABC

, M $\in \overline{AD}$, $\overline{ME} \parallel \overline{AC}$, ME = 3 cm.

, then the length of $\overline{AC}$ = ... cm.

(a) 3 (b) 6 (c) 9 (d) 12

(34) In the opposite figure :



If M is the point of intersection of

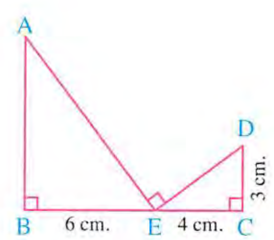
the medians of Δ ABC

, $\overline{MX} \parallel \overline{BC}$, BC = 12 cm.

, then MX = ... cm.

(a) 6 (b) 8 (c) 4 (d) 2

(35) In the opposite figure :

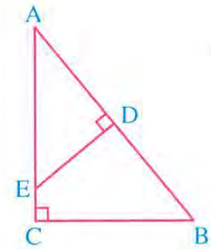


If m ($\angle$ B) = m ($\angle$ C) = m ($\angle$ AED) = 90°

, then the length of $\overline{AB}$ = ... cm.

(a) 12 (b) 8
(c) 10 (d) 15

(36) In the opposite figure :



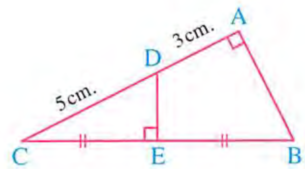
If $\Delta ABC \sim \Delta AED$ and $m(\angle B) = 3X + 20^\circ$

, $m(\angle A) = 60^\circ - 2X$

, then $(\angle AED) = \cdots\cdots\cdots^\circ$

(a) 50 (b) 40 (c) 30 (d) 60

(37) In the opposite figure :

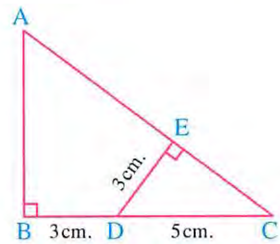


EC = ……… cm.

(a) 3 (b) 4

(c) $2\sqrt{5}$ (d) 5

(38) In the opposite figure :

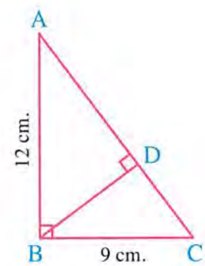


AE = ……… cm.

(a) 5 (b) 6

(c) 7 (d) 8

(39) In the opposite figure :

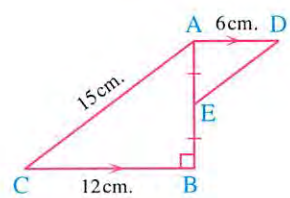


The length of $\overline{BD}$ = ……… cm.

(a) 9.5 (b) 7.2

(c) 7.5 (d) 8

(40) In the opposite figure :



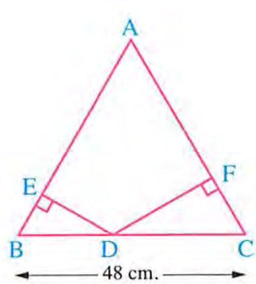
$\overline{AD} \parallel \overline{CB}$, E is the midpoint of $\overline{AB}$

, then the length of $\overline{DE}$ = ……… cm.

(a) 6 (b) 4.5

(c) 3 (d) 7.5

(41) In the opposite figure :



ABC is an isosceles triangle

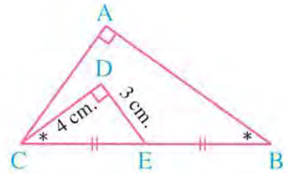
where AB = AC , BC = 48 cm.

, $\frac{DE}{DF} = \frac{5}{7}$, then DC = ……… cm.

(a) 12 (b) 20

(c) 24 (d) 28

(42) In the opposite figure :



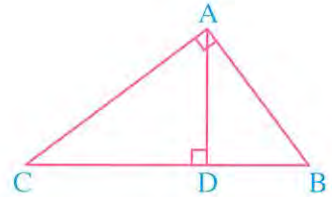
If DE = 3 cm. , DC = 4 cm.

, then area (Δ ABC) = ……… cm^2.

(a) 12 (b) 16

(c) 18 (d) 24

(43) In the opposite figure :



If Δ ABC is a right-angled triangle at A

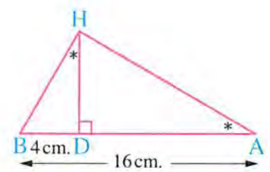
, $\overline{AD} \perp \overline{BC}$, then from the following

the wrong statement is ………

(a) Δ ABC ~ Δ DBA (b) Δ ABC ~ Δ DAC

(c) Δ BAD ~ Δ ACD (d) AD = DB × DC

(44) In the opposite figure :



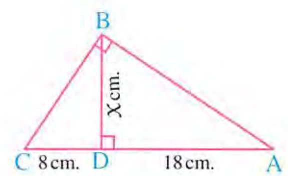
ABH is a triangle , $\overline{HD} \perp \overline{AB}$, m (∠ A) = m (∠ BHD)

, AB = 16 cm. , BD = 4 cm.

, then the length of $\overline{BH}$ = ……… cm.

(a) 4 (b) 8 (c) 12 (d) $8\sqrt{3}$

(45) In the opposite figure :

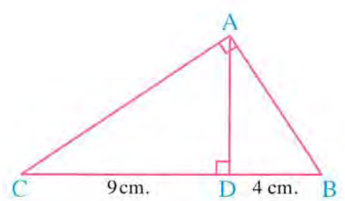


X = ………

(a) $12\sqrt{3}$ (b) 24

(c) 12 (d) $8\sqrt{3}$

(46) In the opposite figure :



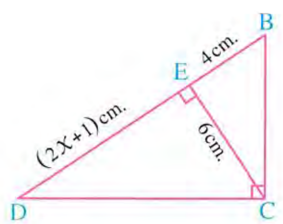
If AD = (X + 2) cm. , BD = 4 cm. , CD = 9 cm.

, then X = ……… cm.

(a) 11 (b) 8

(c) 6 (d) 4

(47) In the opposite figure :

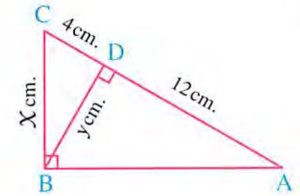


X = ……… cm.

(a) 8 (b) 4

(c) 6 (d) 4.8

(48) In the opposite figure :

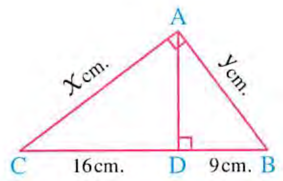


$(X , y) = \dots\dots\dots$

(a) $\left(4\sqrt{3} , 8\right)$ (b) $\left(8 , 4\sqrt{3}\right)$

(c) $\left(4\sqrt{3} , 4\sqrt{3}\right)$ (d) $(8 , 8)$

(49) In the opposite figure :

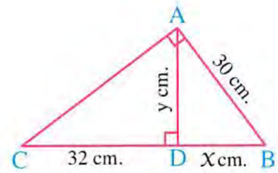


$\frac{y}{X} = \dots\dots$

(a) 1 (b) $\frac{4}{3}$

(c) $\frac{3}{4}$ (d) 2

(50) In the opposite figure :



ABC is a right-angled triangle at A ,

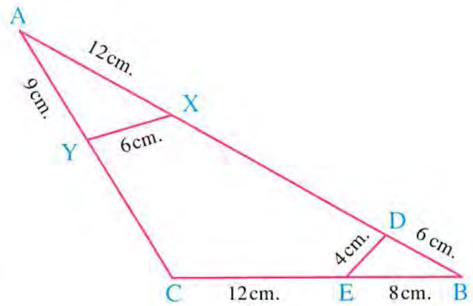
$\overline{AD} \perp \overline{BC}$, AB = 30 cm. , DC = 32 cm.

, then $X + y = \dots\dots$

(a) 36 (b) 48

(c) 42 (d) 52

(51) In the opposite figure :

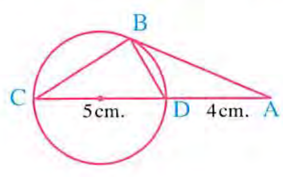


YC = ……… cm.

(a) 9 (b) 10

(c) 11 (d) 12

(52) In the opposite figure :



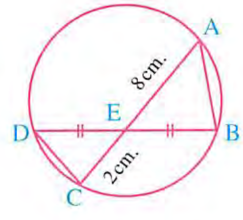
If $\overleftrightarrow{AB}$ is a tangent to the circle

, then AB = ……… cm.

(a) 4 (b) 5

(c) 6 (d) 7

(53) In the opposite figure :



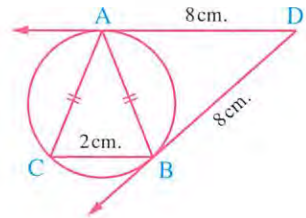
BD = …………… cm.

(a) 8 (b) 4

(c) 16 (d) 2

(54) In the opposite figure :

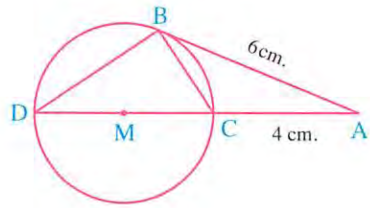
If $\overrightarrow{DA}$, $\overrightarrow{DB}$ are tangents to the circle at A and B respectively , DA = DB = 8 cm. , BC = 2 cm. , then AC = ... cm.



(a) 3 (b) 4 (c) 5 (d) 6

(55) In the opposite figure :

If $\overrightarrow{AB}$ is a tangent to circle M , then the circumference of circle M = ... cm.

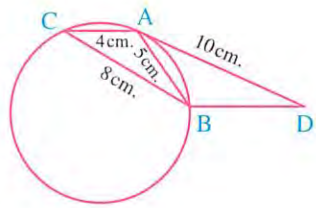


(a) 4π (b) 5π

(c) 6π (d) 9π

(56) In the opposite figure :

$\overline{AD}$ is a tangent to the circle , then the length of $\overline{DB}$ = ... cm.



(a) 5 (b) 4

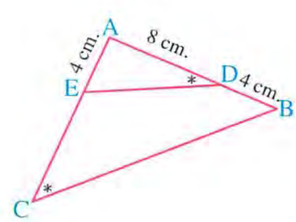
(c) 6 (d) $6\frac{1}{4}$

(57) A person of length 1.6 m. stands beside a light pole if the shadow of the person is 2.4 m. and the length of the shadow of the pole is 6.6 m. , then the length of the light pole equals .... m.

(a) 4.4 (b) 9.9 (c) 8.8 (d) 10.1

(58) By using the opposite figure :

All the following statements is true except ....



(a) BC = 2 DE

(b) DBCE is a cyclic quadrilateral

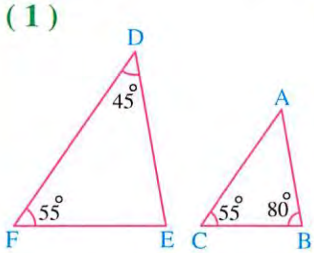
(c) Δ ADE ~ Δ ACB

(d) AD × AB = AE × AC

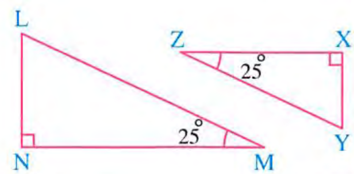
Second Essay questions

1 State in which of the following cases , the two triangles are similar. In case of similarity , state why they are similar :

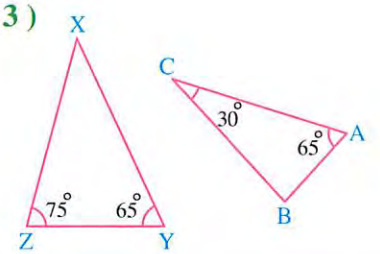
(1)



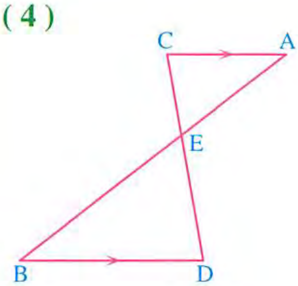
(2)



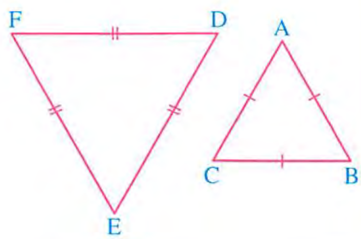
(3)



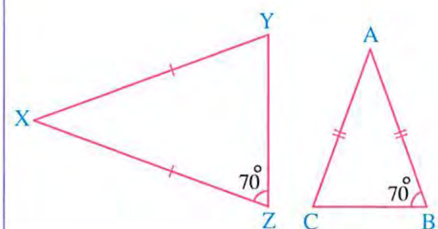
(4)



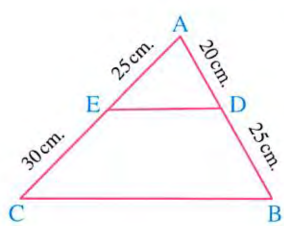
(5)



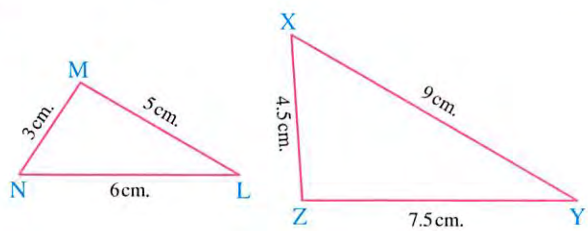
(6)



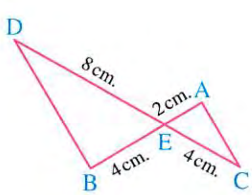
(7)



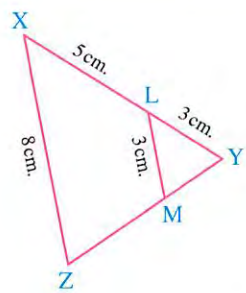
(8)



(9)



(10)

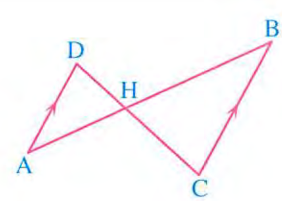


2 In the opposite figure :

$\overline{DA} \parallel \overline{CB}$ Prove that :

(1) Δ AHD $\sim$ Δ BHC

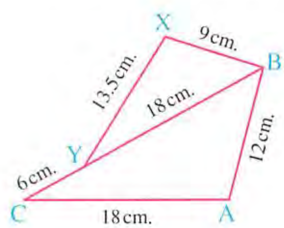
(2) AH $\times$ HC = DH $\times$ HB



3 ABC is a triangle , the lengths of its sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively are 3 cm. , 4.5 cm. , and 6 cm. , DEF is another triangle , the lengths of its sides $\overline{DE}$, $\overline{EF}$ and $\overline{FD}$ respectively are 6 cm. , 4 cm. and 8 cm. Prove that the two triangles are similar , then write them in the same order of corresponding vertices.

---

4 **In the opposite figure :**



B , Y and C are collinear.

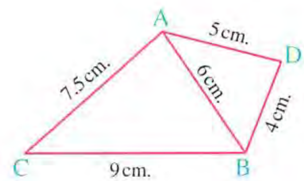
Prove that :

(1) Δ XBY $\sim$ Δ ABC

(2) $\overrightarrow{BC}$ bisects $\angle$ ABX

---

5 **In the opposite figure :**

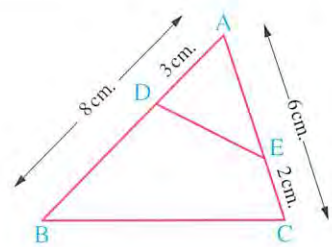


ABC is a triangle in which : AB = 6 cm. , BC = 9 cm. , AC = 7.5 cm. , D is a point outside the triangle ABC where DB = 4 cm. , DA = 5 cm. **Prove that :**

(1) Δ ABC $\sim$ Δ DBA (2) $\overrightarrow{BA}$ bisects $\angle$ DBC

---

6 **In the opposite figure :**



ABC is a triangle in which AB = 8 cm. ,

AC = 6 cm. , D $\in \overline{AB}$,

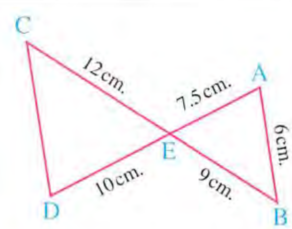
where AD = 3 cm. , E $\in \overline{AC}$,

where EC = 2 cm.

Prove that : Δ AED $\sim$ Δ ABC

---

7 **In the opposite figure :**



$\overline{AD} \cap \overline{BC} = \{E\}$, AE = 7.5 cm. , EC = 12 cm. , BE = 9 cm. ,

ED = 10 cm. , AB = 6 cm.

Prove that : Δ ABE $\sim$ Δ DCE ,

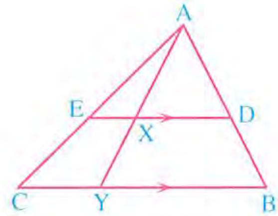
then find the length of : $\overline{CD}$ « 8 cm. »

---

8 In Δ ABC , AC > AB , M $\in \overline{AC}$ where m ($\angle$ ABM) = m ($\angle$ C)

Prove that : $(AB)^2$ = AM $\times$ AC

9 **In the opposite figure :**



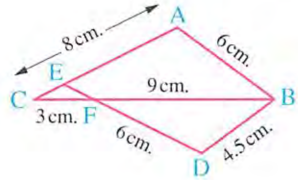
ABC is a triangle , $D \in \overline{AB}$, $\overrightarrow{DE} // \overline{BC}$ and intersects $\overline{AC}$ at E ,

$\overrightarrow{AX}$ is drawn to intersect $\overline{DE}$ and $\overline{BC}$ at X and Y respectively

(1) State three pairs of similar triangles.

(2) **Prove that :** $\frac{DX}{BY} = \frac{XE}{YC} = \frac{DE}{BC}$

10 **In the opposite figure :**



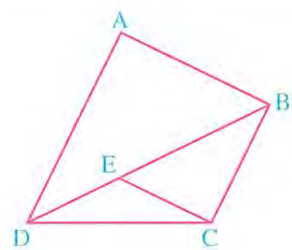
$\overline{BC} \cap \overline{DE} = \{F\}$, AB = 6 cm. ,

BC = 12 cm. , AC = 8 cm. , FC = 3 cm. ,

BD = 4.5 cm. , DF = 6 cm. **Prove that :**

(1) Δ ABC $\sim$ Δ DBF (2) Δ EFC is isosceles.

11 **In the opposite figure :**



ABCD is a quadrilateral ,

$E \in \overline{BD}$ where $\frac{AB}{DA} = \frac{CE}{BC}$, $\frac{BD}{DA} = \frac{EB}{BC}$

Prove that : (1) $\overline{AD} // \overline{BC}$

(2) $\overline{AB} // \overline{CE}$

12 ABC is a triangle in which : AB = 4 cm. , AC = 3 cm. , $D \in \overrightarrow{BA}$ such that AD = 4.5 cm. ,

$E \in \overrightarrow{CA}$ where AE = 6 cm.

Prove that : BCDE is a cyclic quadrilateral.

13 ABC is a triangle , AB = 8 cm. , AC = 10 cm. , BC = 12 cm. , $E \in \overline{AB}$

where AE = 2 cm. , $D \in \overline{BC}$ where BD = 4 cm. **Prove that :**

(1) Δ BDE $\sim$ Δ BAC and deduce the length of $\overline{DE}$ « 5 cm. »

(2) The figure ACDE is a cyclic quadrilateral.

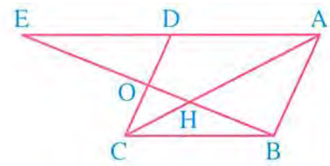
14 XYZ is a right-angled triangle at X , draw $\overrightarrow{XL} \perp \overline{YZ}$ and intersects it at L

Prove that : $\frac{(XY)^2}{(XZ)^2} = \frac{YL}{LZ}$

If XY = 12 cm. and XZ = 16 cm. , **calculate the length of each of :** $\overline{YL}$, $\overline{XL}$

« 7.2 cm. , 9.6 cm. »

15 **In the opposite figure :**



ABCD is a parallelogram , $O \in \overline{DC}$,

$\overrightarrow{BO}$ is drawn intersecting $\overline{AC}$ at H ,

and intersecting $\overrightarrow{AD}$ at E

Prove that : (1) Δ AHE $\sim$ Δ CHB (2) $(HB)^2 = HE \times HO$

---

16 $\overline{AB}$ and $\overline{DC}$ are two chords in a circle , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, where E lies outside the circle , AB = 4 cm. , DC = 7 cm. and BE = 6 cm.

Prove that : Δ ADE $\sim$ Δ CBE **, then find the length of :** $\overline{CE}$ « 12 cm. »

---

17 $\overline{AB}$ is a diameter in a circle , C is a point belonging to the circle , $\overrightarrow{AC}$ is drawn intersecting the tangent to the circle at B at D

Prove that : $(BC)^2 = CA \times CD$

---

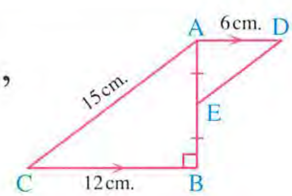
18 ABC is a right-angled triangle at A , $\overrightarrow{AD} \perp \overline{BC}$ to intersect it at D

If $\frac{BD}{DC} = \frac{1}{2}$ and $AD = 6\sqrt{2}$ cm.

, find the length of each of : $\overline{BD}$, $\overline{AB}$ and $\overline{AC}$ « 6 cm. , $6\sqrt{3}$ cm. , $6\sqrt{6}$ cm. »

---

19 **In the opposite figure :**



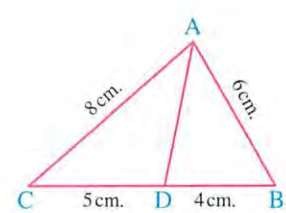
Δ ABC is a right-angled triangle at B , AC = 15 cm. , BC = 12 cm. ,

E is the midpoint of $\overline{AB}$, $\overline{AD}$ // $\overline{BC}$, where AD = 6 cm.

Prove that : Δ ABC $\sim$ Δ EAD and deduce that $\overline{AC}$ // $\overline{DE}$

---

20 **In the opposite figure :**



ABC is a triangle in which : $D \in \overline{BC}$ where BD = 4 cm. ,

DC = 5 cm. If AB = 6 cm. , AC = 8 cm.

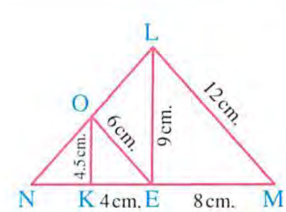
(1) **Prove that :** Δ ABC $\sim$ Δ DBA

(2) **Find the length of :** $\overline{AD}$

(3) **Prove that :** $\overline{AB}$ is a tangent segment for the circle passing through the vertices of Δ ADC « $5\frac{1}{3}$ cm. »

---

21 **In the opposite figure :**



LMN is a triangle , $E \in \overline{MN}$, $K \in \overline{MN}$

, $O \in \overline{LN}$, LM = 12 cm. , ME = 8 cm. ,

LE = 9 cm. , EO = 6 cm. , EK = 4 cm. , KO = 4.5 cm.

Prove that : $\overline{OK}$ // $\overline{LE}$, $\overline{EO}$ // $\overline{ML}$, then find the length of $\overline{NK}$ « 4 cm. »

22 XYZ , LMN are two triangles having equal measures of corresponding angles , YZ = 8 cm. , MN = 12 cm. , $\overrightarrow{XD} \perp \overleftrightarrow{YZ}$ to intersect it at D , and $\overrightarrow{LH} \perp \overleftrightarrow{MN}$ to intersect it at H

If DX = 7 cm. , **find the length of :** $\overline{LH}$ « 10.5 cm. »

---

23 ABC and DEF are two similar triangles , $\overrightarrow{AX} \perp \overline{BC}$ to intersect it at X , $\overrightarrow{DY} \perp \overline{EF}$ to intersect it at Y **Prove that :** BX × YF = CX × YE

---

24 ABC is a triangle , AB = 9 cm. , BC = 12 cm. , CA = 15 cm. , $D \in \overline{BC}$ such that :

$BD = \frac{1}{4} BC$, $\overrightarrow{DH} \perp \overline{BC}$ to intersect $\overline{AC}$ at H

Find tha area of the shape : ABDH « $23\frac{5}{8}$ cm.^{2} »

---

25 ABC is a right-angled triangle at A , $D \in \overline{BC}$ where $\frac{DB}{AB} = \frac{BA}{BC}$

Prove that : (1) $\Delta ABC \sim \Delta DBA$ (2) $\overline{AD} \perp \overline{BC}$

---

26 If $\Delta ABC \sim \Delta DEF$ and X is the midpoint of $\overline{BC}$, Y is the midpoint of $\overline{EF}$, **prove that :** $\Delta ABX \sim \Delta DEY$

---

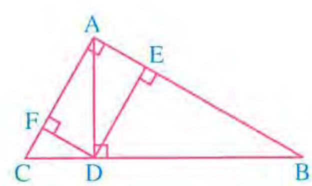
27 ABCD is a quadrilateral inscribed in a circle , its diagonals $\overline{AC}$, $\overline{BD}$ intersect at E ,

If $\frac{BA}{AE} = \frac{BD}{DC}$, **prove that :**

(1) $\Delta ABE \sim \Delta DBC$ (2) $\overrightarrow{BD}$ bisects ∠ ABC

---

28 **In the opposite figure :**



ABC is a right-angled triangle at A

, $\overline{AD} \perp \overline{BC}$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{AC}$

Prove that : (1) $\Delta ADE \sim \Delta CDF$

(2) Area of the rectangle AEDF = $\sqrt{AE \times EB \times AF \times FC}$

---

29 ABCD is a rectangle , draw $\overrightarrow{DF} \perp \overline{AC}$ to intersect $\overline{AC}$ in E and $\overline{BC}$ in F

Prove that : The area of the rectangle ABCD = $\sqrt{AE \times AC \times DE \times DF}$

---

30 ABCD is a trapezium in which : $\overline{AD} \parallel \overline{BC}$, its two diagonals $\overline{AC}$, $\overline{BD}$ intersect at M

Prove that : MA × MB = MC × MD , and if AD = 9 cm. , BC = 12 cm. , AC = 14 cm. , **calculate the length of :** $\overline{MA}$ « 6 cm. »

31 ABC is a triangle , $D \in \overline{BC}$, $\overline{AD}$ is drawn and point H is assumed on it , then $\overrightarrow{HX}$ is drawn // $\overline{AB}$ to intersect $\overline{BD}$ at X , and $\overrightarrow{HY}$ is drawn // $\overline{AC}$ to intersect $\overline{DC}$ at Y

Prove that : (1) $\Delta ABC \sim \Delta HXY$ (2) $XY \times AD = BC \times DH$

---

32 $\overline{AB}$ is a diameter in circle M , $C \in \overrightarrow{AB}$ lying outside the circle , $\overline{CD}$ is drawn tangent to the circle at point D , then $\overrightarrow{DH} \perp \overrightarrow{AB}$ to intersect it at H

Prove that : $(CD)^2 = CH \times CM = CB \times CA$

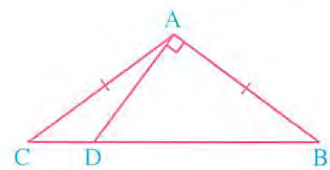
---

33 **In the opposite figure :**

ABC is an obtuse-angled triangle at A ,

AB = AC , $\overrightarrow{AD} \perp \overline{AB}$ and intersects $\overline{BC}$ at D

Prove that : $2\,(AB)^2 = BD \times BC$



---

34 ABCD is a trapezium , $\overline{AD}$ // $\overline{BC}$, m (∠ A) = 90° , $E \in \overline{BD}$

, where AB × EC = DE × BD , CD × BD = DA × EC

Prove that : $(BC)^2 = (AB)^2 + (AD)^2 + (CD)^2$

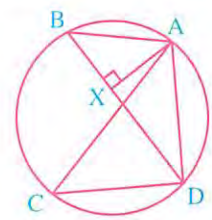
---

35 **In the opposite figure :**

$\overline{AX} \perp \overline{BD}$, $\frac{BX}{CD} = \frac{BA}{CA}$ **Prove that :**

(1) $\Delta BXA \sim \Delta CDA$

(2) $\overline{AC}$ is a diameter in the circle.



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36 ABC is a triangle in which AB = AC , $E \in \overrightarrow{BC}$, $E \notin \overline{BC}$, $D \in \overrightarrow{CB}$, $D \notin \overline{CB}$ where $(AB)^2 = DB \times CE$ **Prove that :** $\Delta ABD \sim \Delta ECA$

Third Higher skills

Choose the correct answer from those given :

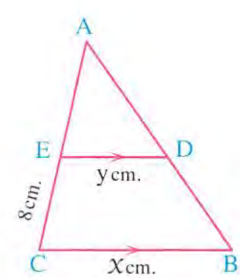
(1) **In the opposite figure :**

If $\frac{X - y}{X + y} = \frac{2}{7}$

, then AE = ... cm.

(a) 16 (b) 15

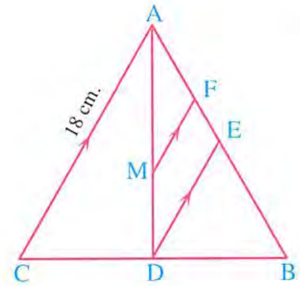
(c) 12 (d) 10



(2) In the opposite figure :

If M is the point of intersection of medians in Δ ABC

, then the length of $\overline{FM}$ = ... cm.



(a) 4 (b) 5

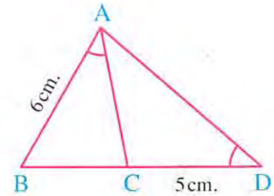
(c) 6 (d) 8

(3) In the opposite figure :

C $\in \overline{BD}$, m ($\angle$ D) = m ($\angle$ BAC)

, AB = 6 cm. , CD = 5 cm.

, then BC = ... cm.



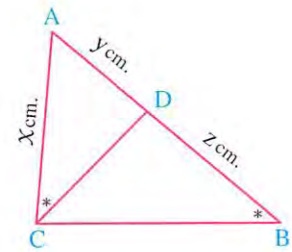
(a) 3 (b) 4

(c) 5 (d) 6

(4) In the opposite figure :

If $x^2 - y^2 = 16$

, then y × z = ... cm^2.



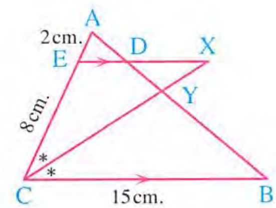
(a) 4 (b) 8

(c) 12 (d) 16

(5) In the opposite figure :

If $\overrightarrow{CX}$ bisects $\angle$ ACB , $\overrightarrow{XD}$ // $\overline{BC}$

, then XD = ... cm.

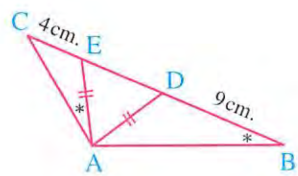


(a) 3 (b) 4

(c) 5 (d) 6

(6) In the opposite figure :

AD = ... cm.



(a) 10 (b) 9

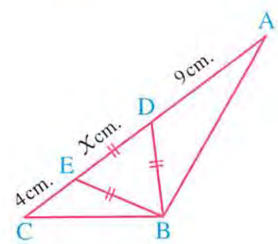
(c) 8 (d) 6

(7) In the opposite figure :

If m ($\angle$ ABC) = 120°

, Δ BDE is an equilateral triangle

, then x = ... cm.



(a) 5 (b) 6

(c) 7 (d) 8

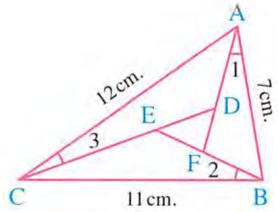
(8) In the opposite figure :

If m (∠ 1) = m (∠ 2) = m (∠ 3)

, then DE : EF : FD = ...

(a) 7 : 11 : 12 (b) 12 : 11 : 7

(c) 12 : 7 : 11 (d) 11 : 12 : 7



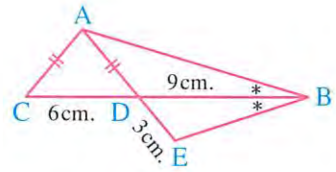
(9) In the opposite figure :

If $\overrightarrow{BD}$ bisects ∠ ABE , BD = 9 cm. , DC = 6 cm.

, DE = 3 cm. , then the perimeter of Δ ADC = ... cm.

(a) 12 (b) 14

(c) 16 (d) 18



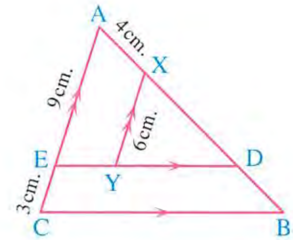
(10) In the opposite figure :

$\overline{XY}$ // $\overline{AC}$, $\overline{DE}$ // $\overline{BC}$

, then DB = ... cm.

(a) 2 (b) 3

(c) 4 (d) 5

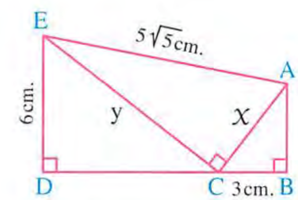


(11) In the opposite figure :

x + y = ... cm.

(a) 12 (b) 15

(c) 18 (d) 21



(12) In the opposite figure :

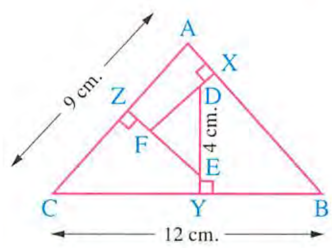
If $\overline{FX} \perp \overline{AB}$, $\overline{DY} \perp \overline{BC}$, $\overline{EZ} \perp \overline{AC}$

, AC = 9 cm. , BC = 12 cm. , DE = 4 cm.

, then EF = ... cm.

(a) 2 (b) 3

(c) 5 (d) 6



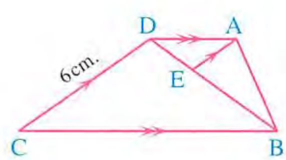
(13) In the opposite figure :

If BE = 2 ED

, then AE = ... cm.

(a) 1 (b) 2

(c) 3 (d) 4

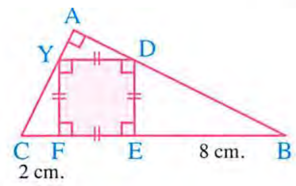


(14) In the opposite figure :

If ABC is a right-angled triangle at A

, DEFY is a square , BE = 8 cm. , FC = 2 cm.

, then the area of the square DEFY = ……… cm^2



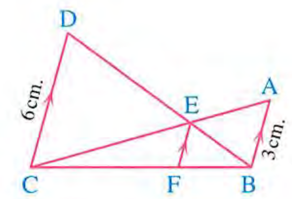
(a) 4 (b) 16

(c) 20 (d) 36

(15) In the opposite figure :

If $\overline{AB}$ // $\overline{EF}$ // $\overline{CD}$

, then EF = ……… cm.



(a) 2.5 (b) 2

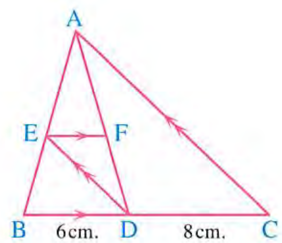
(c) 1.5 (d) 1

(16) In the opposite figure :

$\overline{EF}$ // $\overline{BC}$, $\overline{DE}$ // $\overline{CA}$

If BD = 6 cm. , DC = 8 cm.

, then EF = ……… cm.



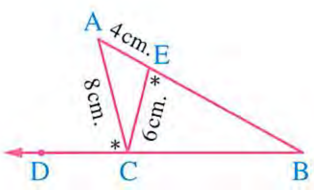
(a) $\frac{12}{7}$ (b) $\frac{18}{7}$

(c) $\frac{24}{7}$ (d) $\frac{28}{7}$

(17) In the opposite figure :

If m (∠ ACD) = m (∠ BEC)

, then BE + BC = ……… cm.



(a) 16 (b) 18

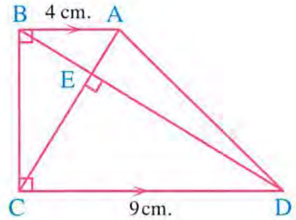
(c) 20 (d) 24

(18) In the opposite figure :

ABCD is a trapezium , m (∠ ABC) = m (∠ DCB) = 90°

, $\overline{AC} \perp \overline{BD}$, then the area of the trapezium

ABCD = ……… cm^2



(a) 13 (b) 26

(c) 39 (d) 60

Exercise **3**

The relation between the areas of two similar polygons

Test yourself

From the school book • Remember • Understand • Apply • Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) The ratio between the perimeters of two similar polygons is 4 : 9 , so the ratio between their areas is ....

(a) 4 : 9 (b) 9 : 4 (c) 2 : 3 (d) 16 : 81

(2) If Δ ABC ~ Δ XYZ , AB = 3 XY , then $\frac{a(\Delta XYZ)}{a(\Delta ABC)}$ = ...

(a) 3 (b) 9 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$

(3) If the ratio between the areas of two similar polygons is 9 : 49 , then the ratio between the lengths of their two corresponding sides is ....

(a) 3 : 7 (b) 9 : 49 (c) 3 : 10 (d) 10 : 3

(4) If the lengths of two corresponding sides in two similar polygons are 7 cm. and 11 cm. , then the ratio between their perimeters is ....

(a) $\frac{49}{121}$ (b) $\frac{7}{18}$ (c) $\frac{7}{11}$ (d) $\frac{11}{18}$

(5) The ratio between the corresponding sides of two similar triangle is 2 : 5 , if the area of the first one is 16 cm^2 , then the area of the second one = ... cm^2.

(a) 40 (b) 80 (c) 100 (d) 120

(6) If the lengths of two corresponding sides in two similar polygons are 12 cm. , 16 cm. and the area of the smaller polygon = 135 cm^2 , then the area of the greater polygon ……… cm^2

(a) 24 (b) 180 (c) 240 (d) 200

(7) If the ratio between perimeters of two similar polygon is 5 : 7 and the area of the greater polygon is 245 cm^2 , then the area of the smaller polygon equals ……… cm^2

(a) 125 (b) 175 (c) 343 (d) 480.2

(8) The ratio between two corresponding sides of two similar squares is 3 : 4 , if the area of the greater square is 48 cm^2 , then the area of the smaller one = ……… cm^2

(a) 16 (b) 12 (c) 20 (d) 27

(9) The ratio between the lengths of the diagonals of two squares is 2 : 5 , if the area of the smaller one is 4 cm^2 , so the area of the greater one is ……… cm^2

(a) 25 (b) 16 (c) 10 (d) 20

(10) The ratio between the areas of two similar polygons is 9 : 25 and the length of one side of the smaller one is 3 cm. , so the length of the corresponding side in the greater one is ……… cm.

(a) $\frac{25}{3}$ (b) $\frac{9}{5}$ (c) 75 (d) 5

(11) If the ratio between areas of two similar triangles equals 9 : 25 and the perimeter of the smaller triangle is 60 cm. , then the perimeter of the greater triangle equals ………

(a) 60 (b) 80 (c) 100 (d) 120

(12) The areas of two similar polygons are 100 cm^2 , 64 $cm.^2$ If the perimeter of the first is 60 cm. , then the perimeter of the other polygon = ……… $cm.^2$

(a) 38.4 (b) 40 (c) 42 (d) 48

(13) If Δ ABC ~ Δ DEF , a (Δ ABC) = 9 a (Δ DEF) and DE = 4 cm. , then AB = ……… cm.

(a) $\frac{4}{3}$ (b) 12 (c) 9 (d) 36

(14) The ratio between the diameters of two circles is 3 : 5 , if the area of the inscribed square in the smaller circle is 27 cm^2 , then the area of the inscribed square in the greater circle equals ……… cm^2

(a) 45 (b) 50 (c) 75 (d) 100

(15) The ratio between two corresponding sides of two similar polygons is 3 : 4 , if the sum of its two areas is 150 cm^2 , then the area of the smaller polygon = ……… cm^2

(a) 54 (b) 96 (c) 75 (d) 52

(16) The ratio between the lengths of two corresponding sides in two similar polygons is 5 : 3 and the difference between their areas is 32 cm^2, then the area of the smaller polygon is ……… cm^2

(a) 18 (b) 50 (c) 32 (d) 16

(17) If the polygon $M_1 \sim$ the polygon M_2 and $\frac{\text{area of polygon } M_1}{\text{area of polygon } M_2} = \frac{9}{16}$

, then it means that ………

(a) the sum of their areas = 25 square units.

(b) the ratio between the two corresponding sides = 9 : 16

(c) the scale factor of the similarity of M_1 to $M_2 = \frac{9}{16}$

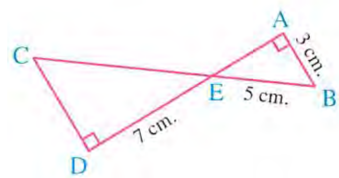
(d) the perimeter of polygon $M_1 = \frac{3}{4}$ the perimeter of polygon M_2

(18) If the polygon ABCD $\sim$ the polygon $A'B'C'D'$, $\frac{AB}{A'B'} = \frac{1}{3}$

, then $\frac{a\text{ (the polygon ABCD)}}{a\text{ (the polygon } A'B'C'D')} + \frac{\text{perimeter of (ABCD)}}{\text{perimeter of } (A'B'C'D')} =$ ………

(a) $\frac{2}{3}$ (b) $\frac{4}{5}$ (c) $\frac{5}{9}$ (d) $\frac{4}{9}$

(19) In the opposite figure :

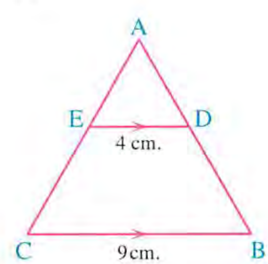


If AB = 3 cm. , BE = 5 cm. , ED = 7 cm.

, then $\frac{a(\Delta ABE)}{a(\Delta CDE)} \times \frac{m(\angle ABE)}{m(\angle DCE)} =$ ………

(a) $\frac{9}{49}$ (b) $\frac{25}{49}$ (c) $\frac{9}{25}$ (d) $\frac{16}{49}$

(20) In the opposite figure :



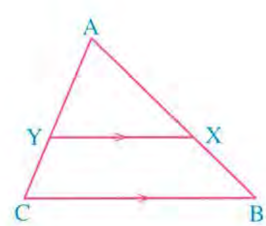
If $\overline{DE} // \overline{BC}$, DE = 4 cm. , BC = 9 cm.

, then $\frac{a(\Delta ADE)}{a(\Delta ABC)} =$ ………

(a) $\frac{16}{81}$ (b) $\frac{81}{65}$

(c) $\frac{65}{81}$ (d) $\frac{16}{65}$

(21) In the opposite figure :



If AX : XB = 5 : 3 , a (Δ ABC) = 25.6 cm^2

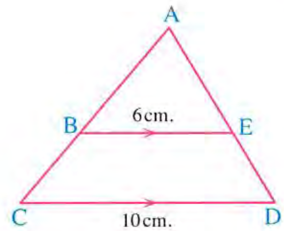
, then a (Δ AXY) = ……… cm^2

(a) 10 (b) 16 (c) 41 (d) 65.5

(22) In the opposite figure :

If $\overline{BE} \parallel \overline{DC}$

, then $\frac{\text{the area of } \Delta \text{ ABE}}{\text{the area of trapezium BCDE}} = \cdots\cdots\cdots$



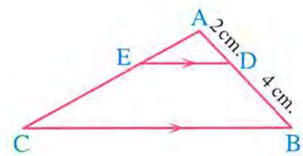
(a) $\frac{25}{81}$ (b) $\frac{3}{5}$

(c) $\frac{9}{16}$ (d) $\frac{9}{25}$

(23) In the opposite figure :

$\overline{DE} \parallel \overline{BC}$, the area of Δ ADE = 8 cm^2.

, then the area of the figure DBCE = ……… cm^2.



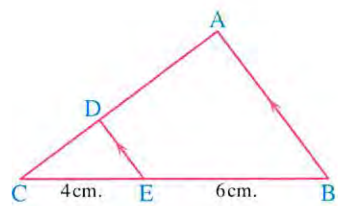
(a) 27 (b) 64

(c) 24 (d) 16

(24) In the opposite figure :

If the area of the figure ABED = 42 cm^2.

, then the area of Δ CED = ……… cm^2.

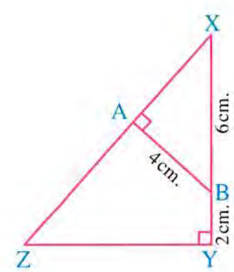


(a) 8 (b) 12

(c) 16 (d) 20

(25) In the opposite figure :

$\frac{a\ (\Delta\ XAB)}{a\ (\Delta\ XYZ)} = \cdots\cdots\cdots$

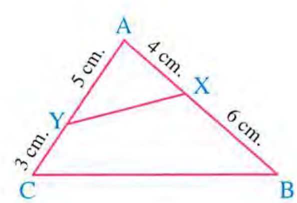


(a) $\frac{3}{5}$ (b) $\frac{5}{16}$

(c) $\frac{9}{25}$ (d) $\frac{4}{5}$

(26) In the opposite figure :

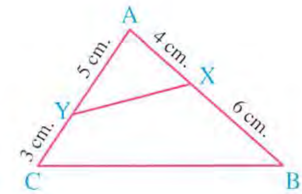
$\frac{a\ (\Delta\ AXY)}{a\ (\Delta\ ACB)} = \cdots\cdots\cdots$



(a) $\frac{5}{8}$ (b) $\frac{2}{5}$

(c) $\frac{5}{2}$ (d) $\frac{1}{4}$

(27) In the opposite figure :



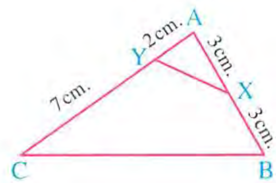
If the area of Δ AXY = 10 cm^2.

, then the area of the shape XBCY = ... cm^2.

(a) 40 (b) 20

(c) 30 (d) 10

(28) In the opposite figure :



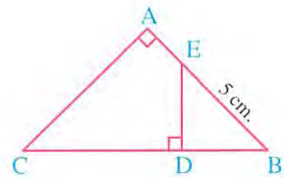
If the area of Δ ABC = 45 cm^2.

, then the area of Δ AXY = ... cm^2.

(a) 22.5 (b) 90

(c) 5 (d) 15

(29) In the opposite figure :

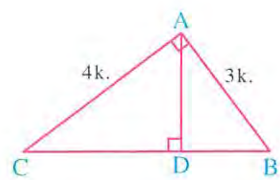


If the area of the shape ACDE = 3 times the area of Δ EBD

, then BC = ... cm.

(a) 7 (b) 8 (c) 9 (d) 10

(30) In the opposite figure :



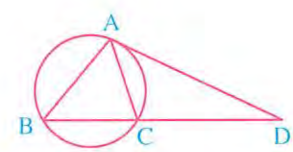
a (Δ ADC) = 160 cm^2.

, then a (Δ ADB) = ... cm^2.

(a) 40 (b) 90

(c) 120 (d) 320

(31) In the opposite figure :

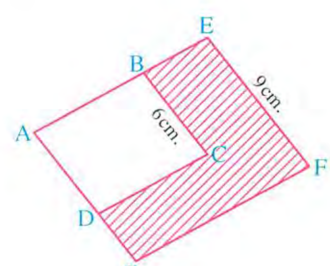


$\overline{AD}$ is a tangent segment to the circle passes through the vertices of Δ ABC , 3 AB = 4 AC

, then $\frac{a(\Delta ACD)}{a(\Delta ACB)}$ = ...

(a) $\frac{9}{7}$ (b) $\frac{9}{16}$ (c) $\frac{7}{16}$ (d) $\frac{3}{4}$

(32) In the opposite figure :



If the polygon ABCD ~ the polygon AEFG

and the area of the polygon ABCD = 32 cm^2.

, then the shaded area = ... cm^2.

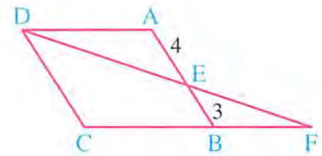
(a) 72 (b) 48

(c) 40 (d) 16

(33) In the opposite figure :

ABCD is a parallelogram , AE : EB = 4 : 3

, a (Δ ADE) = 32 cm^2 , then a (Δ DFC) = ... cm^2



(a) 18 (b) 98

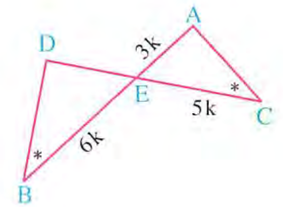
(c) 24 (d) 42

(34) In the opposite figure :

$\overline{AB} \cap \overline{CD} = \{E\}$

, a (Δ ACE) = 900 cm^2

, then area of Δ DEB = ... cm^2



(a) 1080 (b) 1208

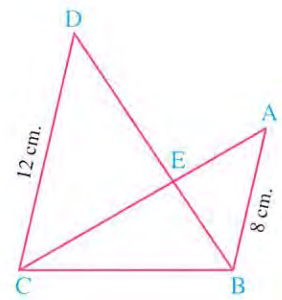
(c) 1296 (d) 1218

(35) In the opposite figure :

ABCD is a cyclic quadrilateral

in which : AB = 8 cm. , CD = 12 cm.

, then a (Δ AEB) : a (Δ DEC) = ...



(a) 3 : 2 (b) 2 : 3

(c) 4 : 9 (d) 9 : 4

Second Essay questions

1 The ratio between the two perimeters of two similar triangles is 3 : 2 and the sum of their areas is 130 cm^2. Find the area of each of them. « 90 cm^2, 40 cm^2 »

---

2 The ratio between the lengths of two corresponding sides in two similar polygons is 1 : 3 Let the difference between their areas be 32 cm^2 , so find the area of each. « 4 cm^2, 36 cm^2 »

---

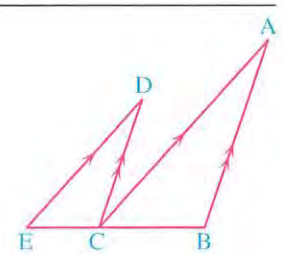
3 **In the opposite figure :**

If $\overline{AB} // \overline{DC}$, $\overline{AC} // \overline{DE}$,

$AB = \frac{3}{2}$ DC

, area of Δ DCE = 16 cm^2

, find the area of : Δ ABC



« 36 cm^2 »

4 ABC is a triangle , D $\in \overline{AB}$ where AD = 2 BD , E $\in \overline{AC}$ where $\overline{DE} \parallel \overline{BC}$

If the area of Δ ADE = 60 cm^2 , find the area of the trapezium DBCE «75 cm^2»

---

5 ABC is a triangle , AB = 8 cm. , AC = 6 cm. , D $\in \overline{AB}$ where AD = 3 cm.

, E $\in \overline{AC}$ where EC = 2 cm. **Find :** $\frac{a(\Delta ADE)}{a(\text{figure DBCE})}$ «$\frac{1}{3}$»

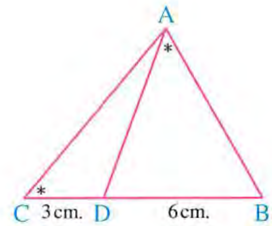
---

6 ABCD , $\grave{A}\grave{B}\grave{C}\grave{D}$ are two similar polygons whose diagonals intersect at X , Y respectively

Prove that : $\frac{a(\text{the polygon ABCD})}{a(\text{the polygon } \grave{A}\grave{B}\grave{C}\grave{D})} = \frac{(BX)^2}{(\grave{B}Y)^2}$

---

7 **In the opposite figure :**



ABC is a triangle where BC = 9 cm.

and D $\in \overline{BC}$ where BD = 6 cm.

If m ($\angle$ BAD) = m ($\angle$ C) ,

then prove that : Δ ABC $\sim$ Δ DBA

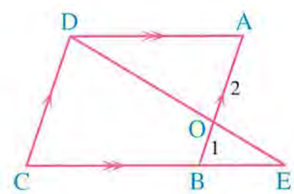
and find the length of : $\overline{AB}$

Find also : The ratio between

the area of Δ ABC and Δ DBA «$3\sqrt{6}$ cm. , 3 : 2»

---

8 **In the opposite figure :**



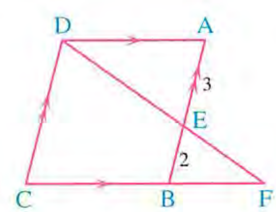
ABCD is a parallelogram , $\frac{BO}{AO} = \frac{1}{2}$

, a (Δ BEO) = 9 cm^2

Find : The area of the parallelogram ABCD «108 cm^2»

---

9 **In the opposite figure :**



ABCD is a parallelogram

, E $\in \overline{AB}$ where $\frac{AE}{EB} = \frac{3}{2}$

, $\overrightarrow{DE} \cap \overrightarrow{CB} = \{F\}$

(1) Prove that : Δ DCF $\sim$ Δ EAD

(2) Find : $\frac{a(\Delta DCF)}{a(\Delta EAD)}$ «$\frac{25}{9}$»

10 ABCD is a parallelogram , $X \in \overrightarrow{AB}$, $X \notin \overline{AB}$ where BX = 2 AB , $Y \in \overrightarrow{CB}$, $Y \notin \overline{CB}$ where BY = 2 BC , the parallelogram BXZY is drawn.

Prove that : $\frac{a \text{ (parallelogram ABCD)}}{a \text{ (parallelogram XBYZ)}} = \frac{1}{4}$

11 ABCD , XYZL are two similar polygons. If M is the midpoint of $\overline{BC}$ and N is the midpoint of $\overline{YZ}$

, **prove that :** a (polygon ABCD) : a (polygon XYZL) = $(MD)^2 : (NL)^2$

12 $\overline{AB}$, $\overline{CD}$ are two non intersecting chords of circle M

If $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, AC = 3 BD

, **find :** $\frac{a (\Delta EBD)}{a (\Delta ECA)}$ «$\frac{1}{9}$»

13 M , N are two touching externally circles at A , the two secants from A are drawn to intersect the circle M at B , D and intersect the circle N at C , E

Prove that : $\frac{a (\Delta ABD)}{a (\Delta ACE)} = \frac{(BD)^2}{(CE)^2}$

14 ABC is a triangle inscribed inside a circle , draw $\overrightarrow{AD}$ to bisect ∠ A and intersect $\overline{BC}$ at D and the circle at E

Prove that : a (Δ ABE) : a (Δ ADC) : a (Δ BDE) = $(EB)^2 : (CD)^2 : (ED)^2$

15 If Δ ABC ~ Δ XYZ , $\overline{AD}$, $\overline{XL}$ are their corresponding heights

, **prove that :** BC × XL = AD × YZ

16 **Prove that :** The ratio between the areas of the two similar triangles equals the square of the ratio between :

(1) Two corresponding heights in them.

(2) The lengths of two corresponding medians in them.

17 ABC is a right-angled triangle at B. The equilateral triangles ABX , BCY , ACZ are drawn. **Prove that :** a (Δ ABX) + a (Δ BCY) = a (Δ ACZ)

18 ABC is an inscribed triangle in a circle where $\frac{AB}{BC} = \frac{4}{3}$, from B a tangent is drawn to the circle to intersect $\overrightarrow{AC}$ at E

Prove that : $\frac{a (\Delta ABC)}{a (\Delta ABE)} = \frac{7}{16}$

19 ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$ Draw $\overleftrightarrow{XY} \parallel \overline{AD}$ to intersect $\overline{AB}$ at X and $\overline{CD}$ at Y such that the trapezium is divided into two similar polygons AXYD and XBCY

Prove that : $\frac{a\text{ (polygon AXYD)}}{a\text{ (polygon XBCY)}} = \frac{a\ (\Delta\text{ ABD})}{a\ (\Delta\text{ BDC})}$

---

20 Δ ABC is right-angled at A , $\overrightarrow{AD} \perp \overline{BC}$ intersecting it at D. The two equilateral triangles ABE , CAF are drawn outside the triangle ABC

Prove that : (1) The polygon ADBE ~ the polygon CDAF

(2) $\frac{a\text{ (the polygon ADBE)}}{a\text{ (the polygon CDAF)}} = \frac{BD}{CD}$

---

21 ABC is a right-angled triangle at B , $\overrightarrow{BD} \perp \overline{AC}$ to intersect it at D. The squares AXYB , BMNC are drawn on $\overline{AB}$, $\overline{BC}$ respectively outside the triangle ABC

(1) **Prove that :** The polygon DAXYB ~ the polygon DBMNC

(2) If AB = 6 cm. , AC = 10 cm.

, **find :** the ratio between areas of the two polygons. « $\frac{9}{16}$ »

---

22 ABC is a triangle in which $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ are corresponding sides to three similar polygons X , Y , Z drawn outside the triangle respectively. If the area of the polygon X = 40 cm^2 , the area of Y = 85 cm^2 , the area of Z = 125 cm^2

, **prove that :** Δ ABC is a right-angled triangle.

---

23 ABCD is a quadrilateral , E $\in \overline{BD}$, draw $\overrightarrow{EF} \parallel \overline{DA}$ to intersect $\overline{AB}$ at F , draw $\overrightarrow{EM} \parallel \overline{DC}$ and intersects $\overline{BC}$ at M

Prove that : a (the polygon BMEF) : a (the polygon BCDA) = $\frac{BF \times BM}{BA \times BC}$

---

24 ABCD is a square , $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ are divided in ratio 1 : 3 by the points X , Y , Z , L respectively.

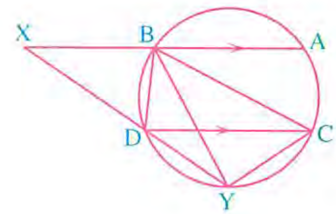
Prove that : (1) XYZL is a square. (2) $\frac{a\text{ (the square XYZL)}}{a\text{ (the square ABCD)}} = \frac{5}{8}$

---

25 **In the opposite figure :**

$\overline{AB}$, $\overline{CD}$ are two parallel chords in a circle , $\overrightarrow{AB} \cap \overrightarrow{YD} = \{X\}$

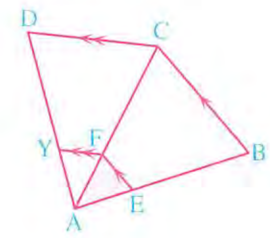
Prove that : $\frac{a\ (\Delta\text{ DBX})}{a\ (\Delta\text{ CYB})} = \frac{(XB)^2}{(BY)^2}$



Third Higher skills

1 Choose the correct answer from those given :

(1) In the opposite figure :



If the area of (polygon DYFC) = 40 cm^2

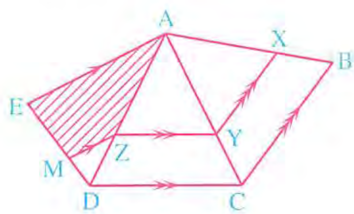
, the area of (polygon FEBC) = 32 cm^2

, the area of (Δ AFY) = 5 cm^2

, then the area of (Δ AEF) = ... cm^2

(a) 3 (b) 4 (c) 5 (d) 6

(2) In the opposite figure :



If the area of (Δ AXY) = 40 cm^2

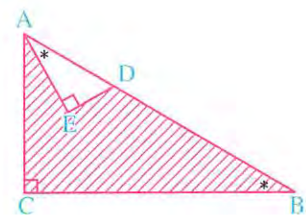
, the area of (Δ DZM) = 13 cm^2

, the area of (the polygon XBCY) = 50 cm^2

Then the shaded area = ... cm^2

(a) 77 (b) 92 (c) 104 (d) 112

(3) In the opposite figure :



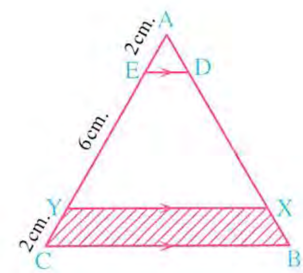
If AB = 3 AD , and the area of Δ ADE = 6 cm^2

, then the shaded area = ... cm^2

(a) 12 (b) 24

(c) 48 (d) 96

(4) In the opposite figure :



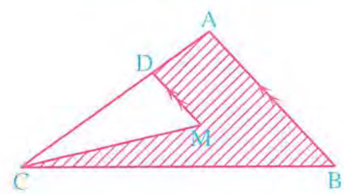
If the area of the polygon DXYE = 30 cm^2

, then the area of the polygon XBCY = ... cm^2

(a) 12 (b) 16

(c) 18 (d) 20

(5) In the opposite figure :



If M is the point of intersection of medians of Δ ABC

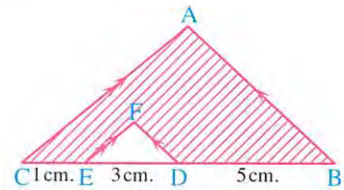
, $\overline{MD}$ // $\overline{AB}$ and the area of Δ ABC = 36 cm^2

, then the shaded area = ... cm^2

(a) 27 (b) 28

(c) 32 (d) 33

(6) In the opposite figure :



If the area of Δ DEF = 6 cm^2

, then the shaded area = ……… cm^2

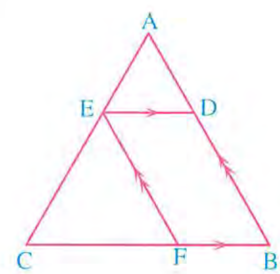
(a) 27 (b) 36

(c) 48 (d) 54

(7) If Δ ABC ~ Δ DEF and AB = x cm. , DE = $(x + 1)$ cm. , the area of Δ ABC = $(x + 2)$ cm^2 , and the area of Δ DEF = $(x + 7)$ cm^2 , then the value of x = ………

(a) 4 (b) 3 (c) 2 (d) 1

(8) In the opposite figure :



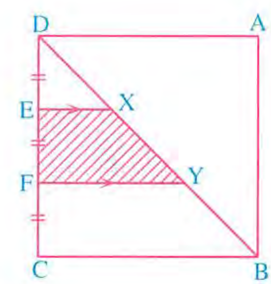
If $\overline{DE} \parallel \overline{BC}$, $\overline{EF} \parallel \overline{AB}$, $\frac{AD}{DB} = \frac{2}{3}$

, then $\frac{\text{Area } (\text{▱ DBFE})}{\text{Area } (\Delta \text{ ABC})}$ = ………

(a) $\frac{21}{25}$ (b) $\frac{16}{25}$

(c) $\frac{12}{25}$ (d) $\frac{13}{25}$

(9) In the opposite figure :



ABCD is a square of side length 6 cm.

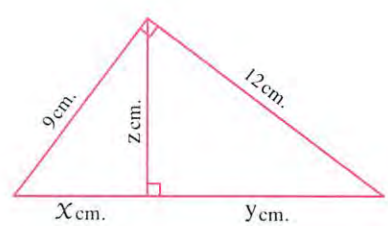
, DE = EF = FC

, then the area of (polygon XYFE) = ……… cm^2

(a) 6 (b) 8

(c) 10 (d) 12

(10) In the opposite figure :

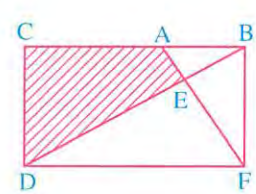


$x + y + z$ = ………

(a) 15 (b) 18.2

(c) 22 (d) 22.2

(11) In the opposite figure :



BCDF is a rectangle , the area of (Δ ABE) = 2 cm^2

, the area of (Δ BEF) = 3 cm^2

, then the shaded area = ……… cm^2

(a) 5 (b) $5\frac{1}{2}$ (c) 6 (d) $7\frac{1}{2}$

(12) If the scale factor of similarity of the polygon P_1 to the polygon P_2 is $\frac{2}{3}$ and the scale factor of similarity of the polygon P_3 to the polygon P_2 is $\frac{1}{3}$, which of the following relations is correct ?

(a) Area (P_1) + Area (P_2) = Area (P_3)

(b) Area (P_1) + Area (P_3) = Area (P_2)

(c) $\sqrt{\text{Area}(P_1)} + \sqrt{\text{Area}(P_2)} = \sqrt{\text{Area}(P_3)}$

(d) $\sqrt{\text{Area}(P_1)} + \sqrt{\text{Area}(P_3)} = \sqrt{\text{Area}(P_2)}$

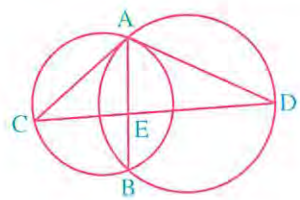
---

2 $\overline{AB}$ is a diameter in a circle , C belongs to the circle , $X \in \overline{AB}$ where AX = BC , draw $\overrightarrow{XY}$ // $\overline{BC}$ and intersects $\overline{AC}$ at Y

Prove that : a (Δ ABC) a (the polygon XBCY) = $(AB)^2 : (AC)^2$

---

3 **In the opposite figure :**



Two intersecting circles at A , B

, $\overline{AC}$ is a chord in one of the two circles and touches the other at A ,

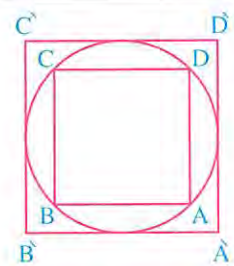
$\overline{AD}$ is a chord in the second circle and touches the first circle at A

If $\overline{AB} \cap \overline{CD} = \{E\}$

, **prove that :** $\frac{CE}{ED} = \frac{(AC)^2}{(AD)^2}$

---

4 **In the opposite figure :**



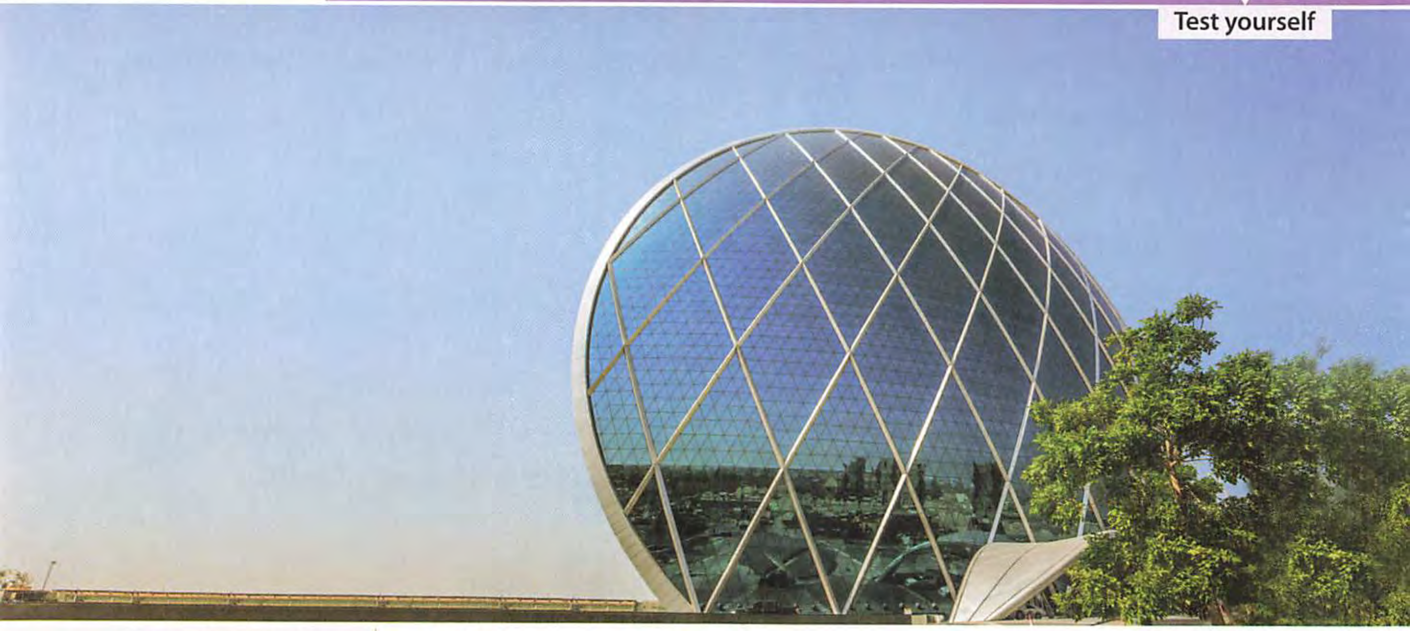
Two squares are drawn , one of them is inside a circle and the other is outside the circle.

Find the ratio between their areas.

« $\frac{1}{2}$ »

Applications of similarity in the circle

Test yourself

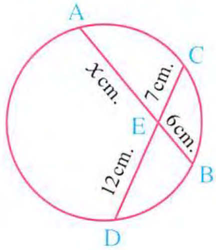


From the school book | Remember | Understand | Apply | Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) In the opposite figure :

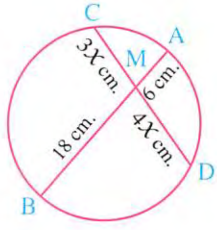


$X = \cdots\cdots$ cm.

(a) 3.5 (b) 14

(c) 6 (d) 12

(2) In the opposite figure :



$\overline{AB} \cap \overline{CD} = \{M\}$, AM = 6 cm.

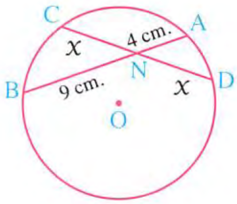
, MB = 18 cm. , CM = 3 X cm.

, DM = 4 X cm. , then CD = $\cdots\cdots$ cm.

(a) 3 (b) 9

(c) 18 (d) 21

(3) In the opposite figure :

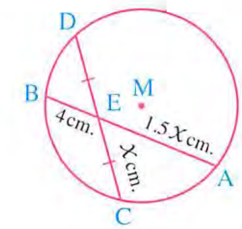


$X = \cdots\cdots$

(a) 6 (b) – 6

(c) ± 6 (d) 36

(4) **In the opposite figure :**

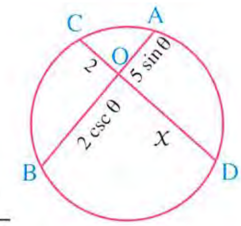


$X =$ .... cm.

(a) 6.5 (b) 13

(c) 6 (d) 36

(5) **In the opposite figure :**



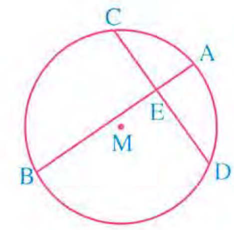
If $\overline{AB}$, $\overline{CD}$ are two chords in the circle ,

$\overline{AB} \cap \overline{CD} = \{O\}$, AO = (5 sin θ) cm.

, OB = (2 csc θ) cm. , OC = 2 cm. , then $X =$ .... cm.

(a) 5 (b) 10 (c) $\frac{\sqrt{3}}{2}$ (d) $10\sqrt{3}$

(6) **In the opposite figure :**



If AE = 5 cm. , CE = 8 cm.

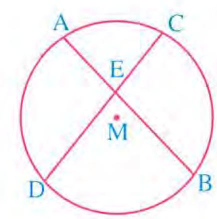
, DE = 10 cm. , BE = $(X + 1)$ cm.

, then $X =$ .... cm.

(a) 12 (b) 14

(c) 16 (d) 15

(7) **In the opposite figure :**



$\overline{AB} \cap \overline{CD} = \{E\}$, AE = 4 cm.

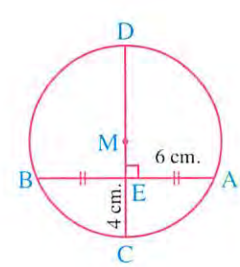
, EB = 6 cm. , DE = $(X + 1)$ cm.

, CE = $(X - 1)$ cm. , then $X =$ .... cm.

(a) 5 (b) 6

(c) 4 (d) 7

(8) **In the opposite figure :**

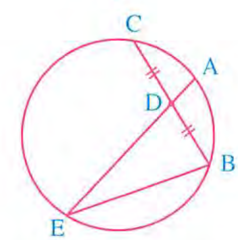


The radius length of the circle = ... cm.

(a) 9 (b) 4.5

(c) 6 (d) 6.5

(9) **In the opposite figure :**



$(BD)^2 =$ ....

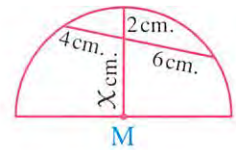
(a) AD × DB (b) AD × DE

(c) AD × BE (d) AC × BD

(10) In the opposite figure :

A semicircle of centre M , then $X =$ .... cm.

(a) 5 (b) 7 (c) 8 (d) 12

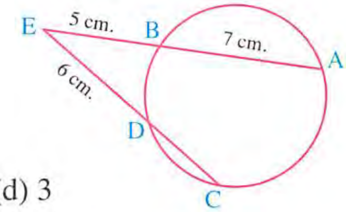


(11) In the opposite figure :

If AB = 7 cm. , BE = 5 cm. , DE = 6 cm.

, then the length of $\overline{CD}$ = ... cm.

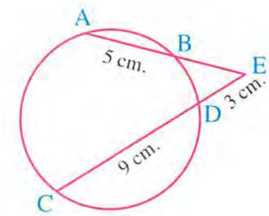
(a) 6 (b) 5 (c) 4 (d) 3



(12) In the opposite figure :

BE = ... cm.

(a) 6 (b) 5 (c) 4 (d) 3



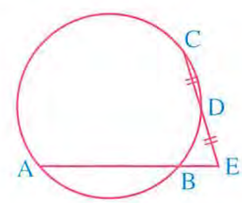
(13) In the opposite figure :

If DE = DC , EB = 2 cm. , AB = 7 cm.

, then the length of $\overline{EC}$ = ... cm.

(a) 6 (b) 4

(c) 5 (d) 3



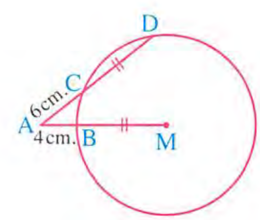
(14) In the opposite figure :

If DC = MB , then the circumference

of circle M = ... cm.

(a) 15 π (b) 18 π

(c) 20 π (d) 24 π

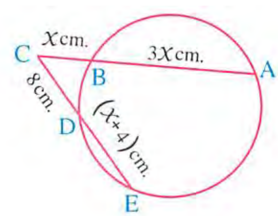


(15) In the opposite figure :

$X =$ ....

(a) 5 (b) 6

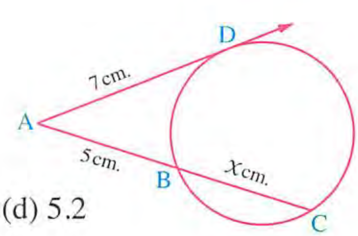
(c) 3 (d) 9



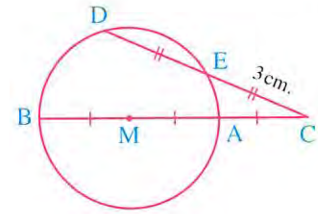
(16) In the opposite figure :

$X =$ ....

(a) 4.8 (b) 5.6 (c) 4.2 (d) 5.2



(17) In the opposite figure :

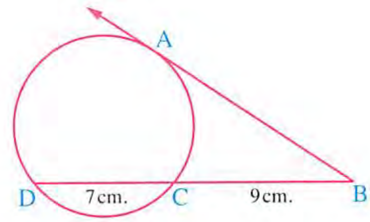


The area of the circle M = ... cm^2.

(a) 6π (b) 18π

(c) $2\sqrt{6}\pi$ (d) $\sqrt{6}\pi$

(18) In the opposite figure :



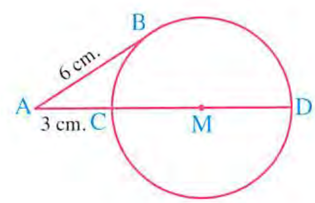
$\overrightarrow{BA}$ is a tangent , BC = 9 cm. , CD = 7 cm.

, then AB = ... cm.

(a) 63 (b) 144

(c) 12 (d) $\frac{9}{16}$

(19) In the opposite figure :



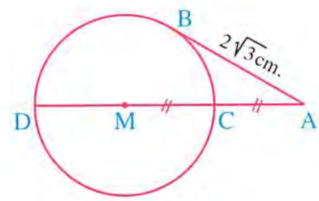
If $\overline{AB}$ is a tangent segment to circle M

, then the circumference of circle M = ...

(a) 6π (b) 9π

(c) 12π (d) 15π

(20) In the opposite figure :

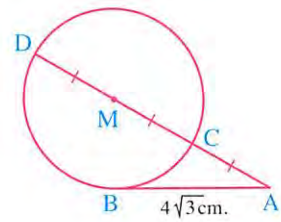


The length of the radius of circle M = ... cm.

(a) 2 (b) 3

(c) 4 (d) 5

(21) In the opposite figure :

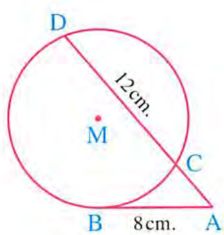


The circumference of the circle = ... cm.

(a) $4\sqrt{3}\pi$ (b) $8\sqrt{3}\pi$

(c) 8π (d) 4π

(22) In the opposite figure :

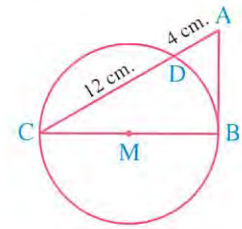


AC = ... cm.

(a) 12 (b) 18

(c) 4 (d) 6

(23) In the opposite figure :



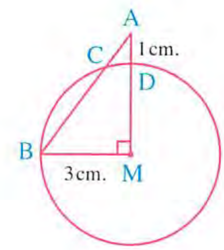
In a circle M , If $\overline{AB}$ is a segment tangent , AD = 4 cm. , DC = 12 cm.

, then the radius length of circle M = ... cm.

(a) $4\sqrt{3}$ (b) $16\sqrt{3}$

(c) $8\sqrt{3}$ (d) $24\sqrt{3}$

(24) In the opposite figure :



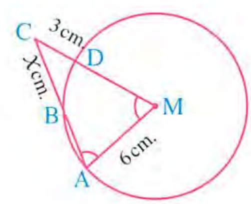
AMB is a right-angled triangle at M

the raduis of the circle = 3 cm. , AD = 1 cm.

, then BC = ... cm.

(a) 3.6 (b) 1.4 (c) 5 (d) 3

(25) In the opposite figure :

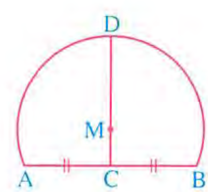


X = ...

(a) 6 (b) 4

(c) 3 (d) 5

(26) In the opposite figure :



A , B , D are three points on a circle whose centre is M

If C is the midpoint of $\overline{AB}$

, D , M , C are collinear ,

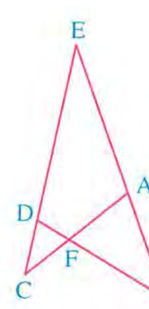
AB = 24 cm. , DC = 18 cm.

, then the radius length of the circle = ... cm.

(a) 9 (b) 8 (c) 12 (d) 13

(27) In the opposite figure :

ABCD is a cyclic quadrilateral if ....



(a) $\frac{EA}{EB} = \frac{ED}{EC}$

(b) $\frac{EA}{AB} = \frac{ED}{DC}$

(c) AF × FD = BF × FC

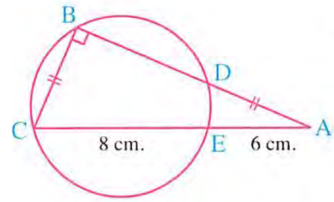
(d) EA × EB = ED × EC

(28) In the opposite figure :

a (Δ ABC) = ·········· cm^2.

(a) 48 (b) 42

(c) 40 (d) 24

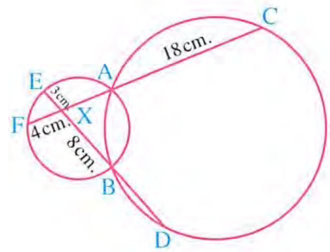


(29) In the opposite figure :

BD = ·········· cm.

(a) 6 (b) 8

(c) 10 (d) 12



(30) In the opposite figure :

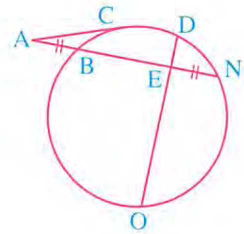
If DE = 2 cm. , OE = 9 cm. ,

BE = 6 cm. , AB = NE ,

$\overline{AC}$ is a segment tangent , then AC = ·········· cm.

(a) 2 (b) 6

(c) 4 (d) 8



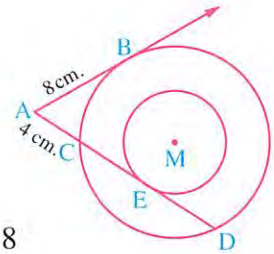
(31) In the opposite figure :

$\overleftrightarrow{AB}$ is a tangent to the greater circle

, $\overleftrightarrow{AD}$ is a tangent to the smaller circle

DE = ·········· cm.

(a) 4 (b) 5 (c) 6 (d) 8



(32) In the opposite figure :

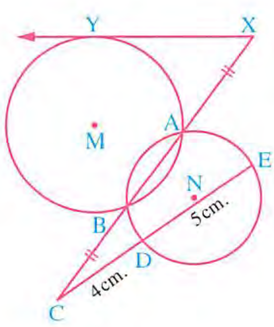
Two circles M and N are intersecting at A and B

, $\overleftrightarrow{XY}$ is a tangent to the circle M , if AX = BC

, then XY = ·········· cm.

(a) 4 (b) 6

(c) 8 (d) 9



(33) In the opposite figure :

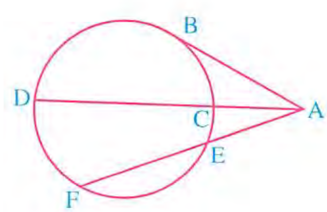
All the following statements are true except ··········

(a) $(AB)^2 = AC \times AD$

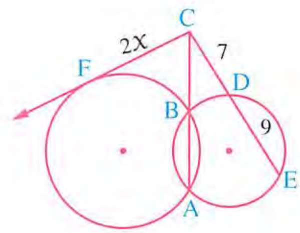
(b) $(AB)^2 = AE \times AF$

(c) $AE \times AF = AC \times AD$

(d) $AC \times CD = AE \times EF$



(34) In the opposite figure :

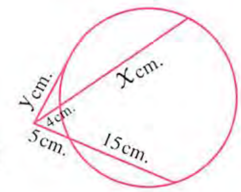


$X =$ ....

(a) $\sqrt{7}$ (b) $2\sqrt{7}$

(c) $3\sqrt{7}$ (d) $4\sqrt{7}$

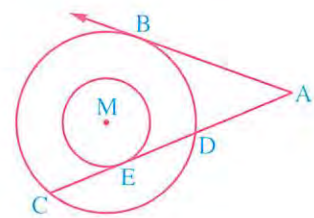
(35) In the opposite figure :



$X + y =$ .... cm.

(a) 9 (b) 18 (c) 22 (d) 31

(36) In the opposite figure :



two concentric circles at M

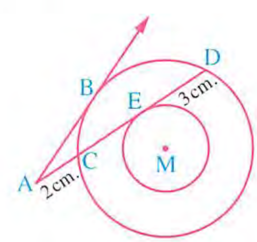
, $\overrightarrow{AB}$ is a tangent to the bigger circle

, $\overrightarrow{AE}$ is a tangent to the smaller one

, AD = 4 cm. and DE = 2.5 cm. , then AB = ... cm.

(a) 6 (b) 5 (c) 4 (d) 8

(37) In the opposite figure :

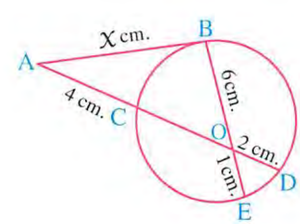


AB = ... cm.

(a) 4 (b) 5

(c) 6 (d) 8

(38) In the opposite figure :

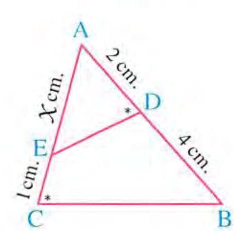


$\overline{AB}$ is a tangent segment to the circle , then $X =$ ....

(a) 8 (b) 6

(c) 4.8 (d) 5

(39) In the opposite figure :

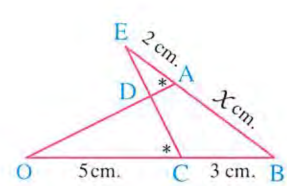


$X =$ ....

(a) 4 (b) 3

(c) 4.5 (d) 5

(40) In the opposite figure :



$X =$ ....

(a) 4 (b) 3.2

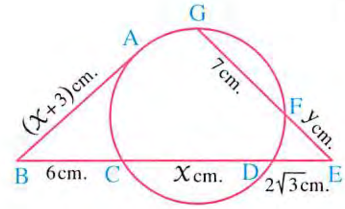
(c) 5 (d) 3

(41) **In the opposite figure :**

$\frac{x}{y} = \dots\dots\dots$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

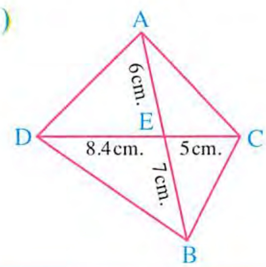
(c) $\sqrt{3}$ (d) 4



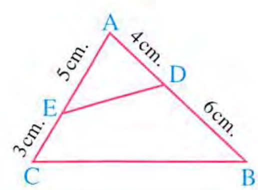
Second Essay questions

1 **In which of the following figures , the points A , B , C and D lie on a circle ? Explain your answer.**

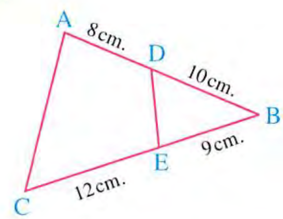
(1)



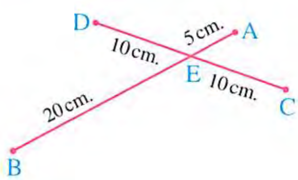
(2)



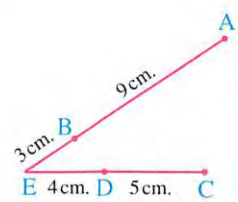
(3)



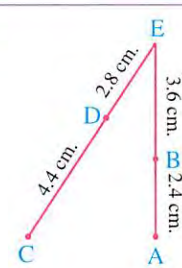
(4)



(5)

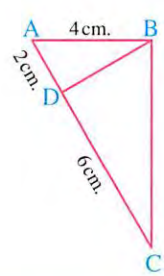


(6)

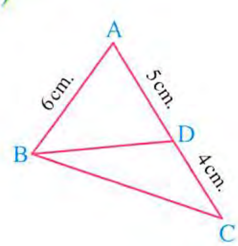


2 **In which of the following figures , $\overline{AB}$ is a tangent segment to the circle which passes through the points B , C and D ?**

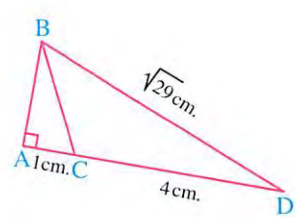
(1)



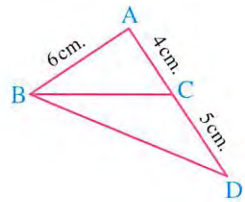
(2)



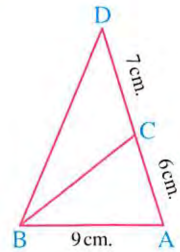
(3)



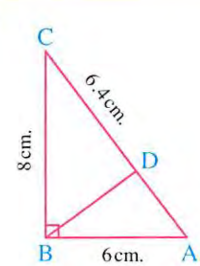
(4)



(5)



(6)

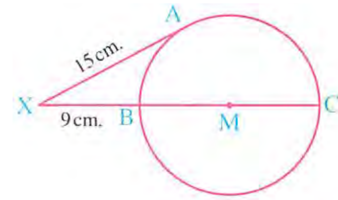


3 **In the opposite figure :**

$\overline{XA}$ is a tangent to the circle M at A

where XA = 15 cm. If XB = 9 cm.

, calculate the length of the radius of the circle.



« 8 cm. »

4 The length of the radius of a circle of center O is 4 cm. Assume a point M such that MO = 6 cm. Let $\overrightarrow{MB}$ be drawn to intersect the circle at A and B , where A $\in \overline{MB}$

If MA = 3 cm. , **so find the length of :** $\overline{AB}$ « $3\frac{2}{3}$ cm. »

5 $\overline{AB}$ and $\overline{CD}$ are two intersecting chords at E in a circle. If the lengths of $\overline{AE}$, $\overline{BE}$, $\overline{CD}$ respectively are 5 cm. , 6 cm. , 11.5 cm. , **calculate the lengths of :** $\overline{EC}$, $\overline{ED}$

« 7.5 cm. , 4 cm. »

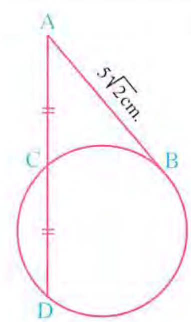
6 **In the opposite figure :**

If $\overline{AB}$ is a tangent segment to the circle at B ,

C is the midpoint of $\overline{AD}$,

AB = $5\sqrt{2}$ cm.

, find the length of : $\overline{AD}$



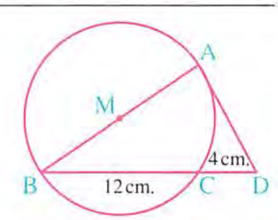
« 10 cm. »

7 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

$\overline{AD}$ is a tangent to the circle at A

Find the area of the circle M



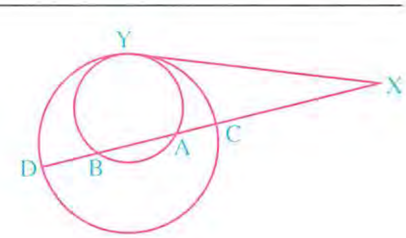
« 48 π cm.^{2} »

8 **In the opposite figure :**

Two circles are touching internally at point Y ,

$\overrightarrow{YX}$ is a common tangent to the two circles.

Prove that : $\frac{XC}{XB} = \frac{XA}{XD}$



9 **In the opposite figure :**

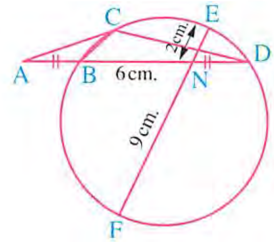
$\overline{AC}$ is a tangent segment to the circle ,

AB = DN , EN = 2 cm. ,

NF = 9 cm. , NB = 6 cm.

Find : (1) The length of $\overline{AC}$

(2) a (Δ ACB) : a (Δ ADC)



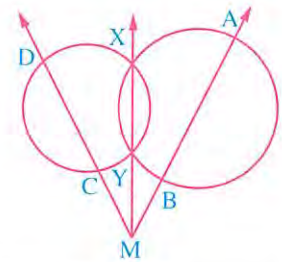
« 6 cm. , $\frac{1}{4}$ »

10 **In the opposite figure :**

Prove that :

One circle passes by

the points A , B , C and D

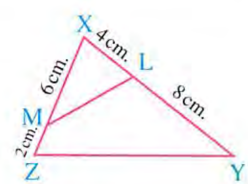


11 **In the opposite figure :**

L $\in \overline{XY}$ where XL = 4 cm. ,

YL = 8 cm. , M $\in \overline{XZ}$

where XM = 6 cm. , ZM = 2 cm.



Prove that : (1) Δ XLM ~ Δ XZY (2) LYZM is a cyclic quadrilateral.

12 $\overline{AB} \cap \overline{CD} = \{E\}$, AE = $\frac{5}{12}$ BE , DE = $\frac{3}{5}$ EC If BE = 6 cm. and CE = 5 cm.

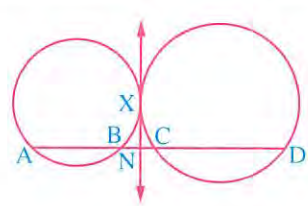
Prove that : The points A , B , C and D lie on one circle.

13 **In the opposite figure :**

The two circles touch each other externally at X ,

$\overleftrightarrow{AD}$ intersects one of the circles at A and B

and the other one at C and D



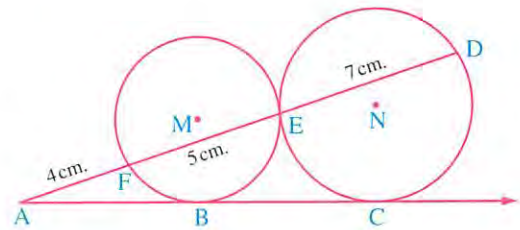
Let the common tangent to the two circles at X intersect $\overleftrightarrow{AD}$ at N

Prove that : $\frac{NB}{NC} = \frac{ND}{NA}$

14 Two circles are intersecting at A and B , C $\in \overleftrightarrow{AB}$ and C $\notin \overline{AB}$, from C the two tangent segments $\overline{CX}$ and $\overline{CY}$ are drawn to touch the circles at X and Y respectively.

Prove that : CX = CY

15 **In the opposite figure :**



M and N are two circles touching externally at E , $\overrightarrow{AC}$ touches the circle M at B and touches the circle N at C , $\overrightarrow{AE}$ intersects the two circles at F and D respectively , where AF = 4 cm. , FE = 5 cm. , ED = 7 cm.

Prove that : B is the midpoint of $\overline{AC}$

---

16 ABC is an acute-angled triangle , $\overline{AD}$, $\overline{BE}$ are two intersecting heights at F

Prove that : $\frac{AE \times AC}{BF \times FE} = \frac{AD}{FD}$

---

17 A circle of centre O and its radius length equals 8 cm. , M is a point where MO = 12 cm. , from M a secant is drawn to intersect the circle at A and B where $A \in \overline{MB}$

If AB = 11 cm.

, **find :** (1) The length of $\overline{MA}$

(2) The length of the tangent segment to the circle from M « 5 cm. , $4\sqrt{5}$ cm. »

---

18 ABC is a triangle $D \in \overline{BC}$ where BD = 5 cm. and DC = 4 cm. If AC = 6 cm.

, **prove that :**

(1) $\overline{AC}$ is a tangent segment to the circle passing through the points A , B and D

(2) Δ ACD ~ Δ BCA

(3) Area of (Δ ABD) : area of (Δ ABC) = 5 : 9

---

19 Two concentric circles at M , the lengths of their radii are 12 cm. and 7 cm.

$\overline{AD}$ is a chord in the larger circle to intersect the smaller circle at B and C respectively.

Prove that : AB × BD = 95

---

20 ABCD is a rectangle in which AB = 6 cm. and BC = 8 cm. , $\overrightarrow{BE} \perp \overline{AC}$ and intersects $\overline{AC}$ at E and $\overline{AD}$ at F

(1) **Prove that :** $(AB)^2$ = AF × AD

(2) **Find the length of :** $\overline{AF}$ « 4.5 cm. »

21 $\overline{AB}$ is a chord of length 8 cm. in a circle of centre M , $\overrightarrow{MC} \perp \overline{AB}$ to intersect it at C and intersect the circle at D. If CD = 2 cm. , calculate the length of the radius of the circle.

« 5 cm. »

22 $\overline{AB}$ is a diameter in a circle , $C \in \overline{AB}$, $\overrightarrow{CX} \perp \overline{AB}$ to intersect the circle at X , $\overline{DE}$ is a chord drawn in the circle passing through point C. **Prove that :** $(XC)^2 = DC \times CE$

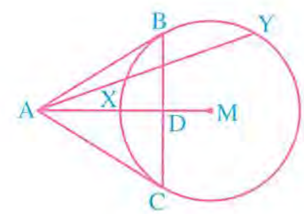
23 $\overline{AB}$ is a diameter in a circle , $\overline{CD}$ is a chord in it perpendicular to $\overline{AB}$ to intersect it at N The two chords $\overline{AE}$ and $\overline{AF}$ are drawn in two different sides from $\overline{AB}$ to intersect $\overline{CD}$ at X and Y respectively. **Prove that :** $AX \times AE = AY \times AF$

24 **In the opposite figure :**

A is a point outside the circle M , $\overline{AB}$ and $\overline{AC}$ are tangents to the circle , $\overline{AY}$ intersects the circle at X and Y ,

$\overline{BC} \cap \overline{MA} = \{D\}$

Prove that : $AX \times AY = AD \times AM$



25 $\overline{AB}$ is a diameter in a circle , $C \in \overrightarrow{AB}$, C is located outside the circle where BC = AB , $\overline{CD}$ is a tangent to the circle at D , $\overrightarrow{AD}$ is drawn to intersect the tangent of the circle from point B at E

Prove that : $(CD)^2 = 2\,AD \times AE$

26 ABC is a triangle , $\overrightarrow{AD}$ bisects $\angle$ BAC and intersects $\overline{BC}$ at D , $E \in \overrightarrow{AD}$ where AD = DE If $(AD)^2 = DB \times DC$

, **prove that :** (1) $\Delta ECD \sim \Delta EAC$

(2) $(EC)^2 = 2\,(ED)^2$

Third Higher skills

1 Choose the correct answer from those given :

(1) **In the opposite figure :**

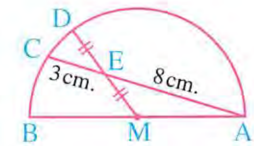
A semicircle M

, ME = ED , EC = 3 cm. , AE = 8 cm.

, then ME = ……… cm.

(a) 2 (b) $\sqrt{2}$

(c) $2\sqrt{2}$ (d) $\frac{8}{3}$



(2) **In the opposite figure :**

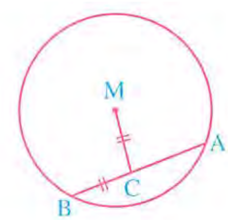
A circle M of diameter length 12 cm.

, MC = CB , AC = (BC + 1) cm.

, then AB = ……… cm.

(a) 4 (b) 6

(c) 8 (d) 9



(3) **In the opposite figure :**

If $\overline{AB}$ is a diameter in circle M

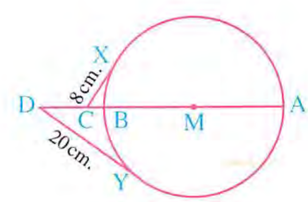
, $\overline{CX}$, $\overline{DY}$ are two tangent segments of circle M

, AB = 30 cm. , CX = 8 cm. , DY = 20 cm.

, then DC = ……… cm.

(a) 2 (b) 6

(c) 8 (d) 10



(4) **In the opposite figure :**

Two intersecting circles at C , E

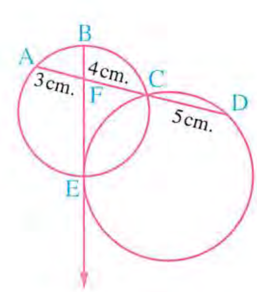
, $\overrightarrow{BE}$ touches the larger circle at E

If AF = 3 cm. , FC = 4 cm. , CD = 5 cm.

, then BE = ……… cm.

(a) 9 (b) 8

(c) 7 (d) 6

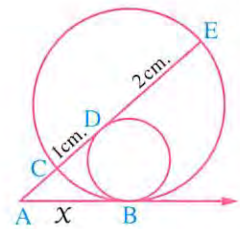


(5) In the opposite figure :

Two circles touching internally at B , $\overrightarrow{AB}$, $\overrightarrow{AD}$ are two tangents to the smaller circle at B , D

If CD = 1 cm. , DE = 2 cm. , AB = x cm.

, then x = ... cm.



(a) 2 (b) 3

(c) 2.5 (d) 3.5

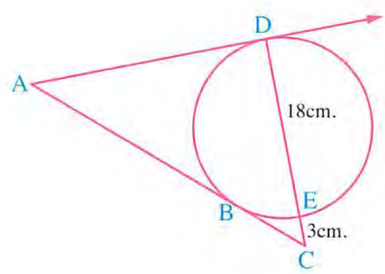
(6) In the opposite figure :

$\overrightarrow{AD}$, $\overrightarrow{AB}$ are two tangents at D , B respectively

$\overrightarrow{CE}$ intersects the circle at E , D

If CE = 3 cm. , ED = 18 cm.

, then (AC – AD) = ... cm.



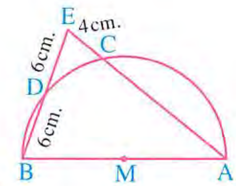
(a) 7 (b) $2\sqrt{7}$ (c) $3\sqrt{7}$ (d) $6\sqrt{7}$

(7) In the opposite figure :

$\overline{AB}$ is a diameter in a semicircle M

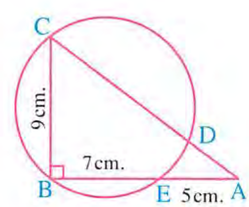
, then r = ... cm.

(a) 9 (b) 12 (c) 18 (d) 24



(8) In the opposite figure :

DC = ... cm.



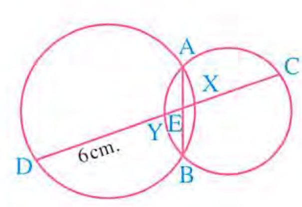
(a) 9 (b) 10

(c) 11 (d) 12

(9) In the opposite figure :

If DY = 6 cm. and $\frac{XE}{EY} = \frac{2}{3}$

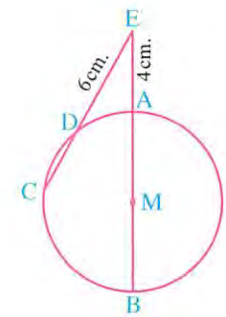
, then CX = ... cm.



(a) 2 (b) 3

(c) 4 (d) 5

(10) In the opposite figure :



$\overline{AB}$ is a diameter in circle M , E $\in \overrightarrow{BA}$ to find the radius length of the circle it is sufficient to have ....

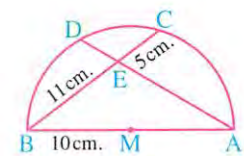
(a) the perimeter of Δ EBC = 26 cm. only.

(b) the perimeter of Δ EMC = 20 cm. only.

(c) (a) , (b) together.

(d) nothing of the previous.

(11) In the opposite figure :

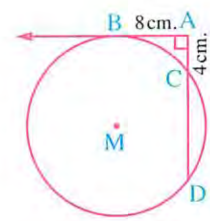


The radius length of semicircle M is 10 cm.

, then ED = ... cm.

(a) $\frac{50}{13}$ (b) $\frac{55}{13}$ (c) $\frac{57}{13}$ (d) $\frac{59}{13}$

(12) In the opposite figure :



$\overrightarrow{AB}$ is a tangent to the circle at B

, AB = 8 cm. , $\overleftrightarrow{AC}$ is a secant to the circle M

at C and D , then the radius length of the circle M is .... cm.

(a) 5 (b) 10 (c) 12 (d) 8

---

2 ABC is a triangle in which : AB = 60 mm. , AC = 40 mm. , BC = 45 mm. , take point D $\in \overline{AB}$ where AD = 16 mm. , E $\in \overline{AC}$ where AE = 24 mm.

(1) **Prove that :** Δ ADE ~ Δ ACB and calculate the length of $\overline{DE}$

(2) If $\overleftrightarrow{DE} \cap \overleftrightarrow{BC} = \{N\}$, **prove that :** Δ DNB ~ Δ CNE and calculate the length of each of : $\overline{EN}$, $\overline{NC}$ « 18 mm. , 21.6 mm. , 14.4 mm. »

Life Applications on Unit Three

From the school book

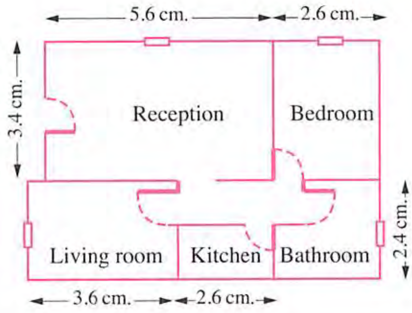
1 The opposite figure shows the floor plan of a house with a drawing scale 1 : 150 **Find :**

(1) The dimensions of the reception.

(2) The dimensions of the bedroom.

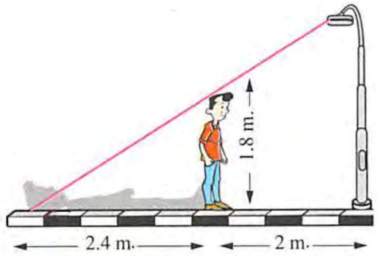
(3) The area of the living room.

(4) The area of the house floor.



2 A man of height 1.8 m. stands against a light pole , at a distance 2 m. from its base. When the light is switched on , the length of the man's shadow is 2.4 m.

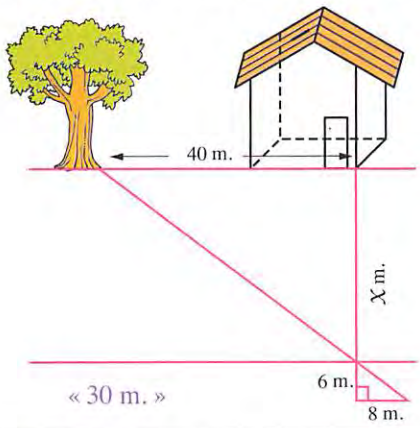
Find the height of the pole.



« 3.3 m. »

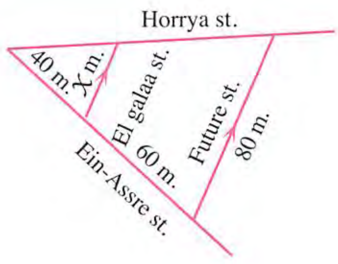
3 **Find the distance X in each of the following :**

(1)



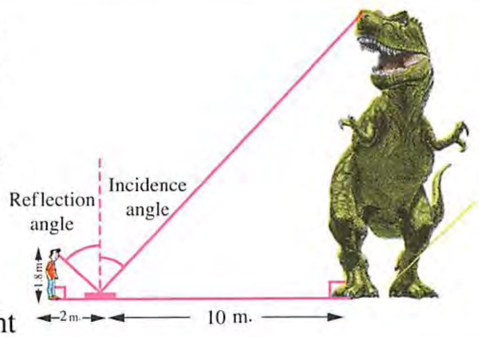
« 30 m. »

(2)



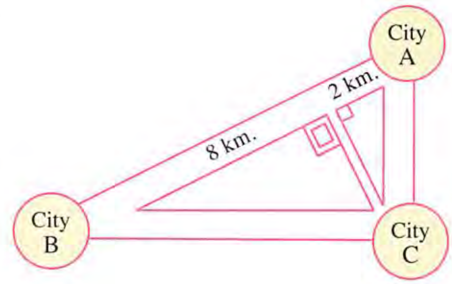
« 32 m. »

4 A man wanted to know the height of a dinosaur in one of the museums , he put a mirror 10 metres away from the foot of the dinosaur , then he moved back until he could see the head of the dinosaur in the mirror. At this moment he measured the distance from the mirror , it was 2 m. and the height of the man was 1.8 m. Given that the measure of the incidence angle equals the measure of the reflection angle , calculate the height of the dinosaur.



« 9 m. »

5 The opposite diagram shows the location of a gas station. It is required to be build on a highway at the intersection of a road that leads to city C and perpendicular to the highway between the two cities A and B , given that the highway between A and C is perpendicular to that between B and C



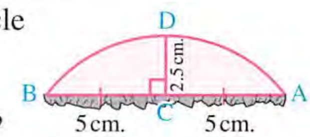
(1) How far is the gas station from city C ?

(2) What is the distance between B and C ?

« 4 km. , $4\sqrt{5}$ km. »

---

6 One of the architects found relics archaeological piece of wood is part of a circular wooden disc , this engineer wanted to know the length of the radius of the disc , so he appointed two points A , B on the circle , he found that AB = 10 cm. , then from the point C which is the midpoint of $\overline{AB}$ he draw $\overline{CD} \perp \overline{AB}$, he found that CD = 2.5 cm. , so he could find the length of the radius geometrically.

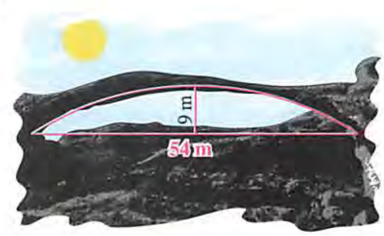


How he could so ?!

« 6.25 cm. »

---

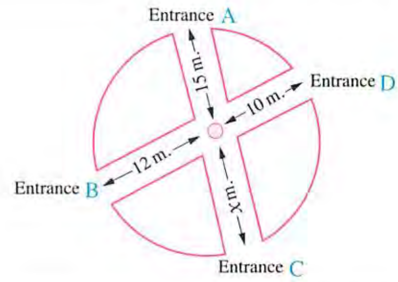
7 In one of the coastal areas , there is a ground layer in the form of a natural arc. The geologists found that , it is an arc of a circle , as in the opposite figure. Find the length of the radius of the circle arc.



« 45 m. »

---

8 The opposite figure illustrates a plan of a circular garden involving two intersected roads at a fountain. How far is the fountain from the entrance C ?

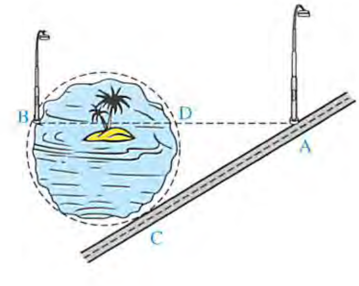


« 8 m. »

---

9 **In the opposite figure :**

A road touches a circular lake , one of the engineers of the electricity company wants to put two light poles , one is on the road and the other lies in other side of the lake and joined between them by an electric wire. Show how to find the length of this wire.



Unit Four | The triangle proportionality theorems

Exercise 5 — Parallel lines and proportional parts.

Exercise 6 — Talis' theorem.

Exercise 7 — Angle bisector and proportional parts.

Exercise 8 — Follow : Angle bisector and proportional parts (Converse of theorem 3).

Exercise 9 — Applications of proportionality in the circle.

At the end of the unit : Life applications on unit four.

Exercise **5**

Parallel lines and proportional parts

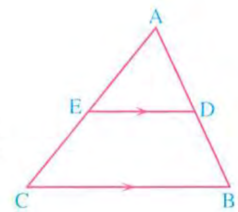
Test yourself

From the school book | Remember | Understand | Apply | Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) In the opposite figure :



First : If $\frac{AD}{DB} = \frac{5}{3}$, then $\frac{AB}{BD} = \cdots\cdots\cdots$

(a) $\frac{3}{5}$ (b) $\frac{8}{3}$ (c) $\frac{3}{8}$ (d) $\frac{5}{8}$

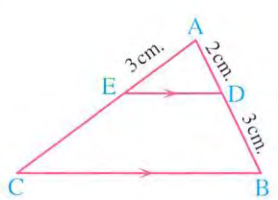
Second : If $\frac{AE}{AC} = \frac{4}{7}$, then $\frac{CE}{EA} = \cdots\cdots\cdots$

(a) $\frac{7}{4}$ (b) $\frac{4}{3}$ (c) $\frac{2}{5}$ (d) $\frac{3}{4}$

Third : If $\frac{DE}{BC} = \frac{3}{5}$, then $\frac{AD}{DB} = \cdots\cdots\cdots$

(a) $\frac{5}{3}$ (b) 1.5 (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

(2) In the opposite figure :



If $\overline{DE} \parallel \overline{BC}$, AD = 2 cm.

and AE = DB = 3 cm.

, then the length of $\overline{EC}$ = ……… cm.

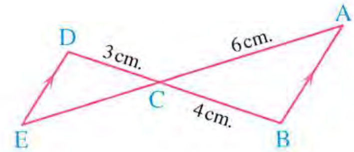
(a) 3 (b) 4 (c) 5 (d) 4.5

(3) **In the opposite figure :**

$\overline{AB} \parallel \overline{DE}$, $\overline{AE} \cap \overline{BD} = \{C\}$

, AC = 6 cm. , BC = 4 cm. and CD = 3 cm.

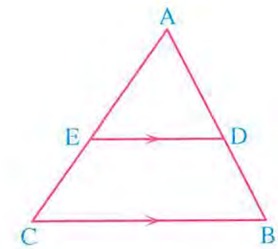
, then the length of $\overline{CE}$ = ... cm.



(a) 5 (b) 4 (c) 4.5 (d) 3.5

(4) **In the opposite figure :**

All the following statements are true except ....

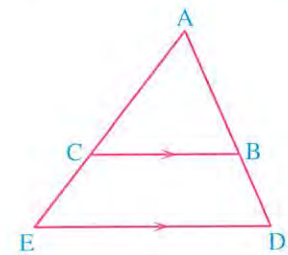


(a) $\frac{AD}{DB} = \frac{AE}{EC}$ (b) $\frac{AD}{DB} = \frac{DE}{BC}$

(c) $\frac{AD}{AB} = \frac{AE}{AC}$ (d) $\frac{AB}{BD} = \frac{AC}{EC}$

(5) **In the opposite figure :**

If $\overline{BC} \parallel \overline{DE}$, then ....



(a) the shape DBCE is a cyclic quadrilateral

(b) Δ ABC $\sim$ Δ ADE

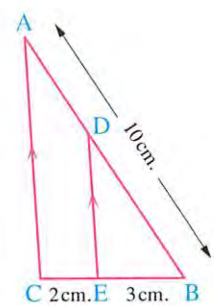
(c) AB $\times$ AD = AC $\times$ AE

(d) $\frac{AB}{BD} = \frac{BC}{DE}$

(6) **In the opposite figure :**

If $\overline{DE} \parallel \overline{AC}$, BE = 3 cm. , EC = 2 cm.

, then AD = ... cm.

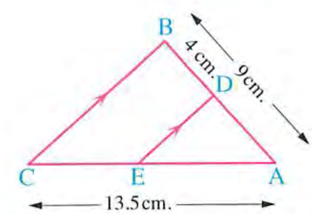


(a) 6 (b) 4

(c) 5 (d) 7

(7) **In the opposite figure :**

$\overline{DE} \parallel \overline{BC}$, then AE = ... cm.



(a) 4 cm. (b) 5 cm.

(c) 6 cm. (d) 7.5 cm.

(8) **In the opposite figure :**

If $\overline{DE} \parallel \overline{BC}$, then

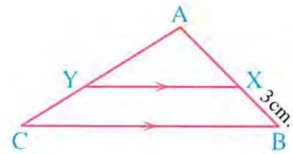
$\frac{a(\Delta ADE)}{a(\Delta ABC)}$ = ...

(a) $\frac{3}{2}$ (b) $\frac{9}{4}$ (c) $\frac{9}{25}$ (d) $\frac{3}{5}$

(9) In the opposite figure :

If $\overline{XY} \parallel \overline{BC}$, $\frac{AX + AY}{AB + AC} = \frac{3}{5}$

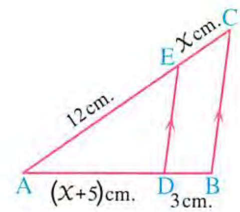
, then AX = ... cm.



(a) 3 (b) 6 (c) 4.5 (d) 4

(10) In the opposite figure :

$\overline{DE} \parallel \overline{BC}$, then $X =$ ....

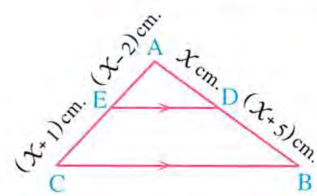


(a) 4 (b) 9

(c) 12 (d) 3

(11) In the opposite figure :

If $\overline{DE} \parallel \overline{BC}$, then $X =$ .... cm.

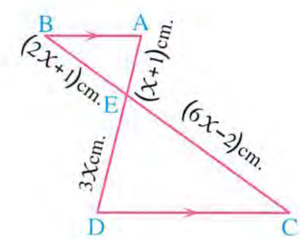


(a) 2 (b) 3

(c) 4 (d) 5

(12) In the opposite figure :

If $\overline{AB} \parallel \overline{CD}$, then $X =$ ....

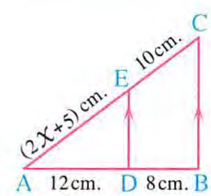


(a) 2 (b) 3

(c) 4.5 (d) 6

(13) In the opposite figure :

If $\overline{DE} \parallel \overline{BC}$, then $X =$ ....



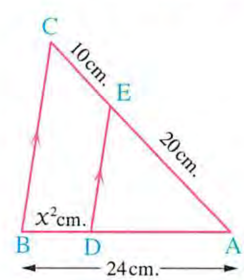
(a) 12 (b) 7

(c) 5 (d) 4

(14) In the opposite figure :

If Δ ABC in which $\overline{DE} \parallel \overline{BC}$

, then $X =$ ....



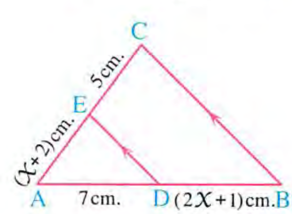
(a) $2\sqrt{2}$ (b) ± 3

(c) 4 (d) $\pm 2\sqrt{2}$

(15) In the opposite figure :

If Δ ABC in which $\overline{DE} \parallel \overline{BC}$

, then $X =$ ....



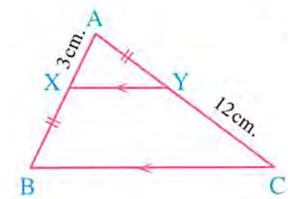
(a) – 5.5 or 3 (b) – 5.5

(c) 3 (d) 2.5

(16) In the opposite figure :

If $\overline{XY} \parallel \overline{BC}$, then

AC = ... cm.



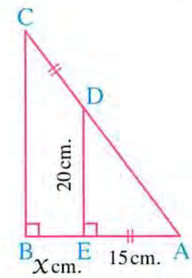
(a) 15 (b) 16

(c) 18 (d) 20

(17) In the opposite figure :

If $\overline{DE} \parallel \overline{BC}$, then

x = ...

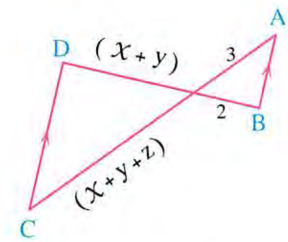


(a) 15 (b) 25

(c) 24 (d) 9

(18) In the opposite figure :

If $\overline{AB} \parallel \overline{CD}$, then z = ...



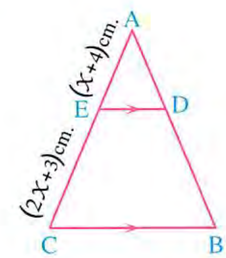
(a) $\frac{x-y}{2}$ (b) $\frac{x+y}{2}$

(c) $5x + 5y$ (d) $\frac{x+y}{5}$

(19) In the opposite figure :

$\overline{ED} \parallel \overline{BC}$, AD : AB = 2 : 5

, then x = ...



(a) 8 (b) 6

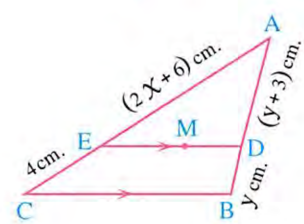
(c) 4 (d) 2

(20) In the opposite figure :

If M is the point of intersection

of medians of Δ ABC

, then $2x + y$ = ... cm.



(a) 2 (b) 3

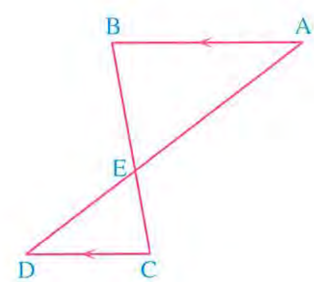
(c) 4 (d) 5

(21) In the opposite figure :

If $\overline{AB} \parallel \overline{CD}$, 2 AE = 3 ED

, BE – CE = 4 cm.

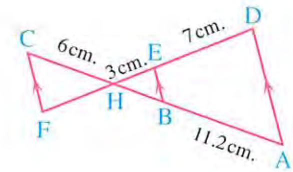
, then BC = ... cm.



(a) 18 (b) 20

(c) 24 (d) 25

(22) In the opposite figure :



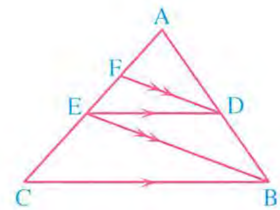
$\overline{AD} \parallel \overline{BE} \parallel \overline{FC}$

, then HF = ... cm.

(a) 3.6 (b) 4.8

(c) 6.3 (d) 3.75

(23) In the opposite figure :



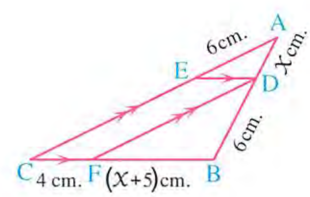
If $\overline{DE} \parallel \overline{BC}$, $\overline{DF} \parallel \overline{BE}$

, then AF × AC = ...

(a) AE (b) $(AE)^2$

(c) $(DE)^2$ (d) FE × EC

(24) In the opposite figure :

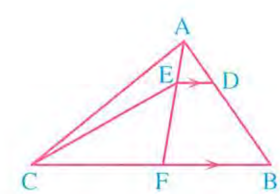


If $\overline{DE} \parallel \overline{BC}$, and $\overline{DF} \parallel \overline{AC}$, then the length of $\overline{EC}$ = ... cm.

(a) 12 (b) 18

(c) 6 (d) 9

(25) In the opposite figure :



$\overline{ED} \parallel \overline{FB}$, a (Δ AEC) = 9 cm²

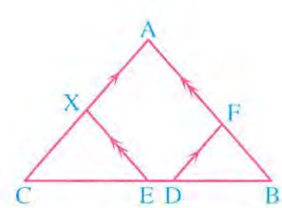
, a (Δ CFE) = 16 cm² , AB = 15 cm.

, then AD = ... cm.

(a) 9.6 (b) 5.4

(c) $8 \frac{4}{7}$ (d) $6 \frac{3}{7}$

(26) In the opposite figure :



If $\overline{FD} \parallel \overline{AC}$ and $\overline{XE} \parallel \overline{AB}$

, BD : DE : EC = 4 : 2 : 5 , AB = AC = 33 cm.

, then AF + AX = ... cm.

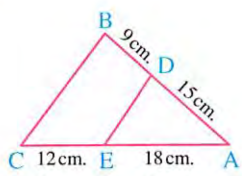
(a) 21 (b) 33

(c) 39 (d) 42

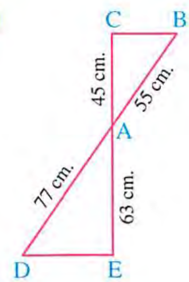
Second Essay questions

1 In each of the following figures , is $\overline{DE} // \overline{BC}$?

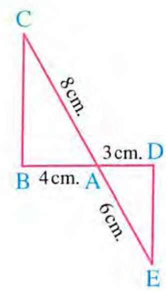
(1)



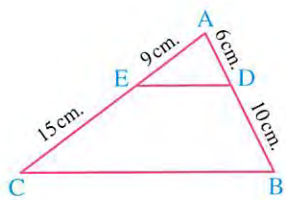
(2)



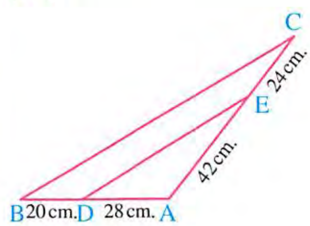
(3)



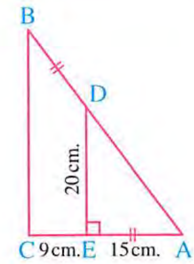
(4)



(5)



(6)

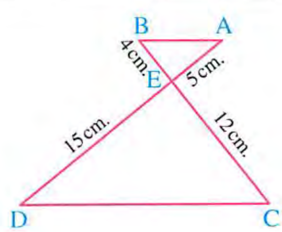


2 In the opposite figure :

$\overline{AD} \cap \overline{BC} = \{E\}$, AE = 5 cm. ,

BE = 4 cm. , CE = 12 cm. and DE = 15 cm.

Prove that : $\overline{AB} // \overline{CD}$



3 $\overline{XY} \cap \overline{ZL} = \{M\}$, where $\overline{XZ} // \overline{LY}$, if XM = 9 cm. , YM = 15 cm. and ZL = 36 cm.

, **find the length of :** $\overline{ZM}$ « 13.5 cm. »

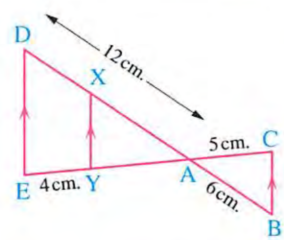
4 In the opposite figure :

$\overline{CE} \cap \overline{BD} = \{A\}$, $X \in \overline{AD}$, $Y \in \overline{AE}$, where

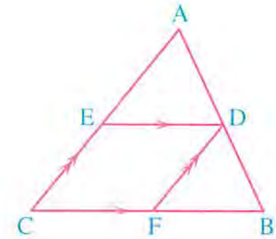
$\overline{XY} // \overline{BC} // \overline{ED}$, if AB = 6 cm. , AC = 5 cm. ,

AD = 12 cm. and EY = 4 cm.

, **find the length of each of :** $\overline{AE}$, $\overline{DX}$ « 10 cm. , 4.8 cm. »



5 **For each of the following , use the opposite figure and the given data to find the value of x (Lengths are measured in centimetres) :**



(1) AD = 4 , BD = 8 , CE = 6 and AE = x

(2) AE = x , EC = 5 , AD = $x - 2$ and DB = 3

(3) AB = 21 , BF = 8 , FC = 6 and AD = x

(4) AD = x , BF = $x + 5$ and 2 DB = 3 FC = 12

---

6 XYZ is a triangle in which XY = 14 cm. , XZ = 21 cm. , L $\in \overline{XY}$, where XL = 5.6 cm. and M $\in \overline{XZ}$ where XM = 8.4 cm. **Prove that :** $\overline{LM} \parallel \overline{YZ}$

---

7 In the triangle ABC , D $\in \overline{AB}$, E $\in \overline{AC}$ and 5 AE = 4 EC. If AD = 10 cm. and DB = 8 cm. , is $\overline{DE} \parallel \overline{BC}$? Explain your answer.

---

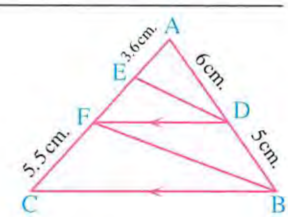
8 ABCD is a trapezium in which $\overline{AD} \parallel \overline{BC}$, its diagonals $\overline{AC}$ and $\overline{BD}$ are intersected at M

If AM = 2.5 cm. , DB = $7\frac{1}{3}$ cm. and MC = 3 cm.

, find the length of each of : $\overline{MD}$ and $\overline{MB}$ « $3\frac{1}{3}$ cm. , 4 cm. »

---

9 **In the opposite figure :**



If $\overline{DF} \parallel \overline{BC}$, AD = 6 cm. ,

BD = 5 cm. , AE = 3.6 cm. and FC = 5.5 cm.

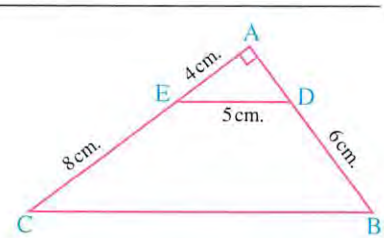
, then prove that : $\overline{DE} \parallel \overline{BF}$

---

10 ABCD is a quadrilateral , its diagonals are intersected at E. If AE = 6 cm. , BE = 13 cm. , EC = 10 cm. and ED = 7.8 cm. , **prove that :** ABCD is a trapezium.

---

11 **In the opposite figure :**



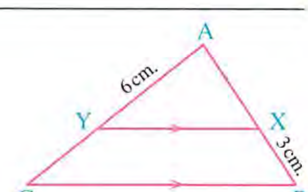
ABC is a right-angled triangle at A

(1) **Prove that :** $\overline{DE} \parallel \overline{BC}$

(2) **Find the length of :** $\overline{BC}$ « 15 cm. »

---

12 **In the opposite figure :**



ABC is a triangle , in which $\overline{XY} \parallel \overline{BC}$

If BX = 3 cm. , AY = 6 cm. and $\frac{AX + AY}{AB + AC} = \frac{3}{5}$

, find the length of each of : $\overline{AX}$, $\overline{CY}$ « 4.5 cm. , 4 cm. »

13 ABC is a triangle , $D \in \overline{AB}$, draw $\overrightarrow{DE} \,//\, \overline{BC}$ to intersect $\overline{AC}$ at E , then draw $\overrightarrow{EF} \,//\, \overline{CD}$ to intersect $\overline{AB}$ at F **Prove that :** $(AD)^2 = AF \times AB$

---

14 ABCD is a quadrilateral , $E \in \overline{AC}$, draw $\overrightarrow{EF} \,//\, \overline{CB}$ to intersect $\overline{AB}$ at F , draw $\overrightarrow{EN} \,//\, \overline{CD}$ to intersect $\overline{AD}$ at N **Prove that :** $\overline{FN} \,//\, \overline{BD}$

---

15 **Prove that :** The line segment drawn between two midpoints of two sides in a triangle is parallel to the third side and its length is equal to a half of the length of this side.

---

16 ABCD is a parallelogram , $E \in \overrightarrow{BA}$, $E \notin \overline{AB}$, draw $\overline{EC}$ to intersect $\overline{AD}$ at F , $\overline{BD}$ at M

Prove that : $(CM)^2 = MF \times ME$

---

17 ABCD is a parallelogram , $E \in \overrightarrow{CB}$, $E \notin \overline{CB}$, draw $\overline{DE}$ to intersect $\overline{AB}$ at N , then draw $\overrightarrow{BG} \,//\, \overline{ED}$ to intersect $\overline{CD}$ at G

Prove that : $\frac{AN}{NB} = \frac{CG}{GD}$

---

18 ABC is a triangle , $D \in \overline{AB}$, where 3 AD = 2 DB and $E \in \overline{AC}$, where 5 CE = 3 AC and $\overline{AX}$ is drawn to intersect $\overline{BC}$ at X , if AF = 8 cm. and AX = 20 cm. where $F \in \overline{AX}$

Prove that : The points D , F and E are collinear.

---

19 ABC is a triangle , $D \in \overline{BC}$, where $\frac{BD}{DC} = \frac{3}{4}$ and $E \in \overline{AD}$, where $\frac{AE}{AD} = \frac{3}{7}$, $\overrightarrow{CE}$ is drawn to intersect $\overline{AB}$ at X , $\overline{DY} \,//\, \overline{CX}$ and intersects $\overline{AB}$ at Y **Prove that :** AX = BY

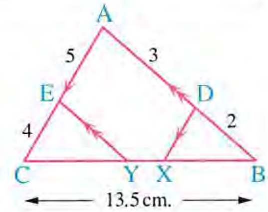
---

20 **In the opposite figure :**

ABC is a triangle in which : $\overline{DX} \,//\, \overline{AC}$, $\overline{EY} \,//\, \overline{AB}$,

BC = 13.5 cm. , $\frac{AD}{DB} = \frac{3}{2}$, $EC = \frac{4}{5}$ AE

Find the length of : $\overline{XY}$



« 2.1 cm. »

---

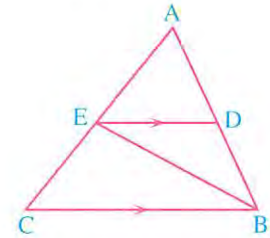
21 ABC is a triangle , D is the midpoint of $\overline{BC}$, $M \in \overline{AD}$, draw $\overrightarrow{ME} \,//\, \overline{AB}$ to intersect $\overline{BC}$ at E , draw $\overrightarrow{MF} \,//\, \overline{AC}$ to intersect $\overline{BC}$ at F

Prove that : D is the midpoint of $\overline{EF}$, if M is the point of intersection of the medians of Δ ABC , **then prove that :** $EF = \frac{1}{3}BC$

22 **In the opposite figure :**

ABC is a triangle in which $\overline{DE} \parallel \overline{BC}$

Prove that : $\frac{\text{The area of } \Delta \text{ ADE}}{\text{The area of } \Delta \text{ ABE}} = \frac{\text{The area of } \Delta \text{ ABE}}{\text{The area of } \Delta \text{ ABC}}$



Third Higher skills

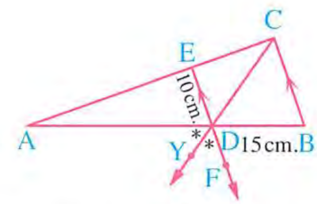
1 **Choose the correct answer from those given :**

(1) In the opposite figure :

If $\overline{ED} \parallel \overline{BC}$, m (∠ ADY) = m (∠ FDY)

and ED = 10 cm. , BD = 15 cm.

, then AD = ... cm.



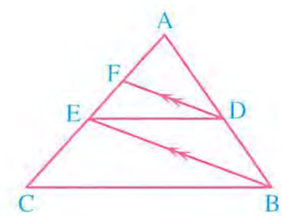
(a) 20 (b) 25 (c) 30 (d) 45

(2) In the opposite figure :

If $\overline{DF} \parallel \overline{BE}$, then to prove that

$\overline{DE} \parallel \overline{BC}$ it is sufficient

to get ....



(a) $\frac{AD}{DB} = \frac{3}{4}$ only (b) $AF \times AC = (AE)^2$ only

(c) (a) , (b) together (d) Nothing of the previous

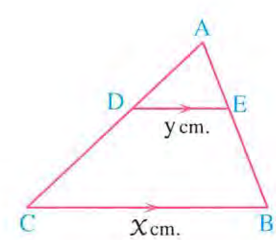
(3) In the opposite figure :

If $\overline{DE} \parallel \overline{BC}$, DE = y cm.

, BC = x cm. , and $2x^2 - 3xy - 5y^2 = 0$

and AB = 10 cm. , then

EB = ... cm.

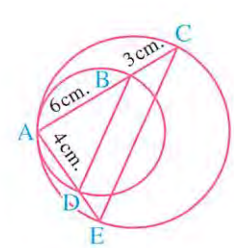


(a) 3 (b) 4 (c) 6 (d) 8

(4) In the opposite figure :

Two circles touching internally at A

, then ED = ... cm.



(a) 2 (b) 3 (c) 3.5 (d) 4

(5) In the opposite figure :

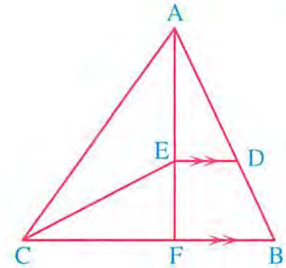
If the area of (Δ AEC) = 15 cm^2

, the area of (Δ EFC) = 9 cm^2

, AB = 16 cm. , then AD = ... cm.

(a) 6 (b) 10

(c) 12 (d) 13



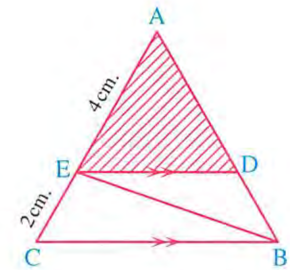
(6) In the opposite figure :

If $\overline{DE}$ // $\overline{BC}$ and the area of (Δ EBC) = 9 cm^2

, then the area of (Δ ADE) = ... cm^2

(a) 6 (b) 12

(c) 18 (d) 27



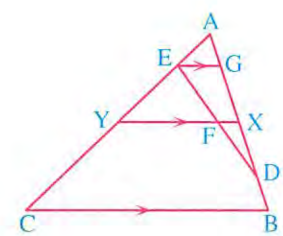
2 **In the opposite figure :**

ABC is a triangle , X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$, D $\in \overline{BX}$,

E $\in \overline{AY}$, where $\frac{AD}{DB} = \frac{CE}{EA}$, $\overline{GE}$ // $\overline{XY}$ // $\overline{BC}$

Prove that : F is the midpoint of $\overline{DE}$



3 ABCD is a rectangle , its diagonals are intersected at M , E is the midpoint of $\overline{AM}$, F is the midpoint of $\overline{MC}$, $\overrightarrow{DE}$ is drawn to intersect $\overline{AB}$ at X and $\overrightarrow{DF}$ is drawn to intersect $\overline{BC}$ at Y

Prove that : $\overline{XY}$ // $\overline{AC}$

Exercise 6

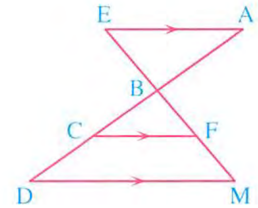
Talis' theorem

Test yourself

From the school book · Remember · Understand · Apply · Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

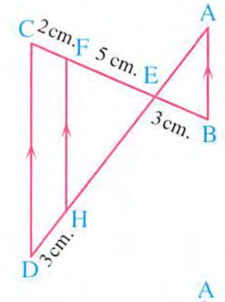


(1) In the opposite figure :

AB : BC : CD = ...

(a) AE : FC : MD (b) EB : BF : FM

(c) EB : BC : CD (d) EB : EF : EM

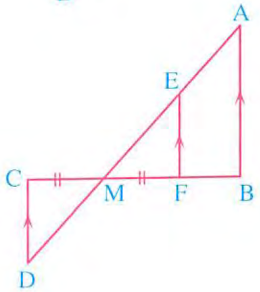


(2) In the opposite figure :

AH = ... cm.

(a) 6 (b) 7.5

(c) 10 (d) 12



(3) In the opposite figure :

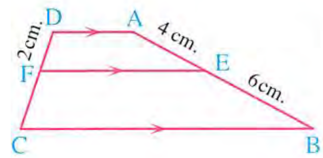
If DA = 21 cm. , MC = 5 cm. , FB = 4 cm.

, then AE = ... cm.

(a) 3 (b) 5

(c) 6 (d) 4

(4) In the opposite figure :



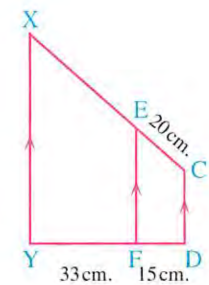
If $\overline{AD}$ // $\overline{EF}$ // $\overline{BC}$, AE = 4 cm.

, EB = 6 cm. , DF = 2 cm.

, then the length of $\overline{CF}$ = ... cm.

(a) 2 (b) 3 (c) 4 (d) 5

(5) In the opposite figure :



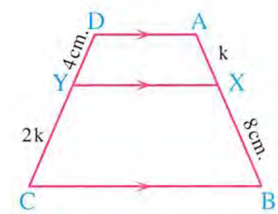
$\overline{CD}$ // $\overline{EF}$ // $\overline{XY}$, CE = 20 cm.

, DF = 15 cm. , FY = 33 cm.

, then the length of $\overline{CX}$ = ... cm.

(a) 48 (b) 64 (c) 44 (d) 21

(6) In the opposite figure :

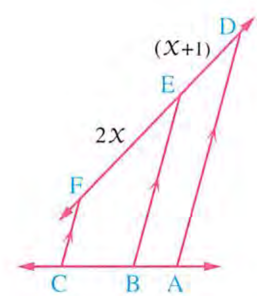


If $\overline{AD}$ // $\overline{XY}$ // $\overline{BC}$, then

AX = ... cm.

(a) $\frac{3}{8}$ (b) 4 (c) 16 (d) 32

(7) In the opposite figure :



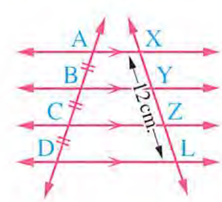
If $\overline{AD}$ // $\overline{BE}$ // $\overline{CF}$, AB = 3 cm.

, BC = 5 cm. , DE = $(X + 1)$ cm.

, EF = 2 X cm. , then X = ... cm.

(a) 3 (b) 4 (c) 5 (d) 8

(8) In the opposite figure :

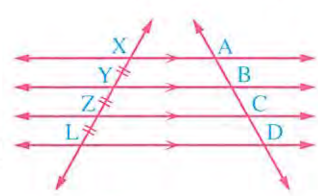


If AB = BC = CD ,

XL = 12 cm. , then XZ = ...

(a) 4 cm. (b) YL (c) AC (d) BC

(9) In the opposite figure :

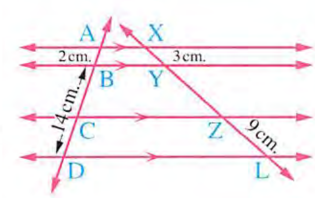


If BD = 14 cm.

, AC = ... cm.

(a) 7 (b) 14 (c) 21 (d) 28

(10) In the opposite figure :

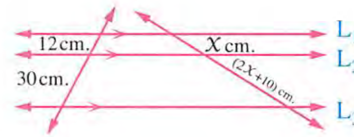


CD = ... cm.

(a) 12 (b) 6 (c) 14 (d) 5

(11) In the opposite figure :

$X =$ .... cm.

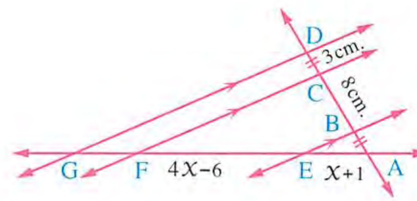


(a) 10 (b) 20

(c) 15 (d) 8

(12) In the opposite figure :

$X =$ ....

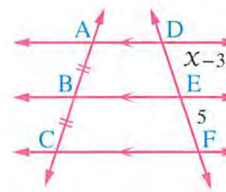


(a) 2 (b) 3.5

(c) 5 (d) 6.5

(13) In the opposite figure :

$X =$ ....

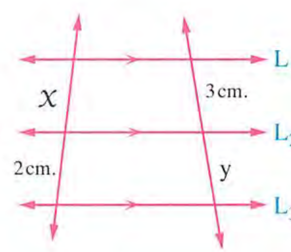


(a) 3 (b) 5

(c) 8 (d) 2

(14) In the opposite figure :

If $X > 2$, then ....



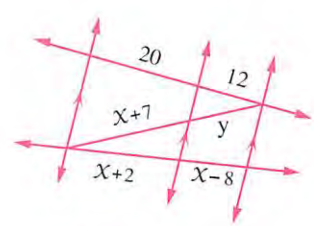
(a) $y = 3$ (b) $y > 3$

(c) $y < 3$ (d) $y \leq 3$

(15) In the opposite figure :

If the given lengths in cm.

, then $X + y$ = ... cm.



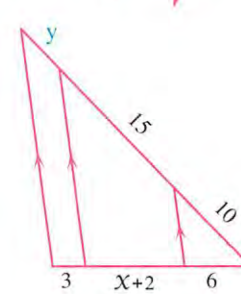
(a) 23 (b) 18

(c) 41 (d) 51

(16) In the opposite figure :

If the given lengths in cm.

, then $X + y$ = ... cm.

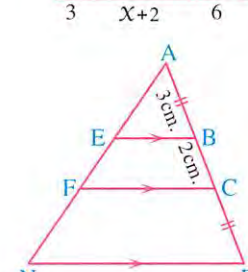


(a) 5 (b) 7

(c) 11 (d) 12

(17) In the opposite figure :

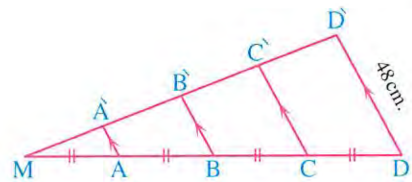
$\frac{BE}{DN} =$ ....



(a) $\frac{3}{8}$ (b) $\frac{3}{4}$

(c) $\frac{3}{5}$ (d) $\frac{3}{2}$

(18) In the opposite figure :

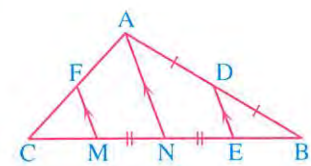


$A\grave{A}$ = ... cm.

(a) 4 (b) 8

(c) 12 (d) 16

(19) In the opposite figure :

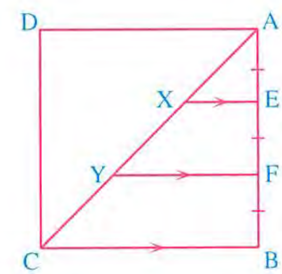


If BC = 35 cm. , $\frac{CF}{FA} = \frac{1}{2}$

, then BE = ... cm.

(a) 5 (b) 7 (c) 10 (d) 14

(20) In the opposite figure :



ABCD is a square of side length 6 cm.

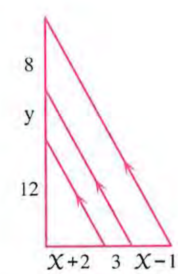
, if AE = FE = FB

, then area of the shape XYFE = ... cm^2.

(a) 8 (b) 10

(c) 12 (d) 6

(21) In the opposite figure :



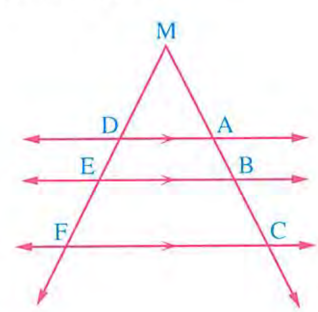
(X , y) = ...

(a) (5 , 7) (b) (4 , 6)

(c) (7 , 4) (d) (11 , 7)

Second Essay questions

1 **Write what each of the following ratios equals using the opposite figure :**



(1) $\frac{AB}{BC} = \frac{DE}{..........}$

(2) $\frac{AC}{BC} = \frac{..........}{EF}$

(3) $\frac{MA}{AB} = \frac{MD}{..........}$

(4) $\frac{AC}{AB} = \frac{..........}{DE}$

(5) $\frac{MB}{AB} = \frac{..........}{DE}$

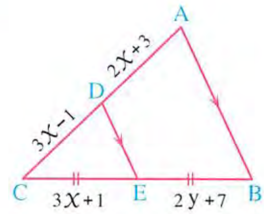
(6) $\frac{MC}{AC} = \frac{MF}{..........}$

(7) $\frac{BC}{MB} = \frac{EF}{..........}$

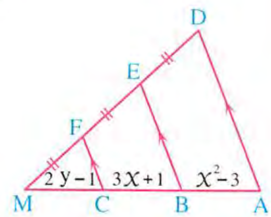
(8) $\frac{DF}{MF} = \frac{AC}{..........}$

2 In each of the following figures , calculate the numerical values of x and y (Lengths are measured in centimetres) :

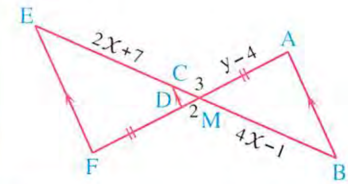
(1)



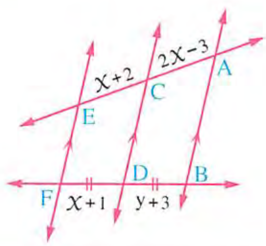
(2)



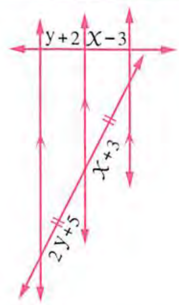
(3)



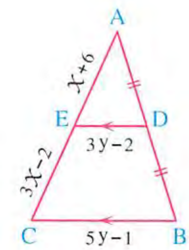
(4)



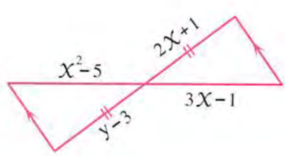
(5)



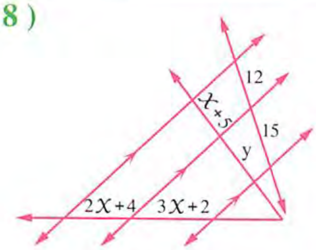
(6)



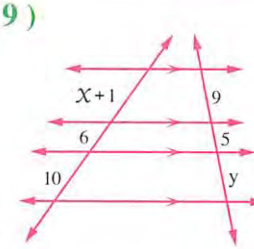
(7)



(8)



(9)



3 **In the opposite figure :**

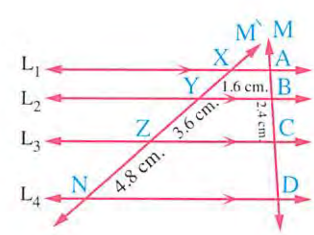
$L_1 // L_2 // L_3 // L_4$,

M , M̀ are two transversals.

If AB = 1.6 cm. , BC = 2.4 cm. ,

YZ = 3.6 cm. , ZN = 4.8 cm.

Calculate the length of each of : $\overline{XY}$ and $\overline{CD}$



« 2.4 cm. , 3.2 cm. »

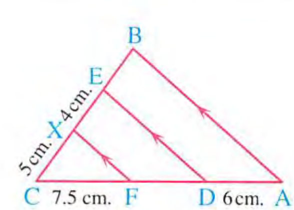
4 **In the opposite figure :**

If $\overline{AB}$ // $\overline{DE}$ // $\overline{FX}$,

AD = 6 cm. , EX = 4 cm. ,

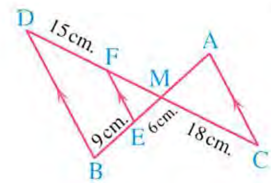
FC = 7.5 cm. , CX = 5 cm.

Find the length of each of : $\overline{DF}$, $\overline{BE}$



« 6 cm. , 4 cm. »

5 **In the opposite figure :**



$\overline{AB} \cap \overline{CD} = \{M\}$, $E \in \overline{MB}$,

$F \in \overline{MD}$ and $\overline{AC} \parallel \overline{FE} \parallel \overline{DB}$

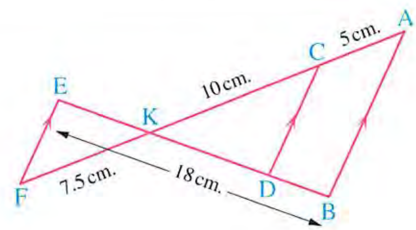
Find : (1) The length of $\overline{MF}$

(2) The length of $\overline{AM}$

« 10 cm. , 10.8 cm. »

---

6 **In the opposite figure :**



If $\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$,

AC = 5 cm. , CK = 10 cm. ,

KF = 7.5 cm. , BE = 18 cm.

Find the length of each of : $\overline{BD}$, $\overline{DK}$ and $\overline{KE}$

« 4 cm. , 8 cm. , 6 cm. »

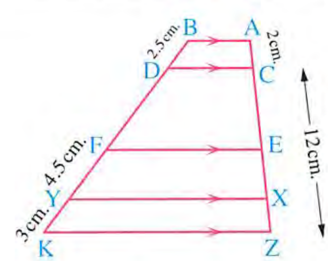
---

7 $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$, $X \in \overline{AB}$, $Y \in \overline{CD}$, and $\overline{XY} \parallel \overline{BD} \parallel \overline{AC}$

Prove that : AX × ED = CY × EB

---

8 **In the opposite figure :**



$\overline{AB} \parallel \overline{CD} \parallel \overline{EF} \parallel \overline{XY} \parallel \overline{ZK}$,

AC = 2 cm. , BD = 2.5 cm. ,

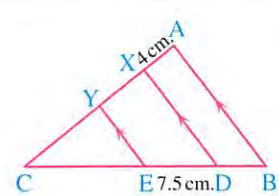
FY = 4.5 cm. , FK = 7.5 cm. , CZ = 12 cm.

Find the length of each of : $\overline{EX}$, $\overline{XZ}$, $\overline{CE}$ and $\overline{DF}$

« 3.6 cm. , 2.4 cm. , 6 cm. , 7.5 cm. »

---

9 **In the opposite figure :**



$\overline{AB} \parallel \overline{DX} \parallel \overline{EY}$,

AX : XY : YC = 2 : 3 : 5

If DE = 7.5 cm. , AX = 4 cm.

, find the length of each of : $\overline{BD}$, $\overline{CE}$ and $\overline{AC}$

« 5 cm. , 12.5 cm. , 20 cm. »

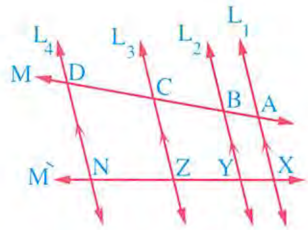
---

10 ABC is a triangle , D , $E \in \overline{AB}$, let $\overrightarrow{DX}$, $\overrightarrow{EY}$ be drawn parallel to $\overline{BC}$ and intersect $\overline{AC}$ at X and Y respectively , if $AD = \frac{1}{2}$ BE , DE = 3 AD , AC = 24 cm.

Find the length of each of : $\overline{AX}$, $\overline{XY}$ and $\overline{YC}$

« 4 cm. , 12 cm. , 8 cm. »

11 **In the opposite figure :**



$L_1 // L_2 // L_3 // L_4$ and M , M` are two transversals.

If $\frac{AB}{BC} = \frac{1}{2}$, $BC = \frac{4}{5}$ CD and XN = 16.5 cm.

Find the length of each of : $\overline{XY}$, $\overline{YZ}$ and $\overline{ZN}$

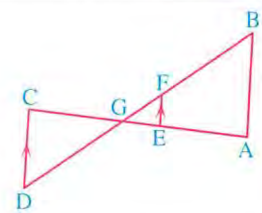
« 3 cm. , 6 cm. , 7.5 cm. »

---

12 ABC is a triangle , D $\in \overline{AB}$ where $\frac{AD}{DB} = \frac{3}{5}$, let E $\in \overrightarrow{BA}$ outside the triangle such that : AE = $\frac{1}{2}$ AB , let $\overrightarrow{DX}$, $\overrightarrow{EY}$ be drawn parallel to $\overline{BC}$ to intersect $\overleftrightarrow{AC}$ at X , Y respectively. If AY = 14 cm. **Find the length of each of :** $\overline{AX}$, $\overline{AC}$ « 10.5 cm. , 28 cm. »

---

13 **In the opposite figure :**



$\overline{EF} // \overline{CD}$, $\frac{AG}{GC} = \frac{DG}{GF}$

Prove that : $(GC)^2 = GA \times GE$

---

14 ABCD is a trapezium in which $\overline{AB} // \overline{DC}$ and M is the midpoint of $\overline{AD}$, draw a straight line passing through the point M and parallel to $\overline{DC}$ to intersect the diagonal $\overline{BD}$ at N , diagonal $\overline{AC}$ at E and the side $\overline{BC}$ at F

(1) Show that the points N , E , F are the midpoints of $\overline{BD}$, $\overline{AC}$ and $\overline{BC}$ respectively.

(2) **Prove that :** MF = $\frac{1}{2}$ (AB + DC)

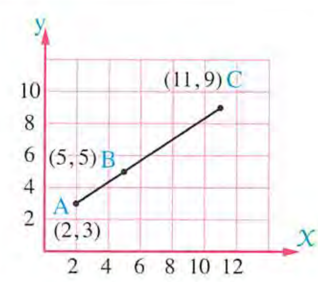
---

15 ABCD is a quadrilateral in which $\overline{AB} // \overline{CD}$, its diagonals intersect at M and E is the midpoint of $\overline{BC}$, $\overrightarrow{EF} // \overline{BA}$ and intersects $\overline{BD}$ at X , $\overline{AC}$ at Y and $\overline{AD}$ at F

Prove that : (1) EY = $\frac{1}{2}$ AB (2) $\frac{AY}{CM} = \frac{BX}{DM}$

---

16 **Logical thinking :**



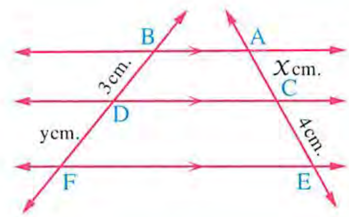
From the figure , find the value of $\frac{AB}{BC}$ in different methods , if possible.

Did you get the same result ?

Third Higher skills

1 **Choose the correct answer from those given :**

(1) In the opposite figure :



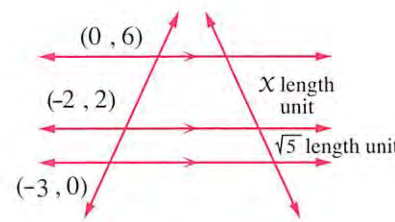
If $X^2 + y^2 = 57$

, then $X + y = \cdots\cdots\cdots$ cm.

(a) 7 (b) 9

(c) 11 (d) 12

(2) In the opposite figure :

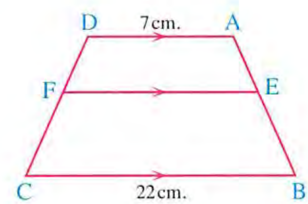


$X = \cdots\cdots\cdots$

(a) $\sqrt{5}$ (b) $2\sqrt{5}$

(c) $3\sqrt{5}$ (d) $4\sqrt{5}$

(3) In the opposite figure :

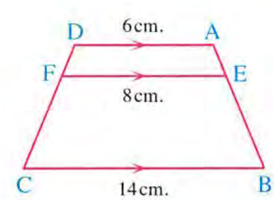


If $\frac{AE}{EB} = \frac{2}{3}$, then EF = ……… cm.

(a) 9 (b) 11

(c) 13 (d) 15

(4) In the opposite figure :



$\frac{AE}{EB} = \cdots\cdots\cdots$

(a) $\frac{3}{4}$ (b) $\frac{4}{7}$

(c) $\frac{3}{7}$ (d) $\frac{1}{3}$

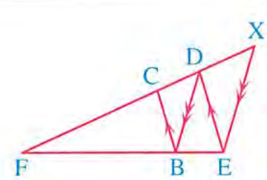
---

2 ABC is a triangle , M is the midpoint of $\overline{BC}$, let K $\in \overline{AM}$, draw $\overrightarrow{KE}$ // $\overline{AB}$ to intersect $\overline{BC}$ at E , draw $\overrightarrow{KG}$ // $\overline{AC}$ to intersect $\overline{BC}$ at G

Prove that : M is the midpoint of $\overline{EG}$, if K is the point of intersection of the medians of Δ ABC , **then prove that :** BE = EG = GC = $\frac{1}{3}$ BC

---

3 **In the opposite figure :**



$\overline{ED}$ // $\overline{BC}$, $\overline{DB}$ // $\overline{EX}$

Prove that : $\left(\frac{FB}{FE}\right)^2 = \frac{FC}{FX}$

---

4 ABCD is a parallelogram , draw $\overrightarrow{DE}$ to intersect $\overline{AC}$, $\overline{AB}$ at X , E respectively , draw $\overrightarrow{DF}$ to intersect $\overline{AC}$, $\overline{BC}$ at Y , F respectively. If AX = CY , **prove that :** $\overline{EF}$ // $\overline{XY}$

Exercise 7

Angle bisector and proportional parts

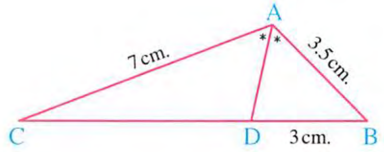
Test yourself

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

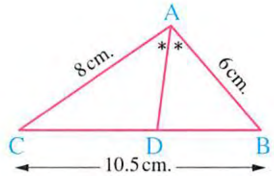
(1) In the opposite figure :



CD = ... cm.

(a) 4.5 (b) 5 (c) 4.9 (d) 6

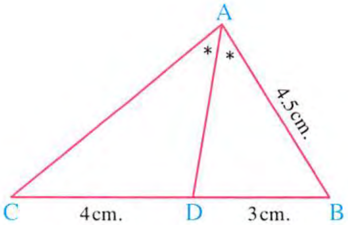
(2) In the opposite figure :



BD = ... cm.

(a) 4 (b) $\frac{2}{3}$ (c) 4.5 (d) 45

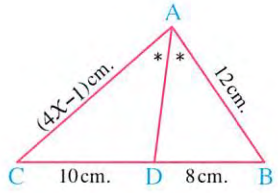
(3) In the opposite figure :



AC = ...

(a) 6 (b) 4.8 (c) 7 (d) 8

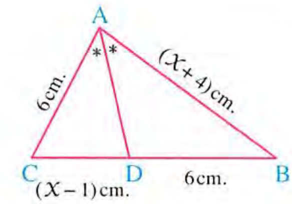
(4) In the opposite figure :



X = ...

(a) 4 (b) 3 (c) 4.5 (d) 6

(5) In the opposite figure :

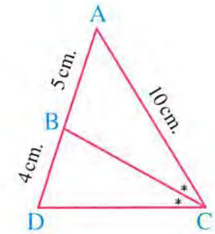


$X =$ ·········· cm.

(a) 6 (b) 5

(c) 8 (d) 10

(6) In the opposite figure :

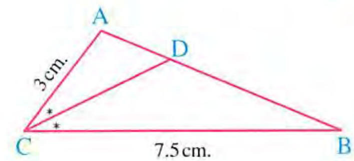


CB = ·········· cm.

(a) 8 (b) $4\sqrt{2}$

(c) $2\sqrt{15}$ (d) 6

(7) In the opposite figure :



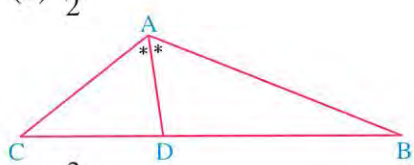
$\overrightarrow{CD}$ bisects $\angle$ C ,

AC = 3 cm. , BC = 7.5 cm.

, then AD : BD = ··········

(a) $\frac{3}{5}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$ (d) $\frac{5}{2}$

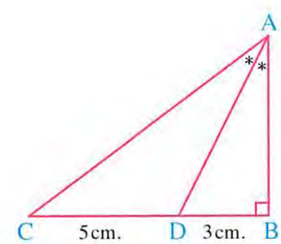
(8) In the opposite figure :



If AB : AC : BC = 5 : 3 : 7 , then BD : DC = ··········

(a) $\frac{5}{3}$ (b) $\frac{5}{7}$ (c) $\frac{3}{5}$ (d) $\frac{3}{7}$

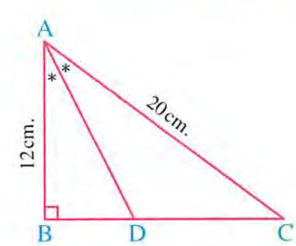
(9) In the opposite figure :



AB = ·········· cm.

(a) 4 (b) 5 (c) 6 (d) 7

(10) In the opposite figure :



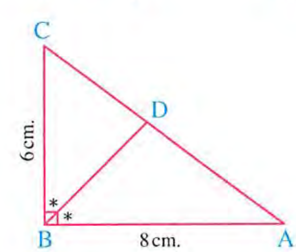
$\overrightarrow{AD}$ bisects $\angle$ BAC , $\angle$ B is a right angle

if AB = 12 cm. , AC = 20 cm. , then CD = ·········· cm.

(a) 6 (b) 8

(c) 10 (d) 9

(11) In the opposite figure :

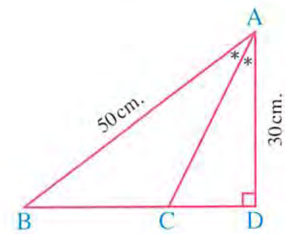


AD = ·········· cm.

(a) $5\frac{5}{7}$ (b) $6\frac{3}{4}$

(c) 5 (d) $\frac{4}{3}$

(12) In the opposite figure :

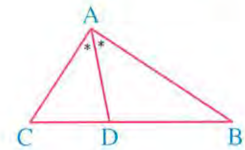


The perimeter of Δ ABC ≈ ·········· cm.

(a) 123.5 (b) 375

(c) 98.5 (d) 108.5

(13) 📖 In the opposite figure :

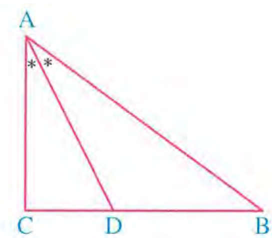


$\overrightarrow{AD}$ bisects $\angle$ A , then AB × CD = ··········

(a) AC × BD (b) $(AD)^2$

(c) AD × BD (d) AC × AB

(14) In the opposite figure :



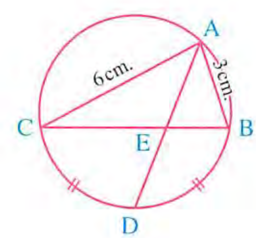
If $\overrightarrow{AD}$ bisects $\angle$ BAC

, then ··········

(a) BD = DC (b) Δ ABD ~ Δ ACD

(c) BA × CD = AC × BD (d) $(AD)^2$ = DB × DC

(15) In the opposite figure :

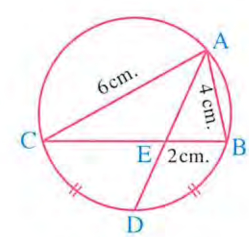


$\frac{BE}{BC}$ = ··········

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{1}{3}$ (d) 3

(16) In the opposite figure :



The length of $\overline{DE}$ = ·········· cm.

(a) 4 (b) 2

(c) $\sqrt{2}$ (d) $3\sqrt{2}$

(17) The exterior bisector of the vertex angle of an isosceles triangle ·········· the base.

(a) bisects (b) perpendicular to

(c) intersect (d) parallel

(18) The bisector of the exterior angle of an equilateral triangle ·········· the side opposite to the vertex of this angle.

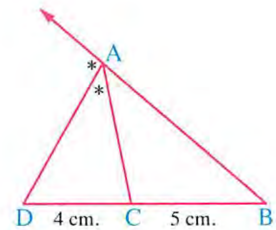
(a) bisects (b) congruent to (c) parallel (d) perpendicular to

(19) The measure of the angle included between the interior and the exterior bisector at any vertex of angles of the triangle equal ....

(a) 45° (b) 90° (c) 135° (d) 180°

(20) In the opposite figure :

AB : AC = ...

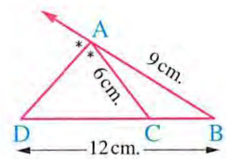


(a) 5 : 4 (b) 5 : 9

(c) 9 : 5 (d) 9 : 4

(21) In the opposite figure :

CD = ... cm.

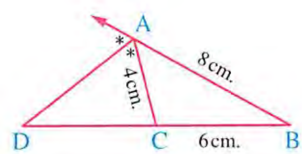


(a) 8 (b) 6

(c) 4.8 (d) 5

(22) In the opposite figure :

CD = ... cm.



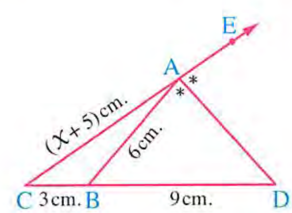
(a) 2 (b) 6 (c) 4 (d) 8

(23) In the opposite figure :

$\overrightarrow{AD}$ bisects $\angle$ BAE , if AC = $(X + 5)$ cm. ,

AB = 6 cm. , BC = 3 cm. , BD = 9 cm.

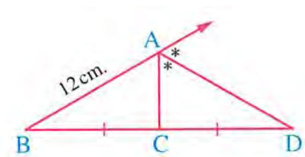
, then X = ... cm.



(a) 4 (b) 3 (c) 2 (d) 6

(24) In the opposite figure :

AC = ... cm.

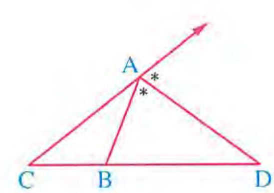


(a) 3 (b) 4 (c) 6 (d) 8

(25) In the opposite figure :

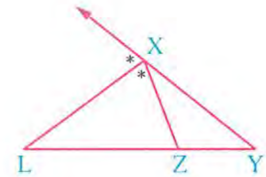
If AB : AC = 2 : 3

, then BD : BC = ...



(a) 2 : 1 (b) $\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$

(26) In the opposite figure :

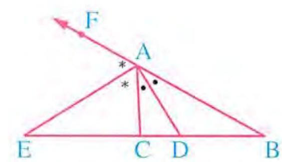


$\overrightarrow{XL}$ bisects the exterior angle X , then $\frac{YL}{YX} = \cdots\cdots$

(a) $\frac{YZ}{ZL}$ (b) $\frac{YL}{LZ}$

(c) $\frac{LZ}{ZX}$ (d) $\frac{XZ}{XY}$

(27) By using the opposite figure :

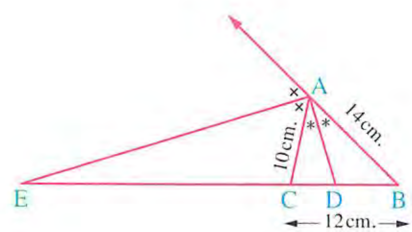


All the following statements are true except $\cdots\cdots$

(a) $\frac{BA}{AC} = \frac{BD}{DC}$ (b) $\frac{BA}{AC} = \frac{BE}{EC}$

(c) $\frac{CA}{AB} = \frac{DA}{AE}$ (d) $\angle$ DAE is a right angle

(28) In the opposite figure :

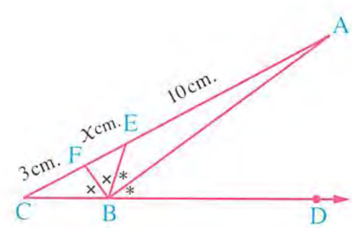


DE $= \cdots\cdots$ cm.

(a) 12 (b) 24

(c) 30 (d) 35

(29) In the opposite figure :

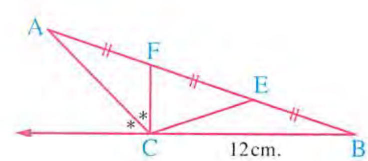


$X = \cdots\cdots$ cm.

(a) 1 (b) 2

(c) 3 (d) 4

(30) In the opposite figure :

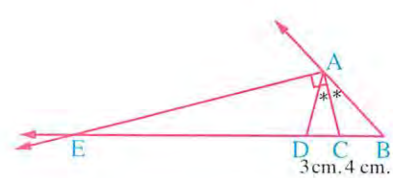


CF $= \cdots\cdots$ cm.

(a) 3 (b) 4

(c) 5 (d) 6

(31) In the opposite figure :



$\overrightarrow{AC}$ is the interior bisector of (Δ ABD) at ($\angle$ A)

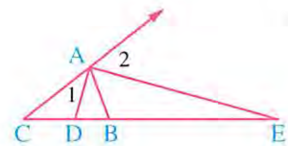
, $\overline{AE} \perp \overline{AC}$, BC = 4 cm. , CD = 3 cm.

, then BE : ED $= \cdots\cdots$

(a) 7 : 4 (b) 7 : 3 (c) 3 : 4 (d) 4 : 3

(32) In the opposite figure :

Δ ABC is a triangle in which $\overrightarrow{AD}$ and $\overrightarrow{AE}$ are the interior and exterior bisectors of the angle at the vertex A respectively , If m ($\angle$ 1) = 36° , then m ($\angle$ 2) = ...°

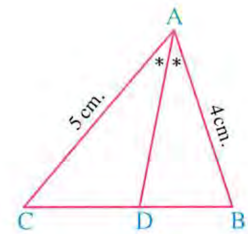


(a) 36 (b) 40 (c) 54 (d) 108

(33) In the opposite figure :

AB = 4 cm. , AC = 5 cm. , $\overrightarrow{AD}$ bisects $\angle$ A

, then a (Δ ABD) : a (Δ ACD) = ...

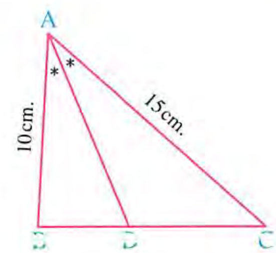


(a) 16 : 25 (b) 25 : 16 (c) 4 : 5 (d) 5 : 2

(34) In the opposite figure :

If a (Δ ABC) = 75 cm^2.

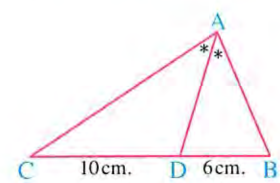
, then a (Δ ADB) = ... cm^2.



(a) 30 (b) $3\frac{1}{13}$ (c) $51\frac{12}{13}$ (d) 45

(35) In the opposite figure :

If AC – AB = 6 cm. , then AC = ... cm.

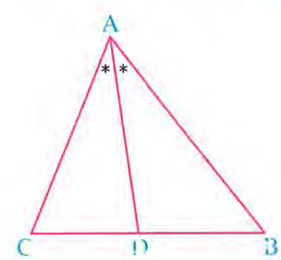


(a) 13 (b) 14 (c) 15 (d) 16

(36) In the opposite figure :

If AB × AC = 8 , BD × DC = 4 and $\overrightarrow{AD}$ bisects $\angle$ BAC

, then AD = ... length units.



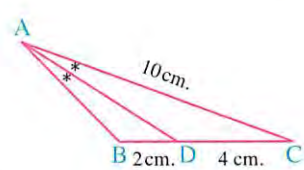
(a) 2 (b) 4 (c) 5 (d) 6

(37) In the opposite figure :

If $\overrightarrow{AD}$ is the interior bisector of $\angle$ BAC , AC = 10 cm.

, DC = 4 cm. , DB = 2 cm.

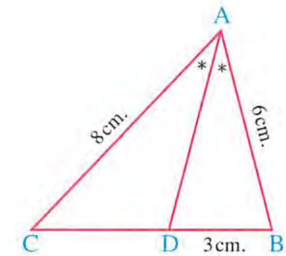
, then the length of $\overline{AD}$ = ... cm.



(a) 9 (b) 5 (c) $\sqrt{42}$ (d) $\sqrt{98}$

(38) In the opposite figure :

If $\overrightarrow{AD}$ bisects $\angle A$, then AD = ... cm.



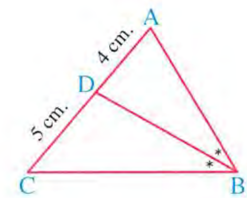
(a) 12 (b) 6

(c) 21 (d) $\frac{6 \times 8}{7}$

(39) In the opposite figure :

If the perimeter of Δ ABC = 27 cm.

, then BD = ... cm.

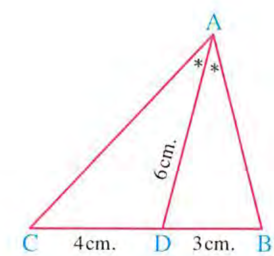


(a) 8 (b) 10

(c) $2\sqrt{15}$ (d) $3\sqrt{15}$

(40) In the opposite figure :

AC = ... cm.

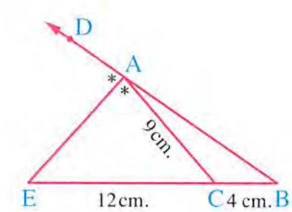


(a) 12 (b) 10

(c) 9 (d) 8

(41) In the opposite figure :

The length of $\overline{AE}$ = ... cm.

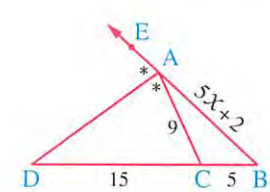


(a) $2\sqrt{15}$ (b) 6

(c) 15 (d) $2\sqrt{21}$

(42) In the opposite figure :

AD = ...



(a) 2 (b) 4

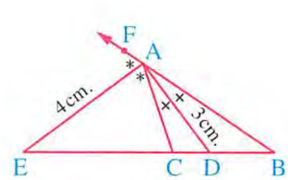
(c) $5\sqrt{3}$ (d) $8\sqrt{3}$

(43) In the opposite figure :

$\overrightarrow{AD}$ bisects $\angle A$ internally , $\overrightarrow{AE}$ bisects $\angle A$ externally ,

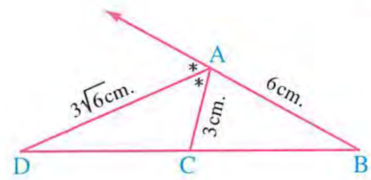
AD = 3 cm. , AE = 4 cm.

, then DE = ... cm.



(a) 3 (b) 4 (c) 5 (d) 6

(44) In the opposite figure :

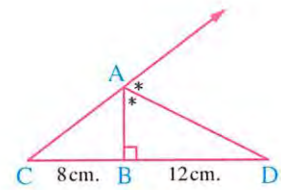


DC = ... cm.

(a) 6 (b) $6\sqrt{3}$

(c) $3\sqrt{6}$ (d) 3

(45) In the opposite figure :

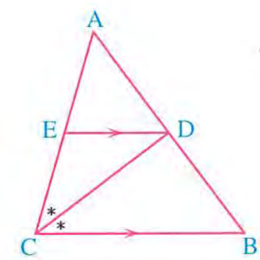


AD = ... cm.

(a) 10 (b) $4\sqrt{5}$

(c) $6\sqrt{5}$ (d) $9\sqrt{2}$

(46) In the opposite figure :

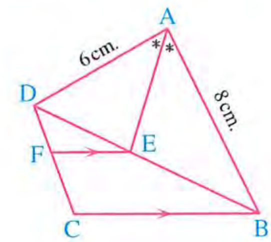


$\frac{AE}{EC}$ = ...

(a) $\frac{DE}{BC}$ (b) $\frac{AD}{AB}$

(c) $\frac{AC}{CB}$ (d) $\frac{AB}{BC}$

(47) In the opposite figure :

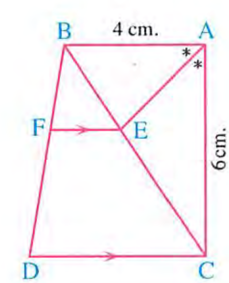


$\frac{DF}{FC}$ = ...

(a) $\frac{4}{3}$ (b) $\frac{8}{7}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

(48) In the opposite figure :

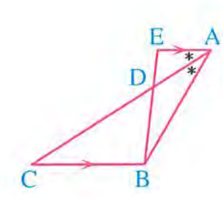


$\frac{EF}{CD}$ = ...

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{3}{2}$

(49) In the opposite figure :



If AC = 3 AD

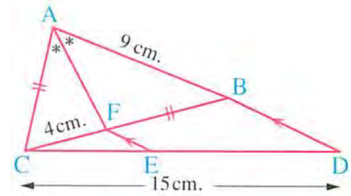
, then AB : AE = ...

(a) 3 : 1 (b) 1 : 2

(c) 4 : 3 (d) 2 : 1

(50) In the opposite figure :

ED = ·········· cm.

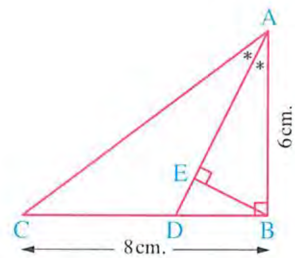


(a) 6 (b) 8

(c) 9 (d) 12

(51) In the opposite figure :

The length of $\overline{DE}$ = ·········· cm.



(a) $\frac{5}{3}\sqrt{5}$ (b) $\frac{3}{5}\sqrt{5}$

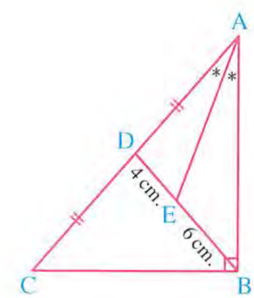
(c) $\frac{5}{3}\sqrt{3}$ (d) $\frac{3}{5}\sqrt{3}$

(52) In the opposite figure :

If m (∠ B) = 90° , D is the midpoint of $\overline{AC}$

, $\overrightarrow{AE}$ bisects ∠ BAD , BE = 6 cm. , ED = 4 cm.

, then the length of $\overline{AB}$ = ·········· cm.

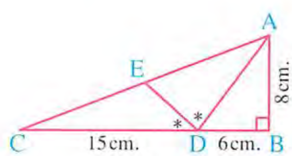


(a) 15 (b) 12 (c) 10 (d) 8

(53) In the opposite figure :

$\overline{AB} \perp \overline{BC}$, $\overrightarrow{DE}$ bisects ∠ ADC

, then the area (Δ ADE) = ·········· cm^2.



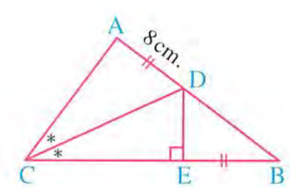
(a) 12 (b) 14 (c) 40 (d) 24

(54) In the opposite figure :

$\overrightarrow{CD}$ bisects ∠ ACB ,

AD = EB = 8 cm.

and $\frac{CB}{CA} = \frac{5}{4}$, then DE = ·········· cm.

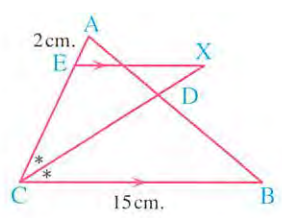


(a) 8 (b) 6 (c) 12 (d) 10

(55) In the opposite figure :

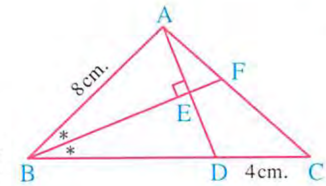
If $\overrightarrow{CX}$ bisects ∠ C , $\overrightarrow{XE}$ // $\overline{BC}$, $\frac{BD}{DA} = \frac{3}{2}$

, then EX = ·········· cm.



(a) 6 (b) 4 (c) 8 (d) 10

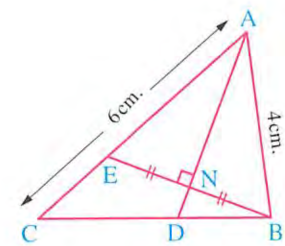
(56) In the opposite figure :



$\frac{AF}{FC} = \dots\dots\dots$

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{1}{2}$

(57) In the opposite figure :



If AC = 6 cm. , AB = 4 cm. , then

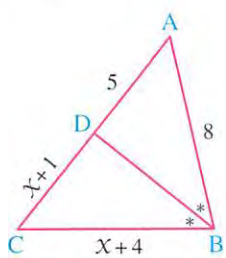
$\frac{BD}{BC} = \dots\dots\dots$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{2}{5}$ (d) $\frac{5}{2}$

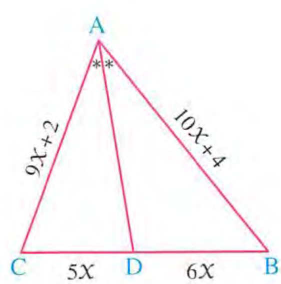
Second Essay questions

1 **In each of the following figures , find the value of X (Lengths are measured in centimetres) :**

(1)

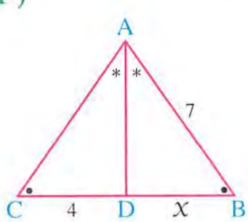


(2)

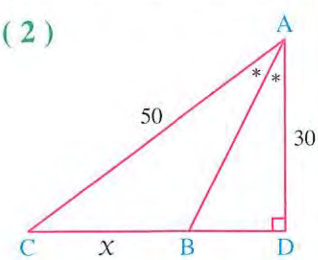


2 **In each of the following figures , find the value of X (Lengths are measured in centimetres) , then find the perimeter of Δ ABC :**

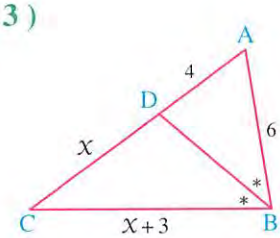
(1)



(2)

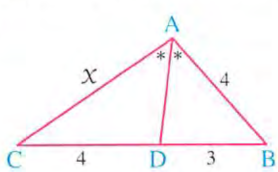


(3)

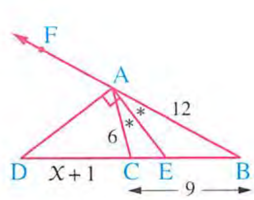


3 **In each of the following figures , calculate the value of X and the length of $\overline{AD}$ (Lengths are measured in centimetres) :**

(1)



(2)



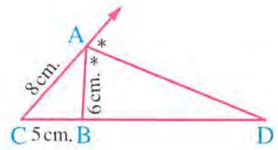
4 ABC is a triangle in which : AB = 4 cm. , BC = 6 cm. , draw $\overrightarrow{BD}$ bisects ∠ ABC and intersects $\overline{AC}$ at D , if AD = 2.4 cm. , **find the length of :** $\overline{AC}$ « 6 cm. »

5 ABC is a triangle in which : AB = 8 cm. , AC = 6 cm. , BC = 7 cm. , $\overrightarrow{AD}$ bisects ∠ BAC and intersects $\overline{BC}$ at D **Find the length of each of :** $\overline{DB}$, $\overline{DC}$ « 4 cm. , 3 cm. »

6 **In the opposite figure :**

ABC is a triangle in which $\overrightarrow{AD}$ bisects the exterior angle at A and intersects $\overrightarrow{CB}$ at D , if AB = 6 cm. , AC = 8 cm. , BC = 5 cm.

Find the length of each of : $\overline{BD}$, $\overline{AD}$



« 15 cm. , $6\sqrt{7}$ cm. »

7 ABC is a triangle in which AB = 3 cm. , BC = 4 cm. , CA = 6 cm. , $\overrightarrow{AD}$ bisects the exterior angle at A and intersects $\overleftrightarrow{BC}$ at D , **find the length of each of :** $\overline{CD}$, $\overline{AD}$ « 8 cm. , $\sqrt{14}$ cm. »

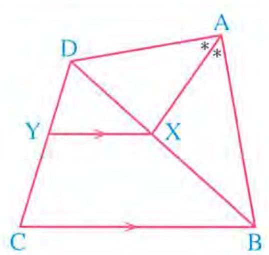
8 ABC is a triangle , its perimeter is 27 cm. , $\overrightarrow{BD}$ bisects ∠ B and intersects $\overline{AC}$ at D If AD = 4 cm. and CD = 5 cm. , **find the length of each of :** $\overline{AB}$, $\overline{BC}$ and $\overline{BD}$

« 8 cm. , 10 cm. , $2\sqrt{15}$ cm. »

9 **In the opposite figure :**

ABCD is a quadrilateral , draw $\overrightarrow{AX}$ bisects ∠ A and intersects $\overline{BD}$ at X , then draw $\overline{XY}$ // $\overline{BC}$ and intersects $\overline{CD}$ at Y

Prove that : $\frac{DY}{YC} = \frac{AD}{AB}$

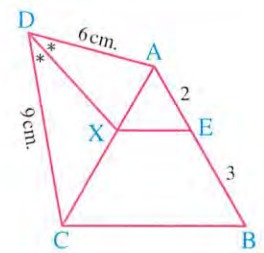


10 **In the opposite figure :**

ABCD is a quadrilateral in which $\overrightarrow{DX}$ bisects ∠ D ,

AE : EB = 2 : 3 , AD = 6 cm. , DC = 9 cm.

, **prove that :** $\overline{EX}$ // $\overline{BC}$



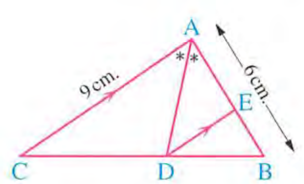
11 **In the opposite figure :**

$\overrightarrow{AD}$ bisects ∠ BAC , $\overline{ED}$ // $\overline{AC}$

Prove that : $\frac{BE}{EA} = \frac{BA}{AC}$

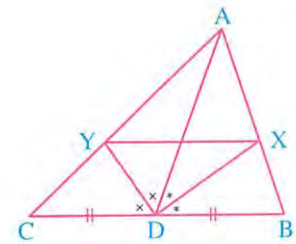
and if AC = 9 cm. , AB = 6 cm.

, **find the length of each of :** $\overline{AE}$ and $\overline{BE}$



« 3.6 cm. , 2.4 cm. »

12 **In the opposite figure :**



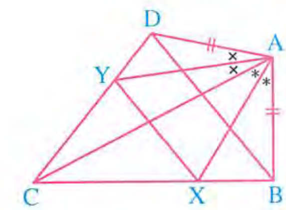
$\overline{AD}$ is a median of ΔABC ,

$\overrightarrow{DX}$ bisects $\angle ADB$, $\overrightarrow{DY}$ bisects $\angle ADC$

Prove that : $\overline{XY} \parallel \overline{BC}$

---

13 **In the opposite figure :**



ABCD is a quadrilateral in which AB = AD ,

$\overrightarrow{AX}$ bisects $\angle BAC$ and intersects $\overline{BC}$ at X ,

$\overrightarrow{AY}$ bisects $\angle DAC$ and intersects $\overline{CD}$ at Y

Prove that : $\overline{XY} \parallel \overline{BD}$

---

14 ABC is a right-angled triangle at B , draw $\overrightarrow{AD}$ bisects $\angle A$, and intersects $\overline{BC}$ at D

If the length of $\overline{BD}$ equals 24 cm. , BA : AC = 3 : 5 , find the perimeter of ΔABC « 192 cm. »

---

15 ABC is a triangle in which AB = 8 cm. , AC = 4 cm. and BC = 6 cm. , $\overrightarrow{AD}$ bisects $\angle A$ and intersects $\overline{BC}$ at D , $\overrightarrow{AE}$ bisects the exterior angle at A and intersects $\overrightarrow{BC}$ at E

Find the length of each of : $\overline{DE}$, $\overline{AD}$ and $\overline{AE}$ « 8 cm. , $2\sqrt{6}$ cm. , $2\sqrt{10}$ cm. »

---

16 ABC is a triangle in which AB = 3 cm. , BC = 7 cm. , CA = 6 cm. , $\overrightarrow{AD}$ bisects $\angle A$ and intersects $\overline{BC}$ at D , $\overrightarrow{AE}$ bisects the exterior angle of the triangle at A and intersects $\overrightarrow{CB}$ at E

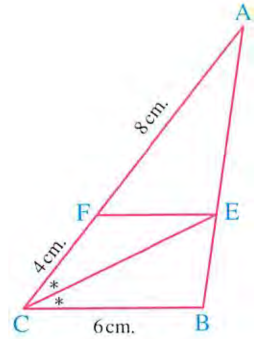
(1) **Prove that :** $\overline{AB}$ is a median in the triangle ACE

(2) **Find the ratio of :** The area of ΔADE to the area of ΔACE « $\frac{2}{3}$ »

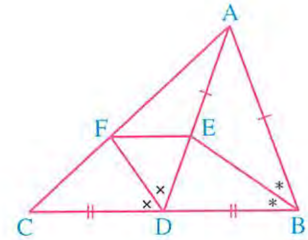
---

17 **In each of the following two figures , prove that $\overline{EF} \parallel \overline{BC}$:**

(1)



(2)



18 ABC is a triangle in which : AB > AC , D ∈ $\overline{AB}$, where BD = AC , draw $\overrightarrow{AE}$ bisects ∠ BAC and intersects $\overline{DC}$ at E , then draw $\overrightarrow{EF}$ // $\overline{BA}$ and intersects $\overline{AC}$ at F

Prove that : $\overline{DF}$ // $\overline{BC}$

19 ABCD is a parallelogram , X ∈ $\overline{AD}$, $\overrightarrow{CX}$ is drawn to intersect $\overrightarrow{BA}$ at Y and ∠ DCX is bisected by $\overrightarrow{CZ}$ which intersected $\overline{AD}$ at Z **Prove that :** $\frac{AY}{YX} = \frac{DZ}{ZX}$

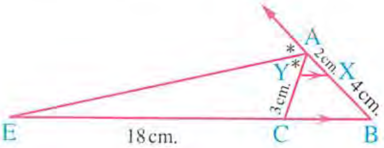
20 ABC is a triangle , $\overrightarrow{AD}$ bisects ∠ BAC and intersects $\overline{BC}$ at D , the two bisectors $\overrightarrow{AE}$, $\overrightarrow{AF}$ bisect the two angles BAD , CAD respectively and intersect $\overline{BC}$ at E and F respectively. **Prove that :** $\frac{BE}{ED} \times \frac{DF}{FC} = \frac{BD}{DC}$

21 ABC is a triangle , draw $\overrightarrow{AD}$, $\overrightarrow{BE}$, $\overrightarrow{CF}$ to bisect ∠ A , ∠ B and ∠ C and to intersect $\overline{BC}$, $\overline{AC}$ and $\overline{AB}$ at D , E and F respectively. **Prove that :** $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$

22 **In the opposite figure :** $\overline{XY}$ // $\overline{BC}$, AX = 2 cm. , XB = 4 cm. , YC = 3 cm. **Find the length of :** $\overline{AY}$

If $\overrightarrow{AE}$ bisects the exterior angle of the triangle at A and intersects $\overrightarrow{BC}$ at E , where CE = 18 cm. ,

find the length of : $\overline{BC}$



« 1.5 cm. , 6 cm. »

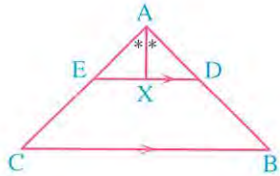
23 ABCD is a quadrilateral in which AB = BD , AD = DC , $\overrightarrow{AE}$ bisects ∠ BAD and intersects $\overline{BD}$ at E , $\overrightarrow{DF}$ bisects ∠ BDC and intersects $\overline{BC}$ at F

Prove that : $\overline{EF}$ // $\overline{DC}$

24 **In the opposite figure :** $\overline{DE}$ // $\overline{BC}$, $\overrightarrow{AX}$ bisects ∠ DAE

Prove that : (1) $\frac{DX}{XE} = \frac{DB}{EC}$

(2) $\frac{\text{The area of } \Delta ADX}{\text{The area of } \Delta AEX} = \frac{AB}{AC}$



25 ABCD is a parallelogram , its diagonals intersect at M , draw $\overrightarrow{AX}$ to bisect ∠ BAD and to intersect $\overline{BD}$ at X , draw $\overrightarrow{DY}$ to bisect ∠ ADC and to intersect $\overline{AC}$ at Y

Prove that : $\overline{XY}$ // $\overline{AD}$

26 $\overline{AB}$ is a chord in a circle , let D $\in$ the major arc $\overset{\frown}{AB}$ such that $\frac{AD}{DB} = \frac{2}{3}$ and let E be the midpoint of the minor arc $\overset{\frown}{AB}$, draw $\overline{DE}$ to intersect $\overline{AB}$ at C , find the ratio between the area of Δ ADE and the area of Δ BDE « $\frac{2}{3}$ »

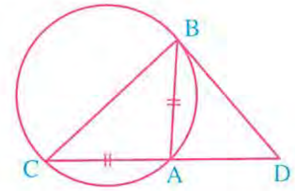
27 $\overline{AB}$ is a diameter of a circle M , C $\in$ this circle , draw a tangent to the circle M at C to intersect $\overrightarrow{AB}$ at E and to intersect the tangent to the circle M from A at D

Prove that : $\frac{AM}{ME} = \frac{DC}{DE}$

28 **In the opposite figure :**

AB = AC , $\overline{BD}$ is a tangent segment to the circle at B

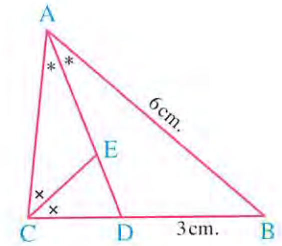
Prove that : DB × BA = DA × BC



Third Higher skills

1 **Choose the correct answer from those given :**

(1) In the opposite figure :

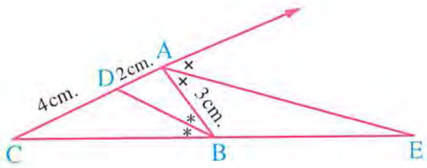


$\frac{AE}{ED} = \cdots\cdots\cdots$

(a) $\frac{1}{2}$ (b) 2

(c) 3 (d) $\frac{2}{3}$

(2) In the opposite figure :

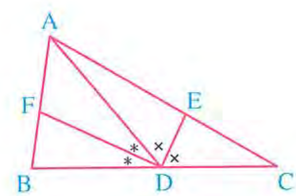


BE = ... cm.

(a) 6 (b) 8

(c) 9 (d) 10

(3) In the opposite figure :

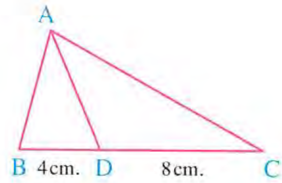


If 3 AE = 4 EC , 2 AF = 3 FB

, BC = 17 cm. , then CD = ... cm.

(a) 7 (b) 8 (c) 9 (d) 10

(4) In the opposite figure :

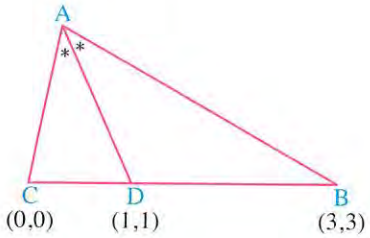


If m (∠ B) = 2 m (∠ DAB) = 2 m (∠ DAC)

, then AB = ……… cm.

(a) 4 (b) 6 (c) 8 (d) 9

(5) In the opposite figure :

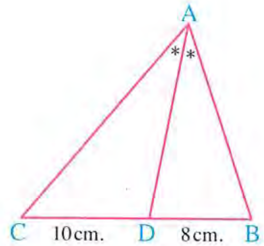


$\frac{AC}{AB}$ = ………

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

(6) In the opposite figure :



If $\overrightarrow{AD}$ bisects ∠ BAC which of the following conditions is sufficient to find the length of $\overline{AB}$?

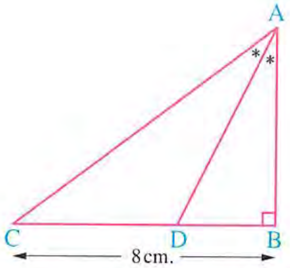
(a) AC – AB = 5 cm.

(b) The perimeter of Δ ABC = 54 cm.

(c) AD = $4\sqrt{15}$ cm.

(d) Anything of the previous.

(7) In the opposite figure :



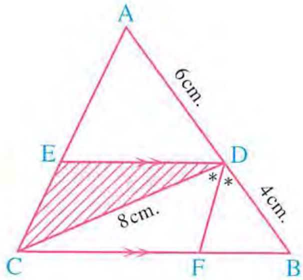
If $\frac{\text{the area of } (\Delta ABD)}{\text{the area of } (\Delta ADC)} = \frac{3}{5}$

, then AB = ……… cm.

(a) 5 (b) 6

(c) 8 (d) 10

(8) In the opposite figure :



If the area of (Δ DBF) = 10 cm^2

, then the area of (Δ DEC) = ……… cm^2

(a) 12 (b) 16

(c) 18 (d) 24

(9) In the opposite figure :

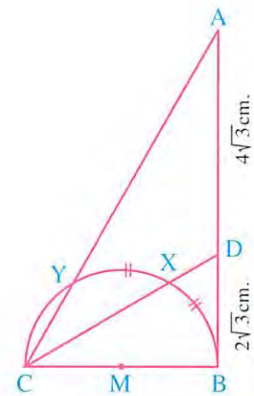
If m $(\widehat{BX})$ = m $(\widehat{XY})$

, BD = $2\sqrt{3}$ cm. , AD = $4\sqrt{3}$ cm.

, then AY = ... cm.

(a) $4\sqrt{3}$ (b) 6

(c) 9 (d) 12

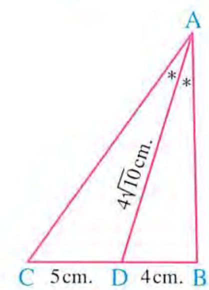


(10) In the opposite figure :

The perimeter of Δ ABC = ... cm.

(a) 36 (b) 32

(c) 28 (d) 24



(11) In the opposite figure :

The area of (Δ ABD) = ... cm^2

(a) 36 (b) 48

(c) 54 (d) 72

(12) In the opposite figure :

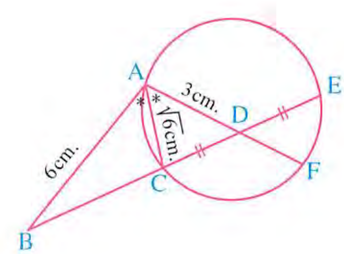
$\overrightarrow{AC}$ bisects $\angle$ BAD , D is the midpoint of $\overline{EC}$

, AC = $\sqrt{6}$ cm. , AD = 3 cm.

, AB = 6 cm. , then DF = ... cm.

(a) 2 (b) 3

(c) 3.5 (d) 4

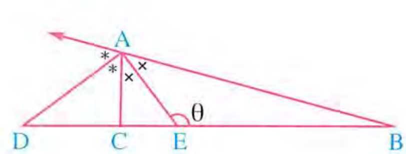


(13) In the opposite figure :

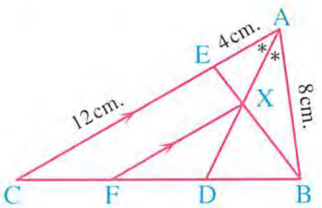
If AD = 8 cm. , AE = 6 cm. , then tan θ = ...

(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$



(14) In the opposite figure :

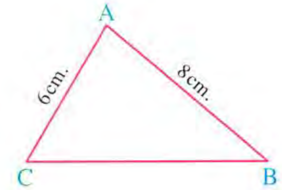


$\frac{DF}{BC} = \cdots\cdots$

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$

(c) $\frac{3}{5}$ (d) $\frac{1}{3}$

(15) In the opposite figure :



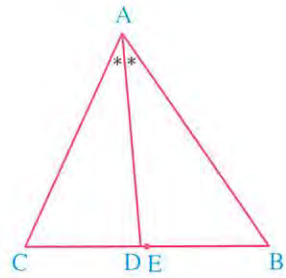
If m (∠ A) = 2 m (∠ B) , then BC = ... cm.

(a) $3\sqrt{10}$ (b) $2\sqrt{21}$

(c) 12 (d) 10

---

2 In the opposite figure :



ABC is a triangle in which : AB > AC

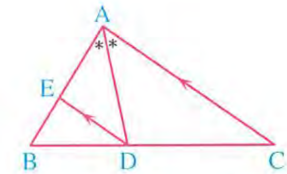
, E is the midpoint of $\overline{BC}$

, $\overrightarrow{AD}$ bisects ∠ A internally.

Prove that : $\frac{ED}{EC} = \frac{AB - AC}{AB + AC}$

---

3 In the opposite figure :



ABC is a triangle , $\overrightarrow{AD}$ bisects ∠ BAC

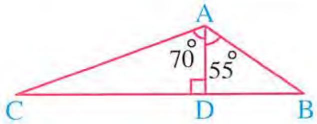
internally , $\overline{DE}$ // $\overline{AC}$

and intersects $\overline{AB}$ at E

Prove that : $DE = \frac{AB \times AC}{AB + AC}$

---

4 In the opposite figure :



If AC × BD = 36 cm.^{2}

Find the area of (Δ ABC)

« 18 cm.^{2} »

Exercise **8**

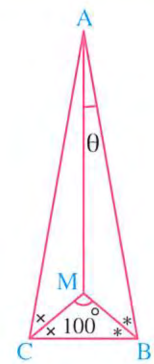
Follow : Angle bisector and proportional parts (Converse of theorem 3)

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

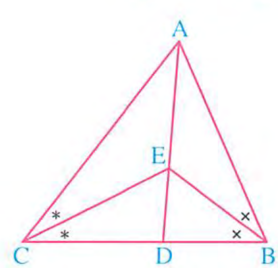
(1) In the opposite figure :



θ = ...

(a) 10° (b) 20° (c) 40° (d) 80°

(2) In the opposite figure :



If $\overrightarrow{BE}$ bisects ∠ ABD , $\overrightarrow{CE}$ bisects ∠ ACD

, then ....

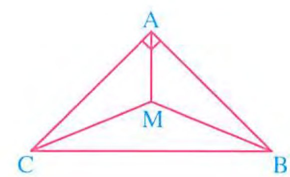
(a) D is a midpoint of $\overline{BC}$

(b) E is the midpoint of $\overline{AD}$

(c) E divides $\overline{AD}$ by the ratio 2 : 1 from the direction of point A

(d) $\overrightarrow{AD}$ bisects ∠ BAC

(3) In the opposite figure :

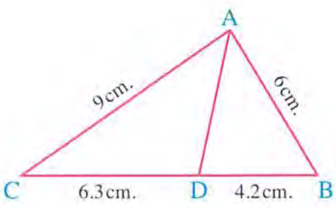


$\overline{AB} \perp \overline{AC}$, M is the point of intersection of the bisectors of the interior angles of Δ ABC

, then m (∠ BMC) = ...

(a) 100° (b) 120° (c) 135° (d) 145°

(4) In the opposite figure :



which of the following statements is true ?

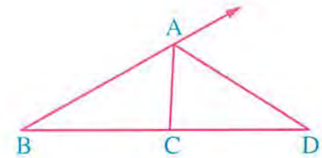
(a) Δ BAD ~ Δ BCA

(b) AB × AC = BD × DC

(c) m ($\angle$ BAD) = m ($\angle$ CAD)

(d) $AD = \sqrt{BD \times DC - AB \times AC}$

(5) In the opposite figure :

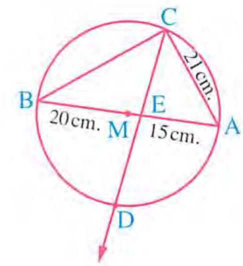


Which of the following conditions is sufficient to prove that $\overrightarrow{AD}$ bisects the exterior angle at the vertex A ?

(a) $\frac{AD}{AC} = \frac{DB}{BC}$ (b) $\frac{AB}{AC} = \frac{BD}{BC}$

(c) $\frac{AB}{AC} = \frac{CD}{BD}$ (d) AB × DC = AC × DB

(6) In the opposite figure :



Circle M in which , $\overline{AB}$ is a diameter , E $\in$ $\overline{AB}$

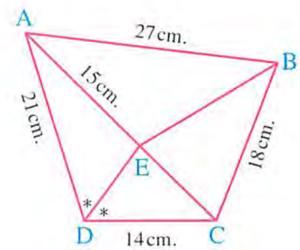
, if AE = 15 cm. , BE = 20 cm. , AC = 21 cm.

, $\overrightarrow{CE}$ intersect circle M at D , then m ($\overset{\frown}{AD}$) = ...°

(a) 45 (b) 90

(c) 22.5 (d) 60

(7) In the opposite figure :

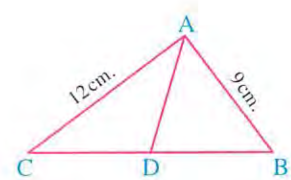


which of the following statements is false ?

(a) CE = 10 cm. (b) $\overrightarrow{BE}$ bisects $\angle$ ABC

(c) $BE = 4\sqrt{21}$ cm. (d) $DE = 12\sqrt{2}$ cm.

(8) In the opposite figure :



If a (Δ ABD) = 30 cm^2 , a (Δ ACD) = 40 cm^2

, then $\overrightarrow{AD}$ is ....

(a) perpendicular to $\overline{BC}$ (b) bisects $\angle$ BAC

(c) passes through the midpoint of $\overline{BC}$ (d) All the previous

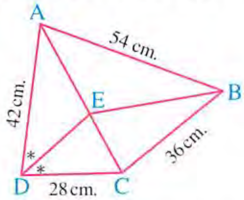
Second Essay questions

1 ABC is a triangle in which : AB = 6 cm. , AC = 9 cm. , BC = 10.5 cm. , D ∈ $\overline{BC}$, where BD = 4.2 cm. **Prove that :** $\overrightarrow{AD}$ bisects ∠ BAC

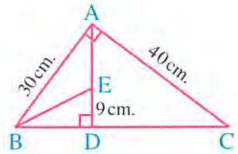
2 ABC is a triangle in which AB = 6 cm. , BC = 4 cm. , CA = 3.6 cm. , D ∈ $\overrightarrow{BC}$ such that CD = 6 cm. **Prove that :** $\overrightarrow{AD}$ bisects the exterior angle of Δ ABC at A

3 **In each of the following figures , prove that :** $\overrightarrow{BE}$ bisects ∠ ABC

(1)



(2)



4 ABCD is a quadrilateral in which AB = 6 cm. , BC = 9 cm. , CD = 6 cm. , AD = 4 cm. , $\overrightarrow{AE}$ bisects ∠ A and intersects $\overline{BD}$ at E

(1) Find the value of the ratio : $\frac{BE}{ED}$

(2) Prove that : $\overrightarrow{CE}$ bisects ∠ BCD « $\frac{3}{2}$ »

5 ABCD is a quadrilateral in which AB = 18 cm. , BC = 12 cm. , E ∈ $\overline{AD}$, where 2 AE = 3 ED , draw $\overrightarrow{EF}$ // $\overline{DC}$ and intersects $\overline{AC}$ at F

Prove that : $\overrightarrow{BF}$ bisects ∠ ABC

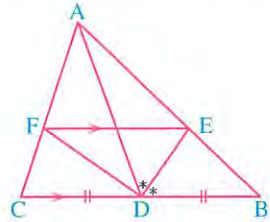
6 **In the opposite figure :**

D is the midpoint of $\overline{BC}$,

$\overrightarrow{DE}$ bisects ∠ ADB , $\overrightarrow{EF}$ // $\overline{BC}$

Prove that : (1) $\overrightarrow{DF}$ bisects ∠ ADC

(2) $\overline{ED} \perp \overline{DF}$



7 ABC is a triangle , X is the midpoint of $\overline{BC}$, BX = 6 cm. , AX = 9 cm. , the bisector of ∠ AXB intersects $\overline{AB}$ at D , take E ∈ $\overline{AC}$, where AE = 6 cm. given that AC = 10 cm.

(1) Find the value of : $\frac{AD}{DB}$ « $\frac{3}{2}$ »

(2) Prove that : $\overline{DE}$ // $\overline{BC}$

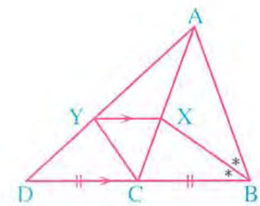
(3) Prove that : $\overrightarrow{XE}$ bisects ∠ AXC

8 **In the opposite figure :**

AB = AC , BC = CD ,

$\overrightarrow{BX}$ bisects ∠ ABC , $\overline{XY}$ // $\overline{BD}$

Prove that : $\overrightarrow{CY}$ bisects ∠ ACD

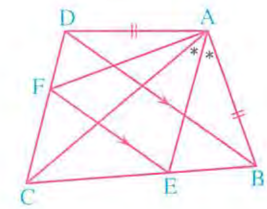


9 **In the opposite figure :**

AB = AD , $\overrightarrow{AE}$ bisects ∠ BAC ,

$\overline{EF}$ // $\overline{BD}$

Prove that : $\overrightarrow{AF}$ bisects ∠ CAD



10 ABC is a triangle , D ∈ $\overrightarrow{BC}$, D ∉ $\overline{BC}$, where CD = AB , draw $\overrightarrow{CE}$ // $\overline{DA}$ and intersects $\overline{AB}$ at E , draw $\overrightarrow{EF}$ // $\overline{BC}$ and intersects $\overline{AC}$ at F

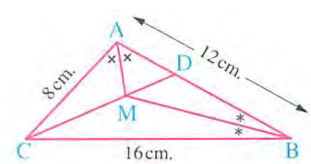
Prove that : $\overrightarrow{BF}$ bisects ∠ ABC

11 **In the opposite figure :**

ABC is a triangle in which AB = 12 cm. ,

AC = 8 cm. , BC = 16 cm. , $\overrightarrow{BM}$ bisects ∠ ABC ,

$\overrightarrow{AM}$ bisects ∠ BAC **Find the length of :** $\overline{AD}$



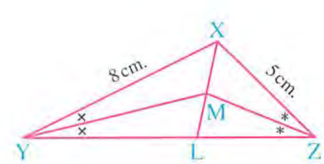
« 4 cm. »

12 **In the opposite figure :**

$\overrightarrow{ZM}$ and $\overrightarrow{YM}$ bisect ∠ Z and ∠ Y respectively

, XY = 8 cm. , XZ = 5 cm.

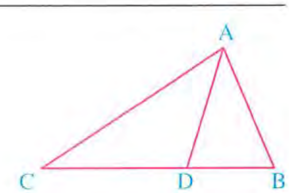
Prove that : 8 LZ = 5 LY



13 **In the opposite figure :**

If AC : CD : AB : BD = 15 : 10 : 9 : 6 ,

Prove that : $\overrightarrow{AD}$ bisects ∠ BAC

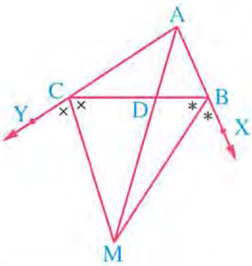


14 ABC is a triangle in which AB = 5 cm. , AC = 10 cm. , BC = 9 cm. , D ∈ $\overline{BC}$ such that BD = 3 cm. , E ∈ $\overrightarrow{CB}$, where $\overline{AE}$ ⊥ $\overline{AD}$

(1) **Prove that :** $\overrightarrow{AD}$ bisects ∠ BAC

(2) **Find the length of :** $\overline{BE}$ « 9 cm. »

15 **In the opposite figure :**



$\overrightarrow{BM}$ bisects $\angle$ CBX ,

$\overrightarrow{CM}$ bisects $\angle$ BCY

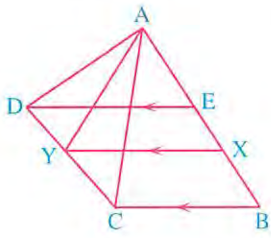
Prove that : $\overrightarrow{AM}$ bisects $\angle$ BAC

---

16 ABC is a triangle in which AB = 6 cm. , BC = 12 cm. , CA = 9 cm. , D $\in \overline{AB}$, where AD = 2 cm. , draw $\overleftrightarrow{DE}$ // $\overline{BC}$ and intersects $\overline{AC}$ at E , find the length of $\overline{AE}$, then **prove that :** $\overrightarrow{BE}$ bisects $\angle$ ABC « 3 cm. »

---

17 **In the opposite figure :**



$\overline{ED}$ // $\overline{XY}$ // $\overline{BC}$

and AD × BX = AC × EX

Prove that : $\overrightarrow{AY}$ bisects $\angle$ CAD

---

18 Two circles M and N are touching externally at A , a straight line is drawn parallel to $\overline{MN}$ and intersects the circle M at B , C and the circle N at D , E respectively. If $\overrightarrow{BM} \cap \overrightarrow{EN} = \{F\}$, **prove that :** $\overrightarrow{FA}$ bisects $\angle$ MFN

---

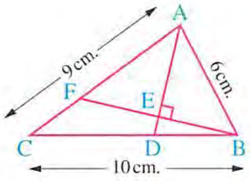
19 $\overline{AB}$ is a diameter of a circle , $\overline{AC}$ is a chord in it , $\overrightarrow{CD}$ is a tangent drawn to the circle at C and intersects $\overrightarrow{AB}$ at D. If E $\in \overline{AB}$, where $\frac{DB}{BE} = \frac{DC}{CE}$

Prove that : (1) $\overrightarrow{CA}$ bisects the exterior angle of Δ CDE at C

(2) $\frac{DA}{DB} = \frac{AE}{BE}$

Third Higher skills

In the opposite figure :



ABC is a triangle in which AB = 6 cm. , AC = 9 cm. , and BC = 10 cm. , D $\in \overline{BC}$, where BD = 4 cm.

$\overrightarrow{BE} \perp \overline{AD}$ and intersects $\overline{AD}$ and $\overline{AC}$ at E and F respectively.

(1) **Prove that :** $\overrightarrow{AD}$ bisects $\angle$ BAC

(2) **Find :** Area of Δ ABF : area of Δ CBF « 2 »

Exercise 9

Applications of proportionality in the circle

Test yourself

From the school book ● Remember ● Understand ● Apply ● Higher Order Thinking Skills

First Multiple choice questions

Choose the correct answer from those given :

(1) If M is a circle of radius length 3 cm. , A is a point lies in its plane where MA = 4 cm. , then P_M (A) = ...

(a) $\sqrt{7}$ (b) 9 (c) 7 (d) – 7

(2) If N is a circle of diameter length 16 cm. , B is a point lies in its plane where NB = 5 cm. , then P_N (B) = ...

(a) 39 (b) – 39 (c) $\sqrt{39}$ (d) – 231

(3) If the power of a point A with respect to the circle M is a negative quantity , then A lies ....

(a) inside the circle. (b) on the centre of the circle.

(c) outside the circle. (d) on the circle.

(4) If M is a circle , A is a point that lies in its plane where P_M (A) = 0 , then A lies ....

(a) inside the circle. (b) on the centre of the circle.

(c) outside the circle. (d) on the circle.

(5) If P_M (A) = 5^{-1} , then A lies .... the circle M

(a) outside (b) inside (c) on (d) on the centre of

(6) If $P_M(A) = r$, then the point A lies ....

(a) outside circle. (b) on the circle.

(c) inside the circle. (d) on the centre of the circle.

(7) If the power of a point with respect to circle M equals -625 , the distance between this point and the centre of the circle = 15 cm. , then the diameter length of this circle equals .... cm.

(a) 400 (b) 20 (c) $5\sqrt{34}$ (d) $10\sqrt{34}$

(8) If M is a circle , A is a point in its plane where MA = 6 cm. , $P_M(A) = -13$, then the area of this circle = ... cm^2. $\left(\pi = \frac{22}{7}\right)$

(a) 154 (b) 44 (c) 144 (d) 7

(9) If M is a circle of radius length 7 cm. , A is a point in its plane 25 cm. apart from the centre of the circle , then the length of the tangent segment to the circle M from A is .... cm.

(a) 5 (b) 49 (c) 24 (d) 12

(10) If M is a circle with diameter length 12 cm. , A is a point in its plane where $P_M(A) = 13$, then distance between the point A and the centre of the circle equal .... cm.

(a) 7 (b) 14 (c) 3.5 (d) 6

(11) If $P_M(A) = 9$, then it means that ....

(a) the point A lies on the circle M

(b) the point A lies inside the circle M

(c) the radius length of the circle M equal 9 length units.

(d) the length of tangent segment drawn from the point A to the circle M equal 3 length units.

(12) If the point A lies outside the circle M , then the length of the tangent segment drawn from the point A to the circle equal ....

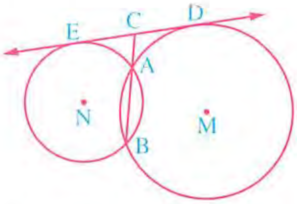
(a) $(AM)^2$ (b) $\left(P_M(A)\right)^2$ (c) $P_M(A)$ (d) $\sqrt{P_M(A)}$

(13) If M , N are two intersecting circles and $P_M(A) = 5$, $2 P_N(A) = 10$, then the point A $\in$ ....

(a) circle M (b) circle N

(c) $\overleftrightarrow{MN}$ (d) the principle axis to the circles.

(14) **In the opposite figure :**

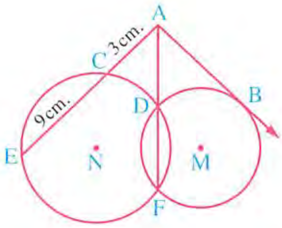


$P_M(C) - P_N(C) =$ ....

(a) Positive quantity. (b) Negative quantity.

(c) Zero (d) Can't be determined.

(15) **In the opposite figure :**



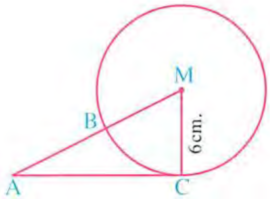
If AC = 3 cm. , CE = 9 cm.

, then $P_M(A) =$ .... cm.

(a) $3\sqrt{3}$ (b) 27

(c) 36 (d) 6

(16) **In the opposite figure :**

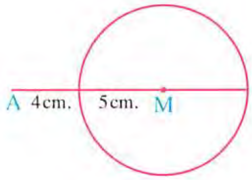


$\overline{AC}$ touches the circle M at C , MC = 6 cm.

, $P_M(A) = 64$, then AB = ... cm.

(a) 3 (b) 4 (c) 5 (d) 6

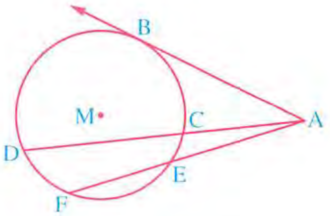
(17) **In the opposite figure :**



$P_M(A) =$ ....

(a) 81 (b) 25 (c) 56 (d) 16

(18) **In the opposite figure :**

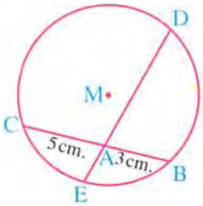


If $\overrightarrow{AB}$ is a tangent , then $(AB)^2 =$ ....

(a) AC × CD (b) AE × EF

(c) $P_M(A)$ (d) $\frac{AC}{AD}$

(19) **In the opposite figure :**

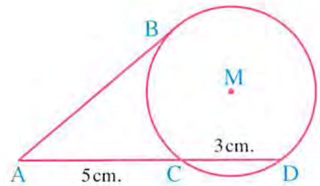


$P_M(A) =$ ....

(a) 15 (b) – 15

(c) 24 (d) – 24

(20) In the opposite figure :



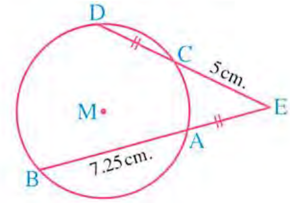
$\overline{AB}$ is a tangent segment to the circle M , if DC = 3 cm.

, CA = 5 cm. , then P_M (A) = ...

(a) 25 (b) $(AB)^2 - r^2$

(c) 40 (d) $(AM)^2 - (AB)^2$

(21) In the opposite figure :

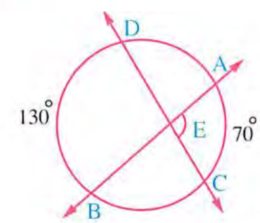


P_M (E) = ...

(a) 20 (b) 29

(c) 25 (d) 45

(22) In the opposite figure :

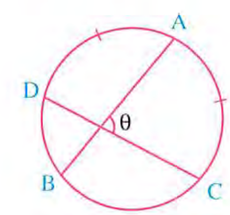


If m $(\widehat{AC})$ = 70° , m $(\widehat{BD})$ = 130°

, then m (∠ DEB) = ...°

(a) 100 (b) 90 (c) 110 (d) 120

(23) In the opposite figure :



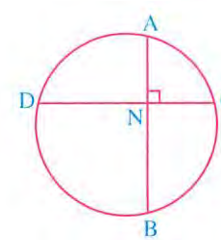
m $(\widehat{AC})$ = m $(\widehat{AD})$ = 2 m $(\widehat{BD})$

, m $(\widehat{BC})$ = 100°

, then θ = ...°

(a) 78 (b) 65 (c) 52 (d) 84

(24) In the opposite figure :

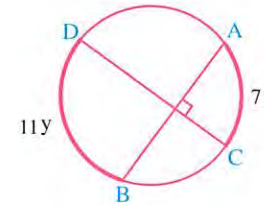


If $\overline{AB} \perp \overline{CD}$, m $(\widehat{AC})$ + m $(\widehat{BD})$ = ...

(a) 45° (b) 90°

(c) 180° (d) 270°

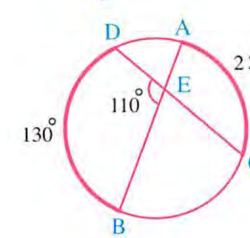
(25) In the opposite figure :



y = ...°

(a) 180 (b) 18 (c) 10 (d) 15

(26) 📖 In the opposite figure :



If $\overline{AB} \cap \overline{CD} = \{E\}$, then Z = ...°

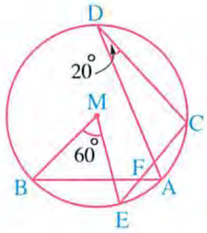
(a) 90 (b) 45 (c) 50 (d) 80

(27) **In the opposite figure :**

A circle M , m (∠ EFB) = ...

(a) 30°　　(b) 40°

(c) 50°　　(d) 60°

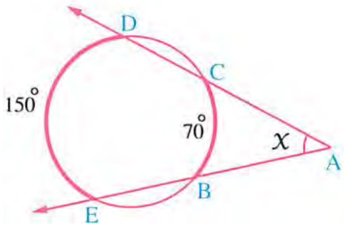


(28) **In the opposite figure :**

x = ...°

(a) 110　　(b) 55

(c) 80　　(d) 40

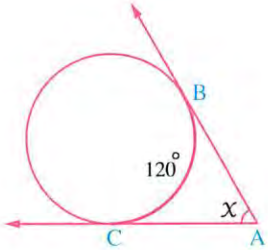


(29) **In the opposite figure :**

x = ...°

(a) 60　　(b) 120

(c) 180　　(d) 240



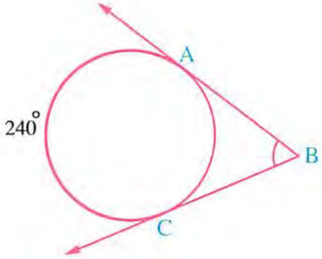
(30) **In the opposite figure :**

If $\overrightarrow{BA}$, $\overrightarrow{BC}$ are two tangents

, then m (∠ B) = ...°

(a) 40　　(b) 60

(c) 80　　(d) 120



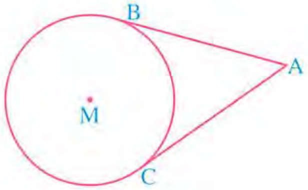
(31) **In the opposite figure :**

If $\overline{AB}$, $\overline{AC}$ are two tangent segment

, m ($\overset{\frown}{BC}$) = 130° + x , then m (∠ A) = ...

(a) 100°　　(b) 65° – x

(c) 50° – x　　(d) 130° – $\frac{x}{2}$

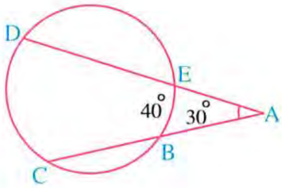


(32) **In the opposite figure :**

If m (∠ A) = 30° , m ($\overset{\frown}{BE}$) = 40° , then m ($\overset{\frown}{CD}$) = ...

(a) 30°　　(b) 40°

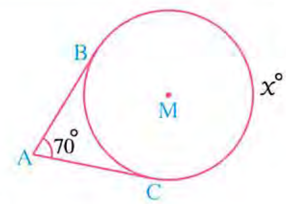
(c) 70°　　(d) 100°



(33) In the opposite figure :

If m (∠ A) = 70° , $\overline{AB}$, $\overline{AC}$ are two tangent segment , m ($\overset{\frown}{BC}$) major = $x°$, then x = ...

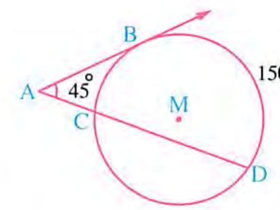
(a) 250° (b) 110° (c) 500° (d) 215°



(34) In the opposite figure :

$\overrightarrow{AB}$ is a tangent to circle M at B , if m (∠ A) = 45° , m ($\overset{\frown}{BD}$) = 150° , then m ($\overset{\frown}{BC}$) = ...

(a) 120° (b) 90° (c) 60° (d) 180°

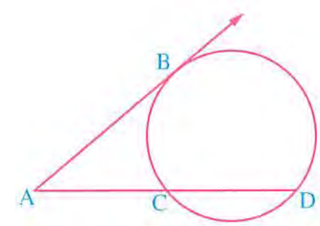


(35) In the opposite figure :

$\overrightarrow{AB}$ touches the circle at B , if m ($\overset{\frown}{BD}$) = (2 x + 50°) , m ($\overset{\frown}{BC}$) = 2 x , then m (∠ A) = ...

(a) 50° (b) 25°

(c) 30° (d) 60°

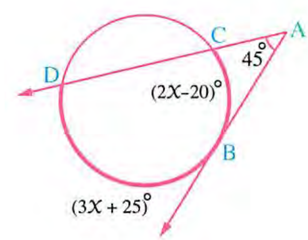


(36) In the opposite figure :

x = ...°

(a) 25 (b) 45

(c) 65 (d) 70

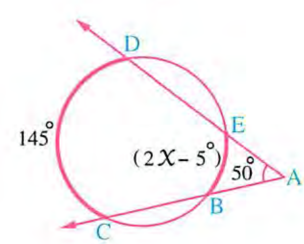


(37) In the opposite figure :

x = ...°

(a) 50 (b) 25

(c) 100 (d) 75

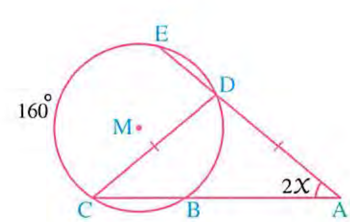


(38) In the opposite figure :

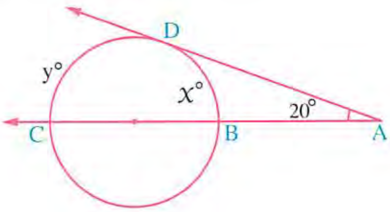
If M is a circle , $\overrightarrow{AE}$ cuts the circle at D and E , $\overrightarrow{AC}$ cuts the circle at B and C , AD = DC , then the value of x = ...°

(a) 40 (b) 30

(c) 20 (d) 10



(39) In the opposite figure :

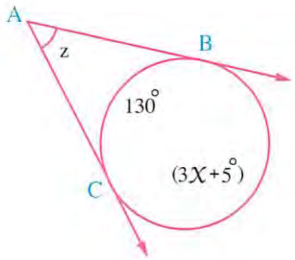


$(x , y) = \cdots\cdots\cdots$

(a) (60° , 120°) (b) (120° , 60°)

(c) (70° , 110°) (d) (110° , 70°)

(40) In the opposite figure :

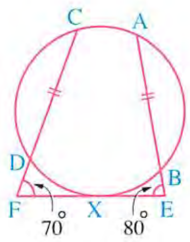


$x + Z = \cdots\cdots\cdots°$

(a) 50 (b) 75

(c) 125 (d) 250

(41) In the opposite figure :



If AB = CD , m (∠ E) = 80° , m (∠ F) = 70°

, then $m(\overset{\frown}{XD}) - m(\overset{\frown}{XB}) = \cdots\cdots\cdots$

(a) 5° (b) 10°

(c) 15° (d) 20°

Second Essay questions

1 **Find the power of the given point with respect to the circle M whose radius length is r :**

(1) The point A where AM = 12 cm. and r = 9 cm.

(2) The point C where CM = 7 cm. and r = 7 cm.

(3) The point D where DM = $\sqrt{17}$ cm. and r = 4 cm.

---

2 Determine the position of each of the following points with respect to the circle M , of radius length 10 cm. , then calculate the distance between each point and the centre of the circle :

(1) P_M (A) = – 36 (2) P_M (B) = 96 (3) P_M (C) = zero

---

3 If the distance between a point and the centre of a circle equals 25 cm. , and the power of this point with respect to the circle equals 400 , find the radius length of this circle.

« 15 cm. »

---

4 If a point A is outside the circle M , $\overline{AD}$ is a tangent to the circle at D where AD = 8 cm. , find the power of point A with respect to circle M « 64 »

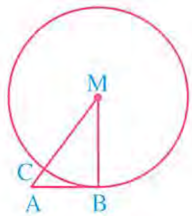
5 **In the opposite figure :**

$\overline{AB}$ is a tangent to the circle M at B

, $\overline{MA}$ intersects the circle M at C

If the radius length of the circle equals 12 cm.

, $P_M(A) = 81$ **, then find :**



(1) The length of $\overline{AB}$ (2) The length of $\overline{AC}$ « 9 cm. , 3 cm. »

---

6 The radius length of circle M equals 31 cm. The point A lies at 23 cm. distant from its centre. Draw the chord $\overline{BC}$ where $A \in \overline{BC}$, AB = 3 AC **Calculate :**

(1) The length of the chord $\overline{BC}$

(2) The distance between the chord $\overline{BC}$ and the centre of the circle. « 48 cm. , 19.6 cm. »

---

7 The radius length of circle N equals 8 cm. The point B lies at 12 cm. distant from its centre , draw a straight line passes through the point B and intersects the circle at C and D where CB = CD Calculate the length of the chord $\overline{CD}$ and its distance from the point N

« $2\sqrt{10}$ cm. , $3\sqrt{6}$ cm. »

---

8 **In the opposite figure :**

M is a circle , $\overline{AB}$ is a diameter in it

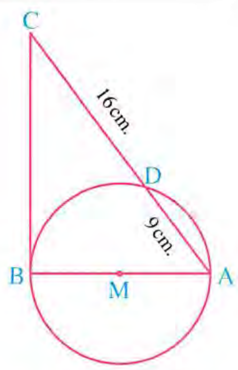
, $\overline{CB}$ is a tangent to the circle M at B

, $\overline{CA}$ intersects the circle M at D , where

CD = 16 cm. , DA = 9 cm. **Find :**

(1) The length of the circle's radius.

(2) The area of triangle ABC



« 7.5 cm. , 150 cm^2. »

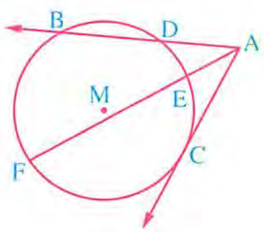
---

9 **In the opposite figure :**

A is a point outside the circle M , $\overrightarrow{AB}$ intersects the circle at D , B , $\overrightarrow{AF}$ intersects the circle at E , F ,

$\overrightarrow{AC}$ is a tangent to the circle at C ,

AD = 8 cm. , EF = 18 cm.



(1) If $P_M(A) = 144$ **, find the length of each of :** $\overline{AC}$, $\overline{DB}$, $\overline{AE}$

(2) If $X \in \overline{BD}$ where DX = 4 cm. **, find :** $P_M(X)$ « 12 cm. , 10 cm. , 6 cm. , – 24 »

10 The two circles M and N are touching each other externally at A , $\overleftrightarrow{AB}$ is a common tangent to the two circles M , N . $\overrightarrow{BC}$ intersects the circle M at C and D. $\overleftrightarrow{BE}$ intersects the circle N at E and F respectively.

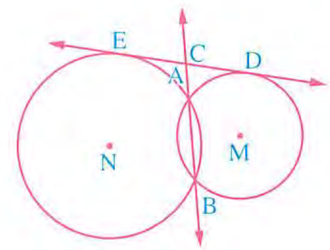
(1) **Prove that :** $\overleftrightarrow{AB}$ is the principle axis of the two circles M and N

(2) If $P_M(B) = 36$, BC = 4 cm. , EF = 9 cm.

Find the length of each of : $\overline{CD}$, $\overline{AB}$ and $\overline{BE}$ « 5 cm. , 6 cm. , 3 cm. »

---

11 **In the opposite figure :**



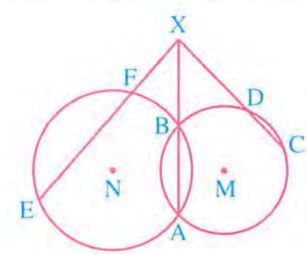
M , N are two intersecting circles at A , B , $\overleftrightarrow{ED}$ is a common tangent to the two circles M , N at D , E respectively. $\overleftrightarrow{AB} \cap \overleftrightarrow{DE} = \{C\}$

(1) **Prove that :** $\overleftrightarrow{BC}$ is the principle axis of the two circles.

(2) If AB = 12 cm. , $P_N(C) = 64$, **find the length of each of :** $\overline{CA}$, $\overline{CD}$ « 4 cm. , 8 cm. »

---

12 **In the opposite figure :**



The two circles M and N are intersecting at A and B where $\overrightarrow{AB} \cap \overrightarrow{CD} \cap \overrightarrow{EF} = \{X\}$,

XD = 2 DC , EF = 10 cm. and $P_N(X) = 144$

(1) **Prove that :** $\overleftrightarrow{AB}$ is the principle axis to the two circles M and N

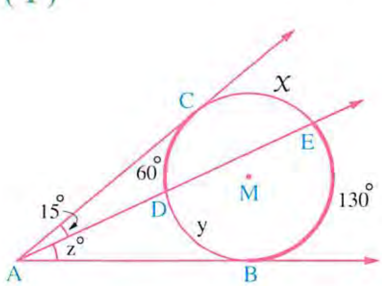
(2) **Find the length of each of :** $\overline{XC}$ and $\overline{XF}$

(3) **Prove that :** CDFE is a cyclic quadrilateral. « $6\sqrt{6}$ cm. , 8 cm. »

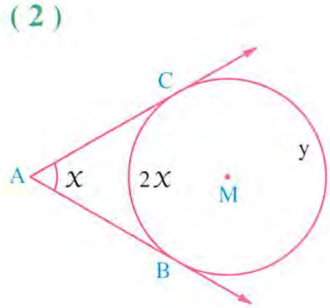
---

13 **Using the given data in each figure , find the value of the symbol used in measurement :**

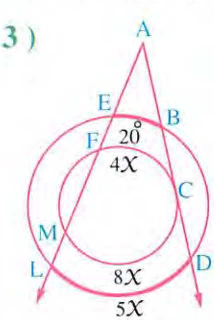
(1)



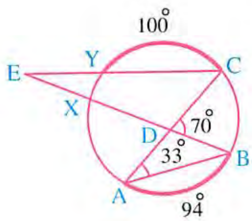
(2)



(3)



14 **In the opposite figure :**



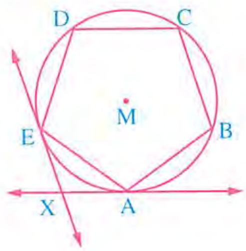
m (∠ BAC) = 33° , m (∠ BDC) = 70° ,

m $(\overset{\frown}{AB})$ = 94° , m $(\overset{\frown}{CY})$ = 100° **Find the measure of each of :**

(1) $\overset{\frown}{XY}$ (2) $\overset{\frown}{AX}$

(3) ∠ BEC « 26° , 74° , 20° »

15 **In the opposite figure :**



ABCDE is a regular pentagon drawn inside the circle M ,

$\overleftrightarrow{AX}$ is a tangent to the circle at A , $\overleftrightarrow{EX}$ is a tangent to the circle at E

where $\overleftrightarrow{AX} \cap \overleftrightarrow{EX} = \{X\}$ **Find :**

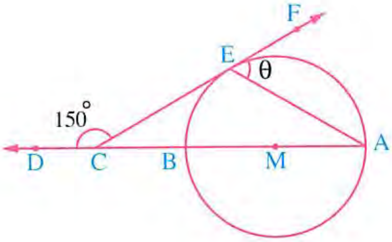
(1) m $(\overset{\frown}{AE})$

(2) m (∠ AXE) « 72° , 108° »

Third Higher skills

Choose the correct answer from those given :

(1) **In the opposite figure :**

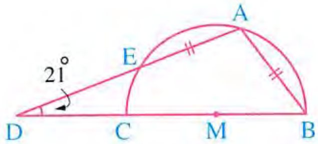


θ = ...

(a) 45° (b) 50°

(c) 55° (d) 60°

(2) **In the opposite figure :**



If AE = AB , $\overline{BC}$ is a diameter , m (∠ D) = 21°

, then m (∠ A) = ...

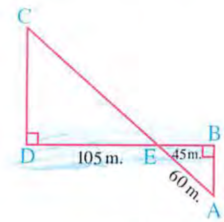
(a) 100° (b) 104°

(c) 106° (d) 110°

Life Applications on Unit Four

From the school book

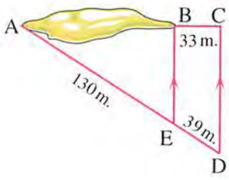
1 To determine the location C ,
surveyors measure and prepare the opposite scheme.
Find the distance between the location C and the location A



« 200 m. »

---

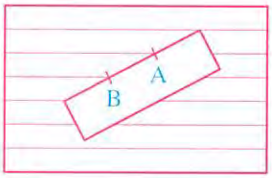
2 A team of pollution control determined the location of an oil spot on one of the beaches as in the opposite figure.
Calculate the length of the oil spot.



« 110 m. »

---

3 Yousef wanted to divide a strip of paper into 3 equal parts in length. He placed it on a paper on his notebook , as in the opposite figure , and determined two points of division A and B
Is the division of Yousef's strip correct ? Explain your answer.
Use your geometric instruments to verify your answer.



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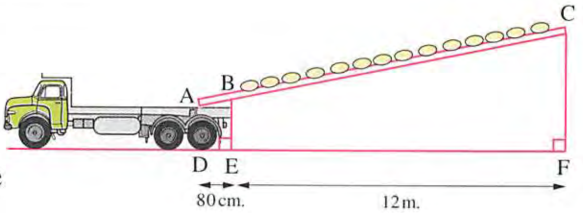
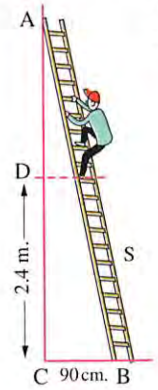
4 Fertilizer packages produced from one of the factories are transfered by sliding on a tube that is inclined and carried on to trucks to the centre of distributions as in the opposite figure.



If D , E and F are the projections of the points A , B and C on the horizontal respectively , AB = 1.2 m. , DE = 80 cm. , EF = 12 m.

Find the length of the tube to the nearest metre. « 19 m. »

5 $\overline{AB}$ is a ladder of length 4.1 metres rests by its upper end A on a vertical wall and with its lower end B on a horizontal rough ground. If the lower end is 90 cm. apart from the wall , calculate the distance which a man ascends on the ladder until it becomes at 2.4 m. high from the ground.



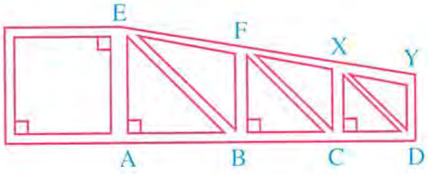
« 2.46 m. »

---

6 If AB = 180 cm. , EF = 2 m. ,

AB : BC : CD = 5 : 4 : 3

Find the length of each of : $\overline{EY}$ and $\overline{CD}$

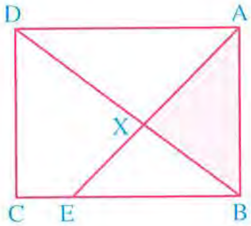


« 480 cm. , 108 cm. »

---

7 The opposite figure shows a rectangular piece of land divided into four different parts by the two lines $\overleftrightarrow{BD}$ and $\overleftrightarrow{AE}$, where $E \in \overline{BC}$, $\overleftrightarrow{BD} \cap \overleftrightarrow{AE} = \{X\}$, if AB = BE = 42 metres , AD = 56 metres

Calculate the area of the piece ABX in square metres and the length of $\overline{AX}$

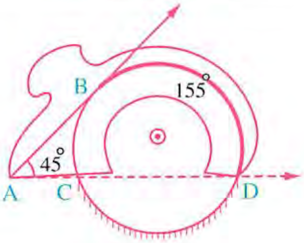


« 504 m.^{2} , $24\sqrt{2}$ m. »

---

8 A circular saw for cutting wood , the radius length of its circle equals 10 cm. It rotates inside a protective container. If m (∠ BAD) = 45° and m $(\overset{\frown}{BD})$ = 155°

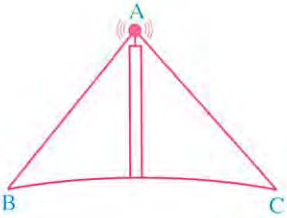
Find the arc length of the disc's saw outside the protective container.



« 24.4 cm. »

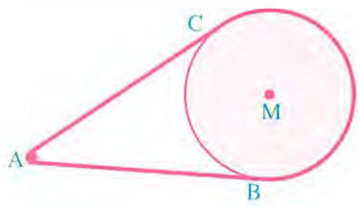
---

9 The signals produced from the communication tower follow a ray in their pathway , its starting point is on the top of the tower and it is a tangent to the surface of the earth , as in the opposite figure. Determine the measure of the arc included by the two tangents supposing that the tower lies at sea level and m (∠ CAB) = 80°



« 100° »

10 A pulley rotates at the axis M by a strap passing over a small pulley at A. If the measure of the angle between the two parts of the strap is 40° Find the length of the major arc $\overset{\frown}{BC}$, given that the radius length of the larger pulley equals 9 cm.



« 34.56 cm. »

11 A satellite revolves in an orbit and keeps in during rotation on a fixed height above the equator. The camera on it can monitor the arc length of 6011 km. on the surface of the earth. If the measure of the arc equals 54° , **find :**

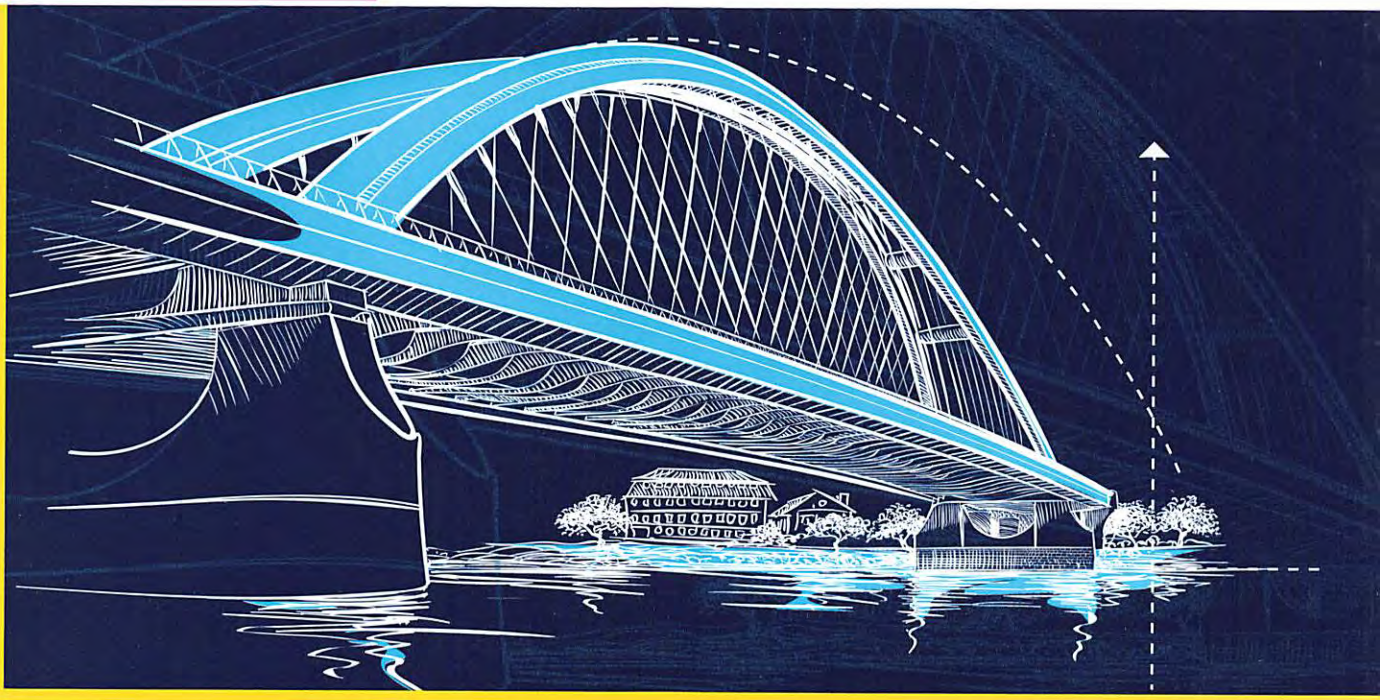
(1) The measure of the angle of the camera placed on the satellite.

(2) The radius length of the Earth of the equator. « 126° , 6378 km. »

Mathematics

By a group of supervisors

Interactive exams using QR CODE



FIRST TERM

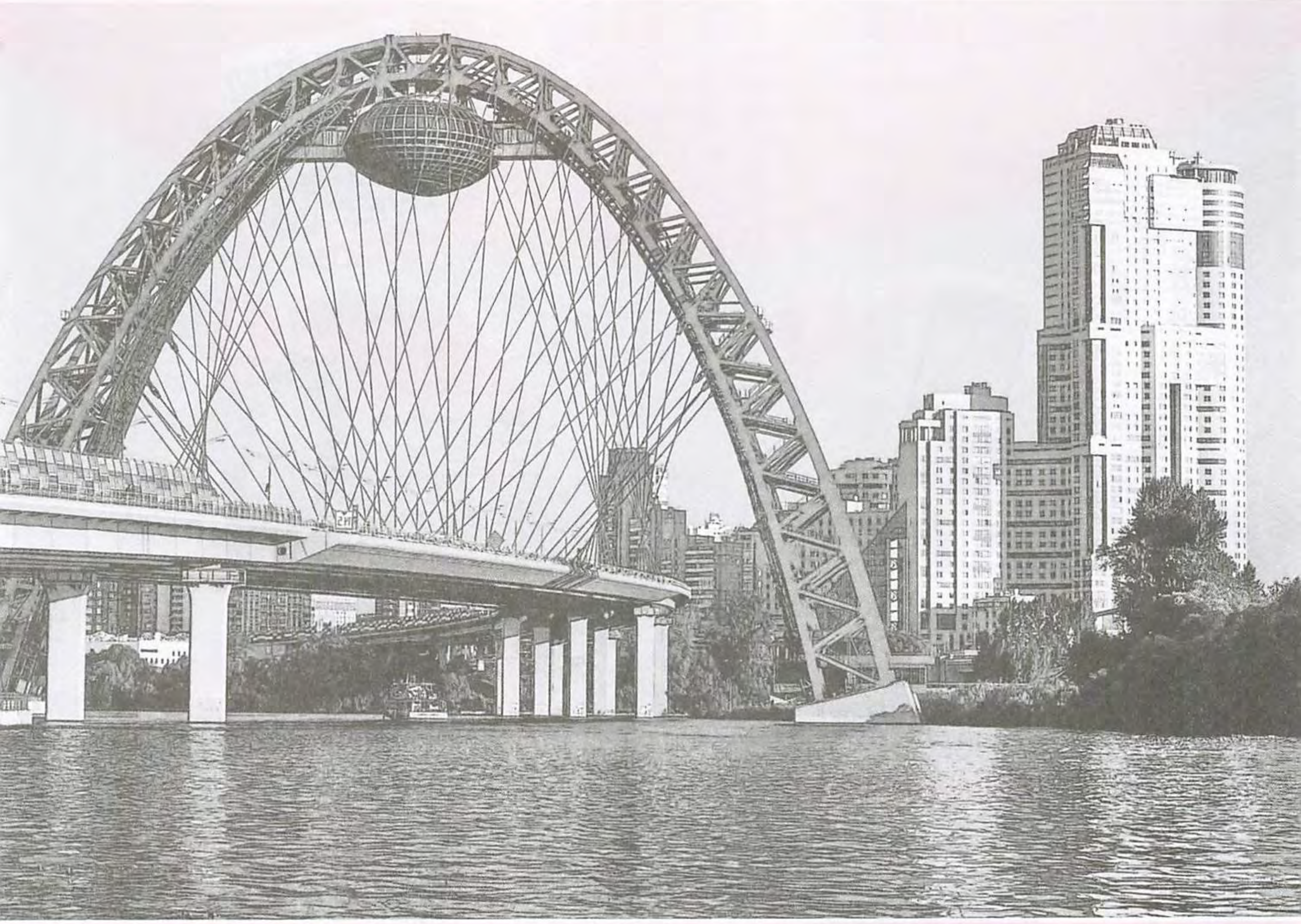
1 SEC.

2023

FINAL REVISION & EXAMINATIONS

EL-MOASSER

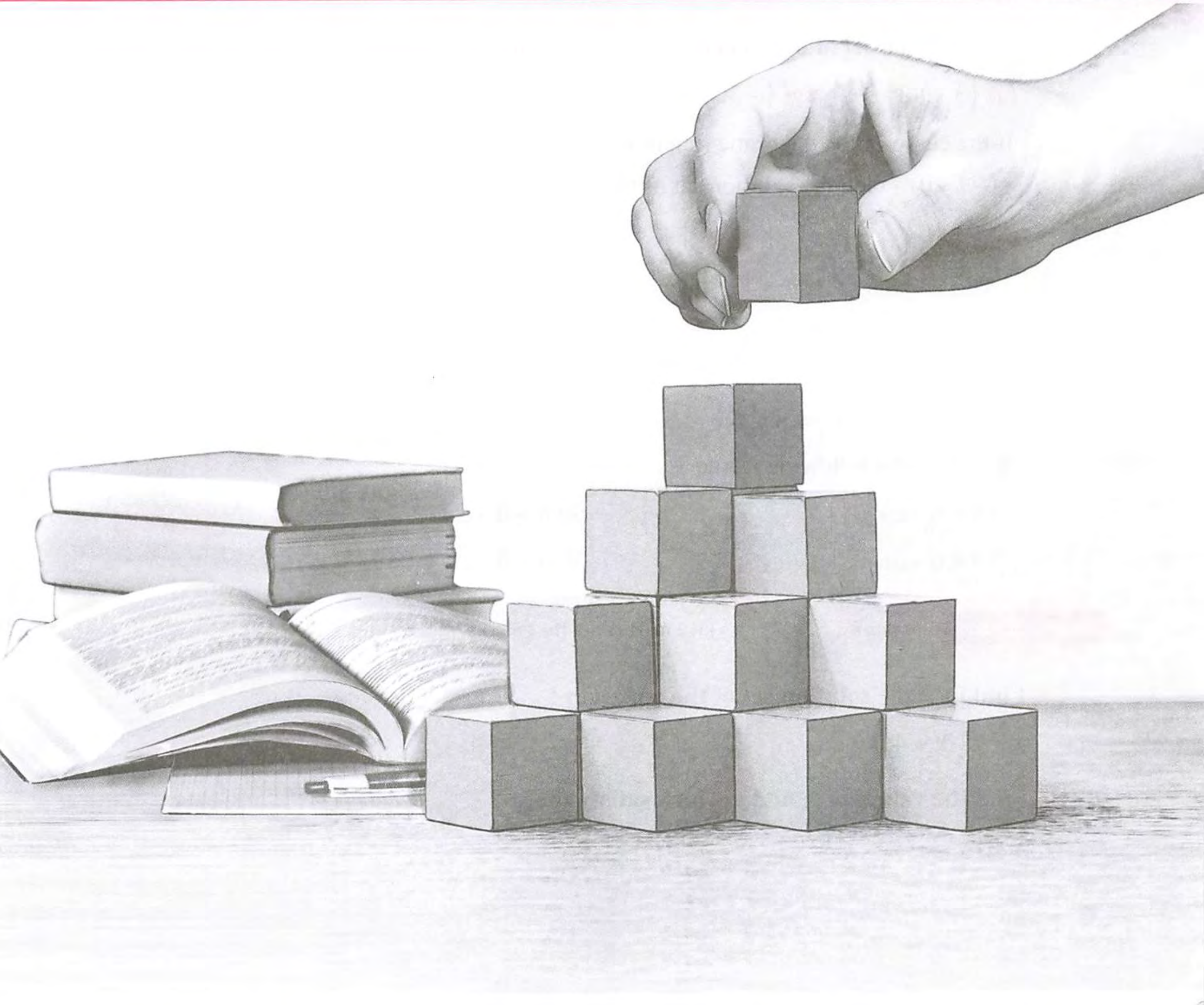
Contents



- Accumulative quizzes.
- Final revision.
- School book examinations.
- Final examinations.
- Answers.

Accumulative quizzes

FIRST	Accumulative quizzes on algebra.
SECOND	Accumulative quizzes on trigonometry.
THIRD	Accumulative quizzes on geometry.



Total mark

....

10

Quiz 1 on lesson 1 – unit 1

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) $\sqrt{-2} \times \sqrt{-8} =$ ....

(a) 4 (b) -4 (c) $4\,i$ (d) -16

(2) The simplest form of the imaginary number i^{42} is ....

(a) -1 (b) 1 (c) i (d) $-i$

(3) The solution set of the equation : $x^2 + 9 = 0$ in $\mathbb{C}$ is ....

(a) $\{3, -3\}$ (b) $\{-3\,i\}$ (c) $\{3\,i, -3\,i\}$ (d) $\varnothing$

(4) If the curve of the quadratic function f intersects the x-axis at the two points $(3, 0)$, $(-1, 0)$, then the solution set of the equation : $f(x) = 0$ in $\mathbb{R}$ is ....

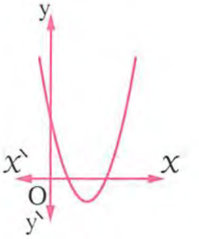
(a) $\{3, 0\}$ (b) $\{-1, 0\}$ (c) $\{-3, 1\}$ (d) $\{3, -1\}$

(5) $1 + i + i^2 + i^3 + i^4 + \cdots + i^{16} =$ ....

(a) i (b) 1 (c) 16 (d) 4

(6) The opposite figure represents the curve $y = a\,x^2 + b\,x + c$

Which of the following it true ?



(a) $a < 0, c < 0$ (b) $a > 0, c < 0$

(c) $a < 0, c > 0$ (d) $a > 0, c > 0$

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] Find in $\mathbb{C}$ the solution set of the equation :

$x^2 - 2\,x + 4 = 0$

[b] Find the values of x and y which satisfy that :

$$x + i\,y = \frac{(2 + i)(2 - i)}{3 + 2\,i}$$

Total mark

10

Quiz 2 till lesson 2 – unit 1

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) If the two roots of the equation : $4x^2 - 12x + c = 0$ are equal , then c = ...

(a) 3 (b) 4 (c) 9 (d) 16

(2) If $x = -1$ is one of the roots of the equation : $x^2 - ax - 2 = 0$, then a = ...

(a) 1 (b) – 1 (c) 3 (d) – 3

(3) If $a = 1 + \sqrt{2}\,i$, $b = 1 - \sqrt{2}\,i$, then a b = ...

(a) – 1 (b) 1 (c) 2 (d) 3

(4) If the two roots of the equation : $x^2 - 6x + k = 0$ are different and real , then k ∈ ....

(a) $]-\infty , 9[$ (b) $]9 , \infty[$ (c) $]-\infty , 9]$ (d) $[9 , \infty[$

(5) If the roots of the equation : $ax^2 + bx + c = 0$ are conjugate complex , which of the following is true ?

(a) $b^2 - 4ac < 0$ (b) $b^2 - 4ac = 0$ (c) $b^2 - 4ac > 0$ (d) $b^2 - 4ac \leq 0$

(6) $(2 + 2i)^{20}$ = ...

(a) 2^{20} (b) 2^{30} (c) $2^{20}\,i$ (d) -2^{30}

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] **Prove that the two roots of the equation :** $3x^2 - 4x + 5 = 0$ are not real , then find the solution set of the equation in $\mathbb{C}$

[b] **Find the values of k which make the equation :** $kx^2 - 4x + 4 = 0$ have two complex and not real roots.

Total mark

Quiz 3 till lesson 3 – unit 1

$\frac{\cdots}{10}$

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) If one of the two roots of the equation : $x^2 - (m - 3)x + 5 = 0$ is the additive inverse of the other root , then m = ...

(a) – 5 (b) – 3 (c) 3 (d) 5

(2) The simplest form of the imaginary number i^{31} is ....

(a) i (b) – i (c) 1 (d) – 1

(3) If one of the two roots of the equation : $a x^2 + 2x + 5 = 0$ is the multiplicative inverse of the other root , then a = ...

(a) – 5 (b) – 2 (c) 2 (d) 5

(4) If the two roots of the equation : $x^2 + 4x + k = 0$ are real , then $k \in$ ....

(a) $[4 , \infty[$ (b) $]4 , \infty[$ (c) $]-\infty , 4]$ (d) $]-\infty , 4[$

(5) If the roots of the quadratic equation : $a x^2 + b x - c = 0$ have different signs , then ....

(a) $b = 0$ (b) $c < 0$ (c) $\frac{c}{a} < 0$ (d) $\frac{c}{a} > 0$

(6) If $(1 + i^8)(1 - i^{11}) = x + y\,i$, then $x + y$ = ...

(a) 4 (b) 3 (c) 2 (d) 1

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] If the two roots of the equation : $x^2 - 3x + 2 + \frac{1}{m} = 0$ are equal

, find the value of : m

[b] **Find the value of k which makes one of the two roots of the equation :**

$x^2 + 3x + k = 0$ double the other root.

Total mark

....

10

Quiz 4 till lesson 4 – unit 1

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) The solution set of the equation : $x^2 - 4x = -4$ in $\mathbb{R}$ is ....

(a) $\{-2\}$ (b) $\{2\}$ (c) $\{-2, 2\}$ (d) $\varnothing$

(2) The quadratic equation whose roots are i , – i is ....

(a) $x^2 - 1 = 0$ (b) $x^2 + 1 = 0$ (c) $(x + 1)^2 = 0$ (d) $(x - 1)^2 = 0$

(3) The two roots of the equation : $x^2 - 2x + k = 0$ are real and different if ....

(a) $k = 1$ (b) $k < 1$ (c) $k > 1$ (d) $k = 4$

(4) The simplest form of the expression : $(1 - i)^4$ is ....

(a) -4 (b) 4 (c) $-4i$ (d) $4i$

(5) If the two roots of the quadratic equation $x^2 + bx + c = 0$ are consecutive odd numbers , then : $b^2 - 4c =$ ....

(a) -1 (b) 2 (c) 3 (d) 4

(6) The product of the roots of the equations :

$ax^2 + bx + c = 0$, $bx^2 + cx + a = 0$, $cx^2 + ax + b = 0$ equals ....

(a) a b c (b) – 1 (c) 1 (d) zero

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] If L , M are the two roots of the equation : $2x^2 + 2x + 3 = 0$,

find the equation whose two roots are : $\frac{2}{L}, \frac{2}{M}$

[b] **Find the simplest form of the expression :** $(3 - 2i)^2 (3 + 2i)$

Total mark

....

10

Quiz 5 till lesson 5 – unit 1

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) The function $f : [-2 , 4] \longrightarrow \mathbb{R}$, $f(X) = 4 - 2X$ is negative in the interval ....

(a) $[-2 , 0[$ (b) $]0 , 4]$ (c) $[2 , 4]$ (d) $]2 , 4]$

(2) If the two roots of the equation : $X^2 - 6X + k = 0$ are equal , then k = ...

(a) 9 (b) 6 (c) 1 (d) 12

(3) The quadratic equation whose two roots are $(1 + i)$, $(1 - i)$ is ....

(a) $X^2 - 2X + 2 = 0$ (b) $X^2 + 2X - 2 = 0$

(c) $X^2 + 2X + 2 = 0$ (d) $X^2 - 2X - 2 = 0$

(4) If one of the two roots of the equation : $aX^2 - 3X + 2 = 0$ is the multiplicative inverse of the other root , then a = ...

(a) $\frac{1}{2}$ (b) 3 (c) 2 (d) -2

(5) If $f : f(X) = aX^2 + bX + c$ is positive for all real values of X , then ....

(a) $b^2 - 4ac < 0$ (b) $b^2 - 4ac > 0$ (c) $b^2 - 4ac = 0$ (d) $b^2 - 4ac \leq 0$

(6) Which of the following are the factors of the expression $(X^2 + 9)$?

(a) $(X - 3)(X + 3)$ (b) $(X + 3)^2$

(c) $(X - 3i)^2$ (d) $(X - 3i)(X + 3i)$

Second question *4 marks* **(1) 2 marks** **(2) 2 marks**

Determine the sign of each of the two functions defined by the following rules , representing your answer on the number line :

(1) $f(X) = (X - 1)(X + 2)$ (2) $f(X) = -X^2 + 9$

Total mark

......

10

Quiz 6 till lesson 6 – unit 1

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) The function $f : f(X) = -3$ is negative in ....

(a) $]-\infty, -3]$ (b) $]-3, 3[$ (c) $]-\infty, \infty[$ (d) $]-\infty, 0[$

(2) The solution set of the inequality : $X(X-2) \geq 0$ in $\mathbb{R}$ is ....

(a) $\{0, 2\}$ (b) $[0, 2]$ (c) $\mathbb{R} - [0, 2]$ (d) $\mathbb{R} -]0, 2[$

(3) The simplest form of the imaginary number i^{52} is ....

(a) i (b) $-i$ (c) 1 (d) -1

(4) If one of the two roots of the equation : $a X^2 + 4X + 7 = 0$ is the multiplicative inverse of the other root , then $a =$ ....

(a) $\frac{1}{7}$ (b) 7 (c) 4 (d) -7

(5) The sum of all integers belonging to the solution set of the inequality $(X-5)(3X-4) \leq 0$ is ....

(a) 7 (b) 14 (c) 15 (d) 9

(6) Which of the following is an imaginary number ?

(a) π (b) $5 - i$ (c) $\sqrt{-5}$ (d) i^2

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] If $1 + i$ is one of the two roots of the equation : $X^2 - 2X + c = 0$ where $c \in \mathbb{R}$, find the other root , then find the value of c

[b] Investigate the sign of the function $f : f(X) = 2X^2 + 7X - 15$ and from this find in $\mathbb{R}$ the solution set of the inequality : $2X^2 + 7X \leq 15$

Total mark

....

10

Quiz 1 on lesson 1 – unit 2

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) The angle of measure 50° in the standard position is equivalent to the angle of measure ....

(a) 130° (b) 310° (c) 140° (d) 410°

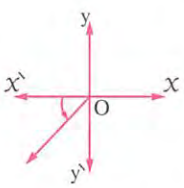
(2) All the following are measures of angles that lie in the second quadrant except ....

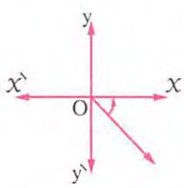
(a) – 210° (b) 120° (c) – 120° (d) 850°

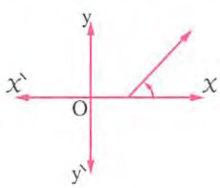
(3) The angle whose measure is (– 750°) lies in the .... quadrant.

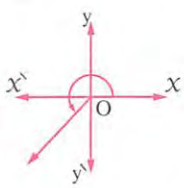
(a) first (b) second (c) third (d) fourth

(4) All the following directed angles are not in the standard position except ....

(a) 

(b) 

(c) 

(d) 

(5) If the terminal side of an angle in the standard position passes through the point (– 1 , 0) , then the terminal side lies in the ....

(a) first quadrant. (b) second quadrant. (c) third quadrant. (d) something else.

(6) If A , B are the measures of two equivalent angles , then : – A , – B are ....

(a) supplementary. (b) equivalent. (c) complementary. (d) their sum is – 360°

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] Determine the quadrant in which each of the following angles lie :

(1) – 52° (2) 220° (3) 1120° 15'

[b] Find two angles , one of them with positive measure and the other with negative measure having common terminal side for each of the following angles :

(1) – 132° (2) 70° (3) – 730°

Total mark .... / 10

Quiz 2 till lesson 2 – unit 2

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) The angle whose measure is $\frac{9\pi}{4}$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(2) The degree measure of a central angle in a circle of radius length 6 cm. and opposite to an arc of length 3π cm. equals ....

(a) 30° (b) 60° (c) 90° (d) 120°

(3) The angle whose measure is -7.3^{rad} is equivalent to the angle whose degree measure is ....

(a) $58^\circ\ 15'\ 33''$ (b) $301^\circ\ 44'\ 27''$ (c) $-233^\circ\ 15'\ 33''$ (d) $211^\circ\ 44'\ 27''$

(4) The radian measure of the central angle subtending an arc of length 3 cm. in a circle whose diameter length is 4 cm. equals ....

(a) $\left(\frac{2}{3}\right)^{rad}$ (b) $\left(\frac{3}{2}\right)^{rad}$ (c) 5^{rad} (d) 6^{rad}

(5) The positive measure of the angle between the hour hand and the minute hand at half past two equals ....

(a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{12}$ (c) $\frac{7\pi}{12}$ (d) $\frac{3\pi}{4}$

(6) If A , – A are measures of two equivalent angles , then one of the values of A is ....

(a) 150° (b) 90° (c) 180° (d) 270°

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] Find the length of the arc which is opposite to an inscribed angle of measure 60° , in a circle whose radius length is 10 cm.

[b] ABC is a triangle in which : $m(\angle A) = 70^\circ$, $m(\angle B) = 60^\circ$, find in radian measure $m(\angle C)$

Total mark

Quiz 3 till lesson 3 – unit 2

$\frac{\cdots}{10}$

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) The radian measure of the central angle which subtends an arc of length 5 cm. in a circle of diameter length 10 cm. equals ....

(a) $\frac{1}{2}^{rad}$ (b) 1^{rad} (c) 2^{rad} (d) π

(2) The measure of the smallest positive angle equivalent to the angle whose measure is $(-870°)$ is ....

(a) 210° (b) 150° (c) – 210° (d) 120°

(3) If θ is the measure of a directed angle drawn in the standard position where $\sin\theta < 0$, in which quadrant does the terminal side of the angle θ lie ?

(a) first. (b) first and second.

(c) second and third. (d) third and fourth.

(4) If $\sec\theta = 2$ where θ is the measure of an acute positive angle , then $\theta =$ ....

(a) 30° (b) 60° (c) 45° (d) 90°

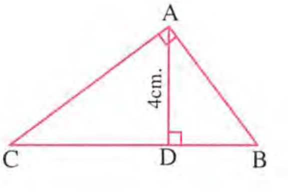
(5) **In the opposite figure :**

If $\tan B + \tan C = \frac{5}{2}$

, then BC = ... cm.

(a) 6 (b) 8

(c) 10 (d) 14



(6) The length of the string of a simple pendulum is 14 cm. and swing through an angle of measure $\frac{1}{10}\pi$, then its arc length $\approx$ .... cm.

(a) 4.6 (b) 4.4 (c) 4.2 (d) 4.8

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] **Without using calculator , find the value of :**

$3\sin 30°\sin^2 60° - \cos 0°\sec 60° + \sin 270°\cos^2 45°$

[b] If $\sin\theta = \frac{3}{5}$, $\theta \in]\frac{\pi}{2}, \pi[$, find all trigonometric functions of the angle whose measure is θ

Total mark

....

10

Quiz 4 till lesson 4 – unit 2

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) The simplest form of the expression : $\tan(180° + \theta) + \cot(270° - \theta)$ is ....

(a) 0 (b) $2\tan\theta$ (c) $2\cot\theta$ (d) 2

(2) If $\sin\theta > 0$, $\tan\theta < 0$, then θ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(3) If θ is the measure of an acute angle , $\cos(\theta + 25°) = \sin 30°$, then $\theta =$ ....

(a) 5° (b) 20° (c) 25° (d) 35°

(4) The degree measure of the central angle which subtends an arc of length 3π cm. in a circle of radius length 4 cm. is ....

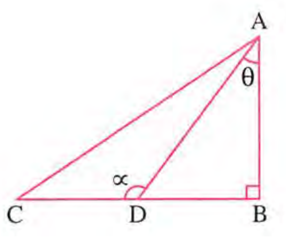
(a) $\frac{3\pi}{4}$ (b) 45° (c) 135° (d) 270°

(5) $\cos 1° \times \cos 2° \times \cos 3° \times \cdots \times \cos 100° =$ ....

(a) $\sin 1° \times \sin 2° \times \sin 3° \times \sin 4° \times \cdots \times \sin 100°$ (b) 1

(c) $1° \times 2° \times 3° \times 4° \times \cdots \times 100°$ (d) zero

(6) **In the opposite figure :**



ΔABC is a right-angled triangle at B

, $\tan\theta = \frac{3}{4}$, then $\cos \propto =$ ....

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$

(c) $-\frac{4}{5}$ (d) $-\frac{3}{5}$

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] If the terminal side of an angle θ drawn in the standard position intersects the unit circle at the point $\left(-\frac{3}{5}, -\frac{4}{5}\right)$, **find in the simplest form the value of the expression :**

$\cos(180° - \theta)\cot(90° - \theta) + \sin(180° - \theta)\tan(-\theta)$

[b] **Find the general solution of the equation :**

$\csc(2\theta - 15°) = \sec(\theta - 30°)$, then find all the values of θ where $\theta \in]0°, 90°[$ which satisfy the equation.

Total mark

......

10

Quiz 5 till lesson 5 – unit 2

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) The maximum value of the function $f : f(\theta) = 4 \sin 2\theta$ is ....

(a) 4 (b) – 4 (c) 2 (d) – 2

(2) The angle of measure 620° lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(3) The radian measure of the angle whose measure is 120° in terms of π is ....

(a) $\frac{1}{3}\pi$ (b) $\frac{2}{3}\pi$ (c) $\frac{3}{2}\pi$ (d) $\frac{1}{2}\pi$

(4) If $\sin\theta = \cos 2\theta$ where $\theta \in]0°, 90°[$, then $\sin 3\theta =$ ....

(a) $\frac{1}{2}$ (b) 1 (c) zero (d) $\frac{\sqrt{3}}{2}$

(5) The function $f : f(\theta) = 3 \cos 2\theta$ is a periodic function and its period equals ....

(a) 2π (b) $\frac{2\pi}{3}$ (c) 6π (d) π

(6) The number of intersections between the curve $y = \sin 3X$ and X-axis on the interval $[0, 2\pi]$ equals ....

(a) 2 (b) 3 (c) 4 (d) 7

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] Find the general solution of the equation : $\tan 4\theta = \cot 2\theta$

[b] If the function $f : f(\theta) = \cos\theta$, find :

(1) Its domain.

(2) Its range.

(3) Its period.

Total mark

$\frac{\cdots\cdots}{10}$

Quiz 6 till lesson 6 – unit 2

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) If $2 \cos \theta = -\sqrt{2}$, then the measure of the smallest positive angle satisfying that is ……………

(a) 45° (b) 135° (c) 225° (d) 315°

(2) The simplest form of the expression : $\tan (360° - \theta) + \cot (270° - \theta)$ is ……………

(a) zero (b) 2 (c) $2 \tan \theta$ (d) $2 \cot \theta$

(3) The degree measure of the central angle which subtends an arc of length 6π cm. in a circle of radius length 9 cm. is ……………

(a) 30° (b) 60° (c) 120° (d) 150°

(4) Which of the following angles whose sine and cosine are negative ?

(a) 50° (b) 150° (c) 210° (d) 300°

(5) $\cos \left(\tan^{-1} \frac{3}{4}\right) =$ ……………

(a) $\frac{3}{4}$ (b) $\frac{4}{5}$ (c) $\frac{3}{5}$ (d) $\sin^{-1} \frac{3}{4}$

(6) If $\sin^2 \theta = \frac{1}{3}$, which of the following can not be an approximate value of θ ?

(a) $215° \, 15' \, 51.8''$ (b) $-35° \, 15' \, 51.8''$

(c) $70° \, 30' \, 50.3''$ (d) $144° \, 44' \, 8.2''$

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] **Find in degree measure the value of θ which satisfies :** $\cos \theta = -0.642$

[b] If the terminal side of a directed angle whose measure is θ in the standard position intersects the unit circle at the point $\left(-\frac{\sqrt{3}}{2} , \frac{1}{2}\right)$, **find the value of :** θ

THIRD Accumulative quizzes on geometry

Total mark

...... / 10

Quiz 1 on lesson 1 – unit 3

Answer the following questions :

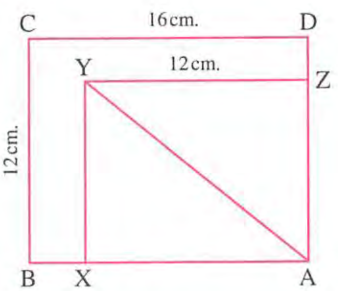
First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) Two similar polygons , the ratio between the lengths of two corresponding sides in them is 2 : 3 , if the perimeter of the smaller is 14 cm. , then the perimeter of the bigger is .... cm.

(a) 14 (b) 28 (c) 15 (d) 21

(2) **In the opposite figure :**



If rectangle ABCD ~ rectangle AXYZ

, DC = 16 cm.

, BC = ZY = 12 cm.

, then AY = ... cm.

(a) 20 (b) 9

(c) 15 (d) 18

(3) Two similar triangles , in which $\frac{AB}{XY} = \frac{AC}{YZ} = \frac{BC}{ZX}$, which of the following is false ?

(a) Δ ABC ~ Δ XYZ (b) m ($\angle$ C) = m ($\angle$ Z)

(c) m ($\angle$ ABC) = m ($\angle$ YXZ) (d) Δ ABC ~ Δ YXZ

(4) Which of the following is always true ?

(a) All regular polygons are similar. (b) All squares are congruent.

(c) All equilateral triangles are similar. (d) All rhombuses are similar.

(5) If Δ LMN ~ Δ XYZ , m ($\angle$ L) = 35° and m ($\angle$ Z) = 75° , then m ($\angle$ M) = ...

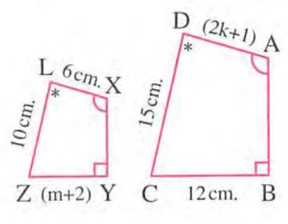
(a) 110° (b) 35° (c) 75° (d) 70°

(6) If k is the scale factor of similarity between two polygons M_1 to M_2 where M_1 is reduction of polygon M_2 , then ....

(a) k > 0 (b) k = 1 (c) k > 1 (d) 0 < k < 1

Second question *4 marks* **(1) 2 marks (2) 2 marks**

In the opposite figure :



Polygon ABCD ~ polygon XYZL

(1) Find the scale factor of similarity between the polygon ABCD and the polygon XYZL

(2) **Find the value of each of :** m , k

Total mark

10

Quiz 2 till lesson 2 – unit 3

Answer the following questions :

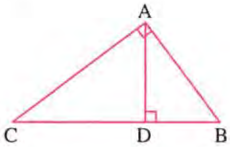
First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) Two similar rectangles , the two dimensions of the first are 12 cm. , 8 cm. and the perimeter of the second is 60 cm. , then the length of the second rectangle is ....

(a) 12 cm. (b) 18 cm. (c) 24 cm. (d) 16 cm.

(2) **In the opposite figure :**

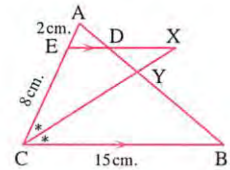


Which of the following expressions is wrong ?

(a) $(AB)^2 = BD \times DC$ (b) $(AC)^2 = CD \times CB$

(c) $(AD)^2 = DB \times DC$ (d) $AB \times AC = BC \times AD$

(3) **In the opposite figure :**

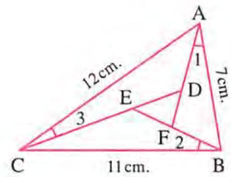


If $\overrightarrow{CX}$ bisects $\angle ACB$, $\overrightarrow{XD} \parallel \overline{BC}$

, then XD = ... cm.

(a) 3 (b) 4 (c) 5 (d) 6

(4) **In the opposite figure :**



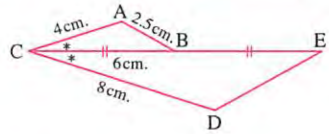
If m ($\angle$ 1) = m ($\angle$ 2) = m ($\angle$ 3)

, then DE : EF : FD = ...

(a) 7 : 11 : 12 (b) 12 : 11 : 7

(c) 12 : 7 : 11 (d) 11 : 12 : 7

(5) **In the opposite figure :**

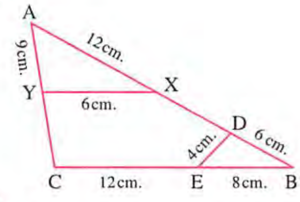


If B is the midpoint of $\overline{CE}$

, then DE = ... cm.

(a) 4 (b) 5 (c) 6 (d) 7

(6) **In the opposite figure :**



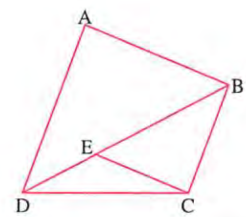
YC = ... cm.

(a) 9 (b) 10

(c) 11 (d) 12

Second question *4 marks* **(1) 2 marks** **(2) 2 marks**

In the opposite figure :



ABCD is a quadrilateral

, $E \in \overline{BD}$ where $\frac{AB}{DA} = \frac{CE}{BC}$, $\frac{BD}{DA} = \frac{EB}{BC}$

Prove that : (1) $\overline{AD} \parallel \overline{BC}$ (2) $\overline{AB} \parallel \overline{CE}$

Total mark

Quiz 3 till lesson 3 – unit 3

10

Answer the following questions :

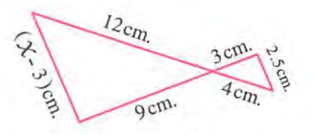
First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) If the ratio between the perimeters of two similar polygons is 4 : 9 , then the ratio between their areas is ....

(a) 4 : 9 (b) 2 : 3 (c) 16 : 81 (d) 8 : 18

(2) **In the opposite figure :**

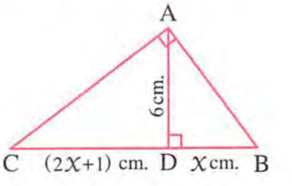


$x =$ ....

(a) $\frac{15}{2}$ (b) 27

(c) 14 (d) $10\frac{1}{2}$

(3) **In the opposite figure :**

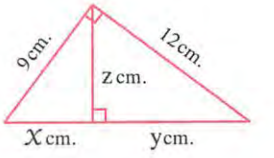


$x =$ ....

(a) 4.5 (b) 4

(c) 6 (d) 36

(4) **In the opposite figure :**

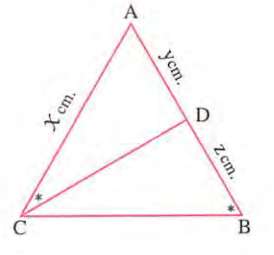


$x + y + z =$ ....

(a) 15 (b) 18.2

(c) 22 (d) 22.2

(5) **In the opposite figure :**



$x^2 - y^2 =$ ....

(a) $(x - y)^2 - 2xy$ (b) z^2

(c) z y (d) zero

(6) If Δ XYZ ~ Δ ABC , a (Δ XYZ) = 3 a (Δ ABC) and XY = 3 cm. , then AB = ... cm.

(a) $\sqrt{3}$ (b) $3\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) 3

Second question *4 marks*

ABCD , XYZL are two similar polygons. If M is the midpoint of $\overline{BC}$, N is the midpoint of $\overline{YZ}$, AM = 4 cm. , XN = 9 cm.

, **prove that :** area of polygon ABCD : area of polygon XYZL = 16 : 81

Total mark 10

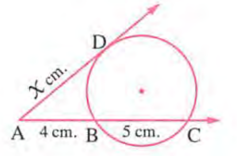
Quiz 4 till lesson 4 – unit 3

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

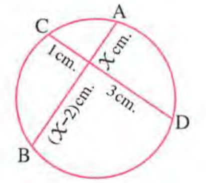
(1) **In the opposite figure :**



X = ...

(a) $2\sqrt{5}$ (b) 36 (c) 20 (d) 6

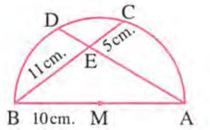
(2) **In the opposite figure :**



X = ...

(a) 5 (b) 2 (c) 3 (d) 7

(3) **In the opposite figure :**



In semicircle M , ED = ... cm.

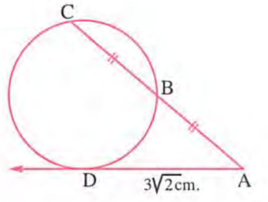
(a) $\frac{50}{13}$ (b) $\frac{55}{13}$ (c) $\frac{57}{13}$ (d) $\frac{59}{13}$

(4) Any two regular polygons with the same number of sides are ....

(a) congruent. (b) equal in area.

(c) equal in perimeter. (d) similar.

(5) **In the opposite figure :**

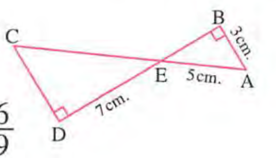


$\overrightarrow{AD}$ is a tangent to the circle

, then AC = ... cm.

(a) $\sqrt{3}$ (b) 3 (c) 18 (d) 6

(6) **In the opposite figure :**



$\frac{a(\Delta ABE)}{a(\Delta CDE)}$ = ...

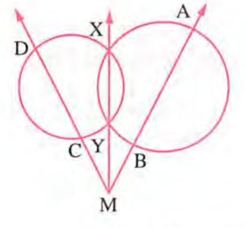
(a) $\frac{9}{49}$ (b) $\frac{25}{49}$ (c) $\frac{9}{25}$ (d) $\frac{16}{49}$

Second question *4 marks* **[a] 2 marks** **[b] 2 marks**

[a] ABC , DEF are two similar triangles , X is the midpoint of $\overline{BC}$ and Y is the midpoint of $\overline{EF}$

Prove that : Δ ABX ~ Δ DEY

[b] **In the opposite figure :**



Prove that : One circle passes by the points A , B , C and D

Total mark

......

10

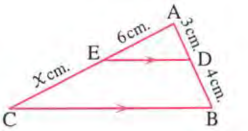
Quiz 5 till lesson 1 – unit 4

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) In the opposite figure :

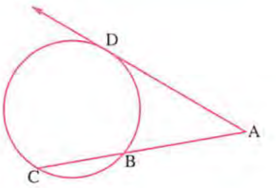


If $\overline{DE} \parallel \overline{BC}$

, then $X =$ ....

(a) 4 (b) 6 (c) 8 (d) 10

(2) In the opposite figure :

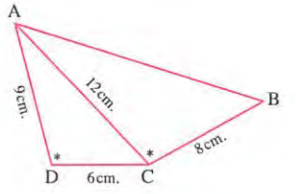


If $\overrightarrow{AD}$ is a tangent to the circle

, then $(AD)^2 =$ ....

(a) $AB \times BC$ (b) $AC \times AB$ (c) $AD \times AB$ (d) $(AC)^2$

(3) In the opposite figure :

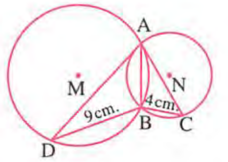


If m ($\angle$ ADC) = m ($\angle$ ACB)

, then AB = ... cm.

(a) 12 (b) 16 (c) 18 (d) 20

(4) In the opposite figure :



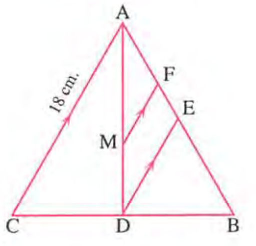
If $\overline{AC}$ is a tangent to the circle M at A

, $\overline{AD}$ is a tangent to the circle N at A

, then AB = ... cm.

(a) 4 (b) 5 (c) 6 (d) 7

(5) In the opposite figure :



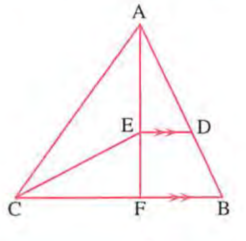
If M is the point of intersection

of the medians of Δ ABC

, the length of $\overline{FM}$ = ... cm.

(a) 4 (b) 5 (c) 6 (d) 8

(6) In the opposite figure :



If the area of Δ AEC = 15 cm^2.

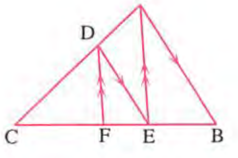
, the area of Δ EFC = 9 cm^2.

, AB = 16 cm. , then AD = ... cm.

(a) 6 (b) 10 (c) 12 (d) 13

Second question *4 marks*

In the opposite figure :



ABC is a triangle , D $\in \overline{AC}$

, $\overline{DE} \parallel \overline{AB}$, $\overline{DF} \parallel \overline{AE}$ **Prove that :** $(CE)^2 = CF \times CB$

Total mark
......
10

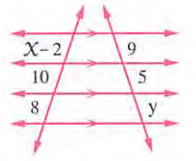
Quiz 6 till lesson 2 – unit 4

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) **In the opposite figure :**



The given lengths are in cm.

$X + y =$ .... cm.

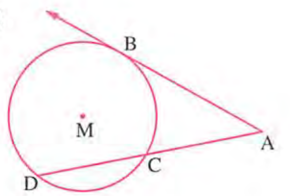
(a) 18 (b) 4 (c) 20 (d) 24

(2) If $\Delta ABC \sim \Delta DEF$, area of $\Delta ABC = 4$ area of ΔDEF and $DE = 6$ cm.

, then $AB =$ .... cm.

(a) 3 (b) 24 (c) 12 (d) 8

(3) **In the opposite figure :**

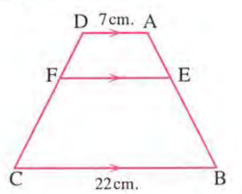


If $\overrightarrow{AB}$ is a tangent to the circle M

, then $(AB)^2 =$ ....

(a) $AC \times CD$ (b) $AC \times AD$ (c) $AB \times AC$ (d) $AB \times CD$

(4) **In the opposite figure :**

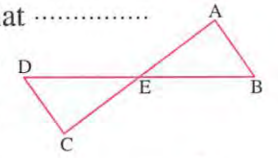


$\frac{AE}{EB} = \frac{2}{3}$

, then $EF =$ .... cm.

(a) 9 (b) 11 (c) 13 (d) 15

(5) **In the opposite figure :**

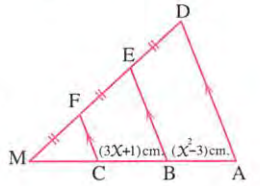


To prove that ABCD is a cyclic quadrilateral you need to prove that ....

(a) $AB \times AC = DB \times DC$ (b) $AE \times AC = BE \times BD$

(c) $m(\angle A) = m(\angle C)$ (d) $AE \times EC = BE \times ED$

(6) **In the opposite figure :**

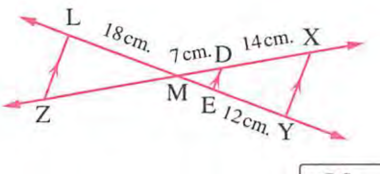


$AM =$ .... cm.

(a) $9X$ (b) $2X^2 + 4$ (c) 39 (d) 26

Second question *4 marks* **(1) 2 marks** **(2) 2 marks**

In the opposite figure :



$\overline{XY} \parallel \overline{DE} \parallel \overline{LZ}$

Find : (1) The length of $\overline{EM}$

(2) The length of $\overline{MZ}$

Total mark

……………

10

Quiz 7 till lesson 3 – unit 4

Answer the following questions :

First question *6 marks* **each item 1 mark**

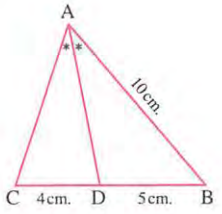
Choose the correct answer from those given :

(1) If Δ ABC ~ Δ XYZ and AB = 3 XY

, then $\frac{\text{the area of } \Delta \text{ XYZ}}{\text{the area of } \Delta \text{ ABC}} =$ ……………

(a) $\frac{1}{3}$ (b) 3 (c) $\frac{1}{9}$ (d) 9

(2) In the opposite figure :

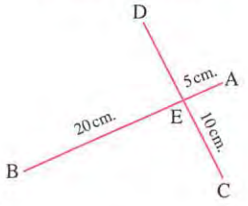


$\overrightarrow{AD}$ bisects $\angle$ BAC

, then AD = …………… cm.

(a) 8 (b) 60 (c) $2\sqrt{15}$ (d) $7\sqrt{3}$

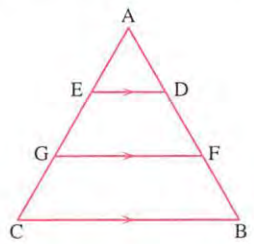
(3) In the opposite figure :



If $\overline{AB} \cap \overline{CD} = \{E\}$, then the points A , C , B and D lie on one circle if ED = ……………

(a) 5 cm. (b) 8 cm. (c) EC (d) EB

(4) In the opposite figure :

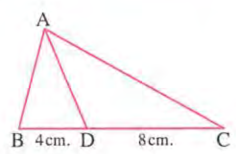


$\frac{DE}{BC} =$ ……………

(a) $\frac{FG}{BC}$ (b) $\frac{AD}{AF}$

(c) $\frac{EG}{EC}$ (d) $\frac{AE}{AC}$

(5) In the opposite figure :

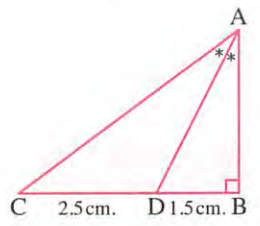


If m ($\angle$ B) = 2 m ($\angle$ DAB) = 2 m ($\angle$ DAC)

, then AB = …………… cm.

(a) 4 (b) 6 (c) 8 (d) 9

(6) In the opposite figure :



AC = …………… cm.

(a) 4 (b) 5

(c) 6 (d) 7

Second question *4 marks*

XYZ is a triangle , $\angle$ XYZ is bisected by a bisector which intersects $\overline{XZ}$ at M

, then draw $\overrightarrow{MN}$ // $\overline{ZY}$ to intersect $\overline{XY}$ at N

Prove that : $\frac{XY}{YZ} = \frac{XN}{YN}$ and if XY = 6 cm. , YZ = 4 cm. , **find the length of :** $\overline{XN}$

Total mark

.... / 10

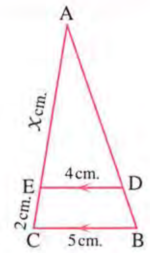
Quiz 8 till lesson 4 – unit 4

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) In the opposite figure :

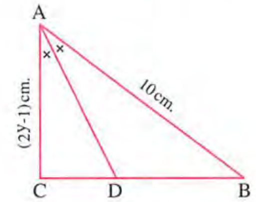


If $\overline{DE} \parallel \overline{BC}$

, then X = ... cm.

(a) 4 (b) 5 (c) 6 (d) 8

(2) In the opposite figure :



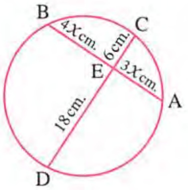
$\overrightarrow{AD}$ bisects $\angle$ A , $\frac{BD}{DC} = \frac{5}{3}$

If AB = 10 cm. , AC = (2 y – 1) cm.

, then y = ... cm.

(a) 35 (b) 25 (c) 3.5 (d) 2.5

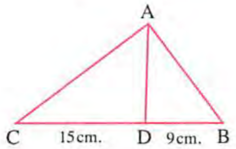
(3) In the opposite figure :



X = ... cm.

(a) 3 (b) 9 (c) 2 (d) 18

(4) In the opposite figure :



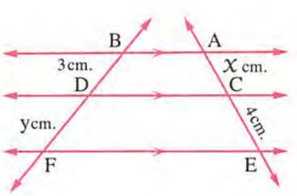
To prove that m ($\angle$ BAD) = m ($\angle$ DAC)

you need to know ....

(a) AB = AC (b) AD = $2\sqrt{30}$ cm.

(c) 3 AC = 5 AB (d) m ($\angle$ B) = m ($\angle$ C)

(5) In the opposite figure :



If $X^2 + y^2 = 57$

, then X + y = ... cm.

(a) 7 (b) 9 (c) 11 (d) 12

(6) In the opposite figure :

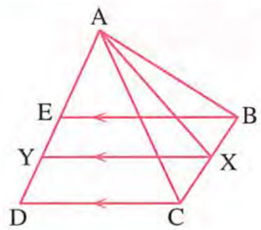
The area of Δ ABD = ... cm^2.

(a) 36 (b) 48

(c) 54 (d) 72

Second question *4 marks*

In the opposite figure :



$\overline{BE} \parallel \overline{XY} \parallel \overline{CD}$, $\frac{AB}{AC} = \frac{EY}{YD}$

Prove that : $\overrightarrow{AX}$ bisects $\angle$ BAC

Total mark

......

10

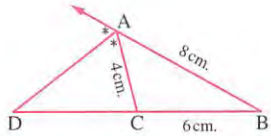
Quiz 9 till lesson 5 – unit 4

Answer the following questions :

First question *6 marks* **each item 1 mark**

Choose the correct answer from those given :

(1) In the opposite figure :

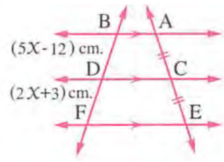


If $\overrightarrow{AD}$ bisects exterior $\angle A$

, then CD = ... cm.

(a) 2 (b) 6 (c) 4 (d) 8

(2) In the opposite figure :

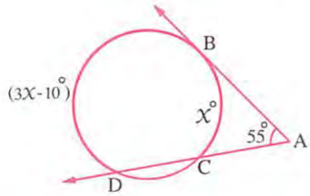


X = ... cm.

(a) 5 (b) 3

(c) 7 (d) 2

(3) In the opposite figure :



If $\overrightarrow{AB}$ is a tangent to the circle

, then X = ...

(a) 60° (b) 30°

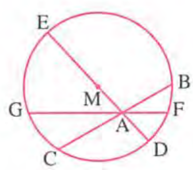
(c) 15° (d) 55°

(4) If AM = 4 cm. , r = 3 cm. , such that A is a point outside the circle M

, then P_M (A) = ...

(a) 16 (b) 9 (c) 25 (d) 7

(5) In the opposite figure :

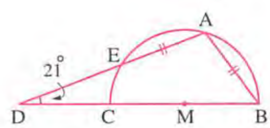


Which of the following is not equal to P_M (A) ?

(a) $(AM)^2 - (DM)^2$ (b) BA × AC

(c) – DA × AE (d) – FA × AG

(6) In the opposite figure :



If AE = AB , $\overline{BC}$ is a diameter , m ($\angle$ D) = 21°

, then m ($\angle$ A) = ...

(a) 100° (b) 104° (c) 106° (d) 110°

Second question *4 marks* **(1) 2 marks** **(2) 2 marks**

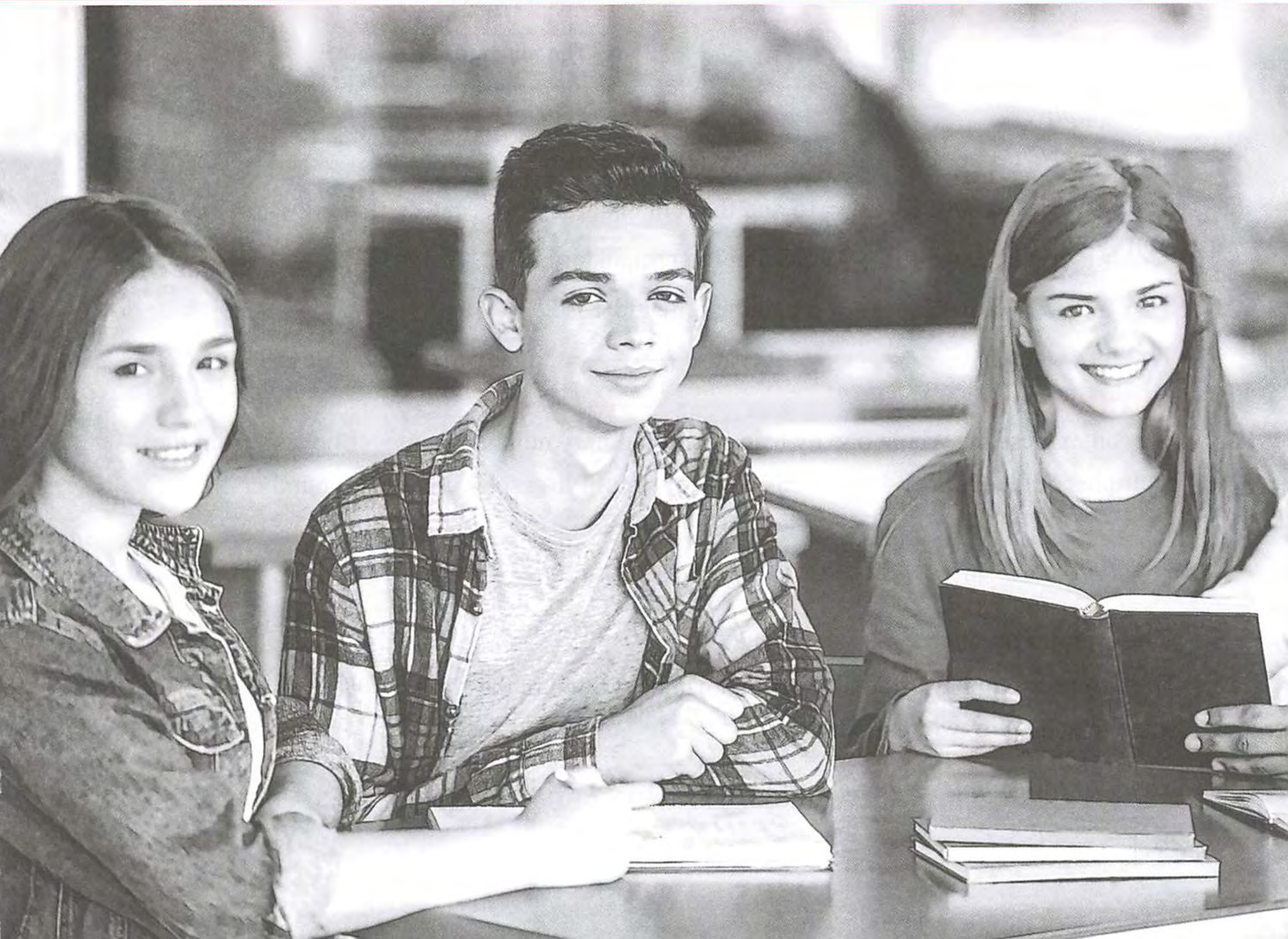
The radius length of circle M is 7 cm. , A is a point at a distance 5 cm. from the centre of the circle , draw the chord $\overline{BC}$ passing through A such that AB = 3 AC

Calculate : (1) The length of $\overline{BC}$

(2) The distance between the chord $\overline{BC}$ and the centre of the circle.

Final revision

FIRST	Final revision on algebra.
SECOND	Final revision on trigonometry.
THIRD	Final revision on geometry.



FIRST Final revision on algebra

Remember The complex numbers

The imaginary number "i"

The imaginary number "i" is defined as the number whose square is – 1 *i.e.* $i^2 = -1$

Notice that

- $i \times i = i^2 = -1$
- $-i \times -i = i^2 = -1$
- $\sqrt{-2} = \sqrt{2\,i^2} = \sqrt{2}\,i$
- $\sqrt{-5} = \sqrt{5}\,i$

Similarly :

- $\sqrt{-9} = 3\,i$

Integer powers of "i"

To find i^m where m is an integer → We find the remainder of $m \div 4$, if

- The remainder = 0 then → $i^m = 1$
- The remainder = 1 then → $i^m = i$
- The remainder = 2 then → $i^m = -1$
- The remainder = 3 then → $i^m = -i$

For example :

- $i^{12} = 1$ "because $12 \div 4 = 3$ and the remainder is 0"
- $i^{63} = -i$ "because $63 \div 4 = 15$ and the remainder is 3"
- $i^{101} = i$ "because $101 \div 4 = 25$ and the remainder is 1"
- $i^{26} = -1$ "because $26 \div 4 = 6$ and the remainder is 2"
- i^{12n+3} "where $n \in \mathbb{Z}$" $= -i$ "because $\frac{12n+3}{4} = 3n$ and the remainder is 3"

Remark

We can express the whole one by using the imaginary number to integer powers from the multiples of the number 4 , and this helps in simplifying some imaginary numbers.

For example : • $\frac{1}{i^{19}} = \frac{i^{20}}{i^{19}} = i$ • $i^{-61} = i^{-61} \times i^{64} = i^3 = -i$

The complex number

The complex number is the number that can be written in the form : $Z = a + bi$ where a and b are two real numbers , $i^2 = -1$

Examples for complex numbers : $13 - 2i$, $7 + \sqrt{5}\,i$, -25 , $8i$, $\sqrt{15}$, $5i - 4$

Equality of two complex numbers

Two complex numbers are equal if and only if the two real parts are equal and the two imaginary parts are equal , and vice versa.

If $Z_1 = -5 + x\,i$, $Z_2 = y + \sqrt{3}\,i$ and $Z_1 = Z_2$, then $y = -5$, $x = \sqrt{3}$

Adding and subtracting complex numbers

When adding and subtracting two complex numbers , we add or subtract real parts together and add or subtract imaginary parts together.

For example : • $(4 + 5\,i) + (-2 - 3\,i) = (4 - 2) + (5 - 3)\,i = 2 + 2\,i$

• $(26 - 4\,i) - (9 - 20\,i) = (26 - 9) + (-4 + 20)\,i = 17 + 16\,i$

Multiplying complex numbers

We use the same properties of multiplying algebraic expressions and multiplying by inspection which we have studied before.

For example : • $2\,i\,(1 - 3\,i) = 2\,i - 6\,i^2$ (where $i^2 = -1$) $= 6 + 2\,i$

• $(3 \underbrace{-5\,i)(2}_{-10\,i} + i) = 6 - 7\,i - 5\,i^2$ (where $i^2 = -1$) $= 11 - 7\,i$; outer product: $3\,i$

• $(4 - i)^2 = 16 - 8\,i + i^2$ (where $i^2 = -1$)

$= 15 - 8\,i$

• $(5 - 3\,i)(5 + 3\,i) = 25 - 9\,i^2$ (where $i^2 = -1$)

$= 25 + 9 = 34$

Remember that

$(a \pm b)^2 = a^2 \pm 2\,ab + b^2$

Remember that

$(a + b)(a - b) = a^2 - b^2$

The two conjugate numbers

The two numbers $a + b\,i$ and $a - b\,i$ are called conjugate numbers and we notice that the complex number and its conjugate differ only in the sign of their imaginary parts , and their sum is a real number and their product is a real number.

For example :

• The two numbers $3 + 4\,i$ and $3 - 4\,i$ are conjugate numbers , while the two numbers $2\,i - 5$ and $2\,i + 5$ are not conjugate because the imaginary part in each of them has the same sign.

• The conjugate of the number $4\,i$ is $-4\,i$ • The conjugate of the number 6 is 6

Remark

To simplify the fraction whose denominator is a complex number not real , we multiply its two terms by the conjugate of denominator.

For example : $\frac{30 + 45\,i}{1 - 2\,i} = \frac{30 + 45\,i}{1 - 2\,i} \times \frac{1 + 2\,i}{1 + 2\,i} = \frac{30 + 105\,i + 90\,i^2}{1 - 4\,i^2} = \frac{-60 + 105\,i}{5} = -12 + 21\,i$

Remember — The quadratic equation in one variable (Determining the type of roots - Finding the solution set)

First — Algebraic method

To determine the type of roots of the quadratic equation and find its solution set in $\mathbb{R}$ or in $\mathbb{C}$ for each of the following equations algebraically :

- $x^2 - 2x - 4 = 0$
- $4x + x^2 + 4 = 0$
- $2 + x^2 = 0$

We will follow the following steps :

Put the given equation on the form : $ax^2 + bx + c = 0$

$\therefore x^2 - 2x - 4 = 0$	$\therefore x^2 + 4x + 4 = 0$	$\therefore x^2 + 2 = 0$

Find the discriminant $(b^2 - 4ac)$

$\therefore$ The discriminant $= (-2)^2 - 4(1)(-4) = 20$	$\therefore$ The discriminant $= (4)^2 - 4(1)(4) = 0$	$\therefore$ The discriminant $= (0)^2 - 4(1)(2) = -8$

Determining the type of roots of the equation

$\therefore$ The two roots are real and different because the discriminant is positive.	$\therefore$ The two roots are real and equal because the discriminant is zero.	$\therefore$ The two roots are complex and non real because the discriminant is negative.

Finding the solution set by factorization or the general formula

$\because x = \frac{2 \pm \sqrt{20}}{2 \times 1}$ $\therefore$ The two roots are $1 + \sqrt{5}$, $1 - \sqrt{5}$ $\therefore$ The S.S. in $\mathbb{R} = \{1 + \sqrt{5} , 1 - \sqrt{5}\}$	$\because x^2 + 4x + 4 = 0$ $\therefore (x + 2)^2 = 0$ $\therefore x = -2$ $\therefore$ The S.S. in $\mathbb{R} = \{-2\}$	$\because x^2 + 2 = 0$ $\therefore x = \pm\sqrt{-2} = \pm\sqrt{2}\,i$ $\therefore$ The S.S. in $\mathbb{R} = \varnothing$, the S.S. in $\mathbb{C} = \{\sqrt{2}\,i , -\sqrt{2}\,i\}$

Second Graphic method

To determine the type of roots of the quadratic equation and find the solution set for each of the following equations graphically :

- $x^2 - 2x - 3 = 0$
- $9 + x^2 - 6x = 0$
- $-x^2 + 3x - 5 = 0$

We will follow the following steps :

Put the given equation on the form : $a x^2 + b x + c = 0$

- $\therefore x^2 - 2x - 3 = 0$
- $\therefore x^2 - 6x + 9 = 0$
- $\therefore -x^2 + 3x - 5 = 0$

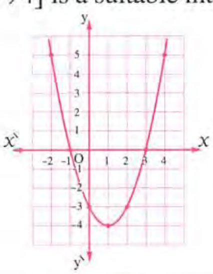
Write the quadratic function f which is related by the equation

- $\therefore f(x) = x^2 - 2x - 3$
- $\therefore f(x) = x^2 - 6x + 9$
- $\therefore f(x) = -x^2 + 3x - 5$

Draw the curve of the function in a suitable interval from real numbers where $\frac{-b}{2a}$ is in its middle

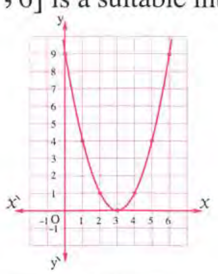
$\because \frac{-b}{2a} = \frac{2}{2} = 1$

$\therefore [-2, 4]$ is a suitable interval.



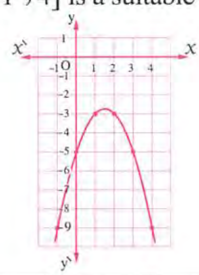
$\because \frac{-b}{2a} = \frac{6}{2} = 3$

$\therefore [0, 6]$ is a suitable interval.



$\because \frac{-b}{2a} = \frac{-3}{-2} = 1\frac{1}{2}$

$\therefore [-1, 4]$ is a suitable interval.



Determining the type of roots of the equation

- The two roots are real and different because the curve intersects x-axis at two points.
- The two roots are real and equal because the curve touches x-axis
- The two roots are complex and non real because the curve does not intersect x-axis.

Finding the solution set in $\mathbb{R}$

- The S.S. in $\mathbb{R} = \{-1, 3\}$
- The S.S. in $\mathbb{R} = \{3\}$
- The S.S. in $\mathbb{R} = \varnothing$

The relation between the two roots of the equation : $a x^2 + b x + c = 0$ and the coefficients of its terms

The sum of the two roots $= \frac{-b}{a}$

The product of the two roots $= \frac{c}{a}$

For example :

Equation of second degree	The sum of the two roots	The product of the two roots
• $2x^2 + 5x - 4 = 0$	$\frac{-5}{2} = -2.5$	$\frac{-4}{2} = -2$
• $3x^2 - 7x + 3 = 0$	$\frac{7}{3}$	$\frac{3}{3} = 1$ (One of the roots is the multiplicative inverse of the other)
• $5x^2 - 7 = 0$	Zero (One of the roots is the additive inverse of the other)	$\frac{-7}{5}$

Remember Forming the quadratic equation

First Forming the quadratic equation whose two roots are known

We find the sum of the two roots and their product , then the equation will be in the form :

$$x^2 - (\text{the sum of the two roots})\, x + \text{the product of the two roots} = 0$$

For example :

If the two roots are	then the sum of the two roots is	the product of the two roots is	Thus , the required equation is
• $3 , -4$	-1	-12	$x^2 + x - 12 = 0$
• $\frac{2}{3} , \frac{3}{2}$	$\frac{13}{6}$	1	$x^2 - \frac{13}{6}x + 1 = 0$ *i.e.* $6x^2 - 13x + 6 = 0$
• $2 + i , 2 - i$	4	5	$x^2 - 4x + 5 = 0$

Second Forming a quadratic equation from another given quadratic equation

First method

This method is used if finding the two roots of the given equation is easy.

For example :

If L and M are the two roots of the equation : $x^2 - x - 6 = 0$ where $L > M$

, form the quadratic equation whose roots are : $L - 2$, $M^2 + 1$

1 We find the two roots of the given equation L and M :

$\because x^2 - x - 6 = 0$ $\quad\quad \therefore (x - 3)(x + 2) = 0$

$\therefore L = 3$, $M = -2$

2 We find the two roots of the required equation D and E :

- $D = L - 2 = 3 - 2 = 1$
- $E = M^2 + 1 = (-2)^2 + 1 = 5$

3 We form the required equation :

$\therefore x^2 - 6x + 5 = 0$

Second method

This method is used if we can find "D + E" , "DE" of the required equation in terms of "L + M" , "LM" of the given equation by one of the following identities :

1 $L^2 + M^2 = (L + M)^2 - 2LM$	2 $(L - M)^2 = (L + M)^2 - 4LM$
3 $L^3 + M^3 = (L + M)\left[(L + M)^2 - 3LM\right]$	4 $L^3 - M^3 = (L - M)\left[(L + M)^2 - LM\right]$
5 $\frac{1}{M} + \frac{1}{L} = \frac{L + M}{LM}$	6 $\frac{L}{M} + \frac{M}{L} = \frac{L^2 + M^2}{LM} = \frac{(L + M)^2 - 2LM}{LM}$

For example :

If L and M are the two roots of the equation : $x^2 - 3x + 1 = 0$

, form the equation whose roots are : $D = \frac{L}{M}$, $E = \frac{M}{L}$

1 **We find L + M , LM from the given equation :**

- $L + M = \frac{-(-3)}{1} = 3$
- $LM = \frac{1}{1} = 1$

2 **We find D + E , DE of the required equation in terms of L and M :**

- $D + E = \frac{L}{M} + \frac{M}{L} = \frac{L^2 + M^2}{ML}$
- $DE = \frac{L}{M} \times \frac{M}{L} = 1$

3 **We use a suitable identity :**

- $D + E = \frac{L^2 + M^2}{ML} = \frac{(L+M)^2 - 2\,LM}{ML} = \frac{(3)^2 - 2\,(1)}{1} = 7$

4 **We form the required equation :**

$\therefore x^2 - (D + E)\,x + DE = 0$

i.e. $x^2 - 7\,x + 1 = 0$

Third method

This method is used only if the relation between D and L is the same relation between E and M

For example :

If L and M are the two roots of the equation : $x^2 - 5\,x + 2 = 0$

, form the equation whose roots are : $D = L - 3$, $E = M - 3$

1 **We find L or M in terms of D or E from the given relation :**

$\because D = L - 3$

$\therefore L = D + 3$

2 **$\because$ L and M are the two roots of the given equation**

$\therefore$ L and M satisfy the given equation

$\therefore (D + 3)^2 - 5\,(D + 3) + 2 = 0$

$\therefore D^2 + 6\,D + 9 - 5\,D - 15 + 2 = 0$

$\therefore D^2 + D - 4 = 0$

3 **We write the required equation :**

$\because$ D is one of the roots of the required equation

$\therefore$ The required equation is : $x^2 + x - 4 = 0$

Remember The sign of the function

The sign of the constant function

The sign of the constant function $f : f(X) = c$, $c \in \mathbb{R}^*$ is the same sign of c for all values of $X \in \mathbb{R}$

For example :

- The sign of the function $f : f(X) = -7$ is negative for all values of $X \in \mathbb{R}$
- The sign of the function $f : f(X) = 2$ is positive for all values of $X \in \mathbb{R}$

The sign of the first degree function (linear function)

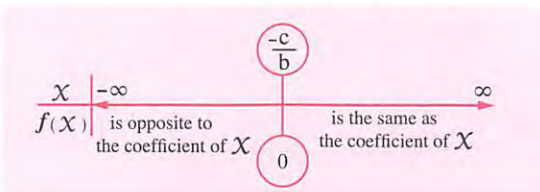
To determine the sign of the linear function $f : f(X) = b\,X + c$, $b \neq 0$

, we put $f(X) = 0$ $\quad \therefore b\,X + c = 0 \quad \therefore X = \frac{-c}{b}$

Then the sign of the function f :

1. Is the same sign of b at $X > \frac{-c}{b}$
2. Is opposite to the sign of b at $X < \frac{-c}{b}$
3. $f(X) = 0$ at $X = \frac{-c}{b}$

And we illustrate this on the number line as in the figure :



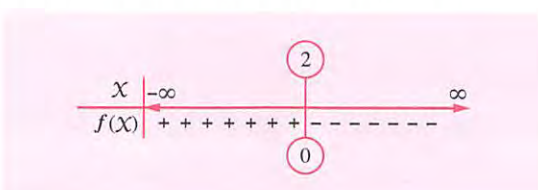
For example :

If $f : f(X) = -3\,X + 6$ $\quad$ Put $-3\,X + 6 = 0$ $\quad \therefore X = 2$

The sign of the function f :

1. Is negative at $X > 2$
2. Is positive at $X < 2$
3. $f(X) = 0$ at $X = 2$

And we illustrate this on the number line as in the figure :



The sign of the second degree function (quadratic function)

To determine the sign of the quadratic function $f : f(X) = a\,X^2 + b\,X + c$, $a \neq 0$, we write the quadratic equation : $a\,X^2 + b\,X + c = 0$ which is related by the function , then do the following steps :

We find the discriminant : $b^2 - 4\,ac$, if

The discriminant is positive	The discriminant $= 0$	The discriminant is negative
Then , the two roots are real and different L , M , L < M	Then , the two roots are real and equal L , L	Then , the two roots are complex and non real.
• f has the opposite of the sign of a for all $X \in \,]L , M[$ • $f(X) = 0$ for all $X \in \{L , M\}$ • f has the same sign of a for all $X \in \mathbb{R} - [L , M]$	• f has the same sign of a for all $X \in \mathbb{R} - \{L\}$ • $f(X) = 0$ at $X = L$	• f has the same sign of a for all values of $X \in \mathbb{R}$

For example :

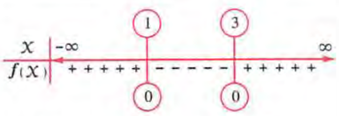
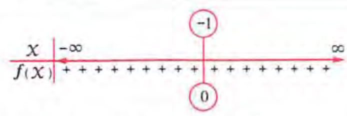
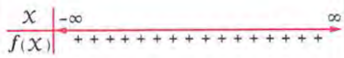
If • $f : f(X) = X^2 - 4\,X + 3$

• $f : f(X) = -\,X^2 - 2\,X - 1$

• $f : f(X) = 2\,X^2 - 3\,X + 5$

, then we can determine the sign of each of the previous functions as the following :

We write the quadratic equations which are related by the previous functions and complete the steps as follows :

$x^2 - 4x + 3 = 0$	$x^2 + 2x + 1 = 0$	$2x^2 - 3x + 5 = 0$
$\because$ The discriminant $= (-4)^2 - 4 \times 1 \times 3$ $= 4$ (positive)	$\because$ The discriminant $= (2)^2 - 4 \times 1 \times 1 = 0$	$\because$ The discriminant $= (-3)^2 - 4 \times 2 \times 5$ $= -31$ (negative)
$\therefore$ The two roots are real and different and they are 3 and 1	$\therefore$ The two roots are real and equal and each of them equals -1	$\therefore$ The two roots are complex and non real
 • f is negative for all $x \in]1 , 3[$ • $f(x) = 0$ for all $x \in \{1 , 3\}$ • f is positive for all $x \in \mathbb{R} - [1 , 3]$	 • f is positive for all $x \in \mathbb{R} - \{-1\}$ • $f(x) = 0$ at $x = -1$	 • f is positive for all values of $x \in \mathbb{R}$

Remember the solving of the quadratic inequalities in $\mathbb{R}$

To find the solution set of the inequality : $x^2 - 5x + 6 > 0$ in $\mathbb{R}$:

1. We write the quadratic function related by the inequality.
2. We study the sign of the quadratic function which we wrote.
3. We determine the intervals which satisfy the inequality.

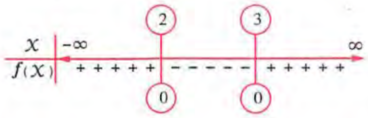
$f : f(x) = x^2 - 5x + 6$

$\because$ The discriminant
$= (-5)^2 - 4 \times 1 \times 6$
$= 1$ (positive)

$\therefore$ The two roots are real and different

, $\because (x-2)(x-3) = 0$

$\therefore x = 2$ or $x = 3$



The solution set of the inequality :
$x^2 - 5x + 6 > 0$
is $\mathbb{R} - [2 , 3]$

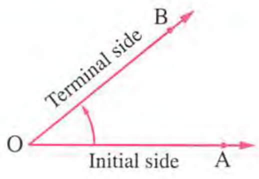
Remember The directed angle

Definition of the directed angle

The directed angle is an ordered pair of two rays called the sides of the angle with a common starting point called the vertex.

For example :

The ordered pair $(\overrightarrow{OA}, \overrightarrow{OB})$ represents the directed angle $\angle AOB$ whose initial side is $\overrightarrow{OA}$ and terminal side is $\overrightarrow{OB}$



Positive and negative measures of a directed angle

If the positive measure of the directed angle = θ
, then the negative measure of the same directed angle = $\theta - 360°$

For example :

The negative measure of the directed angle of measure $210° = 210° - 360° = -150°$

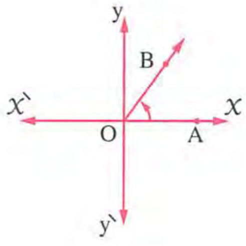
If the negative measure of the directed angle = $-\theta$
, then the positive measure of the same directed angle = $-\theta + 360°$

For example :

The positive measure of the directed angle of measure $(-120°) = -120° + 360° = 240°$

The standard position of the directed angle

A directed angle is in the standard position if the following two conditions are satisfied :



1. Its initial side lies on the positive direction of the x-axis.
2. Its vertex is the origin point of an orthogonal coordinate plane.

Equivalent angles

Several directed angles in the standard position are said to be equivalent when they have one common terminal side.

And we get equivalent angles to the angle whose measure is θ by adding n 360° to it or subtracting n 360° from it where n is an integer.

Determining the quadrant in which the terminal side of the directed angle ∠ AOB whose measure is θ in the standard position lies :

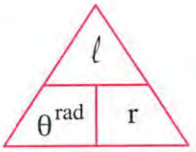
We find α the measure of the smallest positive equivalent angle to the angle θ , then if

- $\alpha \in]0°, 90°[$ then → ∠ AOB lies in the first quadrant.
- $\alpha \in]90°, 180°[$ then → ∠ AOB lies in the second quadrant.
- $\alpha \in]180°, 270°[$ then → ∠ AOB lies in the third quadrant.
- $\alpha \in]270°, 360°[$ then → ∠ AOB lies in the fourth quadrant.
- $\alpha \in \{0°, 90°, 180°, 270°, 360°\}$ then → ∠ AOB is called a quadrantal angle.

Radian measure and degree measure of an angle

- The radian measure of a central angle in a circle $= \dfrac{\text{Length of the arc which the central angle subtends}}{\text{Length of the radius of this circle}}$

i.e. $\boxed{\theta^{rad} = \dfrac{\ell}{r}}$ and from it $\boxed{\ell = \theta^{rad}\, r}$, $\boxed{r = \dfrac{\ell}{\theta^{rad}}}$



- **The relation between the radian measure and the degree measure :**

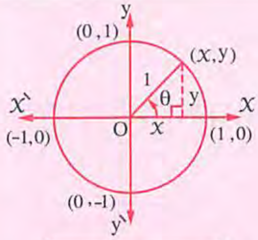
$\boxed{\dfrac{x^\circ}{180°} = \dfrac{\theta^{rad}}{\pi}}$ and from it $\boxed{\theta^{rad} = x^\circ \times \dfrac{\pi}{180°}}$, $\boxed{x^\circ = \theta^{rad} \times \dfrac{180^\circ}{\pi}}$

Notice that

π in radians is equivalent to 180° in degrees.

Remember The trigonometric functions of an acute angle and their reciprocals

- $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = y$
- $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = x$
- $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{y}{x}$



$\boxed{x^2 + y^2 = 1}$

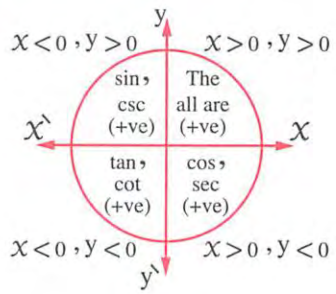
- $\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{1}{y}$
- $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{1}{x}$
- $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{x}{y}$

Notice that

- $x \in [-1 , 1]$ and from it $\cos \theta \in [-1 , 1]$
- $y \in [-1 , 1]$ and from it $\sin \theta \in [-1 , 1]$
- The equivalent angles have the same trigonometric functions.

Remember The signs of trigonometric functions

Quadrant	The interval that θ belongs to	sign of cos , sec	sign of sin , csc	sign of tan , cot
First	$]0 , \frac{\pi}{2}[$	+	+	+
Second	$]\frac{\pi}{2} , \pi[$	−	+	−
Third	$]\pi , \frac{3\pi}{2}[$	−	−	+
Fourth	$]\frac{3\pi}{2} , 2\pi[$	+	−	−



Notice that

The trigonometric functions of the equivalent angles have the same sign.

Remember The trigonometric functions of some special angles

The measure of θ	The point of the intersection of the terminal side with the unit circle	The values of the trigonometric functions		
		sin θ	cos θ	tan θ
0° or 360°	(1 , 0)	0	1	0
90°	(0 , 1)	1	0	undefined
180°	(−1 , 0)	0	−1	0
270°	(0 , −1)	−1	0	undefined
30°	$(\frac{\sqrt{3}}{2} , \frac{1}{2})$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
60°	$(\frac{1}{2} , \frac{\sqrt{3}}{2})$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
45°	$(\frac{1}{\sqrt{2}} , \frac{1}{\sqrt{2}})$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1

Remember The relation between the trigonometric functions of two related angles

To know how to find the relations between the trigonometric functions of two related angles , we will follow the following steps :

First

We determine the quadrant in which the given angle lies

$(90° + \theta)$, $(180° - \theta)$	(θ) , $(90° - \theta)$
$(180° + \theta)$, $(270° - \theta)$	$(270° + \theta)$, $(360° - \theta)$

Second

We put the sign of the given trigonometric function according to the quadrant which is we determined.

Third

In the case of angles of measures θ , $(180° - \theta)$, $(180° + \theta)$, $(360° - \theta)$ or $(- \theta)$, the trigonometric function is written as it is and convert the angle of any form to θ

In the case of angles of measures $(90° - \theta)$, $(90° + \theta)$, $(270° - \theta)$ or $(270° + \theta)$, the trigonometric function is changed as the following :

- sin $\rightleftarrows$ cos
- tan $\rightleftarrows$ cot
- csc $\rightleftarrows$ sec

and convert the angle of any form to θ

For example :

$\cos(180° + \theta)$

$(180° + \theta)$ lies in the third quadrant

The function of cosine in the third quadrant is negative (–ve)

The function as it is because the measure of the angle is $(180° + \theta)$

$- \cos\theta$

$\therefore \cos(180° + \theta) = - \cos\theta$

For example :

$\sin(90° + \theta)$

$(90° + \theta)$ lies in the second quadrant

The function of sine in the second quadrant is positive (+ve)

The function is changed because the measure of the angle is $(90° + \theta)$

$+ \cos\theta$

$\therefore \sin(90° + \theta) = \cos\theta$

For example :

Without using calculator, we can find :

$\cos(-150°)\sin 600° + \cos\frac{2\pi}{3}\sin 330° - \sec\left(-\frac{5\pi}{4}\right)\tan 900°$

$= \cos(210°)\sin(360° + 240°) + \cos 120° \sin(360° - 30°) - \sec 225° \tan(180° + 2 \times 360°)$

$= \underbrace{\cos(180° + 30°)}_{\text{Third quadrant}} \underbrace{\sin(180° + 60°)}_{\text{Third quadrant}} + \underbrace{\cos(180° - 60°)}_{\text{Second quadrant}} \underbrace{\sin(360° - 30°)}_{\text{Fourth quadrant}} - \underbrace{\sec(180° + 45°)}_{\text{Third quadrant}} \underbrace{\tan 180°}_{\text{Quadrantal angle}}$

$= (-\cos 30°)(-\sin 60°) + (-\cos 60°)(-\sin 30°) - (-\sec 45°) \times 0$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} - 0 = \frac{3}{4} + \frac{1}{4} = 1$

Remark

If α and β are the measures of two complementary angles (*i.e.* Their sum is 90°)
, then $\sin\alpha = \cos\beta$, $\tan\alpha = \cot\beta$, $\sec\alpha = \csc\beta$, ...

For example :

20° and 70° are measures of two complementary angles.

$\therefore \sin 20° = \cos 70°$, $\tan 70° = \cot 20°$, ...

Remember **The general solution to solve the equations in the form $\sin\alpha = \cos\beta$ or $\csc\alpha = \sec\beta$ or $\tan\alpha = \cot\beta$**

1. If $\sin\alpha = \cos\beta$

   , then $\alpha \pm \beta = 90° + 360° n$ *i.e.* $\alpha \pm \beta = \frac{\pi}{2} + 2\pi n$ where $n \in \mathbb{Z}$

   i.e. The measure of angle of sine ± the measure of angle of cosine = 90° + 360° n

2. If $\csc\alpha = \sec\beta$

   , then $\alpha \pm \beta = 90° + 360° n$ *i.e.* $\alpha \pm \beta = \frac{\pi}{2} + 2\pi n$ where $n \in \mathbb{Z}$

   , $\alpha \neq n\pi$, $\beta \neq (2n + 1)\frac{\pi}{2}$

3. If $\tan\alpha = \cot\beta$

   , then $\alpha + \beta = 90° + 180° n$ *i.e.* $\alpha + \beta = \frac{\pi}{2} + \pi n$ where $n \in \mathbb{Z}$

   , $\alpha \neq (2n + 1)\frac{\pi}{2}$, $\beta \neq n\pi$

and the following example expresses the previous :

- If $\sin 4\theta = \cos 2\theta$, $\theta \in]0, \frac{\pi}{2}[$

$$\therefore 4\theta \pm 2\theta = \frac{\pi}{2} + 2\pi n, n \in \mathbb{Z}$$

Or

$\therefore 2\theta = \frac{\pi}{2} + 2\pi n$ $\therefore \theta = \frac{\pi}{4} + \pi n$ • At n = 0 $\therefore \theta = \frac{\pi}{4} = 45°$ • At n = 1 $\therefore \theta = \frac{\pi}{4} + \pi$ (refused)	$6\theta = \frac{\pi}{2} + 2\pi n$ $\theta = \frac{\pi}{12} + \frac{\pi}{3} n$ • At n = 0 $\therefore \theta = \frac{\pi}{12} = 15°$ • At n = 1 $\therefore \theta = \frac{\pi}{12} + \frac{\pi}{3} = 75°$ • At n = 2 $\therefore \theta = \frac{\pi}{12} + \frac{2\pi}{3}$ (refused)

$$\therefore \theta = 15°, 45° \text{ or } 75°$$

- If $\tan 3\theta = \cot 2\theta$, $\theta \in]0, \frac{\pi}{2}[$

$$\therefore 3\theta + 2\theta = \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$$

$\therefore 5\theta = \frac{\pi}{2} + \pi n$

$\therefore \theta = \frac{\pi}{10} + \frac{\pi}{5} n$

- At n = 0

$\therefore \theta = \frac{\pi}{10} = 18°$

- At n = 1

$\therefore \theta = \frac{\pi}{10} + \frac{\pi}{5} = \frac{3\pi}{10} = 54°$

- At n = 2

$\therefore \theta = \frac{\pi}{10} + \frac{2\pi}{5} = \frac{1}{2}\pi$ (refused)

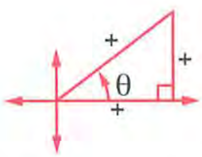
$$\therefore \theta = 18° \text{ or } 54°$$

Remember How to find the measure of an angle (θ) given the value of one of its trigonometric ratios (a)

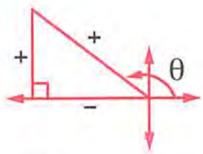
Steps \ Examples	$\sin\theta = -\frac{1}{2}$	$\cos\theta = \frac{1}{\sqrt{2}}$	$\tan\theta = -\sqrt{3}$
1 We determine the quadrant in which θ lies according to the sign of a	The sine function is negative. ∴ θ lies in the third or the fourth quadrant.	The cosine function is positive. ∴ θ lies in the first or the fourth quadrant.	The tangent function is negative. ∴ θ lies in the second or the fourth quadrant
2 We find the measure of the acute angle α whose trigonometric function = | a |	$\sin\alpha = \left\|-\frac{1}{2}\right\| = \frac{1}{2}$ $\therefore \alpha = 30°$	$\cos\alpha = \left\|\frac{1}{\sqrt{2}}\right\| = \frac{1}{\sqrt{2}}$ $\therefore \alpha = 45°$	$\tan\alpha = \left\|-\sqrt{3}\right\| = \sqrt{3}$ $\therefore \alpha = 60°$
3 We put the angle θ in the quadrant that we determined at the first step by using one of the relations : 180° – α , 180° + α or 360° – α	∵ θ lies in the third quadrant. $\therefore \theta = 180° + \alpha$ $= 180° + 30°$ $= 210°$ or θ lies in the fourth quadrant. $\therefore \theta = 360° - \alpha$ $= 360° - 30°$ $= 330°$	∵ θ lies in the first quadrant. $\therefore \theta = \alpha = 45°$ or θ lies in the fourth quadrant $\therefore \theta = 360° - \alpha$ $= 360° - 45°$ $= 315°$	∵ θ lies in the second quadrant. $\therefore \theta = 180° - \alpha$ $= 180° - 60°$ $= 120°$ or θ lies in the fourth quadrant $\therefore \theta = 360° - \alpha$ $= 360° - 60°$ $= 300°$

Remember How to find all the trigonometric functions of an angle given the value of one of its trigonometric functions

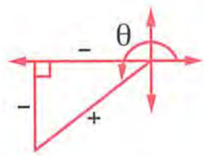
We can find the values of the trigonometric functions of an angle directly if we draw the angle in its standard position and we draw the right-angled triangle that represents it by using the value of the given trigonometric function concerning the signs according to the quadrant in which the angle lies as follows :



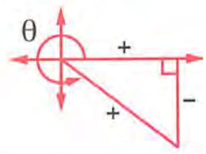
In the 1^{st} quadrant



In the 2^{nd} quadrant

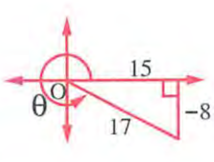
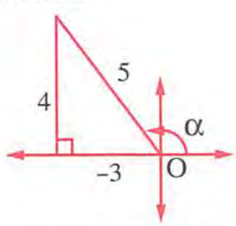
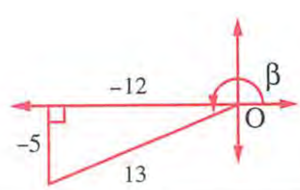


In the 3^{rd} quadrant



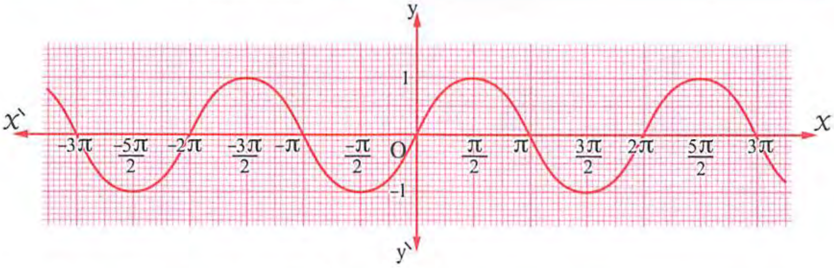
In the 4^{th} quadrant

For example :

$\sin\theta = \frac{-8}{17}$ where $270° < \theta < 360°$	$\cos\alpha = \frac{-3}{5}$ where α is the smallest positive angle.	$\tan\beta = \frac{5}{12}$ where β is the greatest positive angle , $0° < \beta < 360°$
$\because 270° < \theta < 360°$ $\therefore \theta$ lies in the fourth quadrant. 	$\because \cos\alpha$ is negative $\therefore \alpha$ lies in the second or the third quadrant , $\because \alpha$ is the smallest positive angle. $\therefore \alpha$ lies in the second quadrant. 	$\because \tan\beta$ is positive $\therefore \beta$ lies in the first or the third quadrant , $\because \beta$ is the greatest positive angle. $\therefore \beta$ lies in the third quadrant 
$\therefore \cos\theta = \frac{15}{17}$, $\tan\theta = \frac{-8}{15}$, ...	$\therefore \sin\alpha = \frac{4}{5}$, $\tan\alpha = \frac{-4}{3}$, ...	$\therefore \sin\beta = \frac{-5}{13}$, $\cos\beta = \frac{-12}{13}$, ...

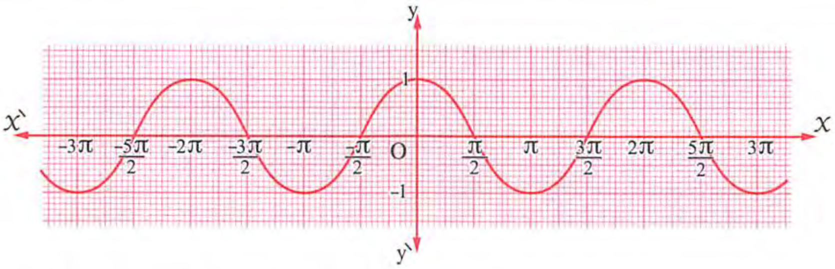
Remember The properties of the sine function and the cosine function

Properties of the sine function $f : f(\theta) = \sin\theta$



1. The domain of the sine function is $]-\infty , \infty[$
2. • The maximum value of the function is 1 and it happens when $\theta = \frac{\pi}{2} + 2n\pi , n \in \mathbb{Z}$
   • The minimum value of the function is – 1 and it happens when $\theta = \frac{3\pi}{2} + 2n\pi , n \in \mathbb{Z}$
3. The range of the function = $[-1 , 1]$
4. The function is periodic and its period is 2π (360°)

Properties of the cosine function $f : f(\theta) = \cos\theta$



1. The domain of the cosine function is $]-\infty , \infty[$
2. • The maximum value of the function is 1 and it happens when $\theta = \pm 2n\pi , n \in \mathbb{Z}$
   • The minimum value of the function is – 1 and it happens when $\theta = \pi \pm 2\pi n , n \in \mathbb{Z}$
3. The range of the function = $[-1 , 1]$
4. The function is periodic and its period is 2π (360°)

Remark

Each of the two functions $f : f(\theta) = a \sin b\theta$, $f : f(\theta) = a \cos b\theta$ is periodic , its period is $\frac{2\pi}{|b|}$ and its range is $[-a , a]$ where a is positive.

For example : • $f : f(\theta) = 5 \sin\theta$ its period is 2π and its range is $[-5 , 5]$

• $f : f(\theta) = 3 \cos 7\theta$ its period is $\frac{2\pi}{7}$ and its range is $[-3 , 3]$

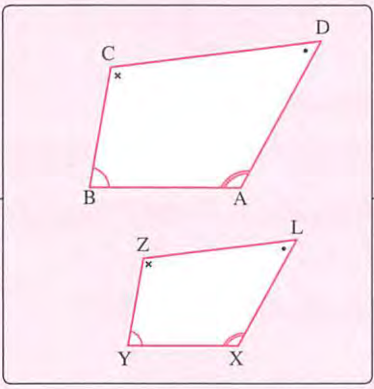
THIRD Final revision on geometry

Remember The similarity of polygons

Two polygons M_1 and M_2 (having the same number of sides) are said to be similar if the following two conditions satisfied together :

1. Their corresponding angles are congruent.

i.e. $m(\angle A) = m(\angle X)$, $m(\angle B) = m(\angle Y)$, $m(\angle C) = m(\angle Z)$, $m(\angle D) = m(\angle L)$



2. The lengths of their corresponding sides are proportional.

i.e. $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX} = K$

In this case , we say that :

- The polygon ABCD $\sim$ the polygon XYZL ,
  that means the polygon ABCD is similar to the polygon XYZL
- K is the scale factor of similarity of the polygon ABCD to the polygon XYZL
- $\frac{1}{K}$ is the scale factor of similarity of the polygon XYZL to the polygon ABCD

Remarks

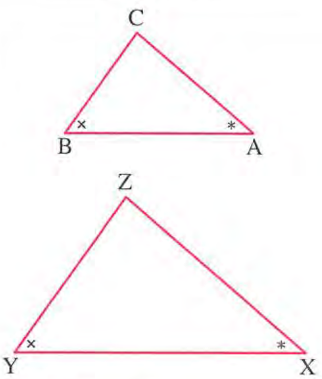
- On writing the similar polygons , write them according to the order of their corresponding vertices.
- If each one of two polygons is similar to a third polygon , then the two polygons are similar.
- All regular polygons which have the same number of sides are similar
  (All equilateral triangles are similar , all squares are similar , all regular pentagons are similar , ...)
- If K is the similarity ratio of polygon M_1 to polygon M_2 , and :
  If $K > 1$, then polygon M_1 is an enlargement of polygon M_2 , where K is called the enlargement ratio.
  If $0 < K < 1$, then polygon M_1 is a shrinking to polygon M_2 , where K is called the shrinking ratio.
  If $K = 1$, then polygon M_1 is congruent to polygon M_2
- The ratio between the perimeters of two similar polygons = the ratio between the lengths of two corresponding sides of them.

Remember The similarity of triangles

Two triangles are similar

First case

If two angles of one triangle are congruent to their corresponding angles of the other triangle.



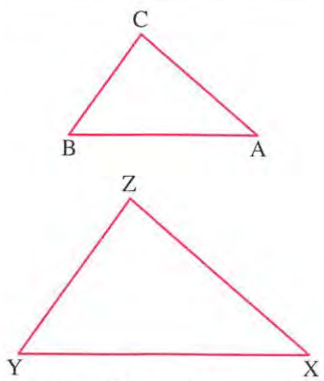
If $\angle A \equiv \angle X$

, $\angle B \equiv \angle Y$

, then $\Delta ABC \sim \Delta XYZ$

Second case

If the side lengths of two triangles are in proportion.

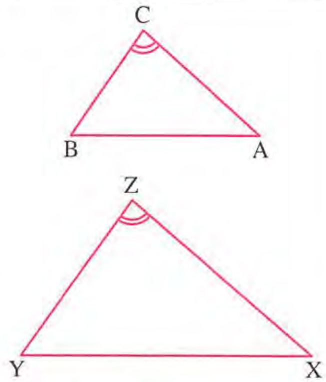


If $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX}$

, then $\Delta ABC \sim \Delta XYZ$

Third case

If an angle of one triangle is congruent to an angle of the other triangle and the lengths of the sides including those angles are in proportion.



If $\angle C \equiv \angle Z$

, $\frac{CA}{ZX} = \frac{CB}{ZY}$

, then $\Delta ABC \sim \Delta XYZ$

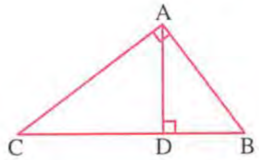
Remarks

- Two isosceles triangles are similar if the measure of an angle in one of them is equal to the measure of the corresponding angle in the other triangle.
- Two right-angled triangles are similar if the measure of an acute angle in one of them is equal to the measure of an acute angle in the other triangle.

Corollary

In any right-angled triangle , the altitude to the hypotenuse separates the triangle into two triangles which are similar to each other and to the original triangle.

In the opposite figure :



If Δ ABC is a right-angled triangle at A and $\overline{AD} \perp \overline{BC}$

, then Δ DBA $\sim$ Δ DAC $\sim$ Δ ABC and from this we can deduce that :

- $(AB)^2 = BD \times BC$
- $(AC)^2 = CD \times CB$
- $(AD)^2 = BD \times DC$
- $AD \times BC = AB \times AC$

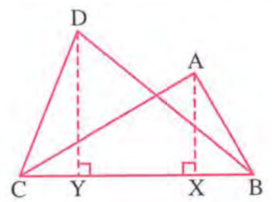
Remember The relation between the areas of two similar polygons

The ratio between the areas of the surfaces of two similar triangles equals the square of the ratio between the lengths of any two corresponding sides of the two triangles.

The ratio between the areas of the surfaces of two similar polygons equals the square of the ratio between the lengths of any two corresponding sides of the two polygons.

The ratio of the areas of two triangles having a common base equals the ratio of the two heights of the two triangles.

In the opposite figure :



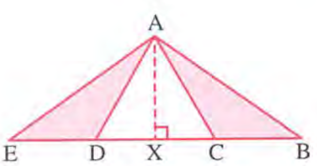
$\overline{BC}$ is a common base of $\Delta\Delta$ ABC , DBC

$$\therefore \frac{a(\Delta ABC)}{a(\Delta DBC)} = \frac{\frac{1}{2} BC \times AX}{\frac{1}{2} BC \times DY} = \frac{AX}{DY}$$

Notice that : It is not necessary that the two triangles are similar.

The ratio of the areas of two triangles having a common height equals the ratio of the lengths of two bases of the two triangles.

In the opposite figure :



AX is a common height for $\Delta\Delta$ ABC , ADE

$$\therefore \frac{a(\Delta ABC)}{a(\Delta ADE)} = \frac{\frac{1}{2} BC \times AX}{\frac{1}{2} DE \times AX} = \frac{BC}{DE}$$

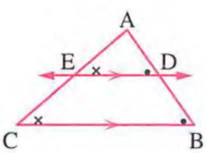
Notice that : It is not necessary that the two triangles are similar.

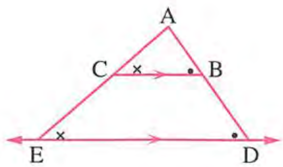
If a line is drawn parallel to one side of a triangle and intersects the other two sides or the lines containing them , then :

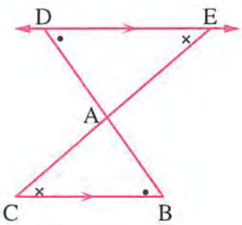
The resulting triangle is similar to the original triangle

It divides them into segments whose lengths are proportional

In each of the following figures :







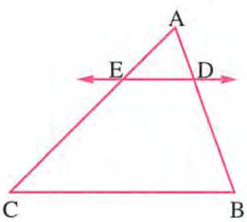
If $\overleftrightarrow{DE} \parallel \overline{BC}$ and intersects $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ at D and E respectively , then :

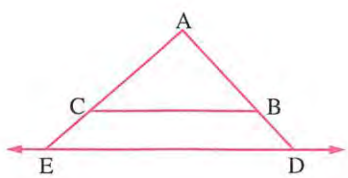
- $\Delta ADE \sim \Delta ABC$
- $\frac{AD}{DB} = \frac{AE}{EC}$ and from the properties of the proportion , we get :

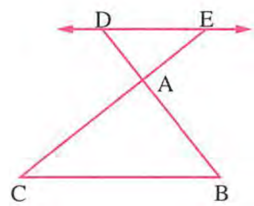
$\frac{AD}{AB} = \frac{AE}{AC}$, $\frac{AB}{DB} = \frac{AC}{CE}$

If a straight line intersects two sides of a triangle and divides them into segments whose lengths are proportional , then it is parallel to the third side of the triangle.

In each of the following figures :



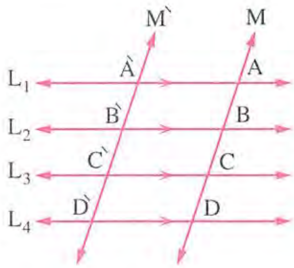


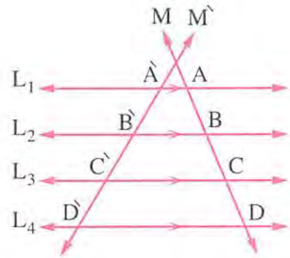


If $\frac{AD}{DB} = \frac{AE}{EC}$, then $\overleftrightarrow{DE} \parallel \overline{BC}$

Remember Talis' theorem

Given several coplanar parallel lines and two transversals , then the lengths of the corresponding segments on the transversals are proportional.





In the previous figures :

If $L_1 // L_2 // L_3 // L_4$ and M , M' are two transversals

, then $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{AC}{A'C'}$

Remember Talis' special theorem

If the lengths of the segments on the transversal are equal , then the lengths of the segments on any other transversal will be also equal.

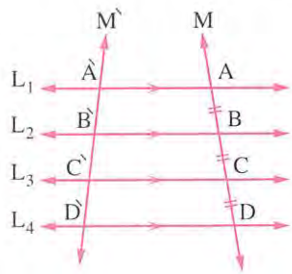
In the opposite figure :

If $L_1 // L_2 // L_3 // L_4$,

M , M' are two transversals to them

and if $AB = BC = CD$

, then $A'B' = B'C' = C'D'$

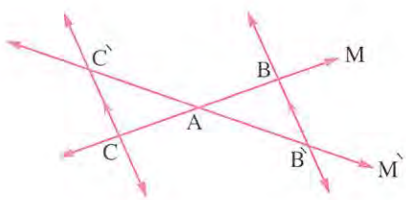


Special case

If the two lines M and M' intersect at the point A and $\overleftrightarrow{BB'} // \overleftrightarrow{CC'}$

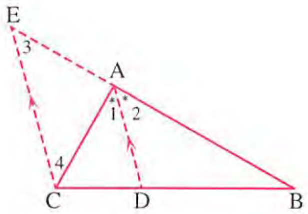
, then $\frac{AB}{AC} = \frac{AB'}{AC'}$

and conversely if $\frac{AB}{AC} = \frac{AB'}{AC'}$, then $\overleftrightarrow{BB'} // \overleftrightarrow{CC'}$



Theorem

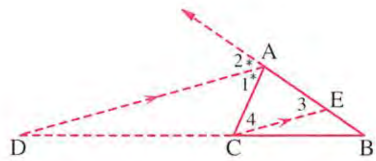
The bisector of the interior or exterior angle of a triangle at any vertex divides the opposite base of the triangle internally or externally into two parts , the ratio of their lengths is equal to the ratio of the lengths of the other two sides of the triangle.



$\because \overrightarrow{AD}$ bisects $\angle$ BAC internally.

$\therefore \boxed{\frac{BD}{DC} = \frac{AB}{AC}}$

, $\boxed{AD = \sqrt{AB \times AC - BD \times DC}}$

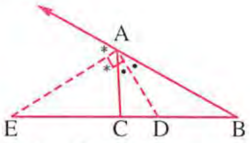


$\because \overrightarrow{AD}$ bisects $\angle$ BAC externally.

$\therefore \boxed{\frac{BD}{DC} = \frac{AB}{AC}}$

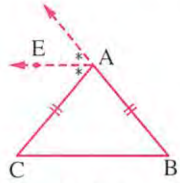
, $\boxed{AD = \sqrt{BD \times DC - AB \times AC}}$

The interior and exterior bisectors of the same angle of the triangle are perpendicular.



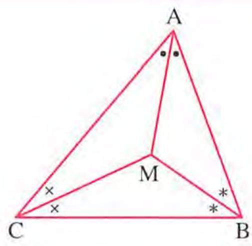
i.e. If $\overrightarrow{AD}$ and $\overrightarrow{AE}$ are the bisectors of the angle A and the exterior angle of Δ ABC at A , then $\boxed{\overline{AD} \perp \overline{AE}}$

The exterior bisector of the vertex angle of an isosceles triangle is parallel to the base.

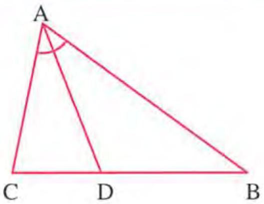


i.e. If AB = AC , $\overrightarrow{AE}$ bisects the exterior angle at A , then $\boxed{\overrightarrow{AE} \,//\, \overline{BC}}$

The bisectors of angles of a triangle are concurrent.



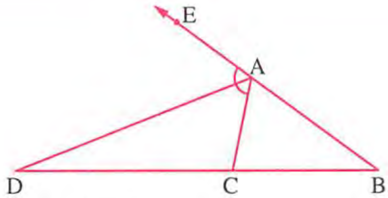
Converse of the theorem



If $D \in \overline{BC}$

such that : $\frac{BD}{DC} = \frac{BA}{AC}$

, then $\overrightarrow{AD}$ bisects $\angle$ BAC



If $D \in \overrightarrow{BC}$, $D \notin \overline{BC}$

such that : $\frac{BD}{DC} = \frac{BA}{AC}$

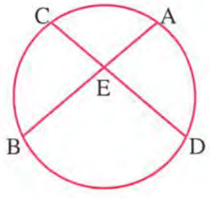
, then $\overrightarrow{AD}$ bisects the exterior angle of Δ ABC at A

Well known problem and a corollary on it

Well known problem

If $\overline{AB}$, $\overline{CD}$ are two chords in a circle

, $\overline{AB} \cap \overline{CD} = \{E\}$

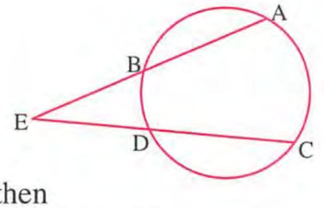


then

EA × EB = EC × ED

If $\overline{AB}$ and $\overline{CD}$ are two chords in a circle

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

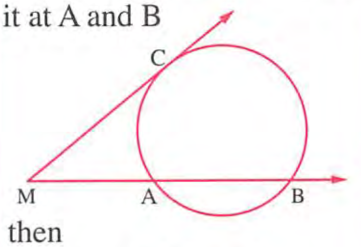


then

EA × EB = EC × ED

Corollary

If M is a point outside the circle , $\overrightarrow{MC}$ touches the circle at C , $\overrightarrow{MB}$ intersects it at A and B



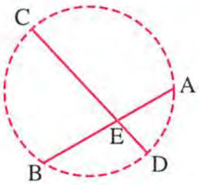
then

$(MC)^2$ = MA × MB

Converse of the well known problem and the corollary

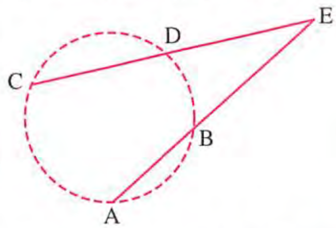
Converse of the well known problem

If $\overline{AB} \cap \overline{CD} = \{E\}$,
A , B , C , D and E are distinct points and
$EA \times EB = EC \times ED$



, then the points A , B , C and D lie on the same circle.

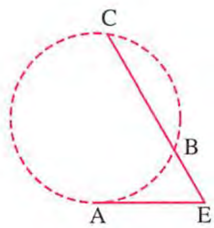
If $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$,
A , B , C , D and E are distinct points and
$EA \times EB = EC \times ED$



, then the points A , B , C and D lie on the same circle.

Converse of the corollary

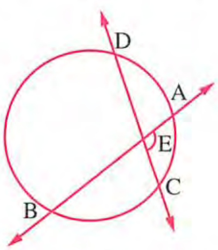
If $E \in \overrightarrow{CB}$, $E \notin \overline{BC}$,
and $(EA)^2 = EB \times EC$



, then $\overline{EA}$ is a tangent segment to the circle which passes through the points A , B and C

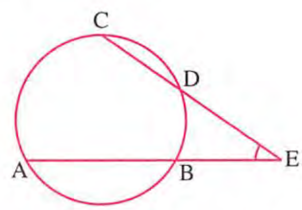
Secant , tangent and measures of angles

1 The measure of an angle formed by two chords that intersect inside a circle is equal to half the sum of the measures of the intercepted arcs.



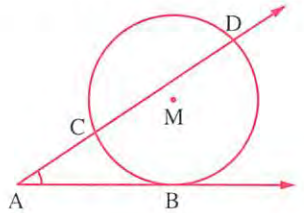
$$m(\angle AEC) = \frac{1}{2}\left[m(\overset{\frown}{AC}) + m(\overset{\frown}{BD})\right]$$

2 The measure of an angle formed by two secants drawn from a point outside a circle is equal to half the positive difference of the measures of the intercepted arcs.



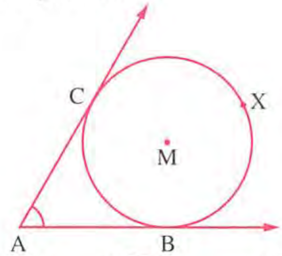
$$m(\angle E) = \frac{1}{2}\left[m(\overset{\frown}{AC}) - m(\overset{\frown}{BD})\right]$$

3 The measure of an angle formed by a secant and a tangent drawn from a point outside a circle is equal to half the positive difference of the measures of the intercepted arcs.



$m(\angle A) = \frac{1}{2}[m(\overset{\frown}{BD}) - m(\overset{\frown}{BC})]$

4 The measure of an angle formed by two tangents drawn from a point outside a circle is equal to half the positive difference of the measures of the intercepted arcs.



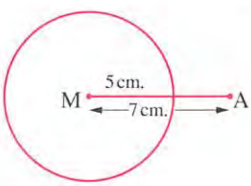
$m(\angle A) = \frac{1}{2}[m(\overset{\frown}{BXC}) - m(\overset{\frown}{BC})]$

Power of a point with respect to a circle

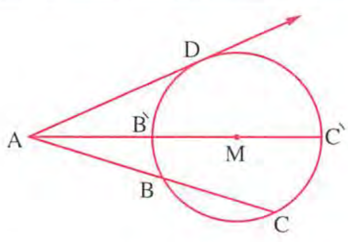
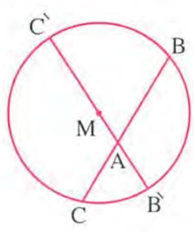
Power of the point A with respect to the circle M in which , the length of its radius r is the real number $P_M(A)$ where $P_M(A) = (AM)^2 - r^2$

For example : **In the opposite figure :**

If A is a point outside the circle M whose radius length equals 5 cm. , where MA = 7 cm. , then $P_M(A) = 7^2 - 5^2 = 24$



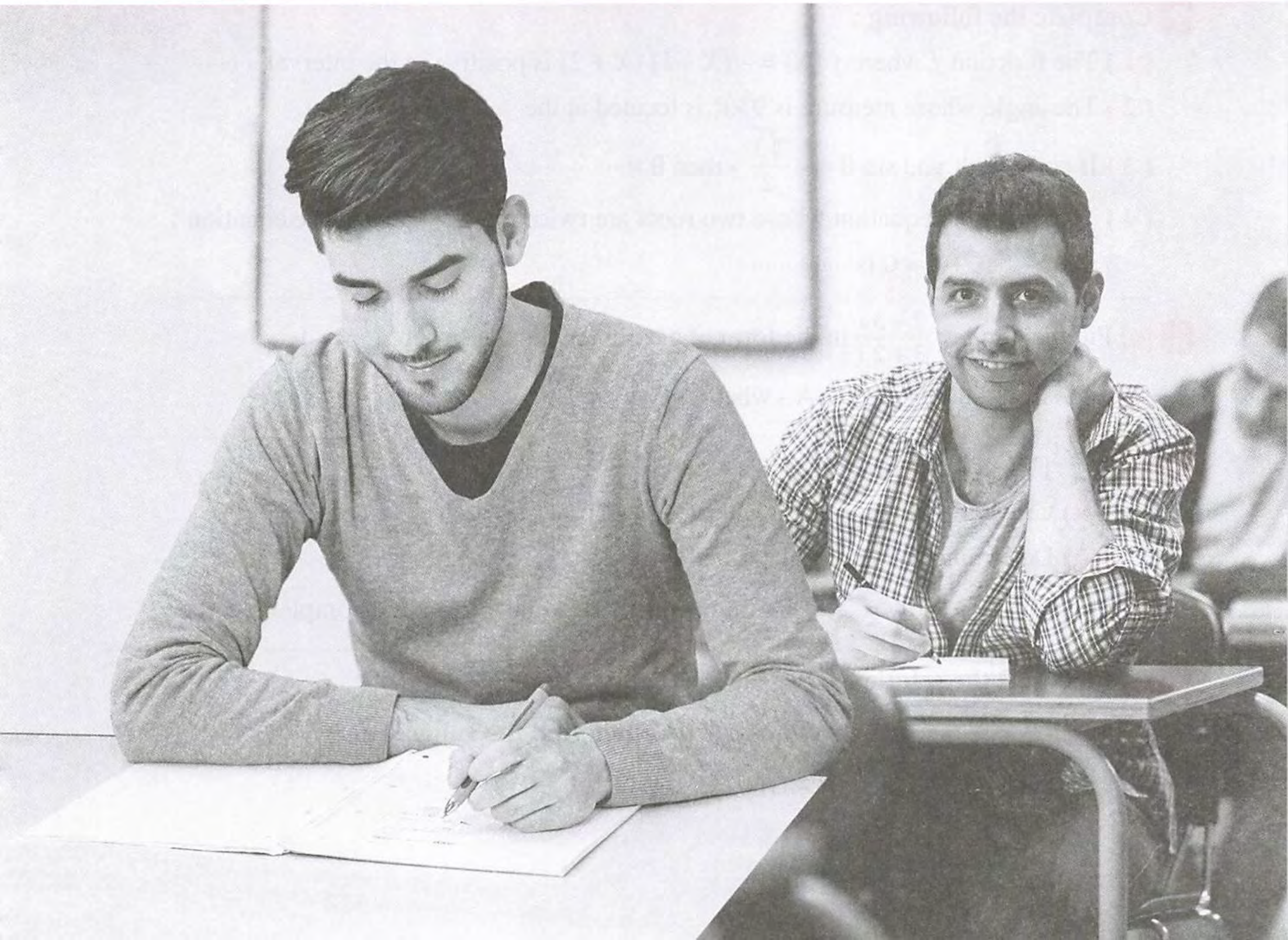
If
- $P_M(A) > 0$, then ⟶ A lies outside the circle M
- $P_M(A) = 0$, then ⟶ A lies on the circle M
- $P_M(A) < 0$, then ⟶ A lies inside the circle M

If A lies outside the circle M , then	If A lies inside the circle M , then
 $P_M(A) = AB \times AC = AB` \times AC` = (AD)^2$	 $P_M(A) = - AB \times AC = - AB` \times AC`$

School book examinations

FIRST | School book examinations in algebra and trigonometry.

SECOND | School book examinations in geometry.



Model 1

1 Choose the correct answer from the given ones :

(1) If L and M are the two roots of the equation : $x^2 - 7x + 3 = 0$, then $L^2 + M^2 =$ ....

(a) 7 (b) 3 (c) 43 (d) 79

(2) If $\sin\theta = -1$ and $\cos\theta =$ zero , then $\theta =$ ....

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π

(3) The quadratic equation whose roots are $2 - 3i$, $2 + 3i$ is ....

(a) $x^2 + 4x + 13 = 0$ (b) $x^2 - 4x + 13 = 0$

(c) $x^2 + 4x - 13 = 0$ (d) $x^2 - 4x - 13 = 0$

(4) If one of the two roots of the equation : $x^2 - (m + 2)x + 3 = 0$ is the additive inverse of the other root , then m = ...

(a) 3 (b) 2 (c) -2 (d) -3

---

2 Complete the following :

(1) The function f where $f(x) = -(x - 1)(x + 2)$ is positive in the interval ....

(2) The angle whose measure is 930° is located at the .... quadrant.

(3) If $\cos\theta = \frac{1}{2}$ and $\sin\theta = -\frac{\sqrt{3}}{2}$, then $\theta =$ ....°

(4) The quadratic equation whose two roots are twice the two roots of the equation : $2x^2 - 8x + 5 = 0$ is ....

---

3 [a] Put the number $\frac{2 - 3i}{3 + 2i}$ in the form of a complex number where $i^2 = -1$

[b] If $4\sin A - 3 = 0$, **find :** A , where $A \in]0 , \frac{\pi}{2}[$

---

4 [a] If $f : \mathbb{R} \longrightarrow \mathbb{R}$ where $f(x) = -x^2 + 8x - 15$

(1) Graph the function in the interval $[1 , 7]$

(2) Determine the sign of the function.

[b] If $x = 3 + 2i$ and $y = \frac{4 - 2i}{1 - i}$, **then find :** $x + y$ in the form of a complex number.

---

5 [a] **Find in $\mathbb{R}$ the solution set of the inequality :** $x^2 + 3x - 4 \leq 0$

[b] If $\tan B = \frac{3}{4}$, where $180° < B < 270°$, **then find the value of :**

$\cos(360° - B) - \cos(90° - B)$

Model 2

1 Complete the following :

(1) The simplest form of the imaginary number i^{43} is ....

(2) If the two roots of the equation : $x^2 - 6x + L = 0$ are real and equal , then L = ...

(3) If $0° < \theta < 90°$ and $\sin 2\theta = \cos 3\theta$, then θ = ...

(4) The range of the function f where $f(\theta) = \frac{3}{2} \sin \theta$ is ....

2 Choose the correct answer :

(1) The equation : $x^2 (x - 1)(x + 1) = 0$ is a .... degree equation.

(a) first (b) second (c) third (d) fourth

(2) If the two roots of the equation : $x^2 + 3x - m = 0$ are real different , then m = ...

(a) -2 (b) -3 (c) -4 (d) -5

(3) If the sum of measures of the angles of a regular polygon equals $180° (n - 2)$ where n is the number of sides , then the measure of the angle of a regular octagon by the radian measure equals ....

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{2\pi}{3}$

(4) If $2 \cos \theta = -\sqrt{3}$ and $\pi < \theta < \frac{3\pi}{2}$, then θ = ...

(a) $\frac{\pi}{3}$ (b) $\frac{6\pi}{7}$ (c) $\frac{4\pi}{3}$ (d) $\frac{7\pi}{6}$

3 [a] **Find the value of k which makes one root of the two roots of the equation :** $4kx^2 + 7x + k^2 + 4 = 0$ be the multiplicative inverse of the other root.

[b] If $\sin \theta = \sin 750° \cos 300° + \sin (-60°) \cot 120°$ where $0° < \theta < 360°$, **find :** θ

4 [a] (1) **Find the two values of a , b which satisfy the equation :** $12 + 3ai = 4b - 27i$

(2) **Find the solution set of the inequality :** $x(x + 1) - 2 \le 0$ in $\mathbb{R}$

[b] A central angle of measure θ is inscribed in a circle of radius length 18 cm. and subtends an arc of length 26 cm. Find θ in degree measure.

5 [a] If the sum of the consecutive integers $(1 + 2 + 3 + \cdots + n)$, where n is the number of integers is given by the relation $S = \frac{n}{2}(1 + n)$, how many consecutive integers starting from number 1 to be summed 210 are there ?

[b] If $\sin x = \frac{4}{5}$ where $90° < x < 180°$

, **find :** $\sin (180° - x) + \tan (360° - x) + 2 \sin (270° - x)$

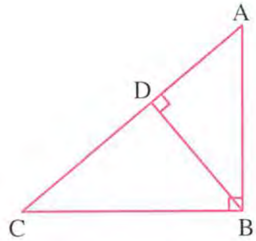
SECOND School book examinations in geometry

Model 1

1 Complete the following :

(1) The two polygons that are similar to a third are ....

(2) **In the opposite figure :**



First : $(AB)^2 = AD \times$ .... and $(CB)^2 = CA \times$ ....

Second : $DA \times DC =$ ....

Third : $AB \times BC =$ .... $\times$ ....

2 Choose the correct answer from the given ones :

(1) Two similar rectangles , the length of the first is 5 cm. and the length of the second is 10 cm., then the ratio between the perimeter of the first to the perimeter of the second equals ....

(a) 1 : 5 (b) 1 : 3 (c) 1 : 2 (d) 2 : 1

(2) Which two triangles of the following are similar ?

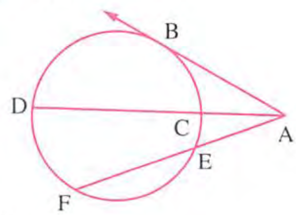
(1) (2) (3) (4)

(a) (3) , (4) (b) (1) , (3) (c) (2) , (4) (d) (1) , (4)

(3) If the ratio between the perimeters of two similar triangles is 1 : 4 , then the ratio between their two surface areas equals ....

(a) 1 : 2 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16

(4) **In the opposite figure :**

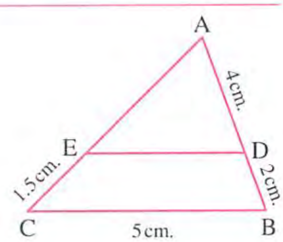


All the following mathematical expressions are correct except the expression ....

(a) $(AB)^2 = AC \times AD$ (b) $(AB)^2 = AE \times AF$

(c) $AC \times AD = AE \times AF$ (d) $AC \times CD = AE \times EF$

3 [a] **In the opposite figure :**



$\Delta ADE \sim \Delta ABC$ **Prove that :** $\overline{DE} \parallel \overline{BC}$

If AD = 4 cm. , DB = 2 cm. , EC = 1.5 cm.

, BC = 5 cm. , **find the lengths of :** $\overline{AE}$ and $\overline{DE}$

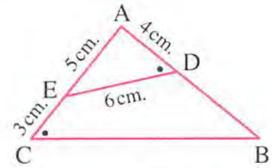
[b] ABC is a triangle , $D \in \overline{BC}$ where BD = 5 cm.

, DC = 3 cm. and $E \in \overline{AC}$ where AE = 2 cm. , CE = 4 cm.

Prove that : Δ DEC $\sim \Delta$ ABC , then find the ratio between their two surface areas.

---

4 **[a] In the opposite figure :**

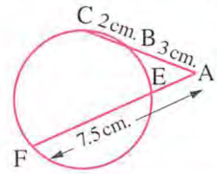


m ($\angle$ ADE) = m ($\angle$ C)

, AD = 4 cm. , AE = 5 cm. , DE = 6 cm. and EC = 3 cm.

Find the lengths of : $\overline{DB}$ and $\overline{BC}$

[b] In the opposite figure :



$\overrightarrow{CB} \cap \overrightarrow{FE} = \{A\}$, AB = 3 cm. , BC = 2 cm. , AF = 7.5 cm.

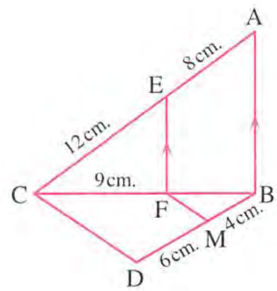
Find the length of : $\overline{EF}$

---

5 **[a]** $\overline{AD}$ is a median in the triangle ABC , $\angle$ ADB is bisected by a bisector to cut $\overline{AB}$ at E , $\angle$ ADC is bisected by a bisector to cut $\overline{AC}$ at F and $\overline{EF}$ is drawn.

Prove that : $\overline{EF} \parallel \overline{BC}$

[b] In the opposite figure :



$\overline{AB} \parallel \overline{EF}$, AE = 8 cm.

, CE = 12 cm. , CF = 9 cm.

, BM = 4 cm. and DM = 6 cm.

(1) Find the length of : $\overline{BF}$

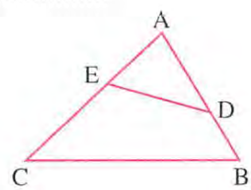
(2) Prove that : $\overline{FM} \parallel \overline{CD}$

Model 2

1 **Complete the following :**

(1) Any two regular polygons that have the same number of sides are ....

(2) In the opposite figure :

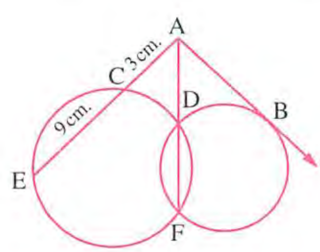


If Δ ADE $\sim \Delta$ ACB

, then m ($\angle$ ADE) = m ($\angle$ ....)

(3) If the two straight lines including the two chords $\overline{DE}$, $\overline{XY}$ intersect at the point N , then ND $\times$ NE = ...

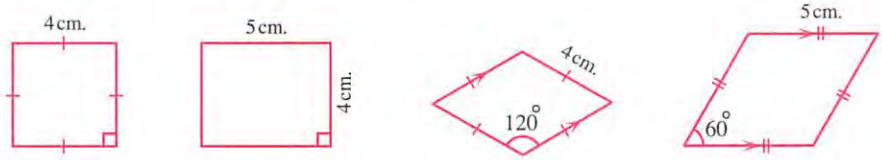
(4) In the opposite figure :



If AC = 3 cm. and CE = 9 cm. , then AB = ...

2 Choose the correct answer from the given ones :

(1) Which two polygons of the following are similar ?



(1) (2) (3) (4)

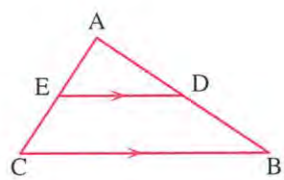
(a) Polygons (1) , (2) (b) Polygons (1) , (3)

(c) Polygons (3) , (4) (d) Polygons (2) , (4)

(2) If the ratio between the surface areas of two similar polygons is 16 : 25 , then the ratio between the lengths of two corresponding sides in the two polygons equals ....

(a) 2 : 5 (b) 4 : 5 (c) 16 : 25 (d) 16 : 41

(3) **In the opposite figure :**

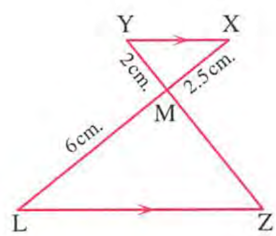


All the following mathematical expressions are correct except ....

(a) $\frac{AD}{DB} = \frac{AE}{EC}$ (b) $\frac{AD}{DB} = \frac{DE}{BC}$

(c) $\frac{AD}{AB} = \frac{AE}{AC}$ (d) $\frac{AB}{BD} = \frac{AC}{EC}$

(4) **In the opposite figure :**



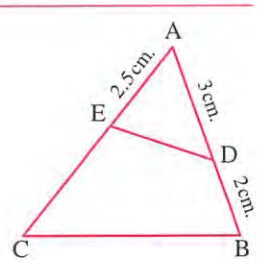
The length of $\overline{MZ}$ equals ....

(a) 3.6 cm. (b) 4 cm.

(c) 4.2 cm. (d) 4.8 cm.

---

3 [a] In the opposite figure :



$\Delta ABC \sim \Delta AED$

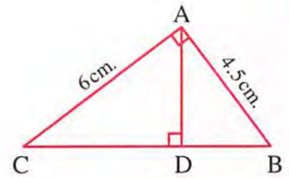
Prove that :

BCED is a cyclic quadrilateral. If AD = 3 cm. , BD = 2 cm. and AE = 2.5 cm. , **find the length of :** $\overline{EC}$

[b] ABCD is a cyclic quadrilateral whose two diagonals intersected at E , $\overrightarrow{EF}$ is drawn parallel to $\overline{CB}$ to intersect $\overline{AB}$ at F , $\overrightarrow{EM}$ is drawn parallel to $\overline{CD}$ to intersect $\overline{AD}$ at M

Prove that : $\overline{FM} \parallel \overline{BD}$

4 [a] **In the opposite figure :**



m ($\angle$ BAC) = 90° , $\overline{AD} \perp \overline{BC}$

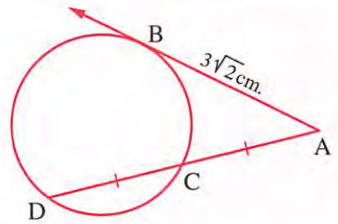
, AB = 4.5 cm. and AC = 6 cm.

Find the length of each of : $\overline{BD}$, $\overline{DC}$ and $\overline{AD}$

[b] ABCD is a cyclic quadrilateral in which : BC = 27 cm. , AB = 12 cm. , AD = 8 cm. , DC = 12 cm. and AC = 18 cm. **Prove that :** Δ BAC ~ Δ ADC and find the ratio between their two surface areas.

---

5 [a] **In the opposite figure :**



$\overrightarrow{AB}$ is a tangent to a circle , C is the midpoint of $\overline{AD}$ and AB = $3\sqrt{2}$ cm.

Find the length of : $\overline{AC}$

[b] ABC is a triangle in which : AB = 8 cm. , AC = 12 cm. , BC = 15 cm. , $\overrightarrow{AD}$ bisects $\angle$ A and intersects $\overline{BC}$ at D , $\overrightarrow{DE}$ // $\overline{BA}$ is drawn to intersect $\overline{AC}$ at E

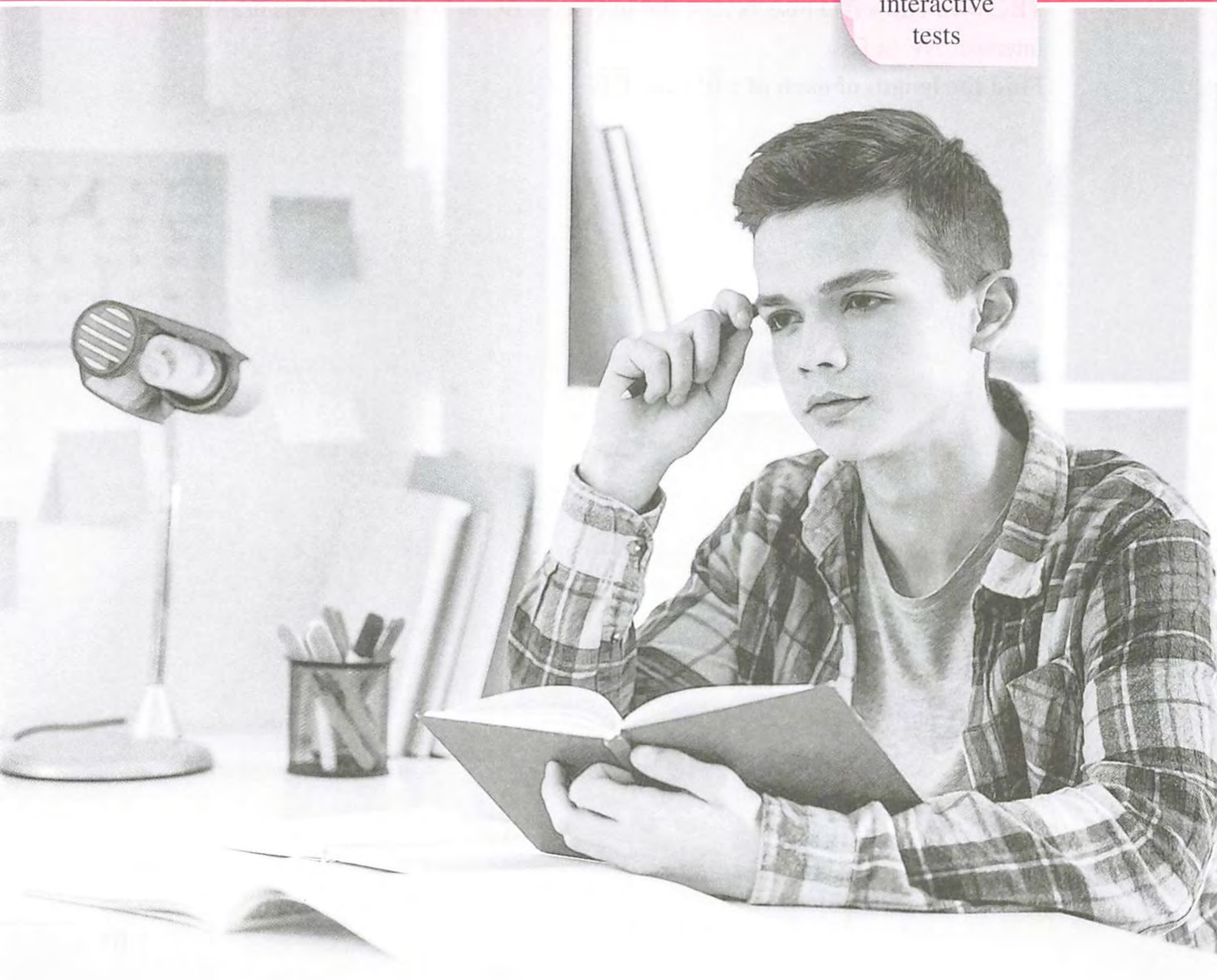
Find the length of each of : $\overline{BD}$ and $\overline{CE}$

Final examinations

FIRST | Examinations of some governorate's schools.

SECOND | Final models.

Scan the QR codes to solve interactive tests



FIRST

Examinations of some governorate's schools

1 Cairo Governorate

El-Sherouk Zone
El-Golf Distinguished Governmental School

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If $(1 + i^4)(1 - i^7) = X + y\,i$, then $X + y =$ ....

(a) -2 (b) 2 (c) 4 (d) 6

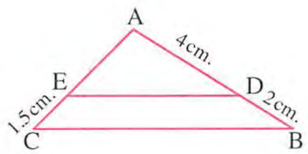
(2) If $\frac{2}{L}$ and $\frac{2}{M}$ are the roots of : $X^2 - 8X + 4 = 0$, then LM = ...

(a) -8 (b) -4 (c) 1 (d) 4

(3) If Δ ADE ~ Δ ABC

, AD = 4 cm. , AB = 6 cm.

and CE = 1.5 cm. , then AE = ... cm.



(a) 3 (b) 5 (c) 6 (b) 7

(4) If L and L^2 are the roots of : $X^2 - bX + 8 = 0$, then b = ...

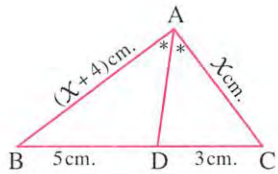
(a) 2 (b) 4 (c) 6 (d) 8

(5) In Δ ABC , $\overrightarrow{AD}$ bisects $\angle$ CAB

, AB = $(X + 4)$ cm. , AC = X cm.

and CD = 3 cm. , DB = 5 cm.

, then $X =$ ....



(a) 4 (b) 6 (c) 8 (d) 10

(6) If $\overrightarrow{AB}$ is tangent to the circle M at the point B and $P_M(A) = 25$ cm^2

, then AB = ... cm.

(a) 5 (b) 16 (c) 20 (d) 25

(7) The range of the function $f : f(X) = 4 \sin 3X$ is ....

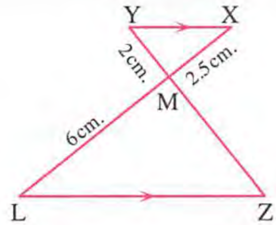
(a) $]-4 , 4[$ (b) $[-4 , 4]$ (c) $\mathbb{R} -]-3 , 4[$ (d) $[-3 , 3]$

(8) If L and M are the roots of the equation : $X^2 + 3X + 3 = 0$, then the equation whose roots are LM and L + M is ....

(a) $X^2 + 9 = 0$ (b) $X^2 = 9$

(c) $X^2 - 3 = 0$ (d) $X^2 + 9X = 0$

(9) In the opposite figure :



If $\overline{LZ} \parallel \overline{YX}$, $\overline{YZ} \cap \overline{XL} = \{M\}$, XM = 2.5 cm. , YM = 2 cm.

, LM = 6 cm. , then MZ = ... cm.

(a) 2.7 (b) 3.6

(c) 4.8 (d) 7.5

(10) The solution set of : $4 - x^2 \geq 0$ is ....

(a) $[-2 , 2]$ (b) $[4 , \infty[$

(c) $\mathbb{R} -]-2 , 2[$ (d) $\mathbb{R} - [-2 , 2]$

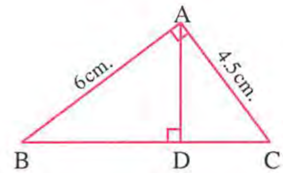
(11) The ratio between the lengths of two corresponding sides of two simillar polygons is 5 : 4 and the difference between thier areas is 27 cm^2 , then the area of the smaller polygon is .... cm^2

(a) 3 (b) 9 (c) 16 (d) 48

(12) $\sin(180° - \theta) \times \sec(270° + \theta) =$ ....

(a) $\tan\theta$ (b) $\csc\theta$ (c) 1 (d) -1

(13) In the opposite figure :

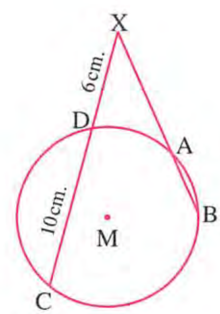


Δ ABC in which m ($\angle$ A) = 90° , $\overline{AD} \perp \overline{BC}$, AB = 6 cm.

and AC = 4.5 cm. , then AD = ... cm.

(a) 2.7 (b) 3.6 (c) 4.8 (d) 7.5

(14) In the opposite figure :



M is a circle where $\overrightarrow{BA} \cap \overrightarrow{CD} = \{X\}$

, if XA = 2 AB , XD = 6 cm. and CD = 10 cm.

, then XB = ... cm.

(a) 4 (b) 8

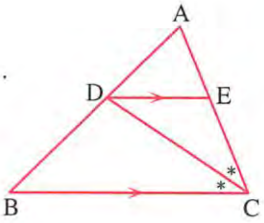
(c) 12 (d) 16

(15) The angle with measure 495° in standard position is equivalent to angle with measure ....

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{4}$ (d) $\frac{7\pi}{4}$

(16) In the opposite figure :

Δ ABC in which $\overline{BC} \parallel \overline{DE}$, $\overrightarrow{CD}$ bisects $\angle$ ACB , then $\frac{AE}{EC}$ = ...



(a) $\frac{AD}{AB}$ (b) $\frac{AD}{AE}$

(c) $\frac{AC}{CB}$ (d) $\frac{DE}{BC}$

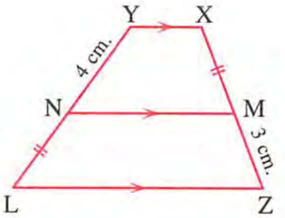
(17) The terminal side of angle θ in the standard position intersects the unit circle at the point $\left(\frac{\sqrt{5}}{3} , \frac{-2}{3}\right)$, then $\cos\left(\frac{\pi}{2} + \theta\right) + \sin(2\pi - \theta)$ = ...

(a) 0 (b) $\frac{4}{3}$ (c) $\frac{-4}{3}$ (d) $\frac{5}{3}$

(18) In the opposite figure :

If $\overline{LZ} \parallel \overline{YX} \parallel \overline{MN}$, XM = NL

, then MX = ... cm.



(a) 3 (b) $2\sqrt{3}$

(c) $3\sqrt{2}$ (d) 12

(19) The simplest form of i^{2022} = ...

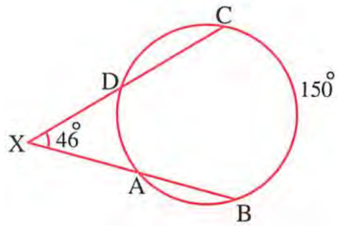
(a) – i (b) – 1 (c) i (d) 1

(20) In the opposite figure :

A circle in which $\overrightarrow{BA} \cap \overrightarrow{CD} = \{X\}$

, if m ($\angle$ X) = 46° and m $(\widehat{BC})$ = 150°

, then m $(\widehat{AD})$ = ...



(a) 58° (b) 92° (c) 103° (d) 196°

(21) The function $f : f(X) = (X - 1)(X + 4)$ is positive at $X \in$ ....

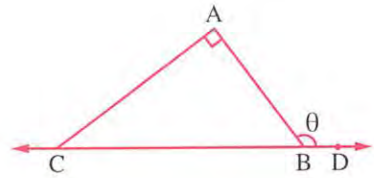
(a) $]-1 , 4[$ (b) $]-4 , 1[$

(c) $\mathbb{R} -]-4 , 1[$ (d) $\mathbb{R} - [-4 , 1]$

(22) If the two roots of the equation : $X^2 + 4X + k = 0$ are real different , then k = ...

(a) $]-\infty , 4[$ (b) $]4 , \infty[$ (c) $]-\infty , 4]$ (d) $\{4\}$

(23) In the opposite figure :



ABC is right-angled triangle at A , D $\in \overleftrightarrow{BC}$

if AB = 12 cm. , AC = 16 cm. , then tan θ = ...

(a) $\frac{3}{4}$ (b) $\frac{-3}{4}$

(c) $\frac{4}{3}$ (d) $\frac{-4}{3}$

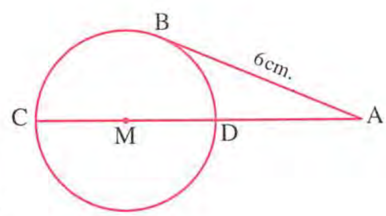
(24) If the two roots of the equation : $4x^2 - 20x + m = 0$ are equal , then m = ...

(a) 5 (b) 16 (c) 20 (d) 25

(25) If one of the two roots of : $x^2 - (b + 4)x - 9 = 0$ is additive inverse of the other , then b = ...

(a) – 4 (b) 0 (c) 4 (d) – 9

(26) $\overrightarrow{AB}$ is a tangent to M at B , AB = 6 cm.



If the radius is 2.5 , then AD = ... cm.

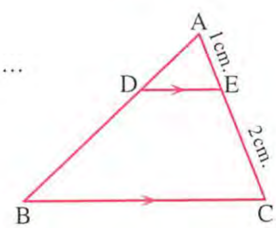
(a) 4 (b) 5

(c) 9 (d) 36

(27) If sin (θ + 10°) = cos (40°) , where $\theta \in \left]\frac{\pi}{2}, \pi\right[$, then θ = ...

(a) 40° (b) 50° (c) 120° (d) 130°

(28) In the opposite figure :



Δ ABC in which $\overline{BC}$ // $\overline{DE}$, then $\frac{\text{area of } \Delta \text{ ADE}}{\text{area of trapezium (BDEC)}}$ = ...

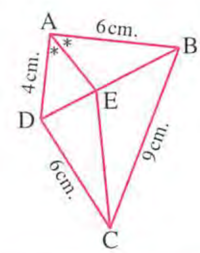
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{9}$ (d) $\frac{1}{8}$

Second | Essay questions

Answer the following questions :

1 In the opposite figure :



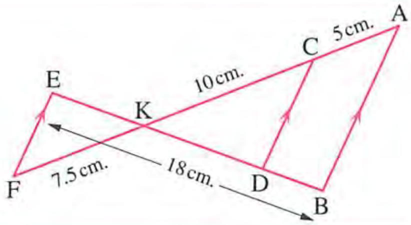
ABCD is a quadrilateral in which AB = 6 cm. , BC = 9 cm. , CD = 6 cm. and AD = 4 cm. If $\overrightarrow{AE}$ bisects ∠ A and intersects $\overline{BD}$ at E

Prove that : $\overrightarrow{CE}$ bisects ∠ BCD

2 $\overline{AB}$ is a diameter of a circle whose radius length is 12 cm. , the chord $\overline{AC}$ is draw such that m (∠ BAC) = 50° , find the length of the arc $\overset{\frown}{AC}$

3 If L + 3 and M + 3 are the roots of the equation $x^2 - 12x + 3 = 0$ find the equation whose roots are L and M

4 **In the opposite figure :**

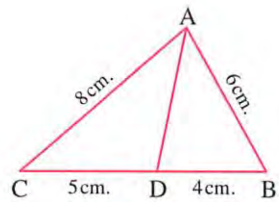


$\overline{BA} \parallel \overline{DC} \parallel \overline{EF}$, where AC = 5 cm.

, EB = 18 cm. , CK = 10 cm. and KF = 7.5 cm.

Find the length of $\overline{DB}$ and $\overline{KE}$

5 **In the opposite figure :**



ABC is a triangle in which D $\in \overline{BC}$ where BD = 4 cm.

, DC = 5 cm. , and AB = 6 cm.

, AC = 8 cm.

Prove that : Δ ABC ~ Δ DBA , then find AD

2 Cairo Governorate

Futures Language Schools
Mathematics department

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If $x = 3$ is one root of the equation : $3x^2 - 8x + m = 0$, then m = ...

(a) 3 (b) – 3 (c) 5 (d) – 5

(2) The quadratic equation whose two roots are 8 , – 13 is ....

(a) $x^2 - 5x + 104 = 0$ (b) $x^2 - 5x - 104 = 0$

(c) $x^2 + 5x - 104 = 0$ (d) $x^2 + 5x + 104 = 0$

(3) The simplest form of the imaginary number i^{15} = ...

(a) i (b) – i (c) 1 (d) – 1

(4) The function $f : f(x) = 12 - 3x$ is negative on the interval ....

(a) $[-4, \infty[$ (b) $]-\infty, 4[$ (c) $]4, \infty[$ (d) $]-\infty, -4]$

(5) The expression (13 – 2 i) – (3 – i) in the form of the number a + b i is ....

(a) 10 i (b) – 10 i (c) 10 + i (d) 10 – i

(6) The two roots of the equation : $x^2 - 4x + k = 0$ are equal if k = ...

(a) 1 (b) 4 (c) 8 (d) 6

(7) The solution set of the equation : $x^2 = x$ in $\mathbb{R}$ is ....

(a) $\{0\}$ (b) $\{1\}$ (c) $\{-1, 1\}$ (d) $\{0, 1\}$

(8) The sign of the function $f : f(X) = X^2 + 2$ is positive in ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\mathbb{R} - \{0\}$ (d) $\mathbb{R} - \{2\}$

(9) If $(2 - i)$ is a root of the equation : $X^2 + b X + 5 = 0$, then b = ...

(a) $2 + i$ (b) 5 (c) -4 (d) $-2i$

(10) The measure of the central angle subtended an arc of length 2π in a circle of diameter length 12 cm. is equal to ....

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

(11) If $\sin X < 0$, $\tan X > 0$, then X lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(12) If $\sin\theta = -1$ and $\cos\theta$ = zero , then θ = ...

(a) 90° (b) 180° (c) 270° (d) 360°

(13) If $0° < \theta < 20°$ and $\sin(5\theta) = \cos(4\theta)$, then θ = ...

(a) 14° (b) 18° (c) 12° (d) 10°

(14) $f(X) = 3\sin X$, for each $X \in \mathbb{R}$, then the maximum possible value of the function $f(X)$ = ...

(a) -3 (b) 3 (c) 1 (d) zero

(15) If $\csc\theta = -2$, $270° < \theta < 360°$, then θ = ...

(a) 30° (b) 300° (c) 330° (d) 210°

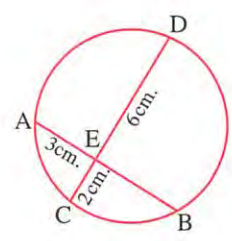
(16) In the opposite figure :

If AE = 3 cm. , EC = 2 cm.

and ED = 6 cm. , then EB = ...

(a) 5 (b) 4

(c) 6 (d) 3



(17) If the ratio between the perimeter of two similar triangles is 1 : 4 , then the ratio between their two areas equals ....

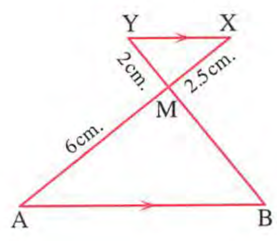
(a) 1 : 2 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16

(18) In the opposite figure :

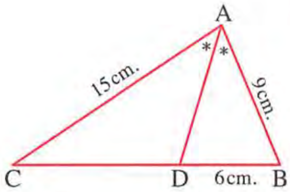
$\overleftrightarrow{AX} \cap \overleftrightarrow{YB} = \{M\}$, $\overleftrightarrow{XY} \parallel \overleftrightarrow{AB}$, then MB = ...

(a) 3.6 cm. (b) 4 cm.

(c) 4.2 cm. (d) 4.8 cm.



(19) In the opposite figure :

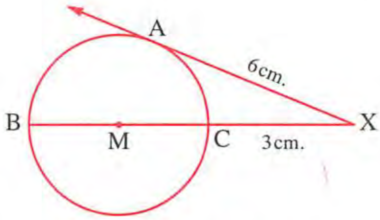


DC = ... cm.

(a) 10 (b) 6

(c) 9 (d) 5

(20) In the opposite figure :



$\overrightarrow{XA}$ is a tangent to circle M

, XA = 6 cm. , XC = 3 cm.

, then the area of the circle = ... cm^2

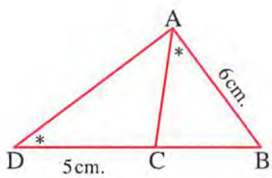
(a) 36π (b) 81π (c) 20.25π (d) 6.25π

(21) If A is a point on the plane of the circle M of radius length 3 cm. and AM = 4 cm.

, then $P_M(A)$ = ...

(a) 16 (b) 9 (c) 25 (d) 7

(22) In the opposite figure :

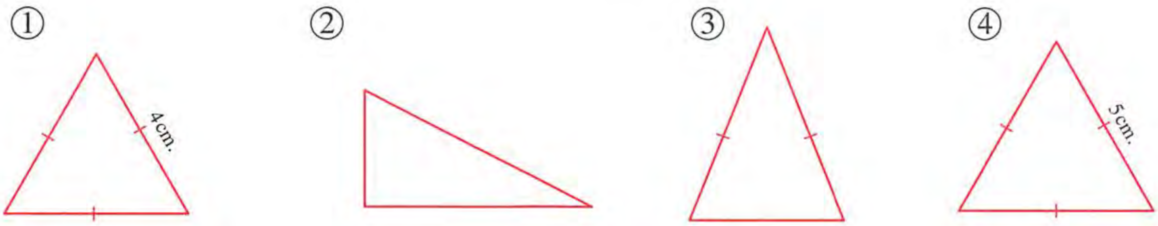


m (∠ BAC) = m (∠ D) , then BC = ...

(a) 3 cm. (b) 4 cm.

(c) 5 cm. (d) 6 cm.

(23) Which of the following triangles are similar ....



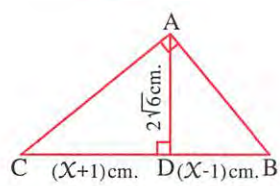
(a) ① and ④ (b) ② and ④ (c) ① and ③ (d) ③ and ④

(24) If $\Delta XYZ \sim \Delta ABC$, a (ΔXYZ) = 3 a (ΔABC) and XY = 3 cm.

, then AB = ... cm.

(a) $\sqrt{3}$ (b) $3\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) 1

(25) In the opposite figure :



X = ... cm.

(a) 6 (b) 7

(c) 5 (d) 8

(26) The exterior bisector at the vertex of an isosceles triangle is ………… to the base.

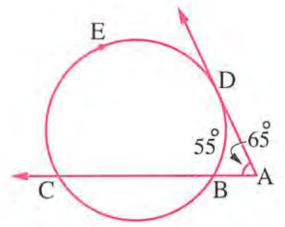
(a) Parallel (b) equal (c) perpendicular (d) bisector

(27) All ………… are similar.

(a) triangles. (b) squares.

(c) rectangles. (d) parallelograms.

(28) In the opposite figure :



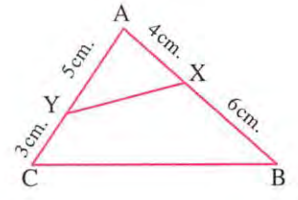
$\overrightarrow{AD}$ is a tangent , $\overrightarrow{AC}$ intersects the circle at B , C , m $(\angle A) = 65°$, m $(\widehat{BD}) = 55°$, m $(\widehat{DEC}) = (3X + 5)°$, then $X =$ …………

(a) 60° (b) 70° (c) 35° (d) 84°

Second Essay questions

Answer the following questions :

1 **In the opposite figure :**

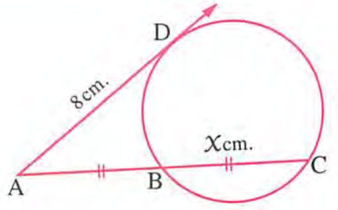


(1) **Prove that :** Δ AXY ~ Δ ACB

(2) If the area of (Δ AXY) = 8 cm^2.

, find the area of the polygon XBCY

2 **In the opposite figure :**



If AD = 8 cm. , AB = BC = X cm.

, then find the value of X

3 State two cases of similarity of two triangles.

4 If L , M are the roots of the equation : $3X^2 - 2X - 7 = 0$

, find the equation whose roots are L^2 , M^2

5 If $4 \tan A - 3 = 0$ where A is the greatest positive angle , $A \in]0 , 2\pi[$, then without using calculator find the value of $\sin(180° - A) + \cos(-A) + \cot(360° - A)$

3 Cairo Governorate

Elkalifa and Elmokattam Educational Zone
Mathematics supervisoin

First Multiple choice questions

Choose the correct answer from the given ones :

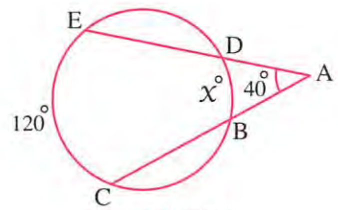
(1) In circle M if MA = 5 cm. , diameter of circle = 6 cm. , then $P_M(A) =$ ....

(a) 16 (b) – 9 (c) 9 (d) – 16

(2) If $x = 4 + 2i$, $y = 4 - 2i$, then $x y =$ ....

(a) 12 (b) 24 (c) 20 (d) 20 i

(3) **In the opposite figure :**



m (∠ A) = 40°

, m $(\widehat{EC}) = 120°$

, then $x =$ ....°

(a) 40 (b) 60 (c) 120 (d) 170

(4) The solution set of the inequality : $x^2 - 3x + 2 \geq 0$ is ....

(a) $[1 , 2]$ (b) $\mathbb{R} -]-2 , -1[$ (c) $\mathbb{R} -]1 , 2[$ (d) $[-2 , -1]$

(5) If the ratio between two corresponding sides of two similar polygons equals 1 : 3 and the difference between their surface areas 200 cm^2 , then area of smaller polygon = ... cm^2

(a) 25 (b) 90 (c) 225 (d) 100

(6) The angle whose measure 1087° lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(7) Two similar triangles the ratio between their perimeters 5 : 3 , then the ratio between their areas is ....

(a) 5 : 3 (b) 3 : 5 (c) 9 : 25 (d) 25 : 9

(8) The simplest form of expression $(1 + i)^8$ is ....

(a) 16 (b) – 16 (c) 16 i (d) – 16 i

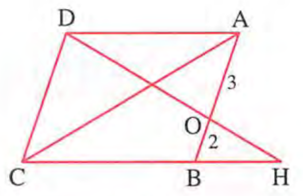
(9) The interior and exterior bisectors of angle of triangle include between them angle of measure ....

(a) 60° (b) 30° (c) 120° (d) 90°

(10) The S.S. of equation : $x^2 + 16 = 0$ in $\mathbb{R}$ is ....

(a) $\{-2\}$ (b) $\{2\}$ (c) $\{-2 , 2\}$ (d) $\varnothing$

(11) In the opposite figure :



ABCD is parallelogram AO : OB = 3 : 2

area of Δ DHC = 100 cm^2.

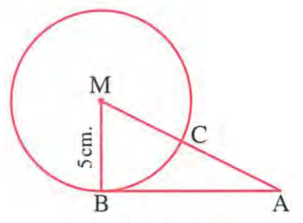
, then area of Δ ODA = ... cm^2.

(a) 36 (b) 48 (c) 60 (d) 90

(12) If $f : f(X) = 4 \sin 2X$, then the greatest possible value of f is ....

(a) 1 (b) zero (c) 4 (d) 8

(13) In the opposite figure :



If $P_M(A) = 144$

, BM = 5 cm.

, then AC = ... cm.

(a) 18 (b) 8 (c) 12 (d) 16

(14) The sign of function $f : f(X) = X - 5$ is positive in the interval ....

(a) $]-\infty , 5[$ (b) $]5 , \infty[$ (c) $[-5 , \infty[$ (d) $]-\infty , -5[$

(15) If L and M are the two roots of the equation : $X^2 + 3X - 4 = 0$, the numerical value of the expression : $L^2 + 3L + 5 =$ ....

(a) -9 (b) -4 (c) -1 (d) 9

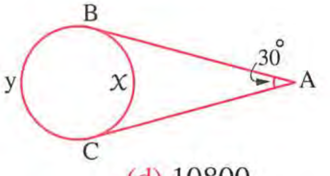
(16) If $\tan(180° + 5\theta) + \tan(270° + 4\theta) = 0$, then value of θ which satisfy the equation where $\theta \in]0 , 2\pi[$ could be equal ....

(a) 5° (b) 10° (c) 20° (d) 90°

(17) If one of the two roots of the equation : $3X^2 - (k + 2)X + k^2 + 2k = 0$ is multiplicative inverse of the other root , then k = ...

(a) $-3 , 1$ (b) $-3 , -1$ (c) $3 , -1$ (d) $3 , 1$

(18) In the opposite figure :

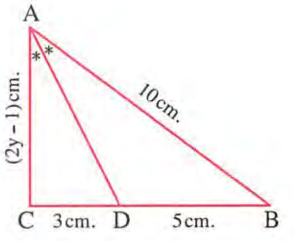


$\overline{AB}$, $\overline{AC}$ are two tangent segments to the circle , m ($\angle$ A) = 30°

, then $y^2 - X^2 =$ ....

(a) 30 (b) 60 (c) 21600 (d) 10800

(19) In the opposite figure :



$\overrightarrow{AD}$ bisects $\angle$ A , $\frac{BD}{DC} = \frac{5}{3}$

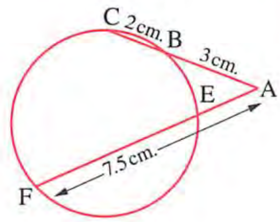
If AB = 10 cm. , AC = (2 y – 1) cm.

, then y = ... cm.

(a) 1.5 (b) 3.5

(c) 6 (d) 10

(20) In the opposite figure :



AB = 3 cm. , BC = 2 cm.

, AF = 7.5 cm.

, then EF = ……………

(a) 2 (b) 3 (c) 5.5 (d) 7.5

(21) All …………… are similar.

(a) triangles (b) rectangles (c) squares (d) parallelograms

(22) The sign of the function f , where : $f(X) = X^2 - 2X - 3$ is negative when $X \in$ ……………

(a) $]-\infty , -1[$ (b) $]-1 , 3[$ (c) $\mathbb{R} - [-1 , 3]$ (d) $]3 , \infty[$

(23) If the two roots of the equation : $k X^2 - 12 X + 9 = 0$ are equal , then ……………

(a) $k < 4$ (b) $k = 4$ (c) $k > 4$ (d) $k = 144$

(24) If the two roots of the equation : $8 X^2 - a X + 3 = 0$ are positive and the ratio between them is 2 : 3 , then a = ……………

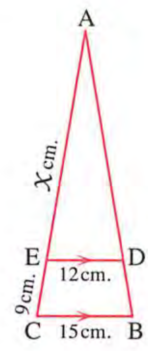
(a) 1 (b) – 1 (c) – 10 (d) 10

(25) If $\theta \in]\frac{\pi}{2} , \pi[$, $\sin \theta = \frac{12}{13}$, then the value of :

$\csc \theta \sin \theta - \tan \theta \cot \theta + \cos^2 \theta =$ ……………

(a) $\frac{169}{25}$ (b) $\frac{144}{169}$ (c) $\frac{25}{169}$ (d) $\frac{169}{144}$

(26) In the opposite figure :



$X =$ ……………

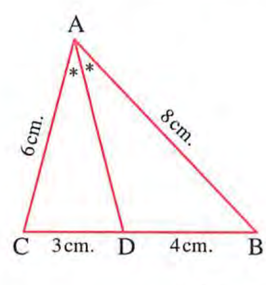
(a) 32

(b) 40

(c) 36

(d) 10

(27) In the opposite figure :



AD = …………… cm.

(a) 4 (b) 8

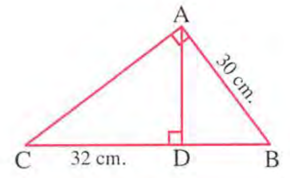
(c) 6 (d) 5

(28) In the opposite figure :

ABC is a right angled triangle at A

, $\overline{AD} \perp \overline{BC}$, AB = 30 cm. , CD = 32 cm.

, then AD = ... cm.



(a) 18 (b) 25 (c) 24 (d) 20

Second Essay questions

Answer the following questions :

1 If L , M are two roots of equation : $x^2 - 3x + 5 = 0$, form equation whose roots are L^2 , M^2

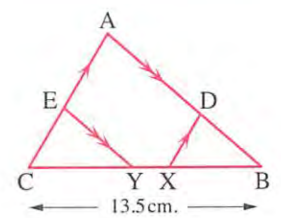
2 Find circumference of circle which contains central angle of measure 120° and subtends arc of length 6 cm.

3 In the opposite figure :

$\overline{DX} \parallel \overline{AC}$, $\overline{EY} \parallel \overline{AB}$

, BC = 13.5 cm. , $\frac{AD}{DB} = \frac{3}{2}$, $\frac{EC}{AE} = \frac{4}{5}$

, then find the length of XY



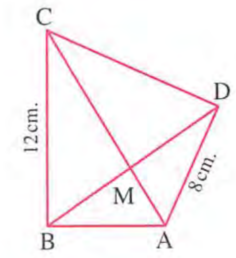
4 In the opposite figure :

ABCD is cyclic quadrilateral

, AD = 8 cm.

, CB = 12 cm.

Find : Area (Δ AMD) : Area (Δ BMC)



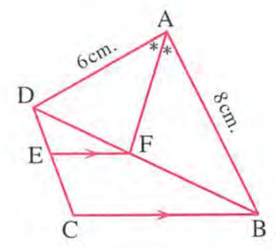
5 In the opposite figure :

$\overrightarrow{AF}$ bisects ∠ BAD

, $\overline{EF}$ parallel to $\overline{BC}$

, AB = 8 cm. , AD = 6 cm.

Find : $\frac{DE}{EC}$



4 Giza Governorate — Omrania Educational Directorate

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If L , 3 – L are the two roots of the equation : $x^2 + a x - 7 = 0$, then a = ...

(a) – 3 (b) 3 (c) – 5 (d) 5

(2) If the length of an arc in a circle equals quarter of its circeumference , then the measure of its inscribed angle subtanded to this arc equals ....

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(3) The maximum value of $f : f(x) = 2 + 3 \sin 2\theta$ is .... where $\theta \in [0 , 2\pi[$

(a) 5 (b) – 5 (c) 3 (d) – 3

(4) If 3 + 2 i is one of the roots of $x^2 - a x + b = 0$, then a + b = ...
where a , b $\in \mathbb{R}$

(a) – 7 (b) 7 (c) 19 (d) 6

(5) If $\sin \theta = -0.6$, $\theta \in]\pi , \frac{3\pi}{2}[$, then $\tan \theta + \cos \theta$ = ...

(a) $\frac{27}{20}$ (b) $\frac{31}{20}$ (c) $\frac{1}{20}$ (d) $-\frac{1}{20}$

(6) If $\tan (2\theta + 15^\circ) = \cot (\theta + 30^\circ)$, $\theta \in]0 , \frac{\pi}{4}[$, then $\sin^2 3\theta + \tan^2 4\theta$ = ...

(a) 3.5 (b) – 3.5 (c) 2.5 (d) – 2.5

(7) If one of the roots of the equation : $(2a - 5) x^2 + 7 x + a = 0$ is multiplicative inverse of the other root , then a = ...

(a) – 6 (b) 6 (c) 5 (d) – 5

(8) The soluation set of the equation : $x^2 + 16 = 0$ is .... where $x \in \mathbb{R}$

(a) {4 i , – 4 i} (b) {4 , – 4} (c) {– 4} (d) ∅

(9) If $13 \sin \theta + 5 = 0$, θ is greatest positive angle in [0 , 360°[
, then $\sin (90^\circ + \theta) \tan (360^\circ - \theta)$ = ...

(a) $\frac{12}{13}$ (b) $-\frac{12}{13}$ (c) $\frac{5}{13}$ (d) $-\frac{5}{13}$

(10) If one of the roots of the equation : $x^2 - 9 x + m = 0$ is double the other root
, then m = ...

(a) 18 (b) 20 (c) 14 (d) 26

(11) Solution set of the equation $2 \cos \theta + \sqrt{2} = 0$ is .... where $\theta \in [0 , 2\pi[$

(a) $\left\{\frac{\pi}{4}\right\}$ (b) $\left\{\frac{\pi}{4} , \frac{5\pi}{4}\right\}$ (c) $\left\{\frac{3\pi}{4} , \frac{5\pi}{4}\right\}$ (d) $\left\{\frac{5\pi}{4}\right\}$

(12) If $(a + i\, b)(3 + 4\, i) = (2 + i)(2 - i)$, then $a^2 + b^2 =$ ....

(a) -1 (b) 1 (c) -2 (d) 2

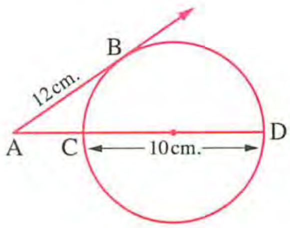
(13) If $\Delta ABC \sim \Delta XYZ$, area $(\Delta ABC) = 9$ area (ΔXYZ) , then $AB =$ ....

(a) 9 XY (b) 3 XY (c) 3 YZ (d) 3 XZ

(14) If $x = 3$ is one of the roots of the equation : $x^2 + 2\, m\, x = 3$, then $m =$ ....

(a) -1 (b) 1 (c) 2 (d) -2

(15) In the opposite figure :



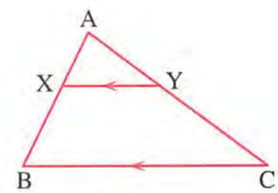
$\overrightarrow{AB}$ is a tangent its length 12 cm.

$\overline{CD}$ is a diameter of length 10 cm.

, then $AC =$ ....

(a) 10 cm. (b) 8 cm. (c) 18 cm. (d) 6 cm.

(16) In the opposite figure :



$\overline{XY} \parallel \overline{BC}$, $AX : XB = 2 : 3$

, area of $\Delta AXY = 16$ cm^2.

, then the area of trapezium XYCB = ... cm^2.

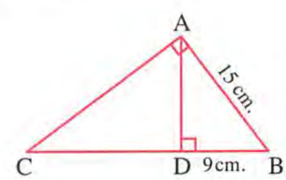
(a) 36 (b) 32 (c) 84 (d) 40

(17) If L , M are the two roots of the equation : $x^2 - 5\, x + 3 = 0$, then the equation whose roots 3 L , 3 M is ....

(a) $x^2 + 15\, x + 27 = 0$ (b) $x^2 - 15\, x + 9 = 0$

(c) $x^2 - 15\, x + 27 = 0$ (d) $x^2 - 9\, x + 15 = 0$

(18) In the opposite figure :



$m(\angle CAB) = m(\angle ADB) = 90°$

, $AB = 15$ cm. , $BD = 9$ cm.

, then $AC + AD =$ .... cm.

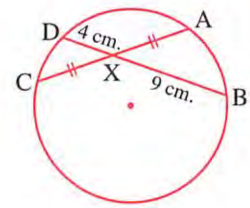
(a) 28 (b) 25 (c) 35 (d) 32

(19) The soluation set of the inequality : $x^2 + 9 < 0$ is ....

(a) $\varnothing$ (b) $[-3 , 3]$

(c) $\mathbb{R} - [-3 , 3]$ (d) $\mathbb{R} - \{-3 , 3\}$

(20) In the opposite figure :



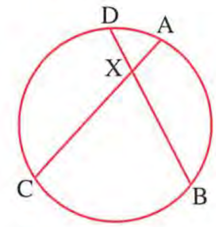
$\overline{AC} \cap \overline{BD} = \{X\}$, AX = XC

, DX = 4 cm. , XB = 9 cm.

, then AC = ... cm.

(a) 6 (b) 13 (c) 12 (d) 18

(21) In the opposite figure :



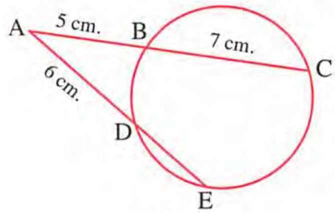
If m $(\overset{\frown}{AB})$ = 100° , m $(\overset{\frown}{CD})$ = 120°

, then m (∠ AXD) = ...

(a) 110° (b) 70°

(c) 140° (d) 180°

(22) In the opposite figure :



AD = 6 cm. , AB = 5 cm.

, BC = 7 cm.

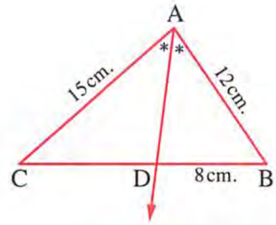
, then DE = ... cm.

(a) 12 (b) 7 (c) 10 (d) 4

(23) In the figure of number (22) If m (∠ A) = 35° , m $(\overset{\frown}{CE})$ = 100° , then m $(\overset{\frown}{BD})$ = ...

(a) 30° (b) 70° (c) 100° (d) 40°

(24) In the opposite figure :



$\overrightarrow{AD}$ is interior bisector of ∠ A

, AB = 12 cm.

, AC = 15 cm. , BD = 8 cm.

, then CD = ... cm.

(a) 12 (b) 10 (c) 8 (d) 15

(25) In the figure of number (24) The ratio between area of Δ ABD : area of Δ ABC = ...

(a) 4 : 9 (b) 4 : 5 (c) 5 : 9 (d) 5 : 10

(26) In the figure of number (24) The length of $\overline{AD}$ = ... cm.

(a) $\sqrt{10}$ (b) 8 (c) 10 (d) $2\sqrt{2}$

(27) If M is a circle , A is a point in its plane where MA = 6 cm. , $P_M(A) = -13$

, then area of circle M = ... cm.^{2} $\left(\pi = \frac{22}{7}\right)$

(a) 154 (b) 44 (c) 144 (d) 7

(28) $\overline{AB}$, $\overline{AC}$ are two tangent to circle M , m $(\overset{\frown}{BC})$ = 120° , then m (∠ A) = ...

(a) 120° (b) 60° (c) 100° (d) 180°

Second Essay questions

Answer the following questions :

1 Find in $\mathbb{R}$ the solution set of the inequality : $x^2 - 4x - 5 > 0$

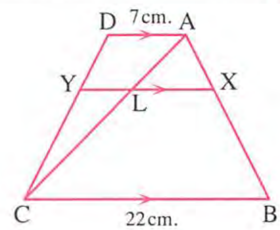
2 Solve the equation : $\cos(\pi + \theta) = \sin(390°) \cos(-60°) + \cos(30°) \sin(120°)$

3 **In the opposite figure :**

$\overleftrightarrow{AD} \parallel \overleftrightarrow{XY} \parallel \overleftrightarrow{BC}$

, $\frac{AX}{XB} = \frac{2}{3}$

Find : XY

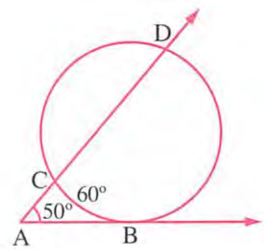


4 **In the opposite figure :**

$m(\angle A) = 50°$

, $m(\overset{\frown}{BC}) = 60°$

Find : $m(\overset{\frown}{BD})$

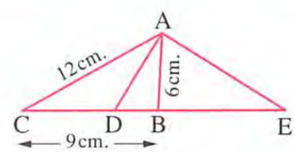


5 **In the opposite figure :**

$\overrightarrow{AD}$ and $\overrightarrow{AE}$ are the interior and exterior bisectors of $\angle$ CAB ,

AC = 12 cm. , AB = 6 cm. , BC = 9 cm.

Find the length of $\overline{AE}$



5 Giza Governorate — Abou El-Nomros Educational Zone

First Multiple choice questions

Choose the correct answer from the given ones :

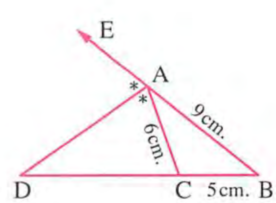
(1) In the opposite figure :

$\overrightarrow{AD}$ bisect ($\angle$ EAC)

, then CD = ... cm.

(a) 5 (b) 10

(c) 12 (d) 18



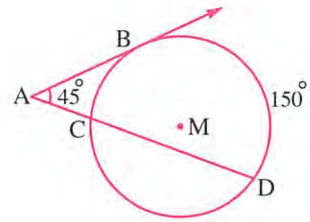
(2) If one root of the quadratic equation : $a x^2 + 4x + 7 = 0$ is a multiplicative inverse of the other , then a = ...

(a) $\frac{1}{7}$ (b) 7 (c) 4 (d) -7

(3) In the opposite figure :

m $(\widehat{BD}) = 150°$, m $(\angle A) = 45°$

, then m $(\widehat{BC})$ = ... °



(a) 60 (b) 120

(c) 90 (d) 195

(4) All the following measure of angles lie in the second quadrant except ....

(a) – 240 (b) 100 (c) – 120 (d) 860

(5) If two roots of quadratic equation : $X^2 - 6X + k = 0$ are equal and real

, then k = ...

(a) 4 (b) – 4 (c) 9 (d) – 9

(6) If $P_M(A) > 0$, then the point A located .... the circle M.

(a) inside (b) outside (c) on (d) on the centre of

(7) The measure of central angle subtended by arc of length π cm. in circle of diameter 8 cm. equal ....

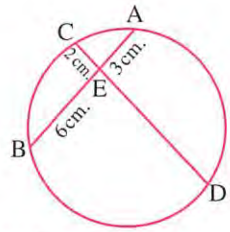
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) 2π

(8) If $\overline{AB} \cap \overline{CD} = \{E\}$,

AE = 3 cm. , CE = 2 cm.

, BE = 6 cm.

, then ED = ... cm.



(a) 9 (b) 8 (c) 7 (d) 6

(9) If the two similar polygons are congruent , then the scale factor is ....

(a) $\frac{1}{2}$ (b) 1 (c) more than 1 (d) less than 1

(10) The sign of function $f : f(X) = 4 - 2X$ positive if ....

(a) $X > 4$ (b) $X < 4$ (c) $X > 2$ (d) $X < 2$

(11) The range of function $f : f(X) = 2 \cos 3X$ is ....

(a) $[-2 , 2]$ (b) $]2 , 3[$ (c) $]-2 , 2[$ (d) $]-3 , 3[$

(12) If a line intersects two sides in a triangle and divides them into segments whose lengths are proportional , then it is .... to the third side.

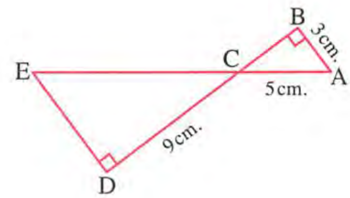
(a) intersect (b) parallel (c) bisect (d) equal

(13) If L and M are two roots of the quadratic equation : $x^2 - 7x + 12 = 0$

, then $L^2 + M^2 =$ ....

(a) 7 (b) 12 (c) 25 (d) 49

(14) In the given figure :



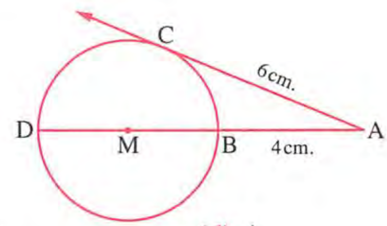
If $\overline{BD} \cap \overline{AE} = \{C\}$

, then : $\dfrac{\text{the area of the smaller triangle}}{\text{the area of the greater triangle}} = \dfrac{\cdots\cdots}{\cdots\cdots}$

(a) $\frac{25}{81}$ (b) $\frac{1}{3}$

(c) $\frac{16}{81}$ (d) $\frac{9}{64}$

(15) In the opposite figure :



$\overline{BD}$ is a diameter , $\overrightarrow{AC}$ is a tangent

, AC = 6 cm. , AB = 4 cm.

, then the radius of circle equal .... cm.

(a) 5 (b) 9 (c) 2.5 (d) 4

(16) The solution set of the inequality : $x^2 \le 5x - 4$ in $\mathbb{R}$ ....

(a) $\mathbb{R} -]1 , 4[$ (b) $\mathbb{R} - [1 , 4]$ (c) $[1 , 4]$ (d) $]1 , 4[$

(17) If the ratio between the perimeter of two similar triangles is 4 : 9 , then the ratio between their areas is ....

(a) 4 : 3 (b) 4 : 9 (c) 16 : 81 (d) 3 : 2

(18) If $\sin \theta = \cos B$ and θ , B are two acute angles , then $\tan (\theta + B) =$ ....

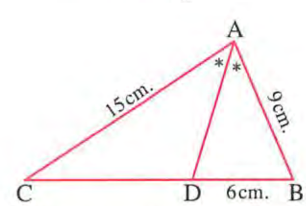
(a) -1 (b) 1 (c) $\sqrt{3}$ (d) undefind.

(19) If $x = 5$ is one of the two roots of the equation : $x^2 + a x = 2a + 4$

, then a = ...

(a) -7 (b) 7 (c) $\frac{29}{3}$ (d) $\frac{-29}{3}$

(20) In the opposite figure :



The length of $\overline{CD} =$ .... cm.

(a) 5 (b) 6

(c) 9 (d) 10

(21) If the terminal side of the angle θ in the standard position intersects the unit circle at the point $(x , -x)$, $x > 0$, then $m(\angle \theta) =$ ....

(a) 225° (b) 315° (c) 135° (d) 45°

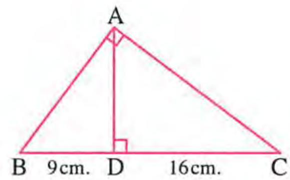
(22) $(1 + i)^4 - (1 - i)^4 =$ ....

(a) zero (b) 8 (c) -8 (d) 4

(23) In the opposite figure :

$\overline{AC} \perp \overline{AB}$, $\overline{AD} \perp \overline{BC}$

, the length of $\overline{AB}$ = ... cm.



(a) 12 (b) 15

(c) 20 (d) 25

(24) If L , M are the roots of the equation : $x^2 - 7x + 3 =$ zero , then the equation whose roots 2 L , 2 M is ....

(a) $x^2 - 14x + 12 = 0$ (b) $x^2 - 14x - 12 = 0$

(c) $x^2 + 14x + 12 = 0$ (d) $x^2 + 14x - 12 = 0$

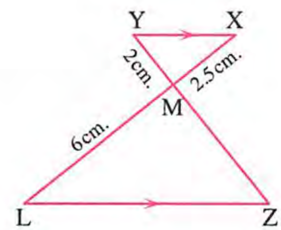
(25) If Δ ABC is right-angled triangle at B , $\cos C = \frac{1}{2}$, then sin (A + B + 2 C) ....

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{-\sqrt{3}}{2}$ (d) zero

(26) In the opposite figure :

$\overleftrightarrow{XL} \cap \overleftrightarrow{YZ} = \{M\}$, $\overleftrightarrow{XY} \parallel \overleftrightarrow{ZL}$

, then the length of $\overline{MZ}$ ....



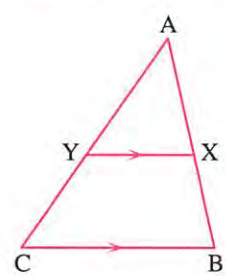
(a) 3.6 (b) 4

(c) 4.2 (d) 4.8

(27) If 2 and 7 are two roots the equation : $x^2 + a\,x + b = 0$, then a + b = ...

(a) 5 (b) – 5 (c) 23 (d) – 23

(28) All of the following mathematical expressions are true except ....



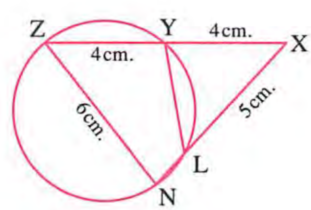
(a) $\frac{AX}{XB} = \frac{XY}{BC}$ (b) $\frac{AX}{AB} = \frac{XY}{BC}$

(c) $\frac{AY}{YC} = \frac{AX}{XB}$ (d) $\frac{AY}{AC} = \frac{AX}{AB}$

Second | Essay questions

Answer the following questions :

1 In the opposite figure :

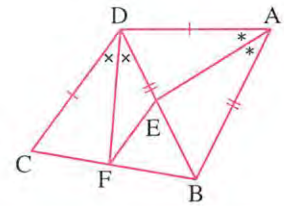


XL = 5 cm. , XY = YZ = 4 cm. , NZ = 6 cm.

Find with proof :

The length of $\overline{LN}$, $\overline{YL}$

2 **In the opposite figure :**



ABCD is a quadrilateral

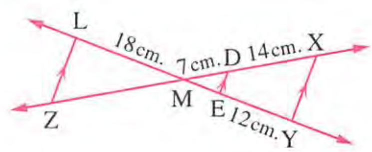
, AB = BD , AD = DC , $\overrightarrow{AE}$ bisects ∠ A , $\overrightarrow{DF}$ bisects ∠ BDC

Prove that : $\overline{EF} \parallel \overline{DC}$

3 Find the solution set of the equation : $x^2 - 2x + 4 = 0$ in $\mathbb{C}$

4 If the difference between two complements angles is $\frac{\pi}{3}$ find the degree and radian measure of the two angles.

5 **In the opposite figure :**



$\overline{XY} \parallel \overline{DE} \parallel \overline{LZ}$

Find with proof :

the length of $\overline{EM}$

, the length of $\overline{MZ}$

6 Giza Governorate — Inspection of Math

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If one of the two roots of the equation : $3x^2 + (a + 3)x + 7 = 0$ is the additive inverse of the other , then : a = ……………

(a) -3 (b) 3 (c) $\frac{1}{3}$ (d) $\frac{-1}{3}$

(2) If $(3x - y) + (x + y)i = \frac{2}{1 + i}$, then $y - x$ = ……………

(a) zero (b) -1 (c) 1 (d) i

(3) If the two roots of the equation : $x^2 + 4x + k = 0$ are real and different , then k ∈ ……………

(a) $]4 , \infty[$ (b) $]-\infty , 4[$ (c) $]-\infty , \infty[$ (d) $[4 , \infty[$

(4) The solution set of the inequality : $(x - 3)(x - 7) < 0$ in $\mathbb{R}$ is ……………

(a) $\{3 , 7\}$ (b) $]3 , 7[$ (c) $[3 , 7]$ (d) $\mathbb{R} - [3 , 7]$

(5) If $(2 + i)$ is one of the two roots of the equation : $x^2 - 4x + c = 0$, then the value of c = ……………

(a) 16 (b) -16 (c) -5 (d) 5

(6) The quadratic equation whose two roots are $(2-3\,i)$, $(2+3\,i)$ is ....

(a) $x^2-4x+13=0$ (b) $x^2+4x+13=0$

(c) $x^2+4x-13=0$ (d) $x^2-4x-13=0$

(7) If L is one of the two root of the equation : $x^2-4x+1=0$, then the numerical value the expression : L^2-4L+5 is ....

(a) 4 (b) -4 (c) 5 (d) -5

(8) If $f(x)=x+2$, where : $x\in]-4\,,3[$, then $f(x)$ is negative when $x\in$ ....

(a) $[-4\,,-2]$ (b) $]-4\,,-2[$ (c) $[-2\,,3]$ (d) $]-2\,,3[$

(9) The conjugate number of : $3\,i-7$ equals ....

(a) $3\,i+7$ (b) $-3\,i-7$ (c) $-3\,i+7$ (d) $\frac{1}{3\,i-7}$

(10) The range of the function $f: f(\theta)=4\sin 2\theta$ where $\theta\in[0\,,2\pi[$ equals ....

(a) $[-4\,,4]$ (b) $]-4\,,4[$ (c) $[-2\,,2]$ (d) $]-2\,,2[$

(11) If the terminal side of the angle of measure 30° in the standard position in the unit circle rotates three and half revolutions clockwise , then the terminal side lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(12) If $\cos\theta>0$, $\sin\theta<0$, then θ lies in the ....quadrant.

(a) first (b) second (c) third (d) fourth

(13) The angle of measure $\frac{-5\pi}{9}$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

(14) The arc of length $8\,k\,\pi$ cm. in a circle whose radius of length $24\,k$ cm. is subtends an inscribed angle of measure ....

(a) 60° (b) 30° (c) 120° (d) $(60\,k)°$

(15) If the ratio between the perimeters of two similar polygons 4 : 9 , then the ratio between their surface areas = ...

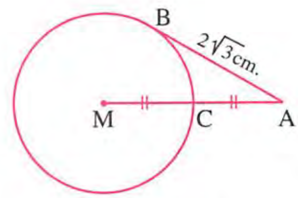
(a) 2 : 3 (b) 4 : 13 (c) 16 : 81 (d) 4 : 9

(16) Any two regular polygons having the same number of sides are ....

(a) congruent. (b) equal in area.

(c) equal in perimeter. (d) similar.

(17) In the opposite figure :

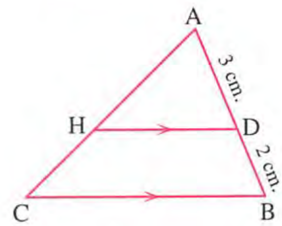


The radius length of circle M = ... cm.

(a) 2 (b) 3

(c) 4 (d) 5

(18) In the opposite figure :

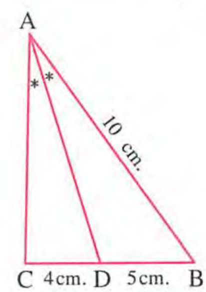


$\frac{\text{The area of } (\Delta \text{ ADH})}{\text{The area of (figure DBCH)}} = \text{.............}$

(a) $\frac{3}{2}$ (b) $\frac{9}{16}$

(c) $\frac{9}{25}$ (d) $\frac{3}{5}$

(19) In the opposite figure :



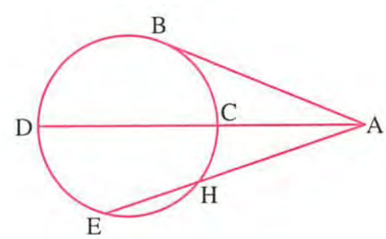
$\overrightarrow{AD}$ bisects $\angle$ BAC

, then AD = ... cm.

(a) 8 (b) 60

(c) $2\sqrt{15}$ (d) $7\sqrt{3}$

(20) In the opposite figure :



All of the following expressions are true

, except ....

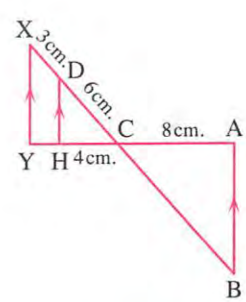
(a) $(AB)^2 = AC \times AD$

(b) $(AB)^2 = AH \times AE$

(c) $AC \times AD = AH \times AE$

(d) $AC \times CD = AH \times HE$

(21) In the opposite figure :



$\overline{AB} \,//\, \overline{DH} \,//\, \overline{XY}$, $\overline{AY} \cap \overline{BX} = \{C\}$, AC = 8 cm.

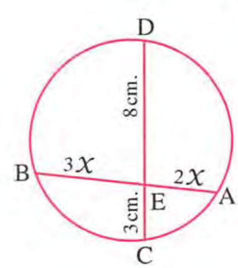
, CH = 4 cm. , CD = 6 cm.

, DX = 3 cm. , then BC + HY = ... cm.

(a) 12 (b) 15

(c) 8 (d) 14

(22) In the opposite figure :



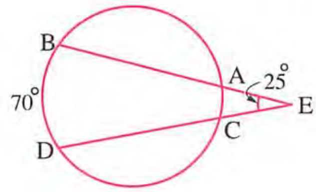
EC = 3 cm. , ED = 8 cm.

, then the value of x = ... cm.

(a) 2 (b) 4

(c) 8 (d) 6

(23) In the opposite figure :

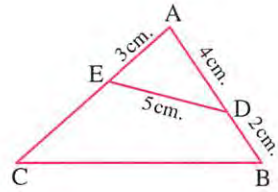


m $(\widehat{AC})$ = ...°

(a) 20 (b) 30

(c) 40 (d) 50

(24) In the opposite figure :



Δ ABC ~ Δ AED , AE = 3 cm. , AD = 4 cm. , BD = 2 cm.

, DE = 5 cm. , then BC = ... cm.

(a) 2.5 (b) 10

(c) 7.5 (d) 7

(25) Two triangles in which the first whose two angles of measure 50° , 60° and the second whose two angles of measure 60° , 70° , then the two triangles are ....

(a) congruent and not similar. (b) similar.

(c) congruent and similar. (d) not congruent and not similar.

(26) If $P_M(A) = -9$, then this means that ....

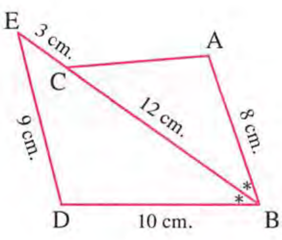
(a) the point A lies outside the circle whose center M.

(b) the point A lies inside the circle whose center M.

(c) the radius length of the circle whose center M equals 9 length unit.

(d) the length of the tangent segment drawn from the point A to the circle whose center M equals 3 length unit.

(27) In the opposite figure :

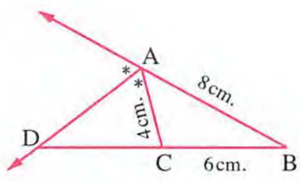


AC = ... cm.

(a) 6.2 (b) 6

(c) 7.2 (d) 7

(28) In the opposite figure :



$\overrightarrow{AD}$ bisects $\angle$ A externally

CD = ... cm.

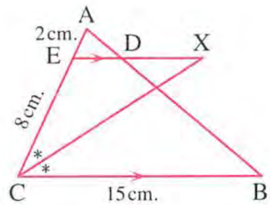
(a) 2 (b) 4

(c) 6 (d) 8

Second | Essay questions

Answer the following questions :

1 **In the opposite figure :**



If $\overrightarrow{CX}$ bisects $\angle$ ACB , $\overline{XE}$ // $\overline{BC}$

AE = 2 cm. , EC = 8 cm.

, BC = 15 cm.

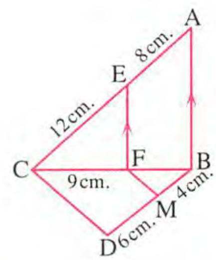
Find the length of XD

2 Find the solution set of the following inequality : $X(X+4) \leq 12$

3 If $\sin\theta = \frac{4}{5}$, where θ is the greatest positive angle in $[0 , 360°[$

, then find : $\sin(180° - \theta) + \tan(90° - \theta)$

4 **In the opposite figure :**



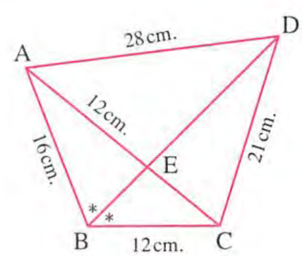
$\overline{AB}$ // $\overline{EF}$, AE = 8 cm. , CE = 12 cm.

CF = 9 cm. , BM = 4 cm. , DM = 6 cm.

, then : (1) **Find :** the length of $\overline{BF}$

(2) **Prove that :** $\overline{FM}$ // $\overline{CD}$

5 **In the opposite figure :**



$\overrightarrow{BE}$ bisects $\angle$ ABC

, BC = AE = 12 cm.

, AB = 16 cm. , DC = 21 cm. , AD = 28 cm.

Prove that : $\overrightarrow{DE}$ bisects $\angle$ ADC

7 El-Kalyoubia Governorate

El-obour Educational Zone
Manart El-Bayan Secondary Schools

First | Multiple choice questions

Choose the correct answer from the given ones :

(1) i^{37} = ...

(a) – 1 (b) 1 (c) – i (d) i

(2) Conjugate 2 i – 5 is ....

(a) 2 i + 5 (b) 2 i – 5 (c) – 2 i – 5 (d) – 5 + 2 i

(3) Form the quadratic equation with the two roots $1 + i$, $1 - i$ ....

(a) $x^2 + x + 2 = 0$ (b) $x^2 + 2x + 2 = 0$ (c) $x^2 - 2x + 2 = 0$ (d) $x^2 + 2 = 0$

(4) L , 2 – L two roots $x^2 - kx + 6 = 0$, then k = ...

(a) 6 (b) 3 (c) 5 (d) 2

(5) L , M are the two roots of the equation : $x^2 - 5x + 3 = 0$, then $L^2 + M^2$ = ...

(a) 9 (b) 5 (c) 19 (d) 22

(6) If $\tan(180° + \theta) = 1$ where θ is the smallest positive angle , then θ = ...

(a) 60° (b) 30° (c) 45° (d) 135°

(7) The solution set of the equation : $x^2 = x$ in $\mathbb{R}$ is ....

(a) $\{0\}$ (b) $\{0 , 1\}$ (c) $\{0 , -1\}$ (d) $\{1\}$

(8) $f(x) = 6 - 2x$ positive at $x \in$ ....

(a) $x \leq 3$ (b) $x \geq 3$ (c) $x > 3$ (d) $x < 3$

(9) If L is one of the two roots of the equation : $x^2 - 5x - 3 = 0$, then $L^2 - 5L + 7$ = ...

(a) 10 (b) 4 (c) 12 (d) – 4

(10) The arc length in a circle of radius 6 cm. opposite to central angle $\frac{\pi}{2}$ is = ... cm.

(a) $\frac{3\pi}{2}$ (b) 2π (c) $\frac{5}{2}\pi$ (d) 3π

(11) If $5 \sin\theta = 4$, $90° < \theta < 180°$, then $3 \cot(90° + \theta)$ = ...

(a) 5 (b) – 5 (c) 4 (d) – 3

(12) The solution set of the inequality : $(x - 3)(x - 7) < 0$ in $\mathbb{R}$ is ....

(a) $\{3 , 7\}$ (b) $]3 , 7[$ (c) $[3 , 7]$ (d) $\mathbb{R} - [3 , 7]$

(13) If $f(x) = 4 \sin x$, $x \in [0 , \pi]$ the rang of function ....

(a) $[0 , 4]$ (b) $]0 , 4[$ (c) $]-4 , 0[$ (d) $[-4 , 4]$

(14) If $\sin\theta = \cos\theta$ where θ is acute angle , then $\tan 2\theta$ = ...

(a) 1 (b) – 1 (c) undefined. (d) $\sqrt{3}$

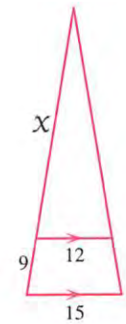
(15) Two similar polygons , ratio between their perimeters equal 4 : 9 , then ratio between the lengths of two corresponding side is ....

(a) 4 : 9 (b) 2 : 3 (c) 16 : 81 (d) 9 : 4

(16) Two similar rectangles , the dimensions of the first are 12 cm. , 8 cm. and the perimeter of the second equal = 60 cm. the area of the second = ... cm^2

(a) 100 (b) 216 (c) 500 (d) 864

(17) In the opposite figure :

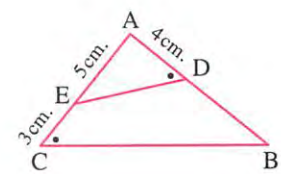


X = ... cm.

(a) 12 (b) 24

(c) 36 (d) 48

(18) In the opposite figure :

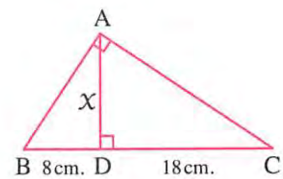


BD = ... cm.

(a) 5 (b) 6

(c) 4 (d) 7

(19) In the opposite figure :



X = ... cm.

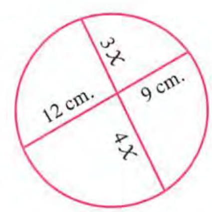
(a) $12\sqrt{3}$ (b) 24

(c) 12 (d) $8\sqrt{3}$

(20) If $\Delta ABC \sim \Delta XYZ$, $AB = 3\,XY$, then $\dfrac{\text{area of } (\Delta XYZ)}{\text{area of } (\Delta ABC)}$ = ...

(a) 3 (b) $\frac{1}{3}$ (c) $\frac{1}{9}$ (d) 9

(21) In the opposite figure :

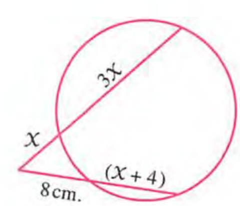


X = ... cm.

(a) 3 (b) 9

(c) 18 (d) 21

(22) In the opposite figure :

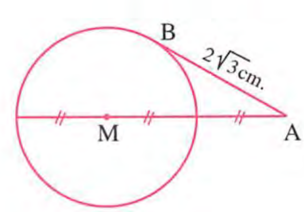


X = ... cm.

(a) 5 (b) 6

(c) 3 (d) 9

(23) In the opposite figure :

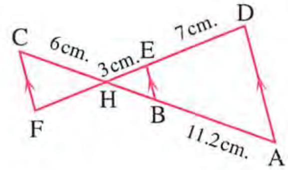


The length of the radius of circle M = ... cm.

(a) 2 (b) 4

(c) 3 (d) 5

(24) In the opposite figure :

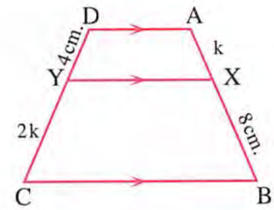


$\overline{AD} \parallel \overline{BE} \parallel \overline{FC}$, then HF = …………… cm.

(a) 3.6 (b) 4.8

(c) 6.3 (d) 3.75

(25) In the opposite figure :

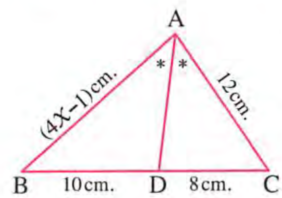


$\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, then K = ……………

(a) $\frac{3}{8}$ (b) 4

(c) 16 (d) 32

(26) In the opposite figure :



X = …………… cm.

(a) 3 (b) 4

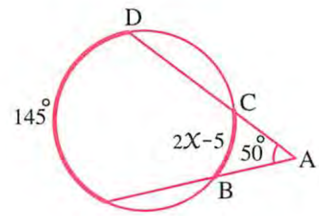
(c) 4.5 (d) 6

(27) If M is a circle of radius length 3 cm. , A is a point lies in its plane where MA = 4 cm.

, then $P_M(A)$ = ……………

(a) $\sqrt{7}$ (b) 9 (c) 7 (d) – 7

(28) In the opposite figure :



X = ……………

(a) 50° (b) 25°

(c) 100° (d) 75°

Second Essay questions

Answer the following questions :

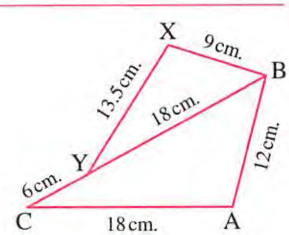
1 Investigate the sign of the function $f(X) = 4 - X^2$ and determine the solution $4 - X^2 \leq 0$

2 Find the general solution of the equation : $\cos 2\theta = \sin 4\theta$

3 If L + 1 , M + 1 are two roots of the equation : $X^2 - 7X + 5 = 0$

, then form the equation whose two roots L^2 , M^2

4 In the opposite figure :

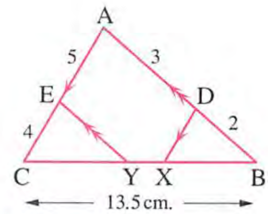


B , Y and C are collinear prove that

$\Delta XBY \sim \Delta ABC$

5 **In the opposite figure :**

The length of $\overline{XY}$ = ...



8 **El-Monoufia Governorate** — Quesna Educational directorate Mathematics Supervision

First | Multiple choice questions

Choose the correct answer from the given ones :

(1) The sign of the function $f : f(X) = 6 - 2X$ is non-positive when ....

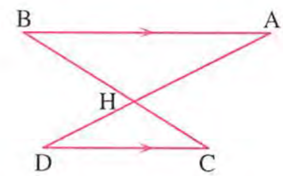
(a) $X > 3$ (b) $X \leq 3$ (c) $X < 3$ (d) $X \geq 3$

(2) In the opposite figure :

$\overline{AB} // \overline{DC}$, $2 AH = 3 HD$

, $BH - CH = 4$ cm.

, then BC = ... cm.

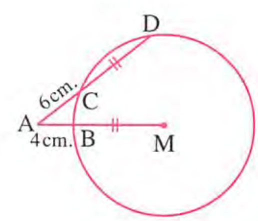


(a) 18 (b) 20 (c) 24 (d) 25

(3) In the opposite figure :

CD = BM , then the circumference

of the circle M = ... cm.



(a) 15π (b) 18π

(c) 20π (d) 24π

(4) If ABC is a right-angled triangle at B , sin A + cos C = 1 , then tan C = ...

(a) 1 (b) -1 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

(5) The solution set of the equation : $X^2 + 9 = 0$ in the set of complex numbers is ....

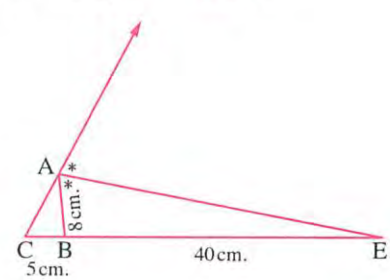
(a) $\{3 , -3\}$ (b) $\{-3i\}$ (c) $\{3i , -3i\}$ (d) $\varnothing$

(6) In the opposite figure :

AE = ... cm.

(a) 32 (b) 45

(c) 48 (d) $24\sqrt{3}$



(7) If $\sin \mathcal{X} = \cos y$, then $\sin (\mathcal{X} + y) =$ ....

(a) 1 (b) zero (c) -1 (d) otherwise.

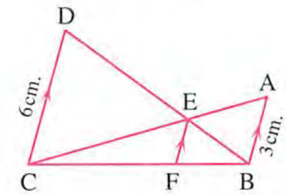
(8) If one of the two roots of the equation : $4 k \mathcal{X}^2 + 7 \mathcal{X} + k^2 + 4 = 0$ is the multiplicative inverse of the other root , then $k =$ ....

(a) ± 2 (b) 3 (c) 4 (d) 2

(9) If $\Delta ABC \sim \Delta XYZ$ and $3 AB = 2 XY$, then area of ΔABC : area of ΔXYZ = ...

(a) 4 : 9 (b) 9 : 4 (c) 2 : 3 (d) 3 : 2

(10) In the opposite figure :



If $\overline{AB} // \overline{EF} // \overline{CD}$, then EF = ... cm.

(a) 2.5 (b) 2

(c) 1.5 (d) 1

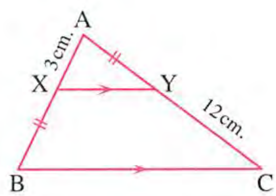
(11) $(1 - i)^{12} =$ ....

(a) $-64 i$ (b) $64 i$ (c) -64 (d) 64

(12) The function $f : f(\theta) = \sin (b \theta)$ is a periodic function and its period $\left(\frac{2\pi}{3}\right)$, then b = ...

(a) $\frac{3}{4}$ (b) $\frac{5}{3}$ (c) 3 (d) 6

(13) In the opposite figure :

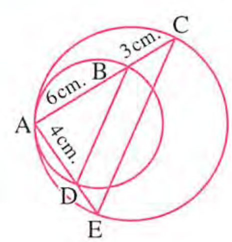


If $\overline{XY} // \overline{BC}$, then AC = ... cm.

(a) 15 (b) 16

(c) 18 (d) 20

(14) In the opposite figure :



AB = 6 cm. , BC = 3 cm. , AD = 4 cm.

the two circles touching internally at A , then ED = ... cm.

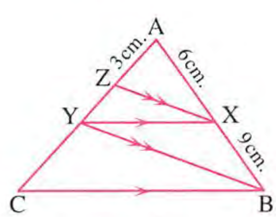
(a) 2 (b) 3

(c) 3.5 (d) 4

(15) If $(2 + 3 i) + (1 - i) = \mathcal{X} + y i$, then $\mathcal{X} + y =$ ....

(a) 2 (b) -4 (c) 5 (d) 7

(16) In the opposite figure :



$\overline{XY} // \overline{BC}$, $\overline{XZ} // \overline{BY}$

, AX = 6 cm. , XB = 9 cm. , AZ = 3 cm.

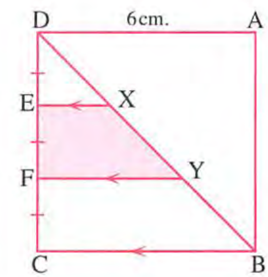
, then the length of $\overline{ZC}$ = ... cm.

(a) 4.5 (b) $15 \frac{3}{4}$ (c) 36 (d) 45

(17) If S_1 is the solution set of the inequality : $x^2 - x - 2 \leq 0$ and S_2 is the solution set of the inequality : $x^2 + x - 2 \leq 0$, then $S_1 \cap S_2 =$ ....

(a) $\varnothing$ (b) $[-2 , 2]$ (c) $[-1 , 1]$ (d) $\mathbb{R} -]-1 , 1[$

(18) In the opposite figure :



ABCD is a square of side length 6 cm.

, DE = EF = FC

, then the area of the polygon XYFE = ... cm^2

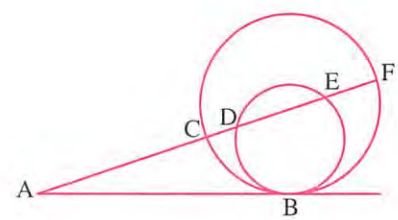
(a) 6 (b) 8

(c) 10 (d) 12

(19) The terminal side of angle θ in standard position intersects the unit circle at point $B\left(x , \frac{3}{5}\right)$ where $x < 0$, then $\sin(90° + \theta) =$ ....

(a) – 0.8 (b) – 0.6 (c) 0.8 (d) 0.6

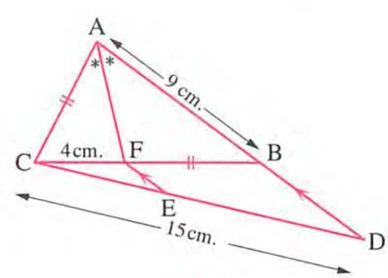
(20) If $\overrightarrow{AB}$ is a common tangent to two circles touching internally at B , then AC : AD = ...



(a) AB : AF (b) 3 : 4

(c) AD : AF (d) AE : AF

(21) In the opposite figure :



ABC is a triangle , $\overrightarrow{AF}$ bisects ∠ A internally

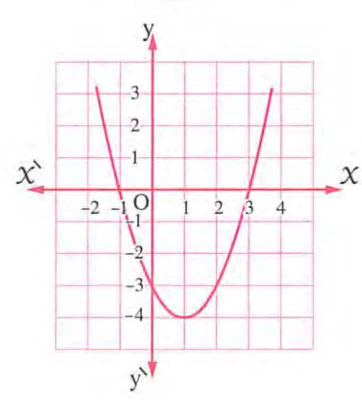
, AC = BF , $\overline{FE} \parallel \overline{BD}$

, CD = 15 cm.

, CF = 4 cm. , AB = 9 cm. , then DE = ... cm.

(a) 4 (b) 6 (c) 9 (d) 11

(22) In the opposite figure :



The curve of the function $f : f(x) = x^2 - 2x - 3$

, then the solution set of the inequality :

$x^2 - 2x - 3 \geq 0$ in $\mathbb{R}$ is ....

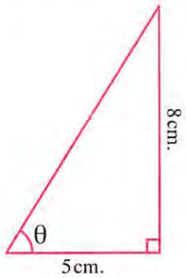
(a) $]-1 , 3[$ (b) $\mathbb{R} - [-1 , 3]$

(c) $]3 , \infty[$ (d) $]-\infty , -1] \cup [3 , \infty[$

(23) The dimensions of a rectangle are 10 cm. , 6 cm. , if the scale factor equals 3 , then the perimeter of another of rectangle similar to it = ... cm.

(a) 96 (b) 69 (c) 15 (d) 30

(24) In the opposite figure :

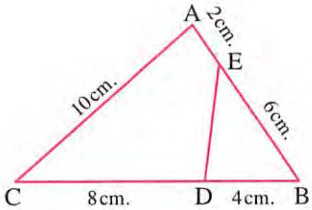


$\theta^{rad} \simeq$ ....

(a) 1.5^{rad} (b) 1.012^{rad}

(c) 2^{rad} (d) 4

(25) In the opposite figure :



If EB = 6 cm. , CD = 8 cm.

, AC = 10 cm

, AE = 2 cm. , BD = 4 cm.

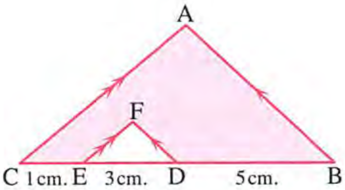
, then ED = ...

(a) 2 (b) 4 (c) 3 (d) 5

(26) If M is a circle with diameter length 12 cm. , A is a point in its plane and the power of the point A with respect to the circle M equals 13 cm. , then MA = ... cm.

(a) 7 (b) 14 (c) 3.5 (d) 6

(27) In the opposite figure :



If the area of Δ DEF = 6 cm^2

, then the area of shaded part = ... cm^2

(a) 27 (b) 36

(c) 48 (d) 54

(28) The range of the function $f : f(\theta) = 3 \sin 2\theta$ is ....

(a) $[-2 , 2]$ (b) $]-2 , 2[$ (c) $[-3 , 3]$ (d) $]-3 , 3[$

Second Essay questions

Answer the following questions :

1 If $\cos X = \frac{3}{5}$, $270° < X < 360°$ Find the value of :

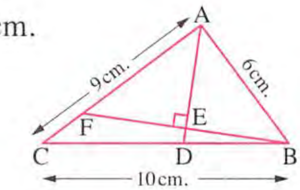
$\sin(180° - X) + \tan(90° - X) + \tan(270° - X)$

2 In the opposite figure :

ABC is a triangle in which AB = 6 cm. , AC = 9 cm. and BC = 10 cm.

, D $\in \overline{BC}$ where BD = 4 cm. , $\overrightarrow{BE} \perp \overline{AD}$

and intersects $\overline{AD}$ and $\overline{AC}$ at E and F respectively.



(1) Prove that : $\overrightarrow{AD}$ bisect $\angle$ A

(2) Find : area of Δ ABF : area of Δ CBF

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3 Find in $\mathbb{R}$ the solution set of the inequality : $(x + 3)^2 < 10 - 3\,(x + 3)$

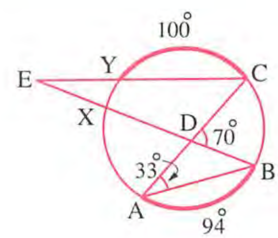
---

4 In the opposite figure :

m ($\angle$ BAC) = 33° , m ($\angle$ BDC) = 70°

, m ($\widehat{AB}$) = 94°

, m ($\widehat{CY}$) = 100°



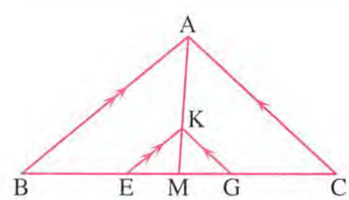
Find : m ($\angle$ BEC)

---

5 ABC is a triangle , M is the midpoint of $\overline{BC}$, let K $\in \overline{AM}$

, draw $\overrightarrow{KE}$ // $\overline{AB}$ to intersect $\overline{BC}$ at E , draw $\overrightarrow{KG}$ // $\overline{AC}$

to intersect $\overline{BC}$ at G.



First : Prove that : M is the midpoint of $\overline{EG}$.

Second : If K is the point of intersection of the medians of Δ ABC

Prove that : BE = EG = GC = $\frac{1}{3}$ BC

9 El-Gharbia Governorate

Central Mathematics Supervision
Official Language Schools

First Multiple choice questions

Choose the correct answer from the given ones :

(1) If tan (180° + θ) = 1 where θ is the smallest positive angle , then θ = ...

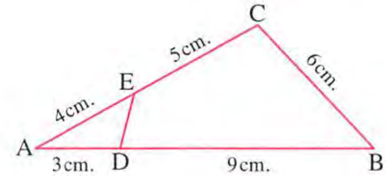
(a) 60° (b) 30° (c) 45° (d) 135°

(2) If L , – L are the two roots of the equation : $x^2 - (k - 7)\,x - 25 = 0$

, then k = ...

(a) 3 (b) 5 (c) 7 (d) 9

(3) In the opposite figure :

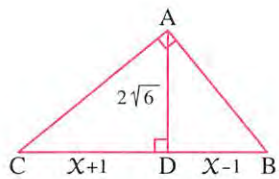


ED = ... cm.

(a) 2 (b) 3

(c) 4 (d) 5

(4) In the opposite figure :



$AD = 2\sqrt{6}$ cm. , then the value of $X =$ ....

(a) 3 (b) 4

(c) 5 (d) 6

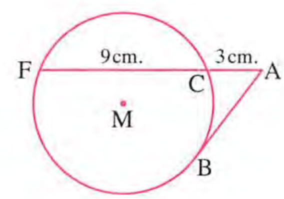
(5) $i^{-24} =$ ....

(a) 1 (b) -1 (c) i (d) $-i$

(6) If the function $y = \sin\left(\frac{\pi}{2} + X\right)$ has the maximum value at $X =$ ....

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) π (d) zero

(7) In the opposite figure :



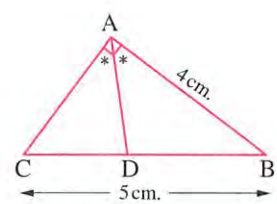
AC = 3 cm. , CF = 9 cm. , $\overline{AB}$ touches the circle M at B

, then $P_M(A) =$ ....

(a) 6 (b) 9

(c) 27 (d) 36

(8) In the opposite figure :



BC = 5 cm. , AB = 4 cm. , $\overline{AB} \perp \overline{AC}$, then $\frac{BD}{DC} =$ ....

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$

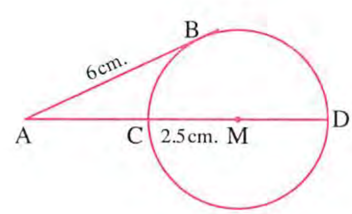
(9) The solution set of the equation : $X^2 + 9 = 0$ in the set of complex number is ....

(a) $\{3 , -3\}$ (b) $\{-3i\}$ (c) $\{3i , -3i\}$ (d) $\varnothing$

(10) The degree measure of the angle whose measure $\frac{7\pi}{6} =$ ....

(a) 105° (b) 210° (c) 420° (d) 840°

(11) In the opposite figure :



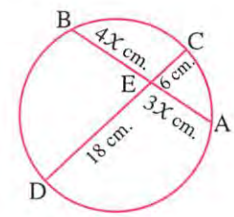
$\overline{AB}$ is a tangent segment to circle M

, AB = 6 cm. , CM = 2.5 cm.

, then AC = ... cm.

(a) 9 (b) 4 (c) 2.5 (d) 5

(12) In the opposite figure :



$X =$ .... cm.

(a) 3 (b) 9

(c) 2 (d) 18

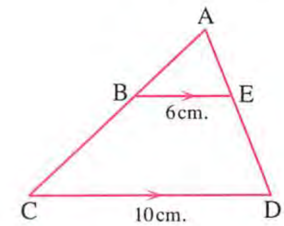
(13) The two roots of the equation : $k X^2 - 12 X + 9 = 0$ are equal if ....

(a) $k > 4$ (b) $k < 4$ (c) $k = 4$ (d) $k = 9$

(14) The angle with measure 585° in standard position is equivalent to the angle with measure ....

(a) $\frac{\pi}{4}$ (b) $\frac{5\pi}{4}$ (c) $\frac{3\pi}{4}$ (d) $\frac{7\pi}{4}$

(15) In the opposite figure :

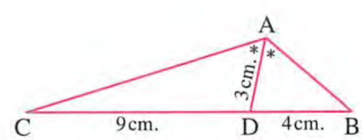


If $\overline{BC} \parallel \overline{DC}$, then $\frac{\text{area of } \Delta ABE}{\text{area of trapezium BCDE}} =$ ....

(a) $\frac{25}{81}$ (b) $\frac{3}{5}$

(c) $\frac{9}{16}$ (d) $\frac{9}{25}$

(16) In the opposite figure :



$AB \times AC =$ .... cm^2

(a) 36 (b) 45

(c) 12 (d) 27

(17) If $\tan(4\theta) = \cot(5\theta)$, then $\sin(3\theta) =$ .... where 3θ is the measure of acute angle.

(a) $\frac{1}{2}$ (b) 1 (c) -1 (d) $\frac{\sqrt{3}}{2}$

(18) If $(2 + 3i) + (1 - i) = X + yi$, then $X + y =$ ....

(a) 2 (b) -4 (c) 5 (d) 6

(19) All .... are simialr.

(a) triangles (b) rectangle (c) parallelograms (d) squares

(20) The length of an arc opposite to a central angle of measure 150° in a circle with radius length 8 cm. equals .... cm.

(a) $\frac{20}{3}\pi$ (b) $\frac{17}{2}\pi$ (c) 8π (d) 20

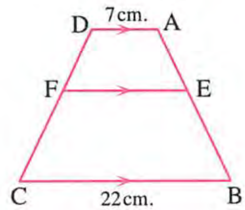
(21) The quadratic equation whose roots $\frac{3}{i}$, $\frac{3+3i}{1-i}$ is ....

(a) $X^2 - 3X + 9 = 0$ (b) $X^2 + 9 = 0$ (c) $X^2 + 9X + 9 = 0$ (d) $X^2 = 9$

(22) If the degree measure of an angle is $64° 48'$, then its radian measure is ....

(a) 0.18^{rad} (b) 0.36^{rad} (c) 11.3^{rad} (d) $\frac{9}{25}\pi$

(23) In the opposite figure :

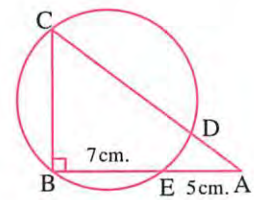


If $\frac{AE}{EB} = \frac{2}{3}$, then FE = …………… cm.

(a) 9 (b) 11

(c) 13 (d) 15

(24) In the opposite figure :



BC = 9 cm.

, then DC = …………… cm.

(a) 9 (b) 10

(c) 11 (d) 12

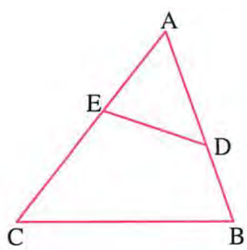
(25) The sign of the function $f : f(X) = 7 - X$ is negative in the interval ……………

(a) $]-\infty , 7[$ (b) $]-\infty , \infty[$ (c) $]7 , \infty[$ (d) $]-7 , 7[$

(26) If $\sin \theta = -\frac{1}{2}$, $\cos \theta = \frac{\sqrt{3}}{2}$, then $\theta =$ ……………

(a) 30° (b) 150° (c) 210° (d) 330°

(27) In the opposite figure :



Δ ABC ~ Δ AED if AD = 3 cm. , BD = 2 cm.

, AE = 2.5 cm. , then EC = …………… cm.

(a) 2.5 (b) 3

(c) 4.5 (d) 3.5

(28) $(1 - i)^{12} =$ ……………

(a) – 64 i (b) 64 i (c) – 64 (d) 64

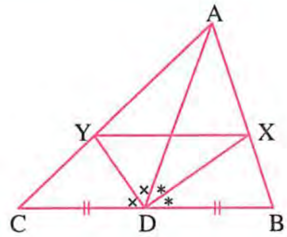
Second Essay questions

Answer the following questions :

1 Find the solution set of the inequality : $X^2 - 5X + 6 \leq 0$ in $\mathbb{R}$

---

2 **In the opposite figure :**



$\overline{AD}$ is the median of Δ ABC

, $\overrightarrow{DX}$ bisects $\angle$ ADB

, $\overrightarrow{DY}$ bisects $\angle$ ADC

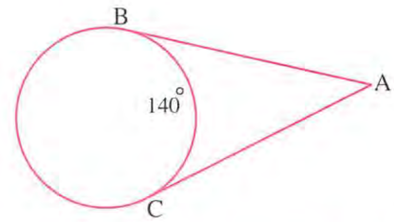
Prove that : $\overline{XY} \parallel \overline{BC}$

---

3 **Prove that :** $\sin 60° \cos 30° - \cos 60° \sin 30° = \sin^2 \frac{\pi}{4}$

4 $\overline{AB}$, $\overline{AC}$ are two tangents to the circle m $(\overset{\frown}{BC}) = 140°$

, find m $(\angle A)$

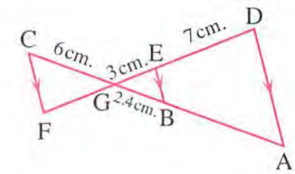


5 **In the opposite figure :**

$\overline{AD} \parallel \overline{BE} \parallel \overline{FC}$ and $\overleftrightarrow{AC}$, $\overleftrightarrow{DF}$

are two transversal intersect at G

Find the length of each of $\overline{GF}$, $\overline{GA}$



10 El-Fayoum Governorate — Directorate of Education

First Multiple choice questions

Choose the correct answer from the given ones :

(1) The conjugate of the number (3 i – 4) is ....

(a) 3 i + 4 (b) – 3 i – 4 (c) – 3 i + 4 (d) 3 i – 4

(2) The two roots of the equation : $x^2 - 5x + 11 = 0$ are ....

(a) two complex and non-real roots. (b) two rational roots.

(c) two different real roots. (d) two equal real roots.

(3) The sum of the two roots of the equation : $4x^2 + 4x - 35 = 0$ is ....

(a) – 1 (b) – 4 (c) 1 (d) $\frac{-35}{4}$

(4) If L and M are the two roots of the equation : $x^2 - 4x + 1 = 0$

, then the value of $L^2 - 4L + 1 =$ ....

(a) 0 (b) – 4 (c) 1 (d) – 1

(5) The sign of the function $f : f(x) = 6 - 2x$ is positive at ....

(a) $x > 3$ (b) $x \leq 3$ (c) $x < 3$ (d) $x \geq 3$

(6) If one of the two roots of the equation : $x^2 - (b - 3)x + 5 = 0$ is the additive inverse of the other root , then b = ...

(a) – 5 (b) – 3 (c) 3 (d) 5

(7) $\sqrt{-16} =$ ....

(a) – 4 (b) 4 (c) 2 i (d) 4 i

(8) The angle of measure 1670° lies in the .... quadrant.

(a) first. (b) second. (c) third. (d) fourth.

(9) In a circle of diameter length 12 cm. , the length of the arc subtended by a central angle of measure 60° equals .... cm.

(a) 5π (b) 4π (c) 3π (d) 2π

(10) If $\csc \theta = 2$, where θ is a positive acute angle , then the measure of angle $\theta =$ ....

(a) 15° (b) 30° (c) 45° (d) 60°

(11) The simplest form of the expression : $\tan(90° - \theta) + \tan(90° + \theta)$ is ....

(a) $2\cot\theta$ (b) $2\tan\theta$ (c) zero (d) $\tan\theta + \cot\theta$

(12) The range of the function $f : f(X) = \cos 5\theta$ is ....

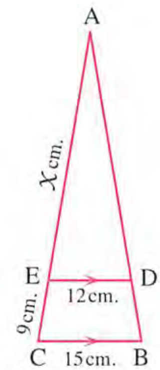
(a) $\{5 , -5\}$ (b) $[-1 , 1]$ (c) $]-5 , 5[$ (d) $[-5 , 5]$

(13) If K is the scale factor of similarity of polygon M_1 to polygon M_2 and $0 < K < 1$, then the polygon M_1 is .... to polygon M_2

(a) congruent to (b) enlargement

(c) minimization (d) of double area

(14) In the opposite figure :



$X =$ .... cm.

(a) 12

(b) 24

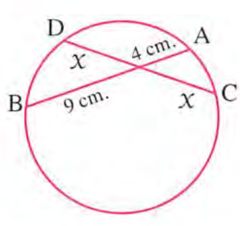
(c) 36

(d) 48

(15) The ratio between the perimeters of two similar polygons is 4 : 9 , so the ratio between their areas is ....

(a) 4 : 9 (b) 9 : 4 (c) 2 : 3 (d) 16 : 81

(16) In the opposite figure :

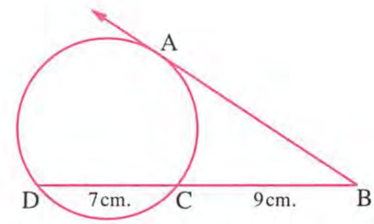


$X =$ ....

(a) 6 (b) – 6

(c) ± 6 (d) 36

(17) In the opposite figure :

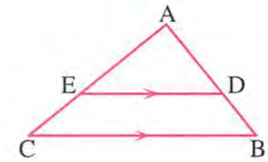


AB = ... cm.

(a) 63 (b) 144

(c) 12 (d) $\frac{9}{16}$

(18) In the opposite figure :

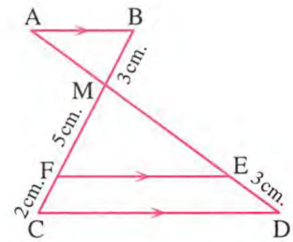


If $\frac{AD}{DB} = \frac{5}{3}$, then $\frac{AB}{BD} =$ ....

(a) $\frac{3}{5}$ (b) $\frac{8}{3}$

(c) $\frac{3}{8}$ (d) $\frac{5}{8}$

(19) In the opposite figure :

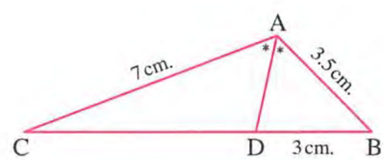


AE = ... cm.

(a) 6 (b) 7.5

(c) 10 (d) 12

(20) In the opposite figure :

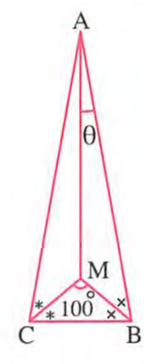


CD = ... cm.

(a) 4.5 (b) 5

(c) 4.9 (d) 6

(21) In the opposite figure :



θ = ...

(a) 10°

(b) 20°

(c) 40°

(d) 80°

(22) If M is a circle of radius length 3 cm. , A is a point lies in its plane where MA = 4 cm. , then $P_M(A) =$ ....

(a) $\sqrt{7}$ (b) 9 (c) 7 (d) – 7

(23) The product of the two roots of the equation : $3x^2 - 4 = 0$ multiplying by the sum of the two roots of the equation : $x^2 - 3x = 0$ is ....

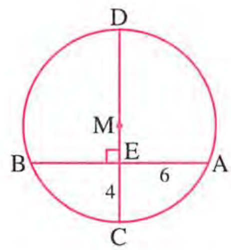
(a) 12 (b) – 3 (c) – 4 (d) 3

(24) The function $f : f(X) = -3$ is negative in the interval ....

(a) $]-\infty , 3[$ only. (b) $]-3 , 3[$ only.

(c) $]-\infty , \infty[$ (d) $]-2 , 2[$ only.

(25) In the opposite figure :



The radius length

of the circle = ... cm.

(a) 9 (b) 4.5

(c) 6 (d) 6.5

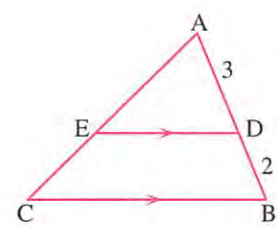
(26) Two similar polygons , the ratio between their perimeters equal 4 : 9 , then the ratio between the lengths of two corresponding sides is ....

(a) 4 : 9 (b) 2 : 3 (c) 16 : 81 (d) 9 : 4

(27) If Δ ABC ~ Δ DEF , BC = 3 EF , then the scale factor of similarity of the two triangle = ...

(a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 3

(28) In the opposite figure :



If $\overline{DE} \parallel \overline{BC}$, then $\frac{\text{The area of } (\Delta \text{ ADE})}{\text{The area of } (\Delta \text{ ABC})} =$ ....

(a) $\frac{3}{2}$ (b) $\frac{9}{4}$

(c) $\frac{9}{25}$ (d) $\frac{3}{5}$

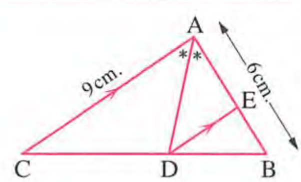
Second Essay questions

Answer the following questions :

1 Find in $\mathbb{R}$ the solution set of the inequality : $X(X + 2) - 3 \leq 0$

2 Find the value of : $\cos 90° \csc 30° + \sec^2 45° \sin 30° - \cos 270° \sin 180°$

3 **In the opposite figure :**



$\overrightarrow{AD}$ bisect $\angle$ BAC , $\overline{ED} \parallel \overline{AC}$

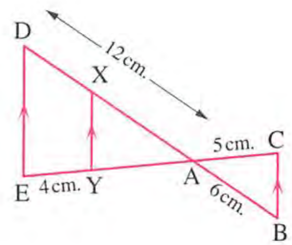
Prove that : $\frac{BE}{EA} = \frac{BA}{AC}$

and if AC = 9 cm. , AB = 6 cm. **find the length of each of :** $\overline{AE}$ and $\overline{BE}$

4 **In the opposite figure :**

$\overline{CE} \cap \overline{BD} = \{A\}$, $X \in \overline{AD}$, $Y \in \overline{AE}$, where $\overline{XY} \parallel \overline{BC} \parallel \overline{ED}$, if AB = 6 cm. , AC = 5 cm. , AD = 12 cm. and EY = 4 cm.

Find the length of each of : $\overline{AE}$ and $\overline{DX}$



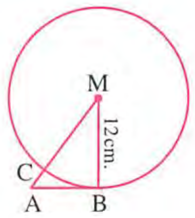
5 **In the opposite figure :**

$\overline{AB}$ is a tangent to the circle M at B , $\overline{MA}$ intersects the circle M at C

If the radius length of the circle equals 12 cm. , $P_M(A) = 81$

, then find : (1) The length of $\overline{AB}$

(2) The length of $\overline{AC}$



Model 1

Interactive test 1

First Multiple choice questions

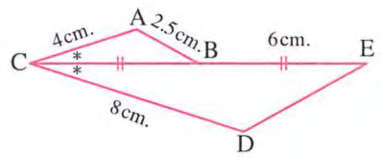
Choose the correct answer from the given ones :

1 If $\tan(180° + \theta) = 1$ where θ is the smallest positive angle , then $\theta =$ ....

(a) 60° (b) 30° (c) 45° (d) 135°

---

2 **In the opposite figure :**



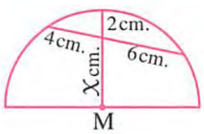
If B is the midpoint of $\overline{CE}$, then DE = ... cm.

(a) 4 (b) 5

(c) 6 (d) 7

---

3 **In the opposite figure :**



M is the centre of semi-circle

, then $X =$ ....

(a) 5 (b) 7 (c) 8 (d) 12

---

4 The solution set of the inequality $(X - 3)(X - 7) < 0$ in $\mathbb{R}$ is ....

(a) $\{3 , 7\}$ (b) $]3 , 7[$ (c) $[3 , 7]$ (d) $\mathbb{R} - [2 , 5]$

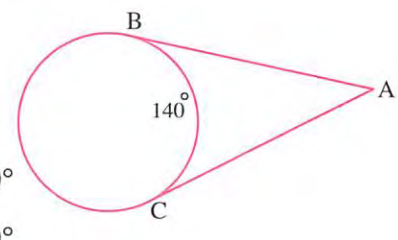
---

5 The exterior bisector at the vertex of an isosceles triangle .... to the base.

(a) parallel (b) perpendicular (c) bisects (d) equal

---

6 **In the opposite figure :**



$\overline{AB}$, $\overline{AC}$ are two tangents to the circle

$m(\widehat{BC}) = 140°$, then $m(\angle A) =$ ....

(a) 30° (b) 40°

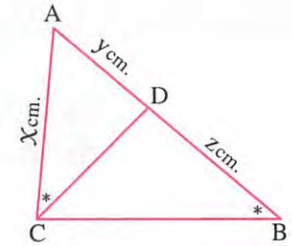
(c) 60° (d) 80°

---

7 The roots of the equation : $k X^2 - 12 X + 9 = 0$ are equal if ....

(a) $k > 4$ (b) $k < 4$ (c) $k = 4$ (d) $k = 9$

8 **In the opposite figure :**



If $X^2 - y^2 = 16$

, then y z = ……………

(a) 4 (b) 8

(c) 12 (d) 16

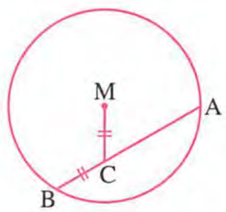
---

9 The simplest form of the imaginary number i^{42} is ……………

(a) 1 (b) – 1 (c) i (d) – i

---

10 **In the opposite figure :**



The diameter of circle M is 12 cm. , MC = CB and AC = (BC + 1) cm.

, then AB = …………… cm.

(a) 4 (b) 6

(c) 8 (d) 9

---

11 The degree measure of the angle whose measure $\frac{7\pi}{6}$ equals ……………

(a) 105° (b) 210° (c) 420° (d) 840°

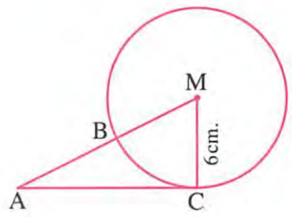
---

12 ABC is a right-angled triangle at A , $\overline{AD} \perp \overline{BC}$ where D $\in \overline{BC}$, then $(AB)^2$ = ……………

(a) BD × BC (b) BD × DC (c) CD × CB (d) AB × AC

---

13 **In the opposite figure :**



$\overline{AC}$ touches the circle M at C , MC = 6 cm.

, $P_M(A) = 64$

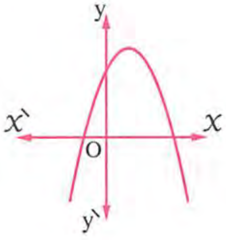
, then AB = …………… cm.

(a) 3 (b) 4

(c) 5 (d) 6

---

14 The opposite figure represents the curve $y = a X^2 + b X + c$ which of the following is true ?



(a) a > 0 , c > 0

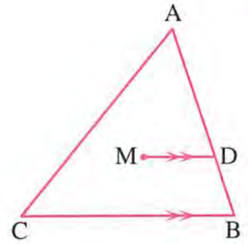
(b) a > 0 , c < 0

(c) a < 0 , c > 0

(d) a < 0 , c < 0

15 In the opposite figure :

If M is the point of concurrence of medians of Δ ABC , and $\overline{DM} \parallel \overline{BC}$, then $\frac{DM}{BC} = \cdots\cdots$



(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{4}$

---

16 If A and B are the measures of two equivalent angles which of the following represents two equivalent angles also where $C \in \mathbb{Z}$?

(a) (A + C) , (B + C) (b) (A – C) , (B – C)

(c) (CA) , (CB) (d) All the previous.

---

17 If the curve $y = x(a - x)$, which of the following statements is true ?

[1] The curve intersects x-axis at (0 , 0) , (a , 0)

[2] The vertex of the curve is $\left(\frac{a}{2} , \frac{a^2}{4}\right)$

[3] The axis of symmetry of the curve is $x = a$

(a) **[1]** , **[2]** only (b) **[1]** , **[3]** only

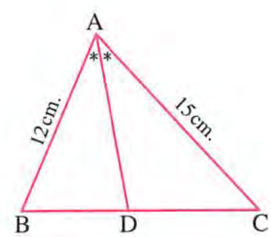
(c) **[2]** , **[3]** only (d) **[1]** , **[2]** and **[3]**

---

18 In the opposite figure :

If area of Δ ABC = 72 cm^2.

, then area of Δ ADB = $\cdots\cdots$ cm^2.



(a) 24 (b) 28

(c) 32 (d) 40

---

19 If L , M are the two roots of the equation : $x^2 - 5x + 6 = 0$, then the quadratic equation whose roots are L + 1 , M + 1 is $\cdots\cdots$

(a) $x^2 - 7x + 8 = 0$ (b) $(x + 1)^2 - 5(x + 1) + 6 = 0$

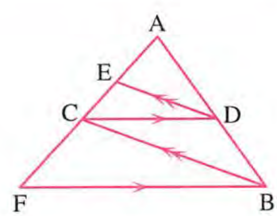
(c) $x^2 - 7x + 12 = 0$ (d) $x^2 + 7x - 10 = 0$

---

20 In the opposite figure :

$\overline{DE} \parallel \overline{BC}$, $\overline{DC} \parallel \overline{BF}$

, then AE × AF = $\cdots\cdots$



(a) $(AC)^2$ (b) AD × AB

(c) AE × AC (d) AC × AB

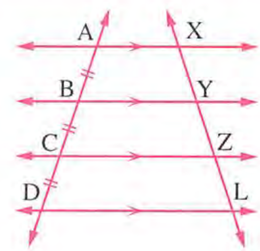
21 ABC is right-angled triangle at B , draw $\overrightarrow{AD}$ to bisect $\angle$ A and intersects $\overline{BC}$ at D , if the length of $\overline{BD} = 24$ cm. , BA : AC = 3 : 5 , then the perimeter of Δ ABC = ... cm.

(a) 177 (b) 192 (c) 213 (d) 184

22 If the ratio between the perimeters of two similar polygons is 4 : 9 , then the ratio between their areas ....

(a) 2 : 3 (b) 4 : 13 (c) 16 : 81 (d) 4 : 9

23 **In the opposite figure :**



$\overleftrightarrow{XA} \,//\, \overleftrightarrow{YB} \,//\, \overleftrightarrow{ZC} \,//\, \overleftrightarrow{LD}$, $\overleftrightarrow{XL}$, $\overleftrightarrow{AD}$ are two transversals

, if XZ = 7 cm.

, then XL = ... cm.

(a) 7 (b) 10

(c) 3.5 (d) 10.5

24 The solution set of the inequality $x(x-1) > 0$ in $\mathbb{R}$ is ....

(a) $\{0 , 1\}$ (b) $]0 , 1[$ (c) $[0 , 1]$ (d) $\mathbb{R} - [0 , 1]$

25 The minimum value of the function $f : f(\theta) = 5 \cos 7\theta$ is ....

(a) 5 (b) zero (c) -5 (d) -7

26 If $\sin \theta = -\frac{1}{2}$, $\tan \theta > 0$, then $\theta =$ ....

(a) 30° (b) 150° (c) 210° (d) 330°

27 If $f : f(x) = a x^2 + b x + c$ is positive for all real values of x , then ....

(a) $b^2 - 4ac < 0$ (b) $b^2 - 4ac > 0$

(c) $b^2 - 4ac = 0$ (d) $b^2 - 4ac \leq 0$

28 If one of the two roots of the equation : $a x^2 - 3x + 2 = 0$ is the multiplicative inverse of the other root , then a = ...

(a) $\frac{1}{2}$ (b) 3 (c) 2 (d) -2

Second | Essay questions

Answer the following questions :

1 In Δ ABC , D $\in \overline{AB}$ where AD = 5 cm. , DB = 3 cm.

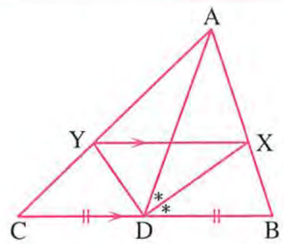
, E $\in \overline{AC}$ where AE = 4 cm. , EC = 6 cm.

Prove that :

[1] Δ ADE ~ Δ ACB **[2]** DBCE is a cyclic quadrilateral.

2 Investigate the sign of the function $f : f(X) = X^2 + 3X - 10$ and illustrate it on a number line , then determine the solution set of the inequality : $X^2 + 3X \le 10$

3 **In the opposite figure :**



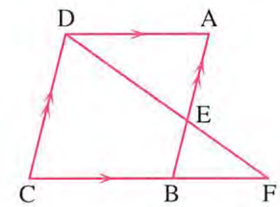
[1] Prove that : $\overrightarrow{DY}$ bisects $\angle$ ADC

[2] Find : m ($\angle$ XDY)

4 If $\cos X = \frac{3}{5}$, $270° < X < 360°$

Find the value of : $\sin(180° - X) + \tan(90° - X) + \tan(270° - X)$

5 **In the opposite figure :**



ABCD is a parallelogram

, E $\in \overline{AB}$ where $\frac{AE}{EB} = \frac{3}{2}$

, $\overrightarrow{DE} \cap \overrightarrow{CB} = \{F\}$

[1] Prove that : Δ DCF ~ Δ EAD

[2] Find : $\frac{a(\Delta DCF)}{a(\Delta EAD)}$

Model 2

Interactive test 2

First | Multiple choice questions

Choose the correct answer from the given ones :

1 The triangle in which the measure of two angles are 50° , 60° is similar to the triangle in which the measure of two angles are 60° , ...

(a) 70° (b) 110° (c) 80° (d) 30°

2 If L , 2 – L are the roots of the equation : $x^2 + k\,x + 6 = 0$, then k = ...

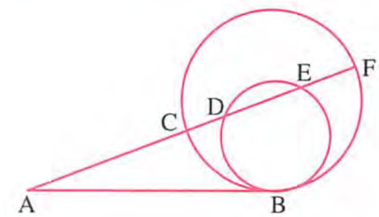
(a) 1 (b) – 2 (c) 3 (d) 5

3 The function $f : f(x) = (x - 1)(x + 3)$ is positive in the interval ....

(a) $[-3 , 1]$ (b) $]-3 , 1[$ (c) $\mathbb{R} - [-3 , 1]$ (d) $\mathbb{R} -]-3 , 1[$

4 **In the opposite figure :**

If $\overline{AB}$ is a common tangent to two circles touching externally at B , then AC : AD = ... : ....

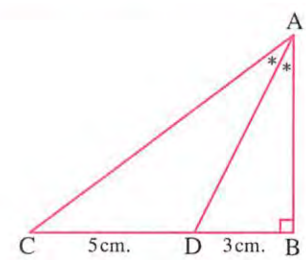


(a) AB : AF (b) AF : AE

(c) AD : AF (d) AE : AF

5 **In the opposite figure :**

AB = ... cm.



(a) 4 (b) 5

(c) 6 (d) 7

6 If a , b are two rational numbers , then the two roots of the equation : $a\,x^2 + b\,x + b - a = 0$ are ....

(a) complex and non-real. (b) complex conjugate.

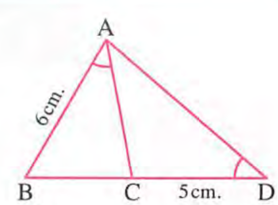
(c) rationals. (d) equal.

7 **In the opposite figure :**

$C \in \overline{BD}$, m (∠ D) = m (∠ BAC)

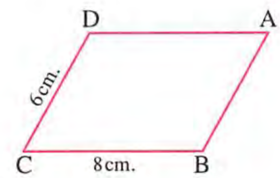
, AB = 6 cm. , CD = 5 cm.

, then BC = ... cm.



(a) 3 (b) 4 (c) 5 (d) 6

8 **In the opposite figure :**



ABCD is a parallelogram

, its area = 40 cm^2.

, then m ($\angle$ A) $\simeq$ ....

(a) 37° (b) 56° (c) 53° (d) 34°

9 If P_M (A) = P_N (A) where M , N are two circles , then ....

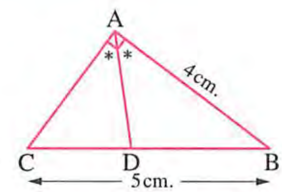
(a) AM = AN

(b) The radius length of M = the radius length of N

(c) A lies on the line of intersection of the two circles.

(d) A lies on the principle axis of the two circles M , N

10 **In the opposite figure :**



BC = 5 cm. , AB = 4 cm. , $\overline{AB} \perp \overline{AC}$, then $\frac{BD}{DC}$ = ...

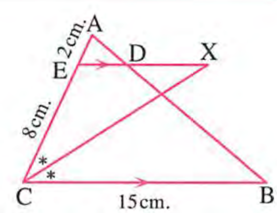
(a) $\frac{4}{5}$ (b) $\frac{3}{5}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$

11 The arc length in a circle of raduis 6 cm. opposite to central angle of measure $\frac{\pi}{2}$ is ....

(a) $\frac{3\pi}{2}$ cm. (b) 2π cm. (c) $\frac{5\pi}{2}$ cm. (d) 3π cm.

12 **In the opposite figure :**



If $\overrightarrow{CX}$ bisects $\angle$ ACB , $\overline{XD}$ // $\overline{BC}$, then XD = ... cm.

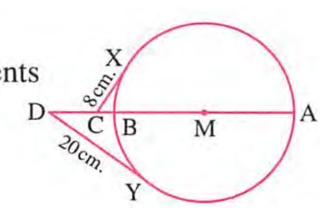
(a) 3 (b) 4

(c) 5 (d) 6

13 If ABC is right-angled triangle at B , sin A + cos C = 1 , then tan C = ...

(a) 1 (b) – 1 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

14 **In the opposite figure :**



If $\overline{AB}$ is a diameter in circle M , $\overline{CX}$, $\overline{YD}$ are two tangent segments to the circle M , AB = 30 cm. , CX = 8 cm.

, DY = 20 cm. , then DC = ... cm.

(a) 2 (b) 6 (c) 8 (d) 10

15 The solution set of the equation : $x^2 + 9 = 0$ in the set of complex numbers is ....

(a) $\{3 , -3\}$ (b) $\{-3i\}$ (c) $\{3i , -3i\}$ (d) $\varnothing$

16 **In the opposite figure :**

The area of Δ ABD = ... cm^2.

(a) 36 (b) 48

(c) 54 (d) 72

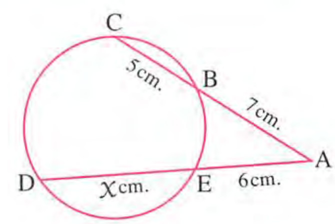
17 If the solution set of the inequality : $x^2 - 4 \le x + k$ is $[-2 , 3]$, then k = ...

(a) – 6 (b) 1 (c) 2 (d) 10

18 The range of the function $f : f(\theta) = 3 \sin 2\theta$ is ....

(a) $[-2 , 2]$ (b) $]-2 , 2[$ (c) $[-3 , 3]$ (d) $]-3 , 3[$

19 **In the opposite figure :**



AB = 7 cm. , BC = 5 cm. , AE = 6 cm.

, DE = x cm. , then the value of x = ...

(a) 5 (b) 14

(c) 12 (d) 8

20 A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B , draw $\overrightarrow{AD}$ to intersect the circle at C and D where C $\in \overline{AD}$, if m $(\widehat{DB}) = 150°$, m $(\widehat{BC}) = 80°$

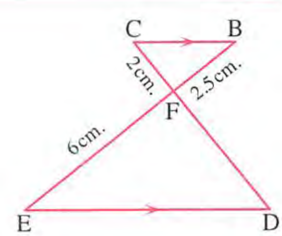
, then m ($\angle$ A) = ... °

(a) 115 (b) 35 (c) 70 (d) 60

21 The terminal side of angle θ in standard position intersects the unit circle at point B $\left(x , \frac{3}{5}\right)$ where $x < 0$, then $\sin(90° + \theta)$ = ...

(a) – 0.8 (b) – 0.6 (c) 0.8 (d) 0.6

22 **In the opposite figure :**



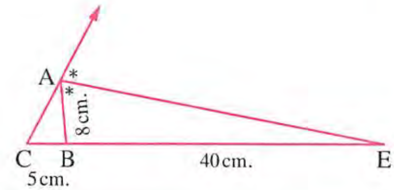
FD = ... cm.

(a) 3.6 (b) 4

(c) 4.2 (d) 4.8

23 **In the opposite figure :**

AE = ... cm.



(a) 32 (b) 45

(c) 48 (d) $24\sqrt{3}$

---

24 If $\sin X = \cos y$, then $\sin (X + y) =$ ....

(a) 1 (b) zero (c) -1 (d) otherwise.

---

25 If one of the roots of the equation : $X^2 - (m + 3) X + 3 = 0$ is additive inverse of the other , then m = ...

(a) 3 (b) -3 (c) zero (d) otherwise.

---

26 The two roots of the equation : $a X^2 + b X + c = 0$ are real equal if $b^2 =$ ....

(a) 2 a c (b) a c (c) 4 a c (d) -4 a c

---

27 If L , M are the two roots of the equation : $X^2 + X + 1 = 0$, then L + M + LM = ...

(a) zero (b) 1 (c) -1 (d) 2

---

28 If $X + y\,i = (2 - 3\,i)^2$, then $X + y =$ ....

(a) $-5 - 12\,i$ (b) -17 (c) 17 (d) 60

Second Essay questions

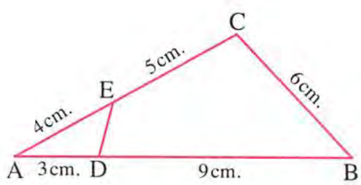
Answer the following questions :

1 **In the opposite figure :**

$E \in \overline{AC}$, $D \in \overline{AB}$ where AD = 3 cm.

, DB = 9 cm. , BC = 6 cm. , EC = 5 cm. , EA = 4 cm.

Prove that : Δ ADE ~ Δ ACB , then find the length of $\overline{ED}$



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2 Find the general solution of the equation : $\tan (\theta + 20°) = \cot (3\,\theta + 30°)$, then find the values of $\theta \in]0° , 90°[$

---

3 In Δ ABC , $\overrightarrow{AD}$ bisects the interior angle and intersects $\overline{BC}$ at D , if AC = 15 cm. , AB = 27 cm. , BD = 18 cm. , calculate the lengths of $\overline{CD}$ and $\overline{AD}$

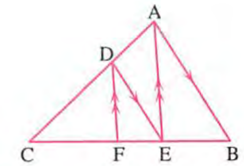
---

4 Find the values of X , y that satisfy the equation : $\frac{(4 - 3\,i)(4 + 3\,i)}{2 + i} = X + y\,i$

5 **In the opposite figure :**

ABC is a triangle , $D \in \overline{AC}$

, $\overline{DE} \parallel \overline{AB}$, $\overline{DF} \parallel \overline{AE}$ **Prove that :** $(CE)^2 = CF \times CB$



Model 3

Interactive test 3

First Multiple choice questions

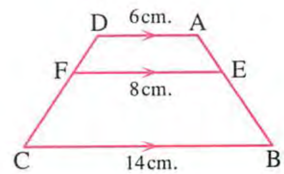
Choose the correct answer from the given ones :

1 The simplest form of the imaginary number i^{73} = ...

(a) – 1 (b) 1 (c) i (d) – i

2 **In the opposite figure :**

$\frac{AE}{EB}$ = ...



(a) $\frac{3}{4}$ (b) $\frac{4}{7}$

(c) $\frac{3}{7}$ (d) $\frac{1}{3}$

3 If one of the two roots of the equation : $x^2 - (m + 2)\, x + 3 = 0$ is additive inverse of the other , then m = ...

(a) – 3 (b) – 2 (c) 2 (d) 3

4 If polygon M_1 is magnification of polygon M_2 and k is the ratio of magnification , then ....

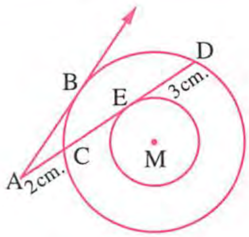
(a) $k > 1$ (b) $k < 1$ (c) $k = 0$ (d) $0 < k < 1$

5 The solution set of the equation $x^2 = x$ in $\mathbb{R}$ is ....

(a) $\{0\}$ (b) $\{1\}$ (c) $\{-1 , 1\}$ (d) $\{0 , 1\}$

6 **In the opposite figure :**

AB = ... cm.



(a) 4 (b) 5

(c) 6 (d) 8

7 If $\overleftrightarrow{AB}$ is a tangent to circle M at point B and $P_M(A) = 25 \text{ cm}^2$, then AB = ... cm.

(a) 5 (b) 10 (c) 15 (d) 25

8 If L , M are the two roots of the quadratic equation $(x - a)(x - b) = k$

, then the quadratic equation whose roots a , b is ....

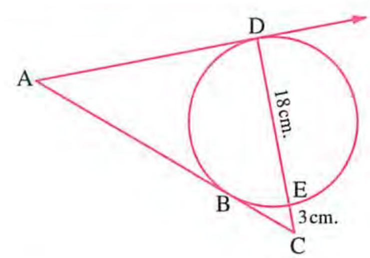
(a) $(x - L)(x - M) = 0$ (b) $(x - L)(x - M) + k = 0$

(c) $(x - L)(x - M) = k$ (d) $x^2 - (L + M)x + k = 0$

9 The radian measure of central angle opposite to an arc of length 3 cm. in a circle its diameter length 4 cm. is ....

(a) $\left(\frac{2}{3}\right)^{rad}$ (b) $\left(\frac{3}{2}\right)^{rad}$ (c) 5^{rad} (d) 6^{rad}

10 **In the opposite figure :**



$\overrightarrow{AD}$, $\overrightarrow{AB}$ are two tangents to the circle at D , B respectively.

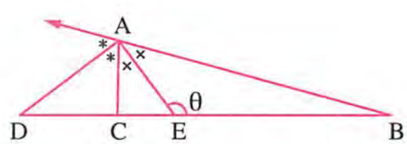
$\overrightarrow{CE}$ intersects the circle at E , D

If CE = 3 cm. , ED = 18 cm.

, then (AC – AD) = ... cm.

(a) $\sqrt{7}$ (b) $2\sqrt{7}$ (c) $3\sqrt{7}$ (d) $6\sqrt{7}$

11 **In the opposite figure :**

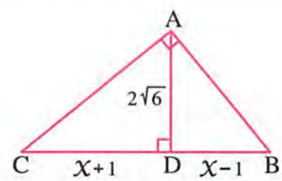


If AD = 8 cm. , AE = 6 cm.

, then tan θ = ...

(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{3}{4}$ (d) $\frac{4}{3}$

12 **In the opposite figure :**



By using the shown givens , then x = ...

(a) 5 (b) 12

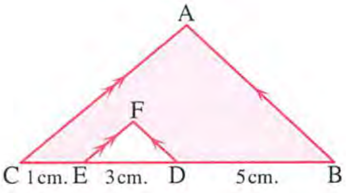
(c) 10 (d) 2.5

13 If sin θ = cos θ where θ is the measure of an acute positive angle

, then tan 2 θ = ...

(a) 1 (b) – 1 (c) undefined. (d) $\sqrt{3}$

14 In the opposite figure :



If the area of Δ DEF = 6 cm^2.

, then the area of the shaded area = ... cm^2.

(a) 27 (b) 36

(c) 48 (d) 54

---

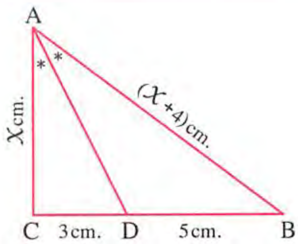
15 The function $f : f(\mathcal{X}) = a\,\mathcal{X}^2 + b\,\mathcal{X} + c$ has one sign in $\mathbb{R}$ when ....

(a) $b^2 - 4ac > 0$ (b) $b^2 - 4ac < 0$

(c) $b^2 - 4ac = 0$ (d) $b^2 - 4ac \geq 0$

---

16 In the opposite figure :



$\mathcal{X}$ = ...

(a) 3 (b) 4

(c) 5 (d) 6

---

17 The simplest form of the expression : $\sin(180° + \theta) \times \sec(270° + \theta)$ = ...

(a) $2 \sin\theta$ (b) 1 (c) -1 (d) $2 \sec\theta$

---

18 If $(3\mathcal{X} - 5)°$ is the smallest positive measure , $(3y - 5)°$ is the greatest negative measure of two equivalent angles in the standerd position , then $\mathcal{X} - y$ = ...

(a) 360° (b) 180° (c) 120° (d) 90°

---

19 $\cos^{-1}\mathcal{X} + \sin^{-1}\mathcal{X}$ = ...

(a) zero (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

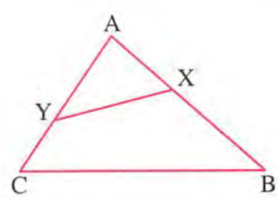
---

20 If $\mathcal{X} + y\,i = (1 + i)^3$, then $\mathcal{X} + y$ = ...

(a) 4 (b) 2 (c) zero (d) 6

---

21 In the opposite figure :



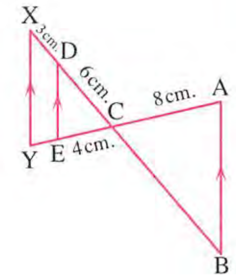
ABC is triangle , $X \in \overline{AB}$, $Y \in \overline{AC}$

If XBCY is a cyclic quadrilateral , then ....

(a) $\frac{AX}{AB} = \frac{AY}{AC}$ (b) $AX \times AB = AY \times AC$

(c) $\frac{AX}{XB} = \frac{AY}{YC}$ (d) $(XY)^2 = AX \times AB$

22 In the opposite figure :



$\overline{AB} // \overline{DE} // \overline{XY}$, AC = 8 cm. , CE = 4 cm.

, CD = 6 cm. , DX = 3 cm. , then BC + EY = ... cm.

(a) 12 (b) 15

(c) 8 (d) 14

---

23 The equation that has the two roots 3 i , – 3 i is ....

(a) $x^2 + 9 = 0$ (b) $x^2 = 9$ (c) $x^2 + 3 = 0$ (d) $x^2 = 3$

---

24 $\sin (90° - \theta) \sec \theta =$ ....

(a) 1 (b) – 1 (c) zero (d) 90°

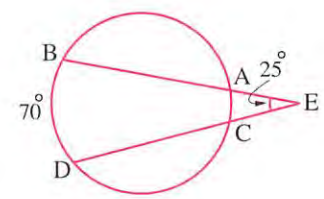
---

25 If k is the scale factor of similarity between two similar polygons , then the two polygons are congruent if ....

(a) k > 1 (b) 0 < k < 1 (c) k = 1 (d) k = 0

---

26 In the opposite figure :



m $(\overset{\frown}{AC})$ = ... °

(a) 20 (b) 30

(c) 40 (d) 50

---

27 If M , (5 – M) are the two roots of the equation : $x^2 - k x + 6 = 0$, then k = ...

(a) – 5 (b) 5 (c) 6 (d) – 8

---

28 The two roots of the equation : $x + \frac{9}{x} = 6$ are ....

(a) two equal real roots. (b) two complex and non real roots.

(c) two different real roots. (d) two equal imaginary numbers.

Second Essay questions

Answer the following questions :

1 The ratio between the length of two corresponding sides of two similar polygons is 5 : 3

If the difference between their areas is 32 cm^2.

Find the area of each polygon.

---

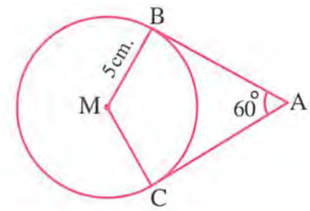
2 Solve the following inequality in $\mathbb{R}$: $(x + 3)^2 \le 10 - 3 (x + 3)$

3 **In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent segments to the circle M at B and C

, m (∠ A) = 60° , MB = 5 cm.

Find the length of the minor arc $\overset{\frown}{BC}$



4 **Prove without using the calculator :**

$\sin (600°) \cos (-30°) + \sin (150°) \cos (240°) = \sin \frac{3\pi}{2}$

5 $\overline{AD}$ is a median in Δ ABC , $\overrightarrow{DX}$ bisects ∠ ADB and intersects $\overline{AB}$ at X

, $\overrightarrow{DY}$ bisects ∠ ADC and intersects $\overline{AC}$ at Y , **prove that :** $\overline{XY}$ // $\overline{BC}$

Model 4

Interactive test 4

First Multiple choice questions

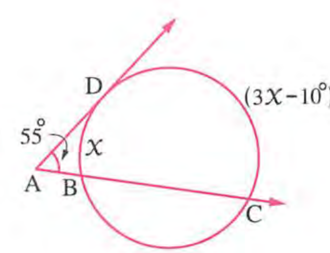
Choose the correct answer from the given ones :

1 **In the opposite figure :**

If $\overrightarrow{AD}$ is a tangent to the circle

, m (∠ A) = 55° , m $(\overset{\frown}{DC}) = (3X - 10°)$

, m $(\overset{\frown}{DB}) = X$, then X = ...°



(a) 120 (b) 60

(c) 30 (d) 15

2 If θ is the measure of an acute angle and sin (θ + 10°) = cos (50°) , then θ = ...

(a) 30° (b) 40° (c) 20° (d) 50°

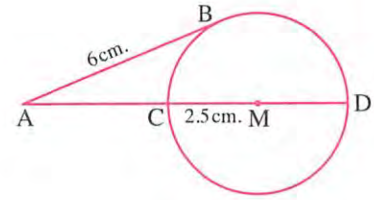
3 The ratio between the length of two radii of two circles is 3 : 5 , if the area of the smaller circle is 27 cm^2 , then the area of the greater circle equals .... cm^2

(a) 45 (b) 50 (c) 75 (d) 100

4 If $X = -1$ is one of the two roots of the equation : $X^2 - kX - 6 = 0$, then k = ...

(a) 5 (b) – 5 (c) 6 (d) – 6

5 **In the opposite figure :**



$\overline{AB}$ is a tangent segment to circle M ,

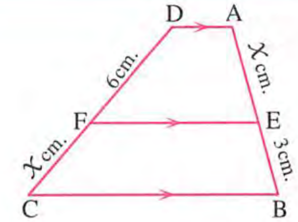
AB = 6 cm. , CM = 2.5 cm.

, then AC = ... cm.

(a) 9 (b) 4 (c) 2.5 (d) 5

---

6 **In the opposite figure :**



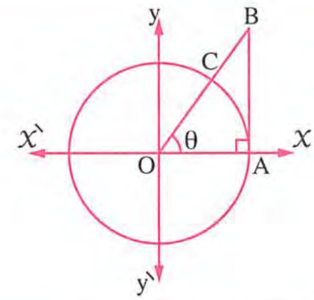
X = ...

(a) 6 (b) $3\sqrt{2}$

(c) $3\sqrt{3}$ (d) 18

---

7 **In the opposite figure :**



$\overline{AB}$ is a tangent segment of a unit circle , then OB = ...

(a) sin θ (b) cos θ

(c) csc θ (d) sec θ

---

8 The function $f : f(X) = 3 - X$ is non-negative at $X \in$ ....

(a) $]-\infty , 3[$ (b) $]-\infty , 3]$ (c) $[3 , \infty[$ (d) $]3 , \infty[$

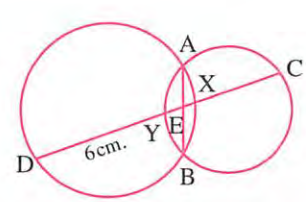
---

9 The degree measure of an inscribed angle opposite an arc whose length 5π cm. in a circle with radius 15 cm. equals ....

(a) 120° (b) 60° (c) 30° (d) 90°

---

10 **In the opposite figure :**



If DY = 6 cm. and $\frac{XE}{EY} = \frac{2}{3}$

, then CX = ... cm.

(a) 2 (b) 3

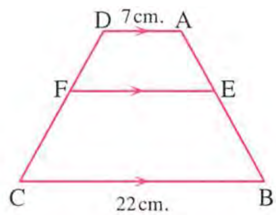
(c) 4 (d) 5

---

11 If the function $f : f(X) = a \cos bX$ where $a > 0$ is a periodic function and its period $\frac{\pi}{2}$ and its range $[-1 , 1]$, then $\left|\frac{a}{b}\right|$ = ...

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

12 **In the opposite figure :**



$\frac{AE}{EB} = \frac{2}{3}$, then FE = ... cm.

(a) 9 (b) 11

(c) 13 (d) 15

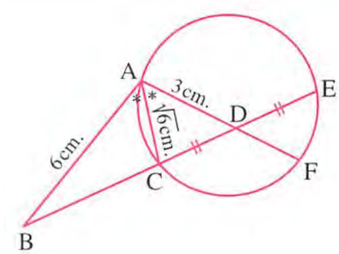
---

13 If Δ ABC $\sim \Delta$ DEF , m ($\angle$ A) = 50° , m ($\angle$ E) = 60° , then m ($\angle$ C) = ...

(a) 110° (b) 70° (c) 100° (d) 120°

---

14 **In the opposite figure :**



$\overrightarrow{AC}$ bisects $\angle$ BAD , D is the midpoint of $\overline{EC}$

, AC = $\sqrt{6}$ cm. , AD = 3 cm.

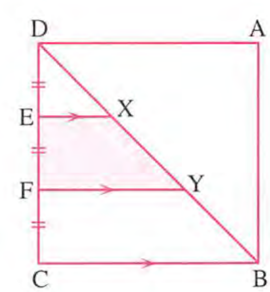
, AB = 6 cm. , then DF = ... cm.

(a) 2 (b) 3

(c) 3.5 (d) 4

---

15 **In the opposite figure :**



ABCD is a square of side length 6 cm.

, DE = EF = FC

, then the area of (polygon XYFE) = ... cm^2.

(a) 6 (b) 8

(c) 10 (d) 12

---

16 If L , M are the two roots of the quadratic equation $X^2 + 1 = 0$

, then $L^{2018} + M^{2018}$ = ...

(a) – 2 i (b) 2 i (c) – 2 (d) 2

---

17 If one of the two roots of the equation $(X + k)^2 - 6X = 0$ is additive inverse of the other

, then k = ...

(a) 6 (b) – 6 (c) 3 (d) 9

---

18 If the solution set of the inequality : $X^2 - 10 < bX$ is $]-2 , 5[$, then b = ...

(a) – 10 (b) – 2 (c) 3 (d) 5

19 The quadratic equation whose roots are : $\frac{3}{i}$, $\frac{3+3i}{1-i}$ is ....

(a) $x^2 - 3x + 9 = 0$ (b) $x^2 + 9 = 0$

(c) $x^2 + 9x + 9 = 0$ (d) $x^2 = 9$

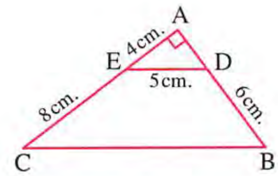
---

20 ABC is a triangle in which AB = 8 cm. , AC = 6 cm. , BC = 7 cm. Draw $\overrightarrow{AD}$ bisects $\angle$ BAC , $\overrightarrow{AD} \cap \overline{BC} = \{D\}$, then BD = ... cm.

(a) 3 (b) 6 (c) 4 (d) $\sqrt{17}$

---

21 **In the opposite figure :**



$\frac{DE}{BC} =$ ....

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

---

22 If one of the roots of the equation : $3x^2 - (k+2)x + k^2 + 2k = 0$ is the multiplicative inverse of the other , then k = ...

(a) – 3 or 1 (b) – 3 or – 1 (c) 3 or – 1 (d) 3 or 1

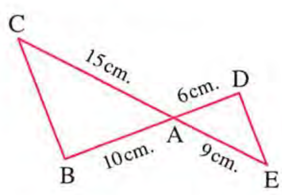
---

23 If $10 \sin x = 6$ where x is the greatest positive angle , $x \in [0 , 2\pi[$, then the numerical value of the expression : sec (540° + x) equals ....

(a) $\frac{3}{5}$ (b) $\frac{-5}{4}$ (c) $\frac{5}{4}$ (d) $\frac{-5}{3}$

---

24 **In the opposite figure :**



$\overline{DB} \cap \overline{EC} = \{A\}$

, AE = 9 cm. , AB = 10 cm. , AC = 15 cm.

, DA = 6 cm. , a (Δ ADE) = 36 cm^2.

, then a (Δ ABC) = ... cm^2.

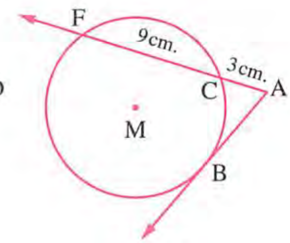
(a) 60 (b) 75 (c) 100 (d) 225

---

25 The range of the function $f : f(x) = 4 \sin x$ where $x \in [0 , \pi]$ equals ....

(a) [0 , 4] (b) [0 , 4[(c) [– 4 , 0] (d) [– 4 , 4]

26 In the opposite figure :



$\overrightarrow{AB}$ touches the circle M at B , $\overrightarrow{AF}$ intersects the circle M at the two points C , F respectively. If AC = 3 cm. , CF = 9 cm. , then P_M (A) = ...

(a) 6 (b) 9 (c) 27 (d) 36

27 If the two roots of the equation : $x^2 - 4x + k = 0$ are real , then $k \in$ ....

(a) $[4 , \infty[$ (b) $]-\infty , 4[$ (c) $]4 , \infty[$ (d) $]-\infty , 4]$

28 If $3x - 2yi = (5 - 2i)^2$, then $y - x =$ ....

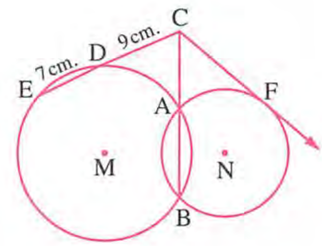
(a) 17 (b) –3 (c) 3 (d) 21 – 20 i

Second Essay questions

Answer the following questions :

1 Investigate in $\mathbb{R}$ the sign of the function $f : f(x) = 8 + 2x - x^2$ showing that on number line , then find in $\mathbb{R}$ the solution set of the inequality : $8 + 2x - x^2 \geq 0$

2 In the opposite figure :



M and N are two intersecting circles at A and B , $C \in \overrightarrow{BA}$, $C \notin \overline{BA}$ Draw $\overrightarrow{CD}$ to intersect circle M at D , E where CD = 9 cm. , DE = 7 cm.

Draw $\overrightarrow{CF}$ to touch circle N at F

[1] Prove that : $P_M(C) = P_N(C)$

[2] If : AB = 10 cm. , find the length of each $\overline{AC}$, $\overline{CF}$

3 In Δ ABC , AB = 8 cm. , AC = 4 cm. , $D \in \overrightarrow{AC}$, $D \notin \overline{AC}$ where CD = 12 cm.

Prove that : $\overline{AB}$ touches the circle passes through the points B , C , D

4 If Δ ABC is right-angled triangle at angle C , sin A + cos B = 1

Find the value of sin 5 A

5 ABC is a triangle , $D \in \overline{AB}$ where AD = 2 BD , $E \in \overline{AC}$ where $\overline{DE} \parallel \overline{BC}$

If the area of Δ ADE = 60 cm^2 , find the area of the trapezium DBCE

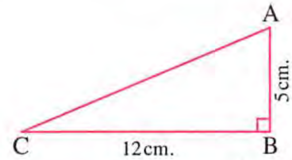
Model 5

Interactive test 5

First Multiple choice questions

Choose the correct answer from the given ones :

1 **In the opposite figure :**



$\sin\left(\tan^{-1}\left(\frac{5}{12}\right)\right) = \ldots\ldots\ldots\ldots$

(a) $\frac{5}{12}$ (b) $\frac{5}{13}$

(c) $\frac{12}{13}$ (d) 13

2 If L , M are the two roots of the equation : $x^2 + 3x - 4 = 0$, then LM = ……………

(a) 3 (b) -3 (c) 4 (d) -4

3 The solution set of the equation : $x^2 + 9 = 0$ in $\mathbb{R}$ is ……………

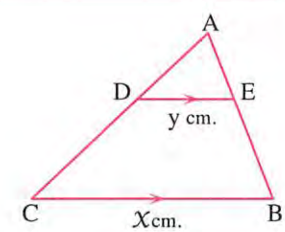
(a) $\{-3\}$ (b) $\{3\}$ (c) $\{-3 , 3\}$ (d) $\varnothing$

4 If S_1 is the solution set of the inequality : $x^2 - x - 2 \le 0$ in $\mathbb{R}$ and S_2 is the solution set of the inequality : $x^2 + x - 2 \le 0$ in $\mathbb{R}$, then $S_1 \cap S_2 = \ldots\ldots\ldots\ldots$

(a) $\varnothing$ (b) $[-2 , 2]$

(c) $[-1 , 1]$ (d) $\mathbb{R} -]-1 , 1[$

5 **In the opposite figure :**



If $\overline{DE} \parallel \overline{BC}$, DE = y cm.

, BC = x cm. and $2x^2 - 3xy - 5y^2 = 0$

, AB = 10 cm. , then EB = …………… cm.

(a) 3 (b) 4 (c) 6 (d) 8

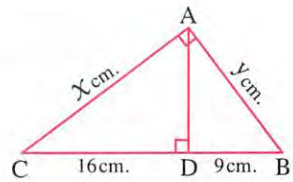
6 The angle with measure 585° in standard position is equivalent to the angle with measure ……………

(a) $\frac{1}{4}\pi$ (b) $\frac{5}{4}\pi$ (c) $\frac{3}{4}\pi$ (d) $\frac{7}{4}\pi$

7 If $\Delta ABC \sim \Delta XYZ$ and AB = 3 XY , then $\frac{a(\Delta XYZ)}{a(\Delta ABC)} = \ldots\ldots\ldots\ldots$

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{4}{1}$ (d) $\frac{9}{1}$

8 **In the opposite figure :**



$\frac{y}{x} = \cdots\cdots$

(a) 1 (b) $\frac{4}{3}$

(c) $\frac{3}{4}$ (d) 2

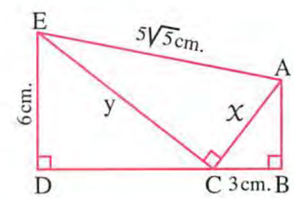
9 The function $y = \sin\left(\frac{\pi}{4} + x\right)$ has maximum value at $x = \cdots\cdots$

(a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$ (c) $\frac{\pi}{4}$ (d) zero

10 The sign of $f : f(x) = -5x$ is negative at $\cdots\cdots$

(a) $x > -5$ (b) $x < -5$ (c) $x > 0$ (d) $x < 0$

11 **In the opposite figure :**



$x + y = \cdots\cdots$ cm.

(a) 12 (b) 15

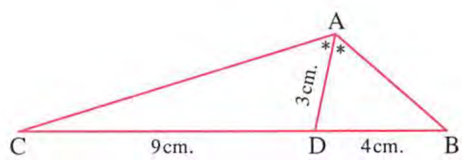
(c) 18 (d) 21

12 If $\overrightarrow{AB}$ is a tangent to a circle at B , $\overrightarrow{AC}$ intersects the circle at C , D where $C \in \overline{AD}$

, AC = 3 cm. AB = 6 cm. , then CD = $\cdots\cdots$ cm.

(a) 6 (b) 9 (c) 12 (d) 15

13 **In the opposite figure :**



$AB \times AC = \cdots\cdots$ cm^2.

(a) 36 (b) 45

(c) 12 (d) 27

14 In circle M , if two chords $\overline{AB}$ and $\overline{CF}$ intersecting at D , then $\cdots\cdots$

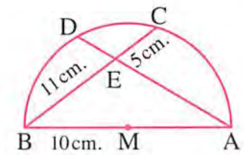
(a) $P_M(D) = (AB)^2 - r^2$ (b) $AD \times DB = AM \times MB$

(c) $P_M(D) + AD \times DB = \text{zero}$ (d) $P_M(D) = CD \times DF$

15 If $\tan(4\theta) = \cot(5\theta)$, then $\sin(3\theta) = \cdots\cdots$ where 3θ is the measure of an acute angle.

(a) $\frac{1}{2}$ (b) 1 (c) -1 (d) $\frac{\sqrt{3}}{2}$

16 In the opposite figure :



The radius length of semicircle (M) = 10 cm.

, then ED = ... cm.

(a) $\frac{50}{13}$ (b) $\frac{55}{13}$

(c) $\frac{57}{13}$ (d) $\frac{59}{13}$

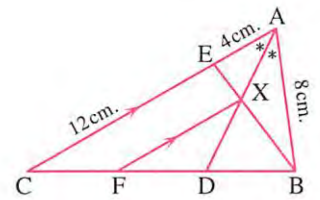
---

17 If the two roots of the equation : a x^2 + b x + c = 0 are equal in value but different in signs , then ....

(a) c = 0 (b) a = 0 (c) b = 0 (d) otherwise.

---

18 In the opposite figure :



$\frac{DF}{BC}$ = ...

(a) $\frac{4}{3}$ (b) $\frac{2}{3}$

(c) $\frac{3}{5}$ (d) $\frac{1}{3}$

---

19 If the distance between point A from the centre of a circle equals 24 cm. and the power of this point with respect to this circle equals 176 , then the radius length of this circle equals .... cm.

(a) $4\sqrt{47}$ (b) 400 (c) 20 (d) 38

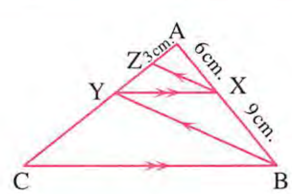
---

20 The length of an arc opposite to a central angle of measure 150° in a circle with radius length 8 cm. equals .... cm.

(a) $\frac{20}{3}\pi$ (b) $\frac{17}{2}\pi$ (c) 8π (d) 20

---

21 In the opposite figure :



$\overline{XY}$ // $\overline{BC}$, $\overline{XZ}$ // $\overline{BY}$

, AX = 6 cm. , XB = 9 cm. , AZ = 3 cm.

, then the length of $\overline{ZC}$ = ... cm.

(a) 4.5 (b) $15\frac{3}{4}$ (c) 15 (d) $12\frac{3}{4}$

---

22 If sin 2 θ = cos θ , then θ could be equal ....°

(a) 18 (b) 30 (c) 36 (d) 45

23 If (2 i) is a root of the quadratic equation : $x^2 + a x + b = 0$ where the coefficients of its terms are real numbers , then all the following are true except ....

(a) the other root of the quadratic equation is (– 2 i)

(b) the sum of the roots = zero

(c) the product of the roots = – 4

(d) the discriminant of the quadratic equation < zero

24 **In the opposite figure :**

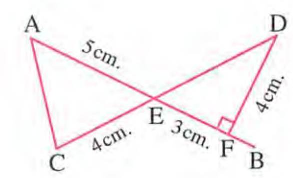
$\overrightarrow{AC}$ bisects $\angle$ A of triangle ABD internally , $\overline{AE} \perp \overline{AC}$, BC = 4 cm. , CD = 3 cm. , then BE : ED = ...

(a) 7 : 4 (b) 7 : 3 (c) 3 : 4 (d) 4 : 3

25 If $f(x) = x + 2$, where $x \in]-4 , 3[$, then $f(x)$ is positive at $x \in$ ....

(a) $]-\infty , -2[$ (b) $]-2 , \infty[$ (c) $]-4 , -2[$ (d) $]-2 , 3[$

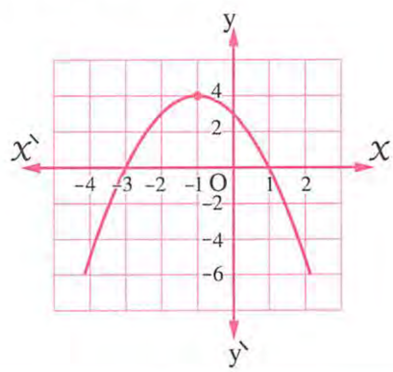
26 **In the opposite figure :**



If $\overline{AB} \cap \overline{DC} = \{E\}$, AE = 5 cm. , EF = 3 cm. , EC = 4 cm. , DF = 4 cm. , $\overline{DF} \perp \overline{BE}$, the points A , B , C , D lie on a circle , then the length of $\overline{FB}$ = ... cm.

(a) 0.5 (b) 1 (c) 1.5 (d) 2

27 If the opposite figure represents a graph of a quadratic function in one variable , then the rule of the function can be written as ....



(a) $f(x) = -x^2 - 2x + 3$

(b) $f(x) = -x^2 + 2x + 3$

(c) $f(x) = x^2 + 2x + 3$

(d) $f(x) = -x^2 + 2x - 3$

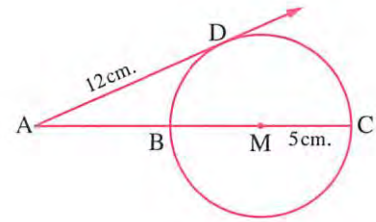
28 If the roots of the equation : $k x^2 - 8x + 16 = 0$ are two complex and non real , then ....

(a) k > 2 (b) k < 2 (c) $k \in]1 , 10[$ (d) k > 1

Second Essay questions

Answer the following questions :

1 In the opposite figure :



The radius of circle M is 5 cm.

, $\overrightarrow{AD}$ is a tangent at D , AD = 12 cm.

Find the length of $\overline{AC}$

---

2 If $\sin \theta = \frac{4}{5}$ where $90° < \theta < 180°$ **Find the value of :**

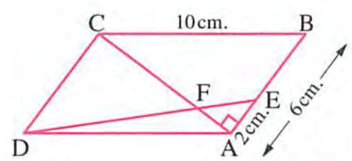
$\sin (180° - \theta) + \tan (360° - \theta) + 2 \sin (270° - \theta)$

---

3 If $X = \frac{13 + 13\,i}{5 + i}$, $y = \frac{5 + i}{1 + i}$, **find :** $X + y$

---

4 In the opposite figure :



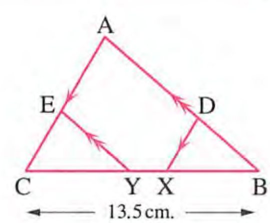
ABCD is a parallelogram in which AB = 6 cm. , BC = 10 cm.

, m ($\angle$ BAC) = 90° , E $\in \overline{AB}$ such that : AE = 2 cm.

, $\overline{DE}$ intersects $\overline{AC}$ at F **Prove that :** Δ AFE is an isosceles triangle.

---

5 In the opposite figure :



ABC is a triangle in which : $\overline{DX} \,//\, \overline{AC}$, $\overline{EY} \,//\, \overline{AB}$,

BC = 13.5 cm. , $\frac{AD}{DB} = \frac{3}{2}$, $EC = \frac{4}{5}$ AE

Find the length of : $\overline{XY}$

Model 6

Interactive test 6

First Multiple choice questions

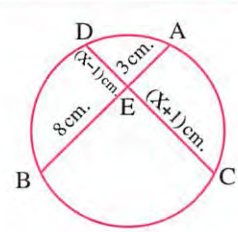
Choose the correct answer from the given ones :

1 If the two roots of the equation : $4X^2 - 12X + c = 0$ are real and equal , then c = ...

(a) 3 (b) 4 (c) 9 (d) 16

---

2 In the opposite figure :



X = ...

(a) 25 (b) 24

(c) 5 (d) 8

3 The solution set of the equation : $(x + 1)^2$ = zero in $\mathbb{R}$ is ....

(a) $\{-1\}$ (b) $\{1\}$ (c) $\{-1 , 1\}$ (d) $\varnothing$

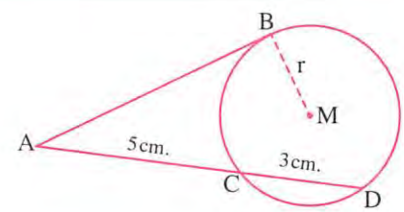
4 If $b^2 - 4ac < 0$ in the equation $a x^2 + b x + c = 0$, then the solution set of the inequality $a x^2 + b x + c < 0$ where a is negative is ....

(a) $\mathbb{R}$ (b) $\varnothing$ (c) $\mathbb{R}^+$ (d) $\mathbb{R}^-$

5 All .... are similar.

(a) triangles (b) rectangles (c) parallelograms (d) squares

6 **In the opposite figure :**

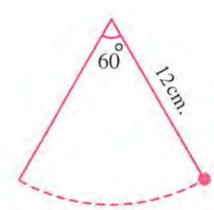


$P_M(A) =$ ....

(a) 25 (b) $(AB)^2 - r^2$

(c) 40 (d) $(AM)^2 - (AB)^2$

7 **In the opposite figure :**



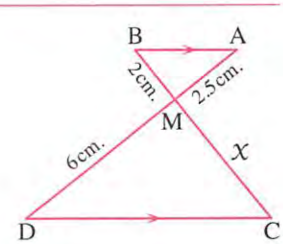
A pendulum swings through an angle of measure 60° if the length of its string is 12 cm.

, then the length of the circular path covered by the pendulum equals ....

(a) 3π cm. (b) 4π cm.

(c) 6π cm. (d) 8π cm.

8 **In the opposite figure :**

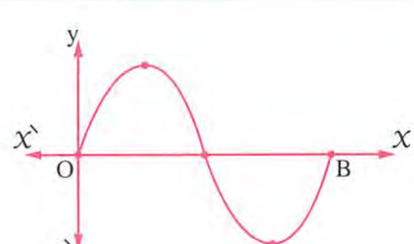


$x =$ .... cm.

(a) 3.6 (b) 4

(c) 4.2 (d) 4.8

9 The opposite figure represents the curve $y = 3 \sin \frac{1}{2} x$, then the x coordinates of the point B is ....



(a) $\frac{\pi}{2}$ (b) π

(c) 2π (d) 4π

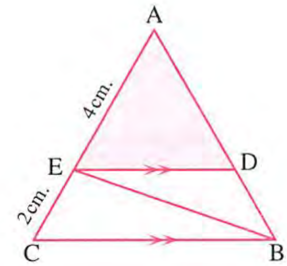
10 sec ($\cos^{-1}$ zero) = ...

(a) 1 (b) – 1 (c) undefind. (d) zero

11 The angle with measure (– 120°) lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

12 **In the opposite figure :**



If $\overline{DE} \parallel \overline{BC}$

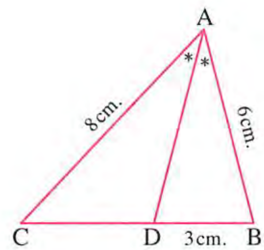
and the area of (Δ EBC) = 9 cm^2.

, then the area of (Δ ADE) = ... cm^2.

(a) 6 (b) 12

(c) 18 (d) 27

13 **In the opposite figure :**



$\overrightarrow{AD}$ bisects $\angle$ BAC , AB = 6 cm.

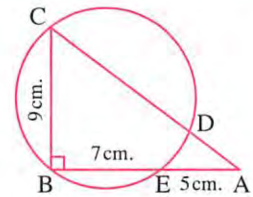
, AC = 8 cm. , BD = 3 cm.

, then AD = ... cm.

(a) 4 (b) 5

(c) 6 (d) 8

14 **In the opposite figure :**



DC = ... cm.

(a) 9 (b) 10

(c) 11 (d) 12

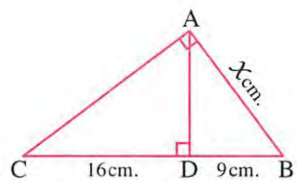
15 If a , b and c are integers , a + b + c = 0 , a ≠ c , then the roots of the equation :

$(b + c - a) x^2 + (c + a - b) x + (a + b - c) = 0$ are ....

(a) real and equal. (b) distinct rational real.

(c) distinct irrational real. (d) not real.

16 **In the opposite figure :**

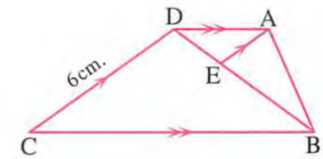


x = ...

(a) 9 (b) 12

(c) 20 (d) 15

17 **In the opposite figure :**



If BE = 2 ED

, then AE = ... cm.

(a) 1 (b) 2 (c) 3 (d) 4

---

18 The sign of function $f : f(X) = 7 - X$ is negative in the interval ....

(a) $]-\infty , 7[$ (b) $]-\infty , \infty[$ (c) $]7 , \infty[$ (d) $]-7 , 7[$

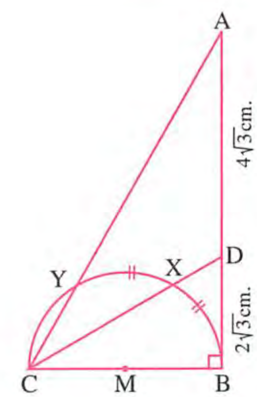
---

19 If $\sin \theta = -\frac{1}{2}$, $\cos \theta = \frac{\sqrt{3}}{2}$, then $\theta =$ ....

(a) 30° (b) 150° (c) 210° (d) 330°

---

20 **In the opposite figure :**



If $m(\widehat{BX}) = m(\widehat{XY})$

and $\overrightarrow{BA}$ is a tangent to the circle M at B

, $BD = 2\sqrt{3}$ cm. , $AD = 4\sqrt{3}$ cm.

, then AY = ... cm.

(a) $4\sqrt{3}$ (b) 6

(c) 9 (d) 12

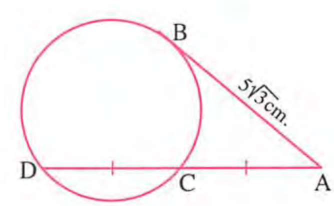
---

21 If $(2 + 3i) + (1 - i) = X + yi$, then $X + y =$ ....

(a) 2 (b) – 4 (c) 5 (d) 7

---

22 **In the opposite figure :**



$\overline{AB}$ is a tangent segment , C is the midpoint of $\overline{AD}$

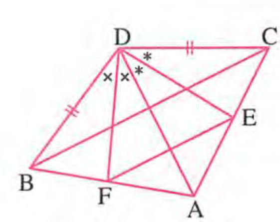
, $AB = 5\sqrt{3}$ cm. , then CD = ... cm.

(a) $2\sqrt{6}$ (b) $5\sqrt{6}$

(c) 5 (d) $2.5\sqrt{6}$

---

23 **In the opposite figure :**



$\frac{CD}{DA} =$ ....

(a) $\frac{AE}{EC}$ (b) $\frac{DE}{DF}$

(c) $\frac{AC}{AB}$ (d) $\frac{BF}{FA}$

24 If $f(x) = x^2 - 7x + 12$, $x \in \mathbb{R}$, then all the following are true except ....

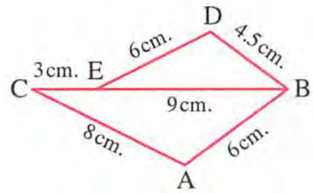
(a) the solution set of the equation $f(x) = 0$ is $\{3 , 4\}$

(b) the solution set of the inequality $f(x) > 0$ is $\mathbb{R} - [3 , 4]$

(c) the solution set of the inequality $f(x) < 0$ is $]3 , 4[$

(d) $f(x)$ is positive in the interval $\mathbb{R} -]3 , 4[$

25 In the opposite figure :



B , E and C are collinear. If CE = 3 cm. , BE = 9 cm. , BD = 4.5 cm. , DE = 6 cm. , BA = 6 cm. , AC = 8 cm. , then the scale factor of the similarity of the two triangles ABC , DBE = ...

(a) 4 : 3 (b) 3 : 4 (c) 16 : 9 (d) 9 : 16

26 If $\tan(180° + 5\theta) + \tan(270° + 4\theta) = 0$, then the value of θ which satisfies the equation , where $\theta \in]0 , \frac{\pi}{2}[$ from the following equals ....°

(a) 5 (b) 10 (c) 20 (d) 90

27 The quadratic equation in which each of its two roots more than the two roots of the equation : $x^2 - 3x + 2 = 0$ by 2 is ....

(a) $x^2 - 3x + 2 = 0$ (b) $x^2 + 7x + 12 = 0$

(c) $x^2 - 7x + 12 = 0$ (d) $x^2 - 7x - 12 = 0$

28 If L is one of the roots of the equation : $x^2 + 4x + 7 = 0$, then $(L + 2)^2 =$ ....

(a) -11 (b) 11 (c) 3 (d) -3

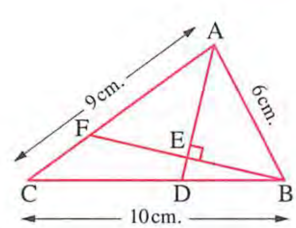
Second Essay questions

Answer the following questions :

1 Find the values of θ where $0° \le \theta \le 90°$ which satisfies :

$\tan(\theta + 20)° = \cot(3\theta + 30°)$

2 In the opposite figure :



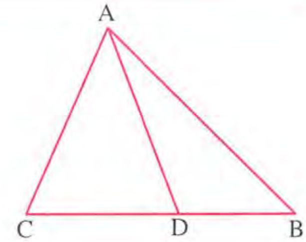
ABC is a triangle in which AB = 6 cm. , AC = 9 cm. and BC = 10 cm. , $D \in \overline{BC}$ where BD = 4 cm. , $\overrightarrow{BE} \perp \overline{AD}$ and intersects $\overline{AD}$ and $\overline{AC}$ at E and F respectively.

[1] Prove that : $\overrightarrow{AD}$ bisects $\angle A$

[2] Find : Area of Δ ABF : Area of Δ CBF

3 If the terminal side of angle θ in the standard position intersects the unit cricle at point $\left(\frac{\sqrt{5}}{3}, \frac{-2}{3}\right)$ **Find the value of :** $\sin\left(\frac{\pi}{2} - \theta\right) + \cot(2\pi - \theta)$

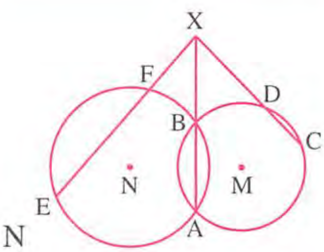
4 In the opposite figure :



If $(AC)^2 = CD \times CB$

Prove that : $\Delta\, ACD \sim \Delta\, BCA$

4 In the opposite figure :



The two circles M and N are intersecting at A and B where $\overrightarrow{AB} \cap \overrightarrow{CD} \cap \overrightarrow{EF} = \{X\}$,

$XD = 2\, DC$, $EF = 10$ cm. and $P_N(X) = 144$

[1] Prove that : $\overleftrightarrow{AB}$ is the principle axis to the two circles M and N

[2] Find the length of each of : $\overline{XC}$ and $\overline{XF}$

[3] Prove that : CDFE is a cyclic quadrilateral.

Model 7

Interactive test 7

First Multiple choice questions

Choose the correct answer from the given ones :

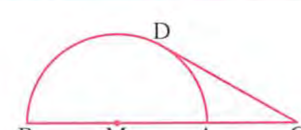
1 If the sum of the measures of interior angles in any convex polygon = 180° (n – 2) where n is the number of sides , then the measure of an interior angle in a regular hexagon in radian = ...

(a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{2}$

2 The angle with measure $\frac{31\pi}{6}$ lies in the .... quadrant.

(a) first (b) second (c) third (d) fourth

3 In the opposite figure :

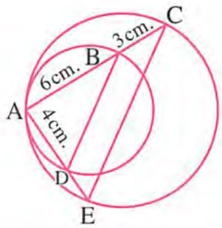


$\overline{CD}$ touches the semicircle M at D

If 2 CA = AB = 6 cm. , then CD = ... cm.

(a) 6 (b) 3 (c) $3\sqrt{3}$ (d) 27

4 **In the opposite figure :**



Two circles touching internally at A

, then ED = ... cm.

(a) 2 (b) 3

(c) 3.5 (d) 4

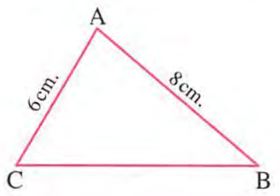
---

5 If $2 \cos \theta = -\sqrt{3}$, $\pi < \theta < \frac{3\pi}{2}$, then $\theta =$ ....

(a) $\frac{\pi}{3}$ (b) $\frac{6\pi}{7}$ (c) $\frac{4\pi}{3}$ (d) $\frac{7\pi}{6}$

---

6 **In the opposite figure :**



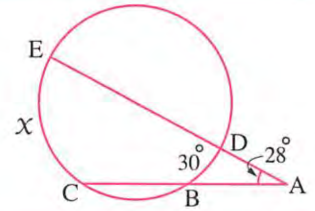
If m (∠ A) = 2 m (∠ B) , then BC = ... cm.

(a) $3\sqrt{10}$ (b) $2\sqrt{21}$

(c) 12 (d) 10

---

7 **In the opposite figure :**



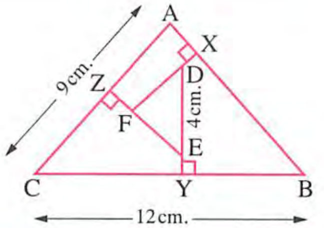
$x =$ ....

(a) 30° (b) 60°

(c) 86° (d) 26°

---

8 **In the opposite figure :**



If $\overline{FX} \perp \overline{AB}$, $\overline{DY} \perp \overline{BC}$, $\overline{EZ} \perp \overline{AC}$, AC = 9 cm.

, BC = 12 cm. , DE = 4 cm. , then EF = ... cm.

(a) 2 (b) 3

(c) 5 (d) 6

---

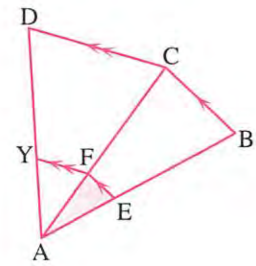
9 Which of the following is a factorization to the expression : $x^2 + 4$?

(a) $(x - 2)(x + 2)$ (b) $(x + 2)^2$

(c) $(x - 2i)^2$ (d) $(x - 2i)(x + 2i)$

---

10 **In the opposite figure :**



If the area of (polygon DYFC) = 40 cm^2.

, the area of (polygon FEBC) = 32 cm^2.

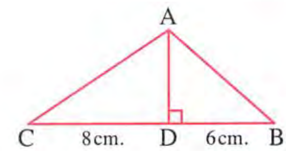
, the area of (Δ AFY) = 5 cm^2.

, then the area of (Δ AEF) = ... cm^2.

(a) 3 (b) 4

(c) 5 (d) 6

11 **In the opposite figure :**

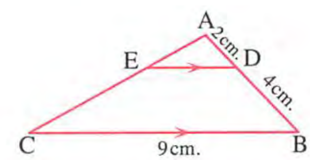


AB cos B + AC cos C = ... cm.

(a) 6 (b) 8

(c) 14 (d) 48

12 **In the opposite figure :**



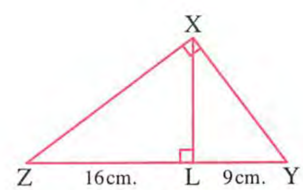
If the area of Δ ADE = 8 cm^2

, then the area of the figure

DBCE = ... cm^2

(a) 27 (b) 64 (c) 24 (d) 16

13 **In the opposite figure :**



XL = ... cm.

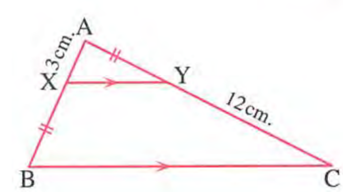
(a) 7 (b) 12

(c) 20 (d) 144

14 The function $f : f(x) = 2x$ is positive in ....

(a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\mathbb{R}^-$ (d) $\mathbb{R} - \{0\}$

15 **In the opposite figure :**

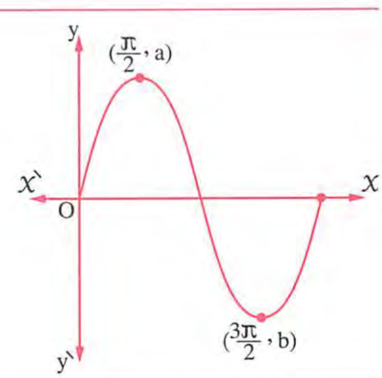


AC = ... cm.

(a) 15 (b) 16

(c) 18 (d) 20

16 The opposite figure show the curve



y = sin x , then | a | + | b | = ...

(a) 1

(b) 2

(c) π

(d) 2 π

17 The product of the roots of the equations :

$a x^2 + b x + c = 0$, $b x^2 + c x + a = 0$, $c x^2 + a x + b = 0$ equals ....

(a) ABC (b) – 1 (c) 1 (d) zero

18 If $x + y\,i = i^{15} + 2\sqrt{-4}$, then $x + y =$ ....

(a) 3 (b) 4 (c) zero (d) – 3

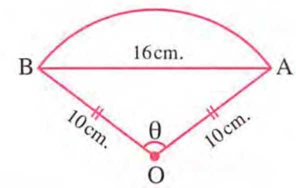
19 If the two roots of the equation : $x^2 + 4x + k = 0$ are distinct real , then $k \in$ ....

(a) $]-\infty , 4[$ (b) $]4 , \infty[$ (c) $]-\infty , 4]$ (d) $\{4\}$

20 If AM = 12 cm. , r = 9 cm. , where A is point outside circle M , then $P_M(A) =$ ....

(a) 65 (b) 63 (c) 49 (d) 7

21 **In the opposite figure :**



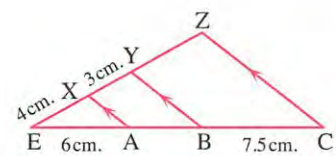
$\overset{\frown}{AB}$ is an arc in a circle whose centre O

, then find the length of $\overset{\frown}{AB} \simeq$ .... cm.

(a) 19 (b) 25

(c) 18 (d) 21

22 **In the opposite figure :**



AB + YZ = ... cm.

(a) 5 (b) 13

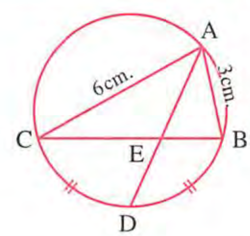
(c) 11 (d) 9.5

23 $(x + 2i)(x - 2i) =$ ....

(a) $x^2 + 4$ (b) $x^2 - 4$

(c) $4xi - 4$ (d) $x^2 - 4xi + 4$

24 **In the opposite figure :**



$\frac{BE}{BC} =$ ....

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) 2 (d) 3

25 The solution set of the equation : $x^2 + 1 = 0$ in $\mathbb{R}$ is ....

(a) $\{1\}$ (b) $\{1 , -1\}$ (c) $\varnothing$ (d) $\{-i , i\}$

26 If the ratio between the areas of two similar polygons is 16 : 25 , then the ratio between their two corresponding sides = ...

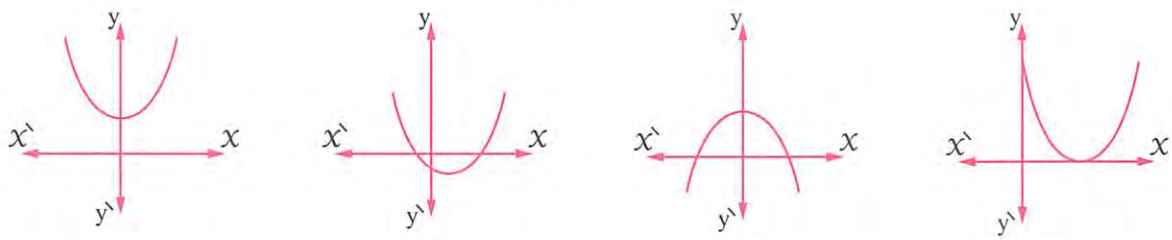
(a) 2 : 5 (b) 4 : 5 (c) 16 : 25 (d) 16 : 41

27 The quadratic equation whose roots are : $2-\sqrt{3}$, $2+\sqrt{3}$ is ....

(a) $x^2+2x+3=0$ (b) $x^2-4x+1=0$

(c) $x^2-4x+7=0$ (d) $x^2+4x+1=0$

28 Each of the following figures represents the curve of the function f :
$f(x)=a x^2+b x+c$ which of these figures does have $b^2-4ac=0$

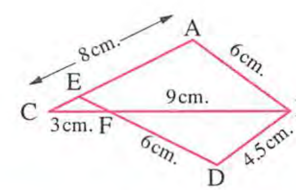


(a) (b) (c) (d)

Second Essay questions

Answer the following questions :

1 **In the opposite figure :**



$\overline{BC} \cap \overline{DE} = \{F\}$, AB = 6 cm. , BC = 12 cm. , AC = 8 cm.
, FC = 3 cm. , BD = 4.5 cm. , DF = 6 cm. **Prove that :**

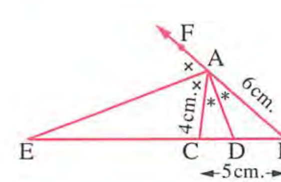
[1] Δ ABC $\sim$ Δ DBF **[2]** Δ EFC is isosceles.

2 If $\sin\theta = \sin 750^\circ \cos 300^\circ + \sin(-60^\circ)\cot 120^\circ$ where $0^\circ < \theta < 360^\circ$

Find : θ

3 Determine the sign of the function f : $f(x)=x^2-x+12$ and hence determine in $\mathbb{R}$ the solution set of the inequality : $x^2+12>x$, represent the solution on the number line.

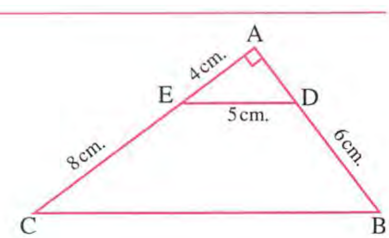
4 **In the opposite figure :**



In Δ ABC : AB = 6 cm. , AC = 4 cm. , BC = 5 cm.
, $\overrightarrow{AD}$ bisects $\angle$ BAC and intersects $\overline{BC}$ at D
, $\overrightarrow{AE}$ bisects $\angle$ A externally and intersects $\overrightarrow{BC}$ at E

Calculate : The length of $\overline{DE}$

5 **In the opposite figure :**



ABC is a right-angled triangle at A

[1] Prove that : $\overline{DE}$ // $\overline{BC}$

[2] Find the length of : $\overline{BC}$

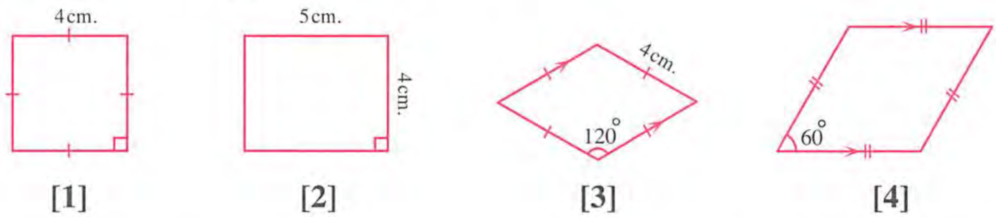
Model 8

Interactive test 8

First Multiple choice questions

Choose the correct answer from the given ones :

1 Which of the following polygons are similar ?



(a) The two polygons [1] , [2]
(b) The two polygons [1] , [3]
(c) The two polygons [3] , [4]
(d) The two polygons [2] , [4]

---

2 If the terminal side of a positive angle $(90° - \theta)$ in standard position intersects the unit circle at point $\left(\frac{-3}{5}, \frac{4}{5}\right)$, then $\sin(90° - \theta) =$ ....

(a) $\frac{-3}{5}$
(b) $\frac{3}{5}$
(c) $\frac{-4}{5}$
(d) $\frac{4}{5}$

---

3 The function $f : f(X) = 4 - 2X$ is non-positive if ....

(a) $X > 2$
(b) $X < 2$
(c) $X \geq 2$
(d) $X \leq 2$

---

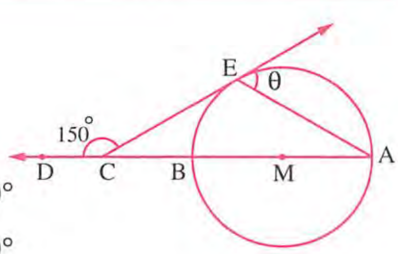
4 The measure of the central angle subtends an arc of length π cm. in a circle with diameter length 8 cm. equals ....

(a) $\frac{\pi}{8}$
(b) $\frac{\pi}{4}$
(c) $\frac{2\pi}{3}$
(d) 2π

---

5 **In the opposite figure :**

If $\overrightarrow{CE}$ is a tangent to the circle , then $\theta =$ ....



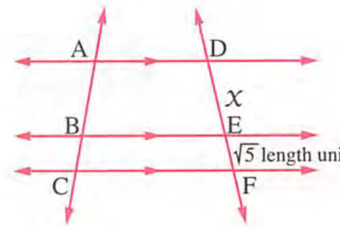
(a) 45°
(b) 50°
(c) 55°
(d) 60°

---

6 The quadratic equation whose terms coefficients are real numbers and one of its roots is $(3 - i)$ is ....

(a) $X^2 - 6X - 10 = 0$
(b) $2X^2 + 6X + 10 = 0$
(c) $X^2 - 6X + 10 = 0$
(d) $X^2 + 6X + 10 = 0$

7 **In the opposite figure :**



If A $(0 , 6)$, B $(-2 , 2)$ and C $(-3 , 0)$, $\overleftrightarrow{AD} // \overleftrightarrow{BE} // \overleftrightarrow{CF}$, EF $= \sqrt{5}$ length unit , then $x =$ .... length unit.

(a) $\sqrt{5}$ (b) $2\sqrt{5}$

(c) $3\sqrt{5}$ (d) $4\sqrt{5}$

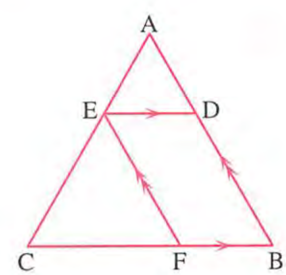
8 If $\cos \theta = \frac{3}{5}$, $0° < \theta < 90°$, then $\sin (90° - \theta) =$ ....

(a) $\frac{3}{4}$ (b) $\frac{5}{3}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

9 The function $f : f(\theta) = \sin (b\,\theta)$ is a periodic function and its period $\left(\frac{2\pi}{3}\right)$, then b = ...

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) 3 (d) 6

10 **In the opposite figure :**



If $\overline{DE} // \overline{BC}$, $\overline{EF} // \overline{AB}$, $\frac{AD}{DB} = \frac{2}{3}$

, then $\frac{\text{area } (\square \text{ DBFE})}{\text{area } (\Delta \text{ ABC})} =$ ....

(a) $\frac{21}{25}$ (b) $\frac{16}{25}$

(c) $\frac{12}{25}$ (d) $\frac{13}{25}$

11 If $4x + 2yi = 8 + 4xi$, then $x + y =$ ....

(a) -2 (b) 5 (c) 6 (d) 4

12 If $x = 4$ is one of the roots of the equation $x^2 + mx = 4$, then ....

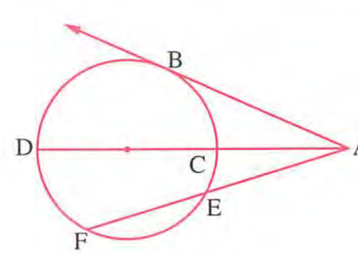
(a) $m = -3$ (b) m is an even.

(c) $(1 - m)$ is a perfect square. (d) (a) , (c) are true.

13 The sum of integers belong to the solution set of the inequality $(x - 2)(3x - 1) \leq 0$ equal ....

(a) -1 (b) 1 (c) 2 (d) 3

14 **In the opposite figure :**

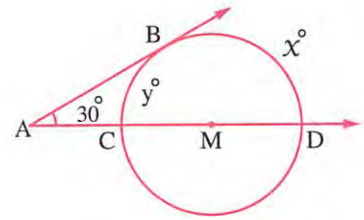


All the following mathematical expressions are true except ....

(a) $(AB)^2 = AC \times AD$ (b) $(AB)^2 = AE \times AF$

(c) $AC \times AD = AE \times AF$ (d) $AC \times CD = AE \times EF$

15 **In the opposite figure :**

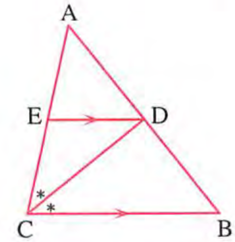


$x^2 - y^2 =$ ....

(a) 30×180 (b) 180×60

(c) 60 (d) 150

16 **In the opposite figure :**

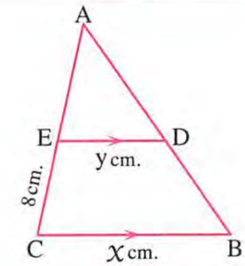


$\frac{AE}{EC} =$ ....

(a) $\frac{DE}{BC}$ (b) $\frac{AD}{AB}$

(c) $\frac{AC}{CB}$ (d) $\frac{AB}{BC}$

17 **In the opposite figure :**



If $\frac{x - y}{x + y} = \frac{2}{7}$, then AE = ... cm.

(a) 16 (b) 15

(c) 12 (d) 10

18 The diameter of circle M is 6 cm. , P_M (B) = zero , then B lies ....

(a) inside the circle. (b) outside the circle.

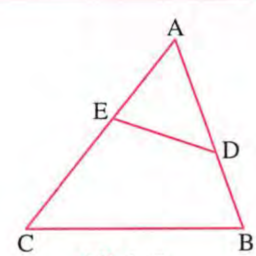
(c) on the circle. (d) at the centre of the circle.

19 If (L – 2) , (M – 2) are roots of the equation : $x^2 - 4x - 4 = 0$

, then $L^2 - 8L + 5 =$ ....

(a) 3 (b) – 3 (c) ± 3 (d) zero

20 **In the opposite figure :**



$\Delta ABC \sim \Delta AED$

If AD = 3 cm. , BD = 2 cm. , AE = 2.5 cm.

, then EC = ... cm.

(a) 2.5 (b) 3 (c) 4.5 (d) 3.5

21 The sum of the areas of two similar polygons is 225 cm² and the ratio between their perimeters 4 : 3 , then the area of the greater polygons. = ... cm²

(a) 81 (b) 144 (c) $128 \frac{4}{7}$ (d) $96 \frac{3}{7}$

22 The function f where $f(\mathcal{X}) = 2 - \mathcal{X}$ is non-negative when $\mathcal{X} \in$ ....

(a) $]-\infty , 2]$ (b) $]-\infty , 2[$ (c) $[2 , \infty[$ (d) $]2 , \infty[$

23 $\tan\left(-\frac{14}{3}\pi\right) =$ ....

(a) $-\sqrt{3}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{-1}{\sqrt{3}}$

24 If $P_M(A) = r$, then A lies .... "where r is the radius length of the circle M"

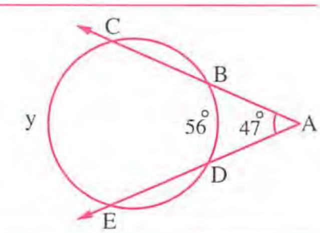
(a) on the circle (b) outside the circle

(c) inside the circle (d) at the centre of the circle

25 If $\sin A = \frac{1}{2}$, then the least positive angle satisfies this trigonometric equation is ....

(a) 150° (b) 30° (c) 60° (d) 330°

26 **In the opposite figure :**



y = ...

(a) 90° (b) 140°

(c) 150° (d) 160°

27 If L , M are the two roots of the equation : $\mathcal{X}^2 - 7\mathcal{X} + 3 = 0$, then $L^2 + M^2 =$ ....

(a) 7 (b) 43 (c) 58 (d) 79

28 The two roots of the equation : $\mathcal{X}(\mathcal{X} - 2) = 5$ are ....

(a) two complex and non real roots. (b) two equal real roots.

(c) two differant real roots. (d) 2 and zero.

Second Essay questions

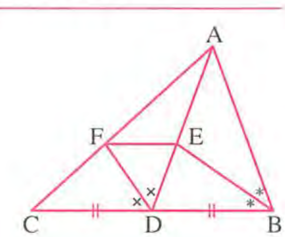
Answer the following questions :

1 ABCD is a rectangle in which AB = 6 cm. , BC = 8 cm.

Draw $\overrightarrow{BE} \perp \overline{AC}$ to intersect $\overline{AC}$ at E , $\overline{AD}$ at F

[1] Prove that : $(AB)^2 = AF \times AD$ **[2] Find :** The length of $\overline{AF}$

2 **In the opposite figure :**



In Δ ABC , D is a midpoint of $\overline{BC}$

, AB = AD , $\overrightarrow{BE}$ bisects $\angle$ B , $\overrightarrow{DF}$ bisects $\angle$ ADC

Prove that : $\overline{EF} \parallel \overline{BC}$

3 **Find the general solution of the equation :** $\csc 6\theta = \sec 3\theta$

4 Prove that the roots of the equation : $7x^2 - 11x + 5 = 0$ are non real conjugate , then find these two roots by using the general formula.

5 ABC is a triangle , $D \in \overline{BC}$ where BD = 5 cm. and DC = 4 cm. If AC = 6 cm.

, prove that :

[1] $\overline{AC}$ is a tangent segment to the circle passing through the points A , B and D

[2] $\Delta ACD \sim \Delta BCA$ **[3]** Area of (ΔABD) : Area of (ΔABC) = 5 : 9

Model 9

Interactive test 9

First Multiple choice questions

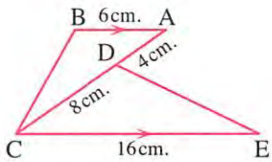
Choose the correct answer from the given ones :

1 The sign of the function f where $f(x) = 6 - 2x$ is positive if ....

(a) $x > 3$ (b) $x \geq 3$ (c) $x < 3$ (d) $x = 3$

2 **In the opposite figure :**

If $\overline{AB} \parallel \overline{EC}$, then $\frac{ED}{BC} =$ ....



(a) $\frac{4}{3}$ (b) $\frac{3}{4}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

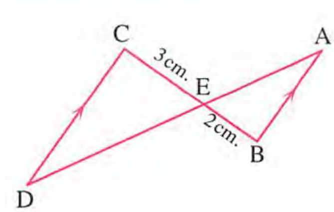
3 If $\cot(90° - \theta) = \cot 2\theta$ where $0° < \theta < 90°$, then $\sin 3\theta =$ ....

(a) -1 (b) zero (c) 1 (d) $\frac{1}{2}$

4 **In the opposite figure :**

$\overline{AB} \parallel \overline{CD}$, BE = 2 cm. , CE = 3 cm. , AD = 10 cm.

, then AE = ... cm.

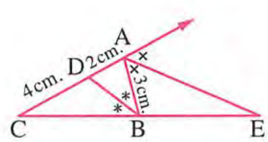


(a) 4 (b) 6

(c) 2 (d) 3

5 **In the opposite figure :**

BE = ... cm.



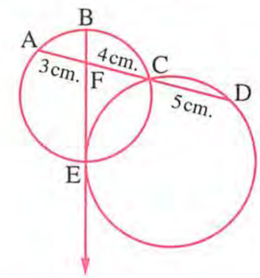
(a) 6 (b) 8

(c) 9 (d) 10

6 $\cos(90° - \theta) \times \csc\theta =$ ....

(a) zero (b) 1 (c) -1 (d) $\cot\theta$

7 **In the opposite figure :**



Two intersecting circles at C , E , $\overrightarrow{BE}$ touches the larger circle at E

If AF = 3 cm. , FC = 4 cm. , CD = 5 cm. , then BE = ... cm.

(a) 9 (b) 8

(c) 7 (d) 6

8 If the terminal side of an angle of measure 30° in standard position rotates three and half revolutions clockwise then the terminal side lies in the .... quadrant.

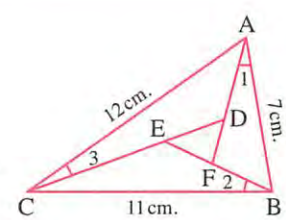
(a) first (b) second (c) third (d) fourth

9 The number of intersections between the curve

$y = \sin 3x$ with x-axis in the interval $[0 , 2\pi]$ equals ....

(a) 2 (b) 3 (c) 4 (d) 7

10 **In the opposite figure :**



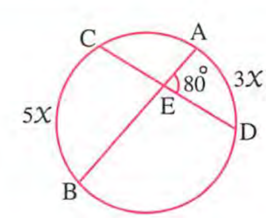
If m (∠ 1) = m (∠ 2) = m (∠ 3)

, then DE : EF : FD = ...

(a) 7 : 11 : 12 (b) 12 : 11 : 7

(c) 12 : 7 : 11 (d) 11 : 12 : 7

11 **In the opposite figure :**



$x =$ ....

(a) 10° (b) 20°

(c) 30° (d) 40°

12 If $\sec 3\theta = 2$ where θ is an acute angle , then $\theta =$ ....

(a) 10° (b) 15° (c) 20° (d) 30°

13 The interior bisector at a vertex of a triangle .... the exterior bisector at this vertex.

(a) parallel (b) perpendicular to

(c) equal (d) coincide with

14 If L , M are the two roots of the equation : $X^2 - 5X - 6 = 0$

the numerical value of the expression : $L^2 - 5L + 3 =$ ....

(a) -6 (b) 6 (c) 9 (d) 3

15 Two similar polygons are congruent if their scale factor of similarity equals ....

(a) $\frac{1}{2}$ (b) 1 (c) more than 1 (d) less than 1

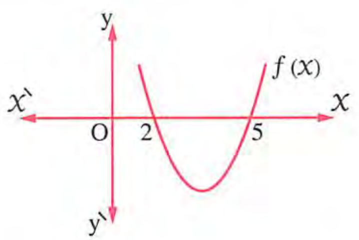
16 If $aX^2 + bX + c = 0$, a , b and c are real numbers and $(b^2 - 4ac)$

is not positive , then the roots of the equation are ....

(a) equal. (b) not real.

(c) conjugate complex. (d) real different.

17 **In the opposite figure :**



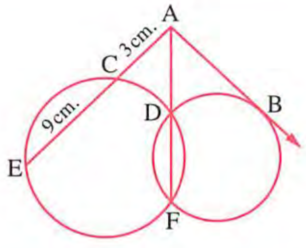
$f(X) = aX^2 + bX + c$

, then $\frac{b+c}{a} =$ ....

(a) 3 (b) 5

(c) 7 (d) 10

18 **In the opposite figure :**



If AC = 3 cm. , CE = 9 cm.

, then AB = ... cm.

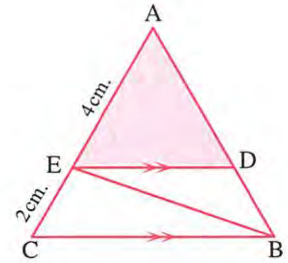
(a) 27 (b) 36

(c) 9 (d) 6

19 The simplest form of the imaginary number $i^{-18} =$ ....

(a) 1 (b) -1 (c) $-i$ (d) i

20 **In the opposite figure :**



If $\overline{DE} \parallel \overline{BC}$ and the area

of (Δ EBC) = 9 cm^2.

, then the area of (Δ ADE) = ... cm^2.

(a) 6 (b) 12

(c) 18 (d) 27

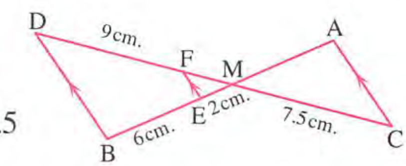
21 The measure of an inscribed angle is 60° subtended by an arc of length 4π cm.

, then the circumference of the circle = ... cm.

(a) 24π (b) 12π (c) 6π (d) 18π

---

22 **In the opposite figure :**



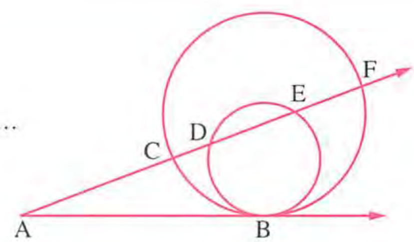
MF + AM = ... cm.

(a) 11 (b) 7.5

(c) 6 (d) 8

---

23 **In the opposite figure :**



$\overrightarrow{AB}$ is a common tangent to the two circles at B

and $\overrightarrow{AF}$ is a secant to both of then , then $(AB)^2$ = ...

(a) AC × CD (b) AD × AE

(a) AD × DF (d) AC × CF

---

24 If the roots of the equation : $4x^2 - 12x + m = 0$ are equal , then m = ...

(a) 3 (b) 4 (c) 9 (d) 16

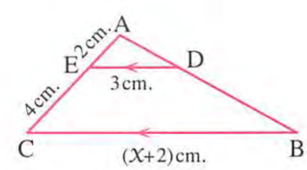
---

25 The sign of $f : f(x) = -2x$ is positive in the interval ....

(a) $]-\infty , \infty[$ (b) $\mathbb{R} - \{2\}$ (c) $]-\infty , 2]$ (d) $]-\infty , 0[$

---

26 **In the opposite figure :**



x = ...

(a) 5 (b) 6

(c) 7 (d) 8

---

27 If L , M are the two roots of the equation : $x^2 + x + 1 = 0$, then L + M + LM = ...

(a) zero (b) 1 (c) – 1 (d) 2

---

28 If L , M are the two roots of the equation : $x^2 - 5x + 7 = 0$, then the equation whose two roots are L^2 and M^2 is ....

(a) $x^2 + 11x + 49 = 0$ (b) $x^2 - 11x + 49 = 0$

(c) $x^2 - 49x + 11 = 0$ (d) $x^2 + 11x - 49 = 0$

Second Essay questions

Answer the following questions :

1 Without using calculator find the value of the following :

$$\sin 420° \cos 330° + \frac{\sin 15°}{\sin 165°} + \tan^2 65° - \cot 25° \tan 65°$$

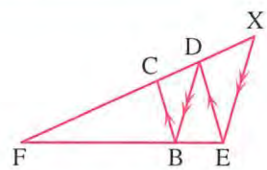
2 ABC is a triangle inscribed in a circle , D is a midpoint of $\overline{BC}$, draw $\overrightarrow{AD}$ to intersect the circle at E

Prove that : [1] $(BD)^2 = AD \times DE$ **[2]** $\Delta EBD \sim \Delta CAD$

3 The perimeter of triangle ABC is 27 cm. , draw $\overrightarrow{BD}$ bisects $\angle$ B and intersect $\overline{AC}$ at D , if AD = 4 cm. , CD = 5 cm. **Find the length of each :** $\overline{AB}$, $\overline{BC}$, $\overline{BD}$

4 If $x = 2 + 3i$, $y = \frac{3+i}{i}$ find the value of the expression : $x^2 + 2xy + y^2$

5 **In the opposite figure :**



$\overline{ED} \parallel \overline{BC}$, $\overline{DB} \parallel \overline{EX}$

Prove that : $\left(\frac{FB}{FE}\right)^2 = \frac{FC}{FX}$

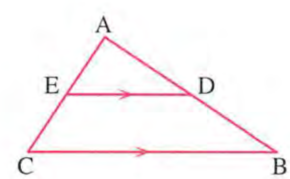
Model 10

Interactive test 10

First Multiple choice questions

Choose the correct answer from the given ones :

1 **In the opposite figure :**



All the following mathematical expressions are true except ....

(a) $\frac{AD}{DB} = \frac{AE}{EC}$ (b) $\frac{AD}{DB} = \frac{DE}{BC}$

(c) $\frac{AD}{AB} = \frac{AE}{AC}$ (d) $\frac{AB}{BD} = \frac{AC}{EC}$

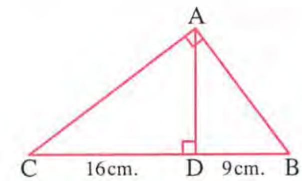
2 If $\sin \alpha = \cos \beta$ where α , β are two acute angles , then $\tan(\alpha + \beta) =$ ....

(a) $\frac{1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{3}$ (d) undefined.

3 The smallest value of the function f , where $f(\theta) = 3 \cos(2\theta)$ is ....

(a) -6 (b) -3 (c) -2 (d) -1

4 In the opposite figure :

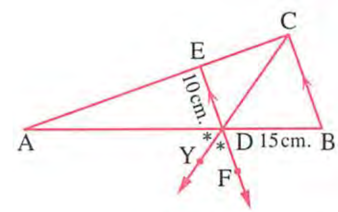


The length of $\overline{AB}$ = ... cm.

(a) 12 (b) 15

(c) 20 (d) 25

5 In the opposite figure :



If $\overline{ED} // \overline{BC}$, m (∠ ADY) = m (∠ FDY)

and ED = 10 cm. , BD = 15 cm.

, then AD = ... cm.

(a) 20 (b) 25 (c) 30 (d) 45

6 The equation whose roots are (2 + 3 i) , (2 – 3 i) is ....

(a) $x^2 + 4x + 13 = 0$ (b) $x^2 - 4x + 13 = 0$

(c) $x^2 + 4x - 13 = 0$ (d) $x^2 - 4x - 13 = 0$

7 $(1 - i)^{12}$ = ...

(a) – 64 i (b) 64 i (c) – 64 (d) 64

8 If the scale factor of similarity of the polygon P_1 to the polygon P_2 is $\frac{2}{3}$ and scale factor of similarity of the polygon P_3 to the polygon P_2 is $\frac{1}{3}$, which of the following relations is correct ?

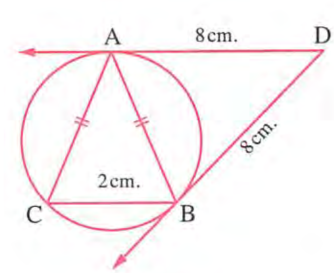
(a) Area (P_1) + Area (P_2) = Area (P_3)

(b) Area (P_1) + Area (P_3) = Area (P_2)

(c) $\sqrt{\text{Area}(P_1)} + \sqrt{\text{Area}(P_2)} = \sqrt{\text{Area}(P_3)}$

(d) $\sqrt{\text{Area}(P_1)} + \sqrt{\text{Area}(P_3)} = \sqrt{\text{Area}(P_2)}$

9 In the opposite figure :



If $\overrightarrow{DA}$, $\overrightarrow{DB}$ are tangents to the circle at A and B respectively

, DA = DB = 8 cm. , BC = 2 cm.

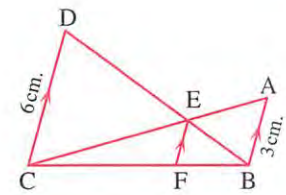
, then AC = ... cm.

(a) 3 (b) 4 (c) 5 (d) 6

10 In the opposite figure :

If $\overline{AB} \parallel \overline{EF} \parallel \overline{CD}$

, then EF = ... cm.

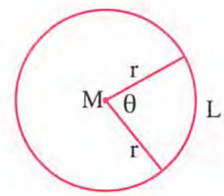


(a) 2.5 (b) 2

(c) 1.5 (d) 1

11 In the opposite figure :

θ^{rad} = ...

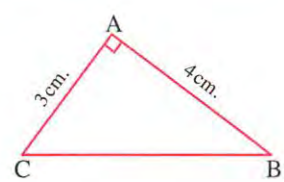


(a) $\frac{L}{r}$ (b) $\frac{r}{L}$

(c) $r \times L$ (d) $L \times 2\,r$

12 In the opposite figure :

m (∠ ABC) = ...

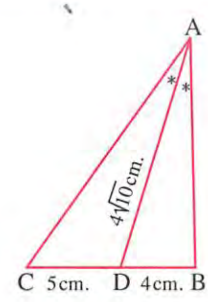


(a) $\sin^{-1}\left(\frac{3}{4}\right)$ (b) $\sin^{-1}\left(\frac{4}{3}\right)$

(c) $\tan^{-1}\left(\frac{3}{4}\right)$ (d) $\cot^{-1}\left(\frac{3}{4}\right)$

13 In the opposite figure :

The perimeter of Δ ABC = ... cm.



(a) 36

(b) 32

(c) 28

(d) 24

14 The roots of the equation : $x^2 - 2\sqrt{5}\,x + 1 = 0$ are ....

(a) rational real. (b) not real.

(c) real equal. (d) irrational real.

15 The sign of the function $f : f(x) = x - 4$ where $x \in \,]4, \infty[$ is ....

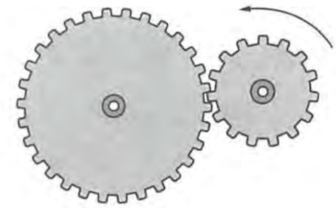
(a) always positive.

(b) always negative.

(c) positive in the interval]4 , 5[and negative in the interval]5 , ∞[

(d) negative in the interval]4 , 5[and positive in the interval]5 , ∞[

16 **In the opposite figure :**



If the greater gear revolves one revolution

, then the smaller gear revolves 3 revolution

If the smaller gear revolves one revolution

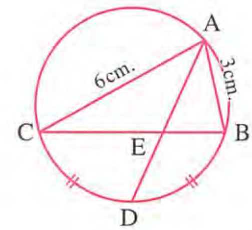
in the direction of the arrow shown on the figure

, then the central angle of revolving the greater gear is ....

(a) $-\frac{\pi}{2}$ (b) $\frac{-2\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) 2π

---

17 **In the opposite figure :**



$\frac{BE}{BC}$ = ...

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{2}$

---

18 The ratio between the length of two corresponding sides of two similar triangles is 1 : 4

, then the ratio between their areas is ....

(a) 1 : 2 (b) 1 : 4 (c) 1 : 8 (d) 1 : 16

---

19 If $L \in \mathbb{R}$, $M \in \mathbb{R}$ are the two roots of the equation : $a x^2 + b x + c = 0$ where $a > 0$, $L < M$

, then the solution set of the inequality : $a x^2 + b x + c < 0$ is ....

(a) $]-\infty , L[$ (b) $]L , M[$ (c) $]M , \infty[$ (d) $\mathbb{R} - [L , M]$

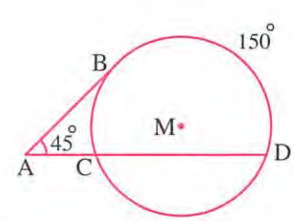
---

20 If one of the roots of the equation : $4 k x^2 + 7 x + k^2 + 4 = 0$ is multiplicative inverse of the other root , then k = ...

(a) ± 2 (b) 3 (c) 4 (d) 2

---

21 **In the opposite figure :**



$\overline{AB}$ is a tangent segment to circle M at B

, $\overrightarrow{AC}$ intersects the circle at C , D

, m ($\angle$ A) = 45° , m ($\overset{\frown}{DB}$) = 150°

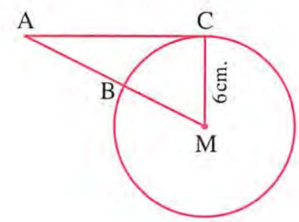
, then m ($\overset{\frown}{BC}$) = ...

(a) 30° (b) 40° (c) 60° (d) 120°

22 In Δ ABC , AB = 8 cm. , AC = 6 cm. , D $\in \overline{AB}$ such that AD = 3 cm. , E $\in \overline{AC}$ such that AE = 4 cm. If the area of Δ AED = 3 cm^2 , then the area of the polygon DBCE = ... cm^2

(a) 12 (b) 9 (c) 6 (d) 8

23 **In the opposite figure :**



$\overline{AC}$ touches the circle M at C , MC = 6 cm.

, P_M (A) = 64 , then AB = ... cm.

(a) 3 (b) 4

(c) 5 (d) 6

24 If Δ ABC ~ Δ XYZ and 3 AB = 2 XY , then area of Δ ABC : area of Δ XYZ = ...

(a) 4 : 9 (b) 9 : 4 (c) 2 : 3 (d) 3 : 2

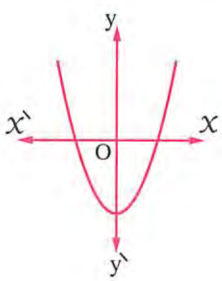
25 The angle of measure $\left(\frac{7\pi}{6}\right)$ radian has degree measure = ...

(a) 225° (b) 210° (c) 840° (d) – 225°

26 $(1 + i)^{10}$ = ...

(a) 32 i (b) – 32 i (c) 32 (d) – 32

27 The opposite figure represents the curve of the function $f : f(x) = a x^2 + b x + c$, then which of the following is true ?



(a) a > 0 , c > 0 (b) a > 0 , c < 0

(c) a < 0 , b > 0 (d) a < 0 , c < 0

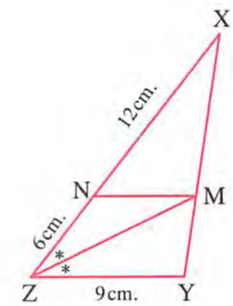
28 If one of the two roots of the equation : $x^2 + k x - 98 = 0$ is twice the additive inverse of the other root , then k = ...

(a) ± 14 (b) ± 7 (c) ± 8 (d) 49

Second | Essay questions

Answer the following questions :

1 **In the opposite figure :**



XN = 12 cm.

, NZ = 6 cm.

, YZ = 9 cm.

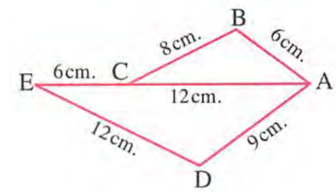
, $\overrightarrow{ZM}$ bisects $\angle$ XZY

Prove that : $\overline{MN} \parallel \overline{YZ}$

2 If $5 \sin \theta - 3 = 0$, $\frac{\pi}{2} < \theta < \pi$

Find the value of : $\cos\left(\frac{\pi}{2} - \theta\right) + \sin(2\pi - \theta) - \cos\left(\frac{3\pi}{2} - \theta\right) + \cos\theta$

3 **In the opposite figure :**



AB = 6 cm. , BC = 8 cm. , AC = 12 cm.

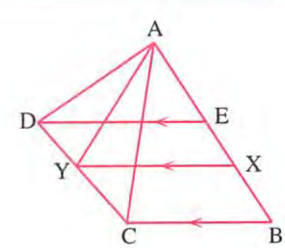
, CE = 6 cm. , AD = 9 cm. , DE = 12 cm.

Prove that :

[1] Δ ABC ~ Δ ADE **[2]** $\overrightarrow{AE}$ bisects $\angle$ BAD

4 Represent graphically the function $f : f(X) = X^2 - 2X - 3$, then determine the sign of the function.

5 **In the opposite figure :**

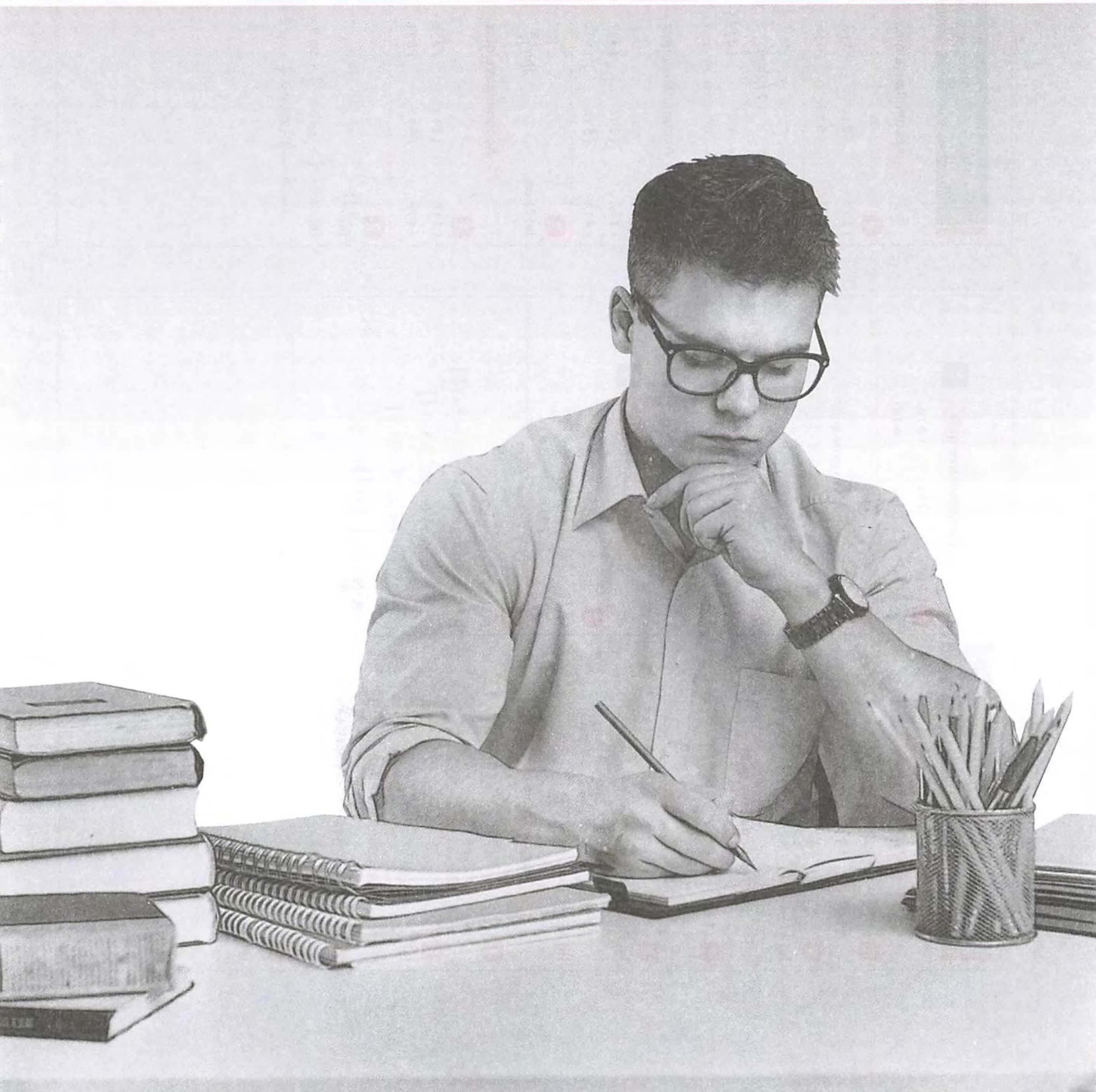


$\overline{ED} \parallel \overline{XY} \parallel \overline{BC}$

and AD × BX = AC × EX

Prove that : $\overrightarrow{AY}$ bisects $\angle$ CAD

Answers



Answers of accumulative quizzes on Algebra

Accumulative quiz 1

1 (1) b (2) a (3) c
(4) d (5) b (6) d

2

[a] $\{1+\sqrt{3}\,i , 1-\sqrt{3}\,i\}$ [b] $\frac{15}{13} , -\frac{10}{13}$

Accumulative quiz 2

1 (1) c (2) a (3) d
(4) a (5) d (6) d

2

[a] Prove by yourself.

, the S.S. $= \{\frac{2}{3} + \frac{\sqrt{11}}{3} i , \frac{2}{3} - \frac{\sqrt{11}}{3} i\}$

[b] $k \in]1 , \infty[$

Accumulative quiz 3

1 (1) c (2) b (3) d
(4) c (5) d (6) a

2

[a] 4 [b] 2

Accumulative quiz 4

1 (1) b (2) b (3) b
(4) a (5) d (6) c

2

[a] $3X^2 + 4X + 8 = 0$ [b] $39 - 26\,i$

Accumulative quiz 5

1 (1) d (2) a (3) a
(4) c (5) a (6) d

2

(1) Draw by yourself , from the graph :

- f is positive when $X \in \mathbb{R} - [-2 , 1]$
- f is negative when $X \in]-2 , 1[$
- $f(X) = 0$ when $X \in \{-2 , 1\}$

(2) Draw by yourself , from the graph :

- f is negative when $X \in \mathbb{R} - [-3 , 3]$
- f is positive when $X \in]-3 , 3[$
- $f(X) = 0$ when $X \in \{-3 , 3\}$

Accumulative quiz 6

1

(1) c (2) d (3) c
(4) b (5) b (6) c

2

[a] $1 - i , 2$

[b] • f is positive when $X \in \mathbb{R} - [-5 , 1\frac{1}{2}]$

• f is negative when $X \in]-5 , 1\frac{1}{2}[$

• $f(X) = 0$ when $X \in \{-5 , 1\frac{1}{2}\}$

• The S.S. $= [-5 , 1\frac{1}{2}]$

Answers of accumulative quizzes on Trigonometry

Accumulative quiz 1

1

(1) d (2) c (3) d
(4) d (5) d (6) b

2

[a] (1) Fourth (2) Third (3) First

[b] (1) 228° , – 492° (2) 430° , – 290°

(3) 350° , – 10° (there are other solutions)

Accumulative quiz 2

1

(1) a (2) c (3) b
(4) b (5) c (6) c

2

[a] 21 cm. [b] $\frac{5\pi}{18}$

Accumulative quiz 3

1

(1) b (2) a (3) d
(4) b (5) c (6) b

2

[a] $-\frac{11}{8}$

[b] $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$

, $\sec\theta = -\frac{5}{4}$, $\csc\theta = \frac{5}{3}$, $\cot\theta = -\frac{4}{3}$

Accumulative quiz 4

1

(1) b (2) b (3) d
(4) c (5) d (6) d

2

[a] $\frac{28}{15}$

[b] $\theta = 45° + 120°\,n$ or $\theta = 75° + 360°\,n$, $n \in \mathbb{Z}$

$\theta = 45°$ or $75°$

Accumulative quiz 5

1

(1) a (2) c (3) b
(4) b (5) d (6) d

2

[a] $15° + 30°\,n$, $n \in \mathbb{Z}$

[b] (1) $]-\infty , \infty[$ (2) $[-1 , 1]$

(3) 2π

Accumulative quiz 6

1

(1) b (2) a (3) c
(4) c (5) b (6) c

2

[a] 129° 56′ 28″ , 230° 3′ 32″

[b] 150°

Answers of accumulative quizzes on Geometry

Accumulative quiz 1

1

(1) d (2) c (3) a
(4) c (5) d (6) d

2

(1) $\frac{3}{2}$ (2) 6 , 4

Accumulative quiz 2

1

(1) b (2) a (3) c
(4) b (5) b (6) c

2

Prove by yourself.

Accumulative quiz 3

1

(1) c (2) d (3) b
(4) d (5) c (6) a

2 Prove by yourself.

Accumulative quiz 4

1

(1) d (2) c (3) b
(4) d (5) d (6) d

2 Prove by yourself.

Accumulative quiz 5

1

(1) c (2) b (3) b
(4) c (5) c (6) b

2 Prove by yourself.

Accumulative quiz 6

1

(1) d (2) c (3) b
(4) c (5) d (6) c

2 (1) 6 cm. (2) 21 cm.

Accumulative quiz 7

1

(1) c (2) c (3) c
(4) d (5) b (6) b

2 Prove by yourself , 3.6 cm.

Accumulative quiz 8

1

(1) d (2) c (3) a
(4) c (5) b (6) d

2 Prove by yourself.

Accumulative quiz 9

1

(1) b (2) a (3) a
(4) d (5) b (6) c

2

(1) $8\sqrt{2}$ cm.
(2) $\sqrt{17}$ cm.

Answers of school book examinations on Algebra & Trigonometry

Model 1

1

(1) c (2) c (3) b (4) c

2

(1) $]-2 , 1[$ (2) third
(3) 300° (4) $x^2 - 8x + 10 = 0$

3

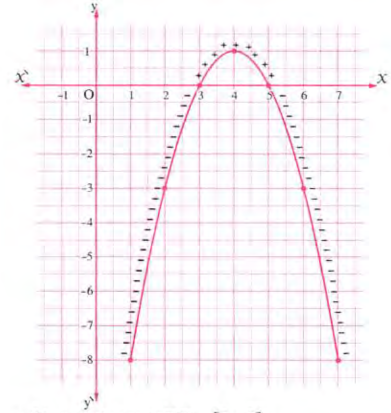
[a] $\frac{2-3i}{3+2i} = \frac{2-3i}{3+2i} \times \frac{3-2i}{3-2i} = \frac{6-13i+6i^2}{9-4i^2}$
$= \frac{-13i}{13} = -i$

[b] $\because \sin A = \frac{3}{4} , A \in]0 , \frac{\pi}{2}[$
$\therefore m(\angle A) \simeq 48° 35' 25''$

4

[a]

x	1	2	3	4	5	6	7
$f(x)$	−8	−3	0	1	0	−3	−8

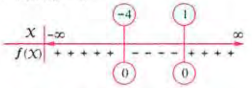


- f is negative at $x \in \mathbb{R} - [3 , 5]$
- f is positive at $x \in]3 , 5[$
- $f(x) = 0$ at $x \in \{3 , 5\}$

[b] $\because y = \frac{4-2i}{1-i} \times \frac{1+i}{1+i} = \frac{4+2i-2i^2}{1-i^2} = \frac{6+2i}{2} = 3 + i$
$\therefore x + y = 3 + 2i + 3 + i = 6 + 3i$

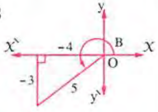
5

[a] $\because x^2 + 3x - 4 \le 0$ Let $f(x) = x^2 + 3x - 4$
Put $x^2 + 3x - 4 = 0$ $\therefore (x+4)(x-1) = 0$
$\therefore x = -4$ or $x = 1$



$\therefore f$ is negative at $x \in]-4 , 1[$
$, f(x) = 0$ at $x \in \{-4 , 1\}$
$\therefore$ The S.S. $= [-4 , 1]$

[b] The expression $= \cos B - \sin B$
$= -\frac{4}{5} + \frac{3}{5}$
$= -\frac{1}{5}$



Model 2

1

(1) $-i$ (2) 9
(3) 18° (4) $[-\frac{3}{2} , \frac{3}{2}]$

2

(1) d (2) a (3) c (4) d

3

[a] $\because$ One root of the equation is the multiplicative inverse of the other root
$\therefore k^2 + 4 = 4k$ $\therefore k^2 - 4k + 4 = 0$
$\therefore (k-2)^2 = 0$ $\therefore k = 2$

[b] $\because \sin\theta = \sin(30° + 2 \times 360°)\cos(360° - 60°)$
$- \sin 60° \cot(180° - 60°)$
$= \sin 30° \cos 60° + \sin 60° \cot 60°$
$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{3}{4}$ (positive)
$\therefore \theta$ lies on first or second quadrant.
$\therefore \theta = 48° 35' 25''$ or $\theta = 131° 24' 35''$

4

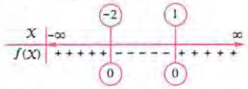
[a] (1) $12 = 4b$ $\therefore b = 3$
$, 3a = -27$ $\therefore a = -9$

(2) $x^2 + x - 2 \le 0$
Let $f(x) = x^2 + x - 2$

put : $x^2 + x - 2 = 0$

$\therefore (x + 2)(x - 1) = 0$

$\therefore x = -2$ or $x = 1$



$\therefore f$ is negative at $x \in]-2 , 1[$

, $f(x) = 0$ at $x \in \{-2 , 1\}$

$\therefore$ The S.S. $= [-2 , 1]$

[b] $\because \theta^{rad} = \frac{l}{r} = \frac{26}{18} = \frac{13}{9}^{rad}$

$\therefore x^{\circ} = \frac{13}{9}^{rad} \times \frac{180^{\circ}}{\pi} = 82^{\circ}\ 45'\ 38''$

5

[a] $210 = \frac{n}{2}(1 + n)$ $\qquad \therefore 420 = n + n^2$

$\therefore n^2 + n - 420 = 0$ $\qquad \therefore (n + 21)(n - 20) = 0$

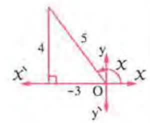
$\therefore n = -21$ (refused) or $n = 20$

$\therefore$ The number of consecutive integers $= 20$

[b] The expression

$= \sin x - \tan x - 2\cos x$

$= \frac{4}{5} + \frac{4}{3} + 2 \times \frac{3}{5} = \frac{10}{3}$



Answers of school book examinations on Geometry

Model 1

1

(1) similar

(2) **First :** AC , CD **Second :** $(BD)^2$ **Third :** BD × AC

2

(1) c (2) a (3) d (4) d

3

[a] $\because \Delta ADE \sim \Delta ABC$

$\therefore m(\angle ADE) = m(\angle B)$ and they are corresponding angles

$\therefore \overline{DE} \parallel \overline{BC}$ (First req.)

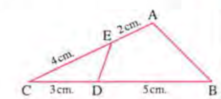
$\because \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ $\qquad \therefore \frac{4}{6} = \frac{DE}{5} = \frac{AE}{AE + 1.5}$

$\therefore 6\,AE = 4\,AE + 6$ $\qquad \therefore 2\,AE = 6$

$\therefore AE = 3$ cm.

, $DE = \frac{5 \times 4}{6} = \frac{10}{3}$ cm. (Second req.)

[b] In ΔΔ DEC , ABC :



$\because \frac{CE}{CB} = \frac{4}{8} = \frac{1}{2}$

, $\frac{CD}{CA} = \frac{3}{6} = \frac{1}{2}$

$\therefore \frac{CE}{CB} = \frac{CD}{CA}$ $\qquad$, ∠ C is common

$\therefore \Delta DEC \sim \Delta ABC$

, $\frac{\text{area of } \Delta DEC}{\text{area of } \Delta ABC} = \left(\frac{CD}{CA}\right)^2 = \frac{1}{4}$ (The req.)

4

[a] In ΔΔ ADE , ACB : $\because m(\angle ADE) = m(\angle C)$

, ∠ A is common

$\therefore \Delta ADE \sim \Delta ACB$ $\qquad \therefore \frac{AD}{AC} = \frac{DE}{CB} = \frac{AE}{AB}$

$\therefore \frac{4}{8} = \frac{6}{CB} = \frac{5}{AB}$ $\qquad \therefore AB = \frac{8 \times 5}{4} = 10$ cm.

$\therefore DB = 10 - 4 = 6$ cm.

, $BC = \frac{8 \times 6}{4} = 12$ cm. (The req.)

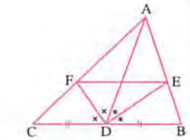
[b] $\because \overrightarrow{CB} \cap \overrightarrow{FE} = \{A\}$ $\qquad \therefore AB \times AC = AE \times AF$

$\therefore 3 \times 5 = AE \times 7.5$ $\qquad \therefore AE = \frac{15}{7.5} = 2$ cm.

$\therefore EF = 7.5 - 2 = 5.5$ cm. (The req.)

5

[a] In Δ ABD :



$\because \overrightarrow{DE}$ bisects ∠ ADB

$\therefore \frac{AE}{EB} = \frac{AD}{DB}$

, in Δ ACD :

$\because \overrightarrow{DF}$ bisects ∠ ADC $\qquad \therefore \frac{AF}{FC} = \frac{AD}{DC}$

, $\because BD = DC$ $\qquad \therefore \frac{AE}{EB} = \frac{AF}{FC}$

$\therefore \overline{EF} \parallel \overline{BC}$ (Q.E.D.)

[b] $\because$ In Δ ABC : $\because \overline{AB} \parallel \overline{EF}$

$\therefore \frac{CE}{EA} = \frac{CF}{FB}$ $\qquad \therefore \frac{12}{8} = \frac{9}{FB}$

$\therefore FB = \frac{8 \times 9}{12} = 6$ cm.

In Δ BCD :

$\because \frac{CF}{FB} = \frac{9}{6} = \frac{3}{2}$ $\qquad$, $\frac{DM}{MB} = \frac{6}{4} = \frac{3}{2}$

$\therefore \frac{CF}{FB} = \frac{DM}{MB}$ $\qquad \therefore \overline{FM} \parallel \overline{CD}$ (Q.E.D.)

Model 2

1

(1) similar (2) ACB

(3) NX × NY (4) 6 cm.

2

(1) c (2) b (3) b (4) d

3

[a] $\because \Delta ABC \sim \Delta AED$ $\qquad \therefore m(\angle ADE) = m(\angle ACB)$

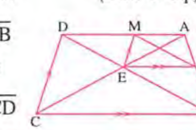
$\therefore$ BCED is a cyclic quadrilateral (First req.)

, $\frac{AB}{AE} = \frac{AC}{AD}$ $\qquad \therefore \frac{5}{2.5} = \frac{AC}{3}$

$\therefore AC = \frac{3 \times 5}{2.5} = 6$ cm.

$\therefore EC = 6 - 2.5 = 3.5$ cm. (Second req.)

[b] In Δ ABC : $\because \overline{EF} \parallel \overline{CB}$



$\therefore \frac{AF}{FB} = \frac{AE}{EC}$ (1)

, in Δ ACD : $\because \overline{EM} \parallel \overline{CD}$

$\therefore \frac{AM}{MD} = \frac{AE}{EC}$ (2)

From (1) , (2) : $\therefore \frac{AF}{FB} = \frac{AM}{MD}$

$\therefore \overline{FM} \parallel \overline{BD}$ (Q.E.D.)

4

[a] ∵ ΔABC is right - angled at A

∴ $BC = 7.5$ cm. (Pythagoras)

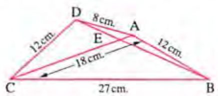
, ∵ $\overline{AD} \perp \overline{BC}$ ∴ $(AB)^2 = DB \times BC$

∴ $(4.5)^2 = BD \times 7.5$ ∴ $BD = \frac{20.25}{7.5} = 2.7$ cm.

, $DC = 7.5 - 2.7 = 4.8$

, $AD = \frac{AB \times AC}{BC} = \frac{4.5 \times 6}{7.5} = 3.6$ cm. (The req.)

[b] ∵ $\frac{BA}{AD} = \frac{12}{8} = \frac{3}{2}$



, $\frac{AC}{DC} = \frac{18}{12} = \frac{3}{2}$

, $\frac{BC}{AC} = \frac{27}{18} = \frac{3}{2}$

∴ $\frac{BA}{AD} = \frac{AC}{DC} = \frac{BC}{AC}$ ∴ $\Delta BAC \sim \Delta ADC$

∴ $\frac{\text{Area of } \Delta BAC}{\text{Area of } \Delta ADC} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$ (The req.)

5

[a] ∵ C is the midpoint of $\overline{AD}$ ∴ $AD = 2\,AC$

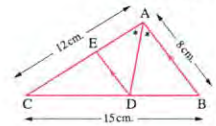
, ∵ $\overleftrightarrow{AB}$ is a tangent to a circle

∴ $(AB)^2 = AC \times AD$ ∴ $\left(3\sqrt{2}\right)^2 = AC \times 2\,AC$

∴ $18 = 2\,(AC)^2$ ∴ $(AC)^2 = 9$

∴ $AC = 3$ cm. (The req.)

[b] In ΔABC :



∵ $\overrightarrow{AD}$ bisects $\angle A$

∴ $\frac{BA}{AC} = \frac{BD}{DC}$

∴ $\frac{8}{12} = \frac{BD}{15 - BD}$

∴ $12\,BD = 120 - 8\,BD$ ∴ $20\,BD = 120$

∴ $BD = 6$ cm. ∴ $DC = 15 - 6 = 9$ cm.

, ∵ $\overleftrightarrow{ED} \,//\, \overleftrightarrow{AB}$ ∴ $\frac{CE}{EA} = \frac{CD}{DB}$

∴ $\frac{CE}{12 - CE} = \frac{9}{6}$ ∴ $6\,CE = 108 - 9\,CE$

∴ $15\,CE = 108$ ∴ $CE = \frac{108}{15} = 7.2$ cm.

(The req.)

Answers of School examinations

1 Cairo

First Multiple choice questions

(1) (c)	(2) (c)	(3) (a)	(4) (c)
(5) (b)	(6) (a)	(7) (b)	(8) (b)
(9) (c)	(10) (a)	(11) (d)	(12) (c)
(13) (b)	(14) (c)	(15) (b)	(16) (c)
(17) (b)	(18) (b)	(19) (b)	(20) (a)
(21) (d)	(22) (a)	(23) (d)	(24) (d)
(25) (a)	(26) (a)	(27) (c)	(28) (d)

Second Essay questions

1

In ΔABD : $\overrightarrow{AE}$ bisects $\angle BAD$

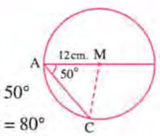
∴ $\frac{BE}{ED} = \frac{BA}{AD}$ ∴ $\frac{BE}{ED} = \frac{6}{4} = \frac{3}{2}$

In ΔBCD : ∵ $\frac{BC}{CD} = \frac{9}{6} = \frac{3}{2}$

∴ $\frac{BC}{CD} = \frac{BE}{ED}$ ∴ $\overrightarrow{CE}$ bisects $\angle BCD$

2

In ΔMAC :



∵ $MC = MA = 12$ cm. (radii)

∴ $m(\angle MAC) = m(\angle MCA) = 50°$

∴ $m(\angle AMC) = 180° - 50° \times 2 = 80°$

∴ The length of $(\overset{\frown}{AC}) = \frac{80°}{360°} \times 2\pi r$

$= \frac{80°}{360°} \times 2\pi(12) = \frac{16}{3}\pi$ cm.

3

The sum of the two roots of the given equation

$(L + 3) + (M + 3) = 12$

∴ $L + M + 6 = 12$ ∴ $\boxed{L + M = 6}$ (1)

The product of the two roots of the given equation

$(L + 3)(M + 3) = 3$

∴ $LM + 3L + 3M + 9 = 3$

∴ $LM + 3(L + M) + 9 = 3$

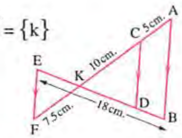
from (1) : $LM + 3(6) + 9 = 3$

∴ $LM = -24$

∴ The required equation is : $X^2 - 6X - 24 = 0$

4

∵ $\overleftrightarrow{EF} \,//\, \overleftrightarrow{CD} \,//\, \overleftrightarrow{AB}$, $\overleftrightarrow{AF} \cap \overleftrightarrow{BE} = \{k\}$



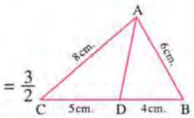
∴ $\frac{EK}{FK} = \frac{KD}{KC} = \frac{DB}{CA} = \frac{EB}{FA}$

∴ $\frac{EK}{7.5} = \frac{DB}{5} = \frac{18}{22.5}$

∴ $EK = 6$ cm. , $DB = 4$ cm.

5

In ΔABC , ΔDBA



∵ $\angle B$ is common angle.

, ∵ $\frac{BA}{BD} = \frac{6}{4} = \frac{3}{2}$, $\frac{BC}{BA} = \frac{9}{6} = \frac{3}{2}$

∴ $\frac{BA}{BD} = \frac{BC}{BA}$ ∴ $\Delta ABC \sim \Delta DBA$

∴ $\frac{BA}{BD} = \frac{AC}{DA}$ ∴ $\frac{6}{4} = \frac{8}{DA}$ ∴ $DA = \frac{16}{3}$ cm.

2 Cairo

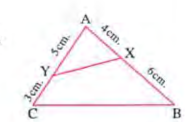
First Multiple choice questions

(1) (b)	(2) (c)	(3) (b)	(4) (c)
(5) (d)	(6) (b)	(7) (d)	(8) (a)
(9) (c)	(10) (c)	(11) (c)	(12) (c)
(13) (d)	(14) (b)	(15) (c)	(16) (b)
(17) (d)	(18) (d)	(19) (a)	(20) (c)
(21) (d)	(22) (b)	(23) (a)	(24) (a)
(25) (c)	(26) (a)	(27) (b)	(28) (a)

Second Essay questions

1

(1) In ΔAXY , ΔACB



∴ $\angle A$ is a common angle

, ∵ $\frac{AX}{AC} = \frac{4}{8} = \frac{1}{2}$

, $\frac{AY}{AB} = \frac{5}{10} = \frac{1}{2}$

∴ $\frac{AX}{AC} = \frac{AY}{AB}$ ∴ $\Delta AXY \sim \Delta ACB$

(2) $\frac{a(\Delta AXY)}{a(\Delta ACB)} = \left(\frac{AX}{AC}\right)^2$

∴ $\frac{8}{a(\Delta ACB)} = \left(\frac{4}{8}\right)^2 = \frac{1}{4}$

∴ $a(\Delta ACB) = 32$ cm².

∴ $a(\text{polygon } XBCY) = 32 - 8 = 24$ cm².

2

∵ $\overrightarrow{AD}$ tangent to the circle

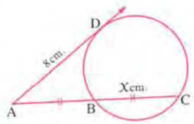
∴ $(AD)^2 = (AB) \times (AC)$

∴ $(8)^2 = (X)(2X)$

∴ $2X^2 = 64$

∴ $X^2 = 32$

∴ $X = 4\sqrt{2}$



3

Theoretical

4

∵ L and M are the roots of the given equation

∴ $L + M = \frac{2}{3}$, $LM = \frac{-7}{3}$

∴ L^2 and M^2 are the roots of the required equation

∴ $L^2 + M^2 = (L + M)^2 - 2\,LM = \left(\frac{2}{3}\right)^2 - 2\left(\frac{-7}{3}\right) = \frac{46}{9}$

, $L^2 M^2 = (LM)^2 = \left(\frac{-7}{3}\right)^2 = \frac{49}{9}$

∴ The required equation is :

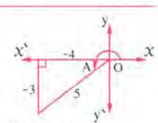
$X^2 - \frac{46}{9} X + \frac{49}{9} = 0$ (multiply by 9)

∴ $9X^2 - 46X + 49 = 0$

5

∵ $4 \tan A - 3 = 0$

$\tan A = \frac{3}{4}$



∴ $\sin(180° - A) + \cos(-A) + \cot(360° - A)$

$= \sin A + \cos A - \cot A = \frac{-3}{5} + \frac{-4}{5} - \frac{4}{3} = \frac{-41}{15}$

3 Cairo

First Multiple choice questions

(1) (a)	(2) (c)	(3) (a)	(4) (c)
(5) (a)	(6) (a)	(7) (d)	(8) (a)
(9) (d)	(10) (d)	(11) (a)	(12) (c)
(13) (b)	(14) (b)	(15) (d)	(16) (b)
(17) (a)	(18) (c)	(19) (b)	(20) (c)
(21) (c)	(22) (b)	(23) (b)	(24) (d)
(25) (c)	(26) (c)	(27) (c)	(28) (c)

Second Essay questions

1

∵ L and M are the roots of the given equation.

∴ L + M = 3 and LM = 5

, ∵ L^2 and M^2 are the roots of the required equation.

∴ $L^2 + M^2 = (L + M)^2 - 2\,LM = (3)^2 - 2(5) = -1$

, $L^2 M^2 = (LM)^2 = (5)^2 = 25$

∴ The required equation is : $X^2 + X + 25 = 0$

2

$\theta^{rad} = 120° \times \frac{\pi}{180°} = \frac{2\pi}{3}$

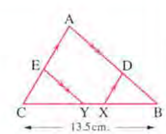
∴ $r = \frac{l}{\theta^{rad}} = \frac{6}{\left(\frac{2\pi}{3}\right)} = \frac{9}{\pi}$ cm.

∴ The circumference of the circle $= 2\pi r$

$= 2\pi\left(\frac{9}{\pi}\right) = 18$ cm.

3

In Δ ABC :



∵ $\overline{XD} \parallel \overline{AC}$

∴ $\frac{BD}{BA} = \frac{BX}{BC}$

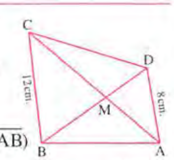
∴ $\frac{2}{5} = \frac{BX}{13.5}$ ∴ BX = 5.4 cm.

, ∵ $\overline{EY} \parallel \overline{AB}$ ∴ $\frac{CE}{CA} = \frac{CY}{CB}$

∴ $\frac{4}{9} = \frac{CY}{13.5}$ ∴ CY = 6 cm.

∴ YX = 13.5 – 6 – 5.4 = 2.1 cm.

4



∵ ABCD is cyclic quadrilateral

∴ m (∠ BDA)

= m (∠ BCA) (Subtended by $\overline{AB}$)

, ∵ m (∠ DMA) = m (∠ CMB) (V.O.A)

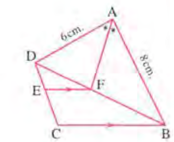
∴ Δ DMA ~ Δ CMB

∴ $\frac{a(\Delta DMA)}{a(\Delta CMB)} = \left(\frac{DA}{CB}\right)^2$

∴ $\frac{a(\Delta DMA)}{a(\Delta CMB)} = \left(\frac{8}{12}\right)^2 = \frac{4}{9}$

5

In Δ ADB :



∵ $\overrightarrow{AF}$ bisects ∠ DAB

∴ $\frac{BF}{FD} = \frac{BA}{AD}$

∴ $\frac{BF}{FD} = \frac{8}{6} = \frac{4}{3}$

In Δ DCB :

, ∵ $\overline{EF} \parallel \overline{CB}$ ∴ $\frac{BF}{FD} = \frac{EC}{ED}$

∴ $\frac{EC}{ED} = \frac{4}{3}$ ∴ $\frac{ED}{EC} = \frac{3}{4}$

4 Giza

First Multiple choice questions

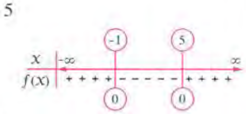
(1) (a)	(2) (c)	(3) (a)	(4) (c)
(5) (d)	(6) (a)	(7) (c)	(8) (d)
(9) (c)	(10) (a)	(11) (c)	(12) (b)
(13) (b)	(14) (a)	(15) (b)	(16) (c)
(17) (c)	(18) (d)	(19) (a)	(20) (c)
(21) (b)	(22) (d)	(23) (a)	(24) (b)
(25) (a)	(26) (c)	(27) (a)	(28) (b)

Second Essay questions

1

Let $f(X) = X^2 - 4X - 5$

, put $f(X) = 0$



∴ $X^2 - 4X - 5 = 0$

∴ $(X - 5)(X + 1) = 0$

∴ $X = 5$ or $X = -1$

The solution set $= \mathbb{R} - [-1 , 5]$

2

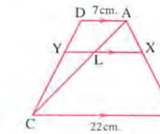
$\cos(\pi + \theta) = \sin(390°)\cos(-60°) + \cos(30°)\sin(120°)$

∴ $-\cos\theta = \sin(30°)\cos(60°) + \cos(30°)\sin(60°)$

∴ $-\cos\theta = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

∴ $\cos\theta = -1$ ∴ $\theta = 180°$

3



∵ $\overleftrightarrow{AD} \parallel \overleftrightarrow{XY} \parallel \overleftrightarrow{BC}$

∴ $\frac{AX}{XB} = \frac{DY}{YC} = \frac{2}{3}$

In Δ ABC :

∵ $\overline{XL} \parallel \overline{BC}$ ∴ $\frac{AX}{AB} = \frac{XL}{BC}$

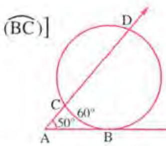
∴ $\frac{2}{5} = \frac{XL}{22}$ ∴ XL = 8.8 cm.

In Δ ACD : ∵ $\overline{YL} \parallel \overline{AD}$ ∴ $\frac{CY}{CD} = \frac{YL}{DA}$

∴ $\frac{3}{5} = \frac{YL}{7}$ ∴ YL = 4.2 cm.

∴ XY = 8.8 + 4.2 = 13 cm.

4



∵ $m(\angle A) = \frac{1}{2}[m(\overset{\frown}{BD}) - m(\overset{\frown}{BC})]$

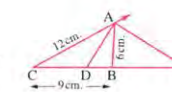
∴ $50 = \frac{1}{2}[m(\overset{\frown}{BD}) - 60°]$

∴ $100 = m(\overset{\frown}{DB}) - 60°$

∴ $m(\overset{\frown}{BD}) = 160°$

5

In Δ ABC :



∵ $\overrightarrow{AE}$ is the exterior bisector of ∠ CAB

∴ $\frac{BA}{AC} = \frac{BE}{EC}$ ∴ $\frac{6}{12} = \frac{BE}{BE + 9}$

∴ $\frac{1}{2} = \frac{BE}{BE + 9}$ ∴ BE + 9 = 2 BE

∴ BE = 9 cm.

∴ $AE = \sqrt{CE \times EB - BA \times AC} = \sqrt{18 \times 9 - 6 \times 12}$

$= 3\sqrt{10}$ cm.

5 Giza

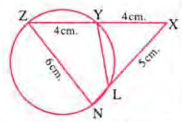
First Multiple choice questions

(1) (b)	(2) (b)	(3) (a)	(4) (c)
(5) (c)	(6) (b)	(7) (b)	(8) (a)
(9) (b)	(10) (d)	(11) (a)	(12) (b)
(13) (c)	(14) (c)	(15) (c)	(16) (c)
(17) (c)	(18) (d)	(19) (a)	(20) (d)
(21) (b)	(22) (a)	(23) (b)	(24) (a)
(25) (c)	(26) (d)	(27) (a)	(28) (a)

Second | Essay questions

1

In ΔXYL , ΔXNZ :



$\because \angle X$ is common angle

, $m(\angle XYL) = m(\angle XNZ)$

(exterior angle of cyclicquad. YLNZ)

$\therefore \Delta XNZ \sim \Delta XYL$ $\quad \therefore \frac{XN}{XY} = \frac{XZ}{XL} = \frac{NZ}{YL}$

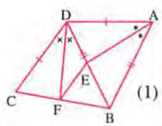
$\therefore \frac{5+NL}{4} = \frac{8}{5} = \frac{6}{YL}$ $\quad \therefore 5(5+NL) = 4 \times 8$

$\therefore 25 + 5\,NL = 32$ $\quad \therefore 5\,NL = 7$

$\therefore NL = 1.4$ cm. $\quad , YL = \frac{5 \times 6}{8} = 3.75$

2

In ΔABD :



$\because \overrightarrow{AE}$ bisects $\angle BAD$

$\therefore \frac{BA}{AD} = \frac{BE}{ED}$ (1)

In ΔBDC : $\because \overrightarrow{DF}$ bisects $\angle BDC$

$\therefore \frac{BD}{DC} = \frac{BF}{FC}$ (2)

, $\because BA = BD$, $AD = DC$ (3)

from (1) , (2) and (3) : $\because \frac{BE}{ED} = \frac{BF}{FC}$ $\quad \therefore \overline{FE} \parallel \overline{DC}$

3

$a = 1$, $b = -2$, $c = 4$

$\therefore x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$, $x = \frac{2 \pm 2\sqrt{3}\,i}{2}$

$\therefore x = 1 \pm \sqrt{3}\,i$

The solution set is $\{1 + \sqrt{3}\,i , 1 - \sqrt{3}\,i\}$

4

Let the two complement angles be x and y

$\therefore x + y = \frac{\pi}{2}$ (1)

and $x - y = \frac{\pi}{3}$ (2)

By adding : $\therefore 2x = \frac{5\pi}{6}$ $\quad \therefore x = \frac{5\pi}{12}$

By substitution in (1) : $\frac{5\pi}{12} + y = \frac{\pi}{2}$

$\therefore y = \frac{1}{12}\pi$

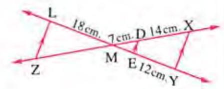
So the two angles (in radians) are $\frac{\pi}{12}$, $\frac{5\pi}{12}$

and in degrees : $\frac{\pi}{12} \times \frac{180°}{\pi} = 15°$

, and $\frac{5\pi}{12} \times \frac{180°}{\pi} = 75°$

5

In ΔMXY :



, $\because \overline{DE} \parallel \overline{XY}$

$\therefore \frac{MD}{DX} = \frac{ME}{EY}$

$\therefore \frac{7}{14} = \frac{ME}{12}$ $\quad \therefore ME = 6$ cm.

In ΔLMZ : $\because \overline{DE} \parallel \overline{LZ}$ $\quad \therefore \frac{MD}{MZ} = \frac{ME}{ML}$

$\therefore \frac{7}{MZ} = \frac{6}{18}$ $\quad \therefore MZ = 21$ cm.

6 Giza

First | Multiple choice questions

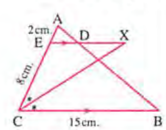
(1) (a)	(2) (b)	(3) (b)	(4) (b)
(5) (d)	(6) (a)	(7) (a)	(8) (b)
(9) (b)	(10) (a)	(11) (c)	(12) (d)
(13) (c)	(14) (b)	(15) (c)	(16) (d)
(17) (a)	(18) (b)	(19) (c)	(20) (d)
(21) (d)	(22) (a)	(23) (a)	(24) (b)
(25) (b)	(26) (b)	(27) (c)	(28) (c)

Second | Essay questions

1

$\because \overline{EX} \parallel \overline{CB}$

$\therefore m(\angle EXC) = m(\angle XCB)$ (alternate angles)



$\because \overrightarrow{CX}$ bisects $\angle ACB$

$\therefore m(\angle ECX) = m(\angle XCB)$

$\therefore m(\angle EXC) = m(\angle ECX)$

In ΔEXC : $\therefore EX = EC = 8$ cm.

In ΔABC : $\because \overline{ED} \parallel \overline{BC}$

$\therefore \frac{AE}{AC} = \frac{ED}{CB}$ $\quad \therefore \frac{2}{10} = \frac{ED}{15}$

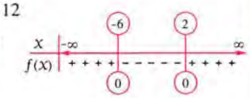
$\therefore ED = 3$ cm. $\quad \therefore XD = 8 - 3 = 5$ cm.

2

$\because x(x+4) \le 12$ $\quad \therefore x^2 + 4x - 12 \le 0$

let $f(x) = x^2 + 4x - 12$

Put $x^2 + 4x - 12 = 0$



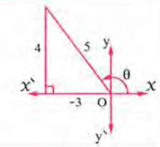
$(x+6)(x-2) = 0$

$x = -6$ or $x = 2$

$\therefore$ The solution set $= [-6 , 2]$

3

$\sin(180° - \theta) + \tan(90° - \theta)$

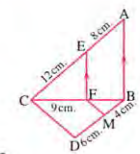


$= \sin\theta + \cot\theta$

$= \frac{4}{5} + \left(\frac{-3}{4}\right) = \frac{1}{20}$

4

First : In ΔACB : $\because \overline{AB} \parallel \overline{EF}$



$\therefore \frac{AE}{EC} = \frac{BF}{FC}$

$\therefore \frac{8}{12} = \frac{BF}{9}$

$\therefore BF = 6$ cm.

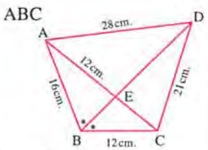
Second : In ΔBDC : $\because \frac{BF}{FC} = \frac{6}{9} = \frac{2}{3}$

, $\because \frac{BM}{MD} = \frac{4}{6} = \frac{2}{3}$ $\quad \therefore \frac{BF}{FC} = \frac{BM}{MD}$

$\therefore \overline{FM} \parallel \overline{DC}$

5

In ΔABC : $\because \overrightarrow{BE}$ bisects $\angle ABC$



$\therefore \frac{AB}{BC} = \frac{AE}{EC}$

$\therefore \frac{16}{12} = \frac{12}{EC}$

$\therefore EC = 9$ cm.

In ΔADC : $\because \frac{AD}{DC} = \frac{28}{21} = \frac{4}{3}$, $\frac{AE}{EC} = \frac{12}{9} = \frac{4}{3}$

$\therefore \frac{AD}{DC} = \frac{AE}{EC}$ $\quad \therefore \overrightarrow{DE}$ bisects $\angle ADC$

7 El-Kalyoubia

First | Multiple choice questions

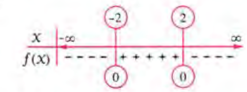
(1) (d)	(2) (c)	(3) (c)	(4) (d)
(5) (c)	(6) (c)	(7) (b)	(8) (d)
(9) (a)	(10) (d)	(11) (c)	(12) (b)
(13) (a)	(14) (c)	(15) (a)	(16) (b)
(17) (c)	(18) (b)	(19) (c)	(20) (c)
(21) (a)	(22) (b)	(23) (a)	(24) (d)
(25) (b)	(26) (b)	(27) (c)	(28) (b)

Second | Essay questions

1

Put $4 - x^2 = 0$ $\quad \therefore x^2 = 4$ $\quad \therefore x = \pm 2$

$\therefore$ The sign of the function f is



• Positive at $x \in]-2 , 2[$

• $f(x) = 0$ at $x \in \{-2 , 2\}$

• Negative at $x \in \mathbb{R} - [-2 , 2]$

, then the solution set of $4 - x^2 \le 0$ is $\mathbb{R} -]-2 , 2[$

2

$\because \sin 4\theta = \cos 2\theta$ $\quad \therefore 4\theta \pm 2\theta = 90° + 360°\,n$

Either $6\theta = 90° + 360°\,n$ $\quad \therefore \theta = 15° + 60°\,n$

or $2\theta = 90° + 360°n$ $\quad \therefore \theta = 45° + 180°n$

The general solution is $15° + 60°\,n$ or $45° + 180°n$

where $n \in \mathbb{Z}$

3

The sum of the roots of the given equation

is $L + 1 + M + 1 = 7$ $\quad \therefore L + M = 5$

The product of the roots of the given equation

is $(L+1)(M+1) = LM + L + M + 1 = 5$

$\therefore LM + 5 + 1 = 5$ $\quad \therefore LM = -1$

The sum of the roots of the required equation

is $L^2 + M^2 = (L+M)^2 - 2\,LM = (5)^2 - 2(-1) = 27$

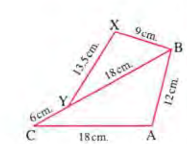
The product of the two roots of the required equation

is $L^2 M^2 = (LM)^2 = (-1)^2 = 1$

$\therefore$ The required equation is $x^2 - 27x + 1 = 0$

4

In ΔXBY , ΔABC :



$\because \frac{XB}{AB} = \frac{9}{12} = \frac{3}{4}$

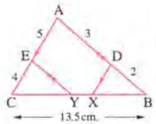
, $\frac{BY}{BC} = \frac{18}{24} = \frac{3}{4}$

, $\frac{XY}{AC} = \frac{13.5}{18} = \frac{3}{4}$

$\therefore \frac{XB}{AB} = \frac{BY}{BC} = \frac{XY}{AC}$ $\quad \therefore \Delta XBY \sim \Delta ABC$

5

In Δ ABC : ∵ $\overline{DX} \parallel \overline{AC}$



$\therefore \frac{BD}{BA} = \frac{BX}{BC}$

$\therefore \frac{2}{5} = \frac{BX}{13.5}$

$\therefore BX = 5.4$ cm.

$\because \overline{EY} \parallel \overline{AB}$ $\quad \therefore \frac{CY}{CB} = \frac{CE}{CA}$ $\quad \therefore \frac{CY}{13.5} = \frac{4}{9}$

$\therefore CY = 6$ cm. $\quad \therefore XY = 13.5 - (6 + 5.4) = 2.1$ cm.

8 El-Monofia

First Multiple choice questions

(1) (d)	(2) (b)	(3) (c)	(4) (d)
(5) (c)	(6) (d)	(7) (a)	(8) (d)
(9) (a)	(10) (b)	(11) (c)	(12) (c)
(13) (c)	(14) (a)	(15) (c)	(16) (b)
(17) (c)	(18) (a)	(19) (a)	(20) (d)
(21) (c)	(22) (d)	(23) (a)	(24) (b)
(25) (d)	(26) (a)	(27) (c)	(28) (c)

Second Essay questions

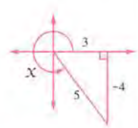
1

$\sin(180° - x) + \tan(90° - x)$

$+ \tan(270° - x) = \sin x$

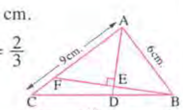
$+ \cot x + \cot x$

$= \frac{-4}{5} + 2\left(\frac{-3}{4}\right) = \frac{-23}{10}$



2

In Δ ABC : ∵ CD = 10 − 4 = 6 cm.



$\therefore \frac{BD}{DC} = \frac{4}{6} = \frac{2}{3}$, $\frac{BA}{AC} = \frac{6}{9} = \frac{2}{3}$

$\therefore \frac{BD}{DC} = \frac{BA}{AC}$

$\therefore \overrightarrow{AD}$ bisects ∠ BAC (First req.)

$\because \overline{AE} \perp \overline{BF}$

, ∵ m (∠ BAE) = m (∠ FAE) $\quad$ ∴ Δ AEB ≡ Δ AEF

∴ Δ ABF is an isosceles triangle

∴ AB = AF = 6 cm. $\quad$ ∴ CF = 9 − 6 = 3 cm.

, ∵ ΔΔ BAF , BCF have a common vertex B , F ∈ $\overline{AC}$

$\therefore \frac{\text{Area of } (\Delta ABF)}{\text{Area of } (\Delta CBF)} = \frac{AF}{FC} = \frac{6}{3} = 2$ (Second req.)

3

$\because x^2 + 6x + 9 < 10 - 3x - 9$

$\therefore x^2 + 9x + 8 < 0$

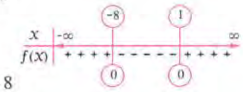
let $f(x) = x^2 + 9x + 8$

Put $x^2 + 9x + 8 = 0$

$\therefore (x + 1)(x + 8) = 0$

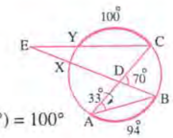
$\therefore x = -1$ or $x = -8$

The solution set is $]-8 , -1[$



4

∵ m (∠ CAB) = 33°



∴ m ($\widehat{CB}$) = 66°

∴ m ($\widehat{YXA}$) = 360°

− (100° + 66° + 94°) = 100°

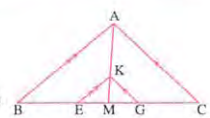
∴ m (∠ ACY) = 100° ÷ 2 = 50°

∵ ∠ CDB is an exterior angle of Δ EDC

∴ m (∠ BEC) = 70° − 50° = 20°

5

First : In Δ MAC :



$\because \overline{GK} \parallel \overline{AC}$

$\therefore \frac{MG}{MC} = \frac{MK}{MA}$ (1)

, in Δ MAB : $\because \overline{EK} \parallel \overline{AB}$ $\quad \therefore \frac{MK}{MA} = \frac{ME}{MB}$ (2)

$\therefore \frac{MG}{MC} = \frac{ME}{MB}$

, ∵ M is the midpoint of $\overline{BC}$ $\quad$ ∴ MC = MB

∴ MG = ME $\quad$ ∴ M is midpoint of $\overline{EG}$

Second : ∵ K is the point of intersection of the medians of Δ ABC

$\therefore \frac{MK}{MA} = \frac{MG}{MC} = \frac{1}{3}$ $\quad \therefore MG = \frac{1}{3} MC$

, $\frac{MK}{MA} = \frac{ME}{MB} = \frac{1}{3}$ $\quad \therefore ME = \frac{1}{3} MB$

$\because MC = MB = \frac{1}{2} BC$

$\therefore MG = \frac{1}{3} \times \frac{1}{2} BC = \frac{1}{6} BC$

, $ME = \frac{1}{3} \times \frac{1}{2} BC = \frac{1}{6} BC$ (by adding)

$\therefore GE = \frac{1}{3} BC$ $\quad$, ∵ GC = 2 MG

$\therefore GC = 2 \times \frac{1}{6} BC = \frac{1}{3} BC$

also BE = 2 ME

$\therefore BE = 2 \times \frac{1}{6} BC = \frac{1}{3} BC$

$\therefore BE = EG = GC = \frac{1}{3} BC$

9 El-Gharbia

First Multiple choice questions

(1) (c)	(2) (c)	(3) (a)	(4) (c)
(5) (a)	(6) (d)	(7) (d)	(8) (d)
(9) (c)	(10) (b)	(11) (b)	(12) (a)
(13) (c)	(14) (b)	(15) (c)	(16) (b)
(17) (a)	(18) (c)	(19) (d)	(20) (a)
(21) (b)	(22) (d)	(23) (c)	(24) (c)
(25) (c)	(26) (d)	(27) (d)	(28) (c)

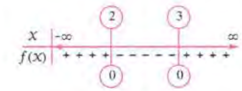
Second Essay questions

1

Let $x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$\therefore x = 3$ or $x = 2$

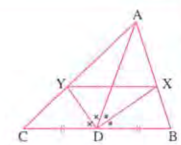


$\therefore f$ is smaller than or equal zero at $x \in [2 , 3]$

$\therefore$ S.S. $= [2 , 3]$

2

In Δ ADB



$\because \overrightarrow{DX}$ bisects ∠ ADB

$\therefore \frac{AD}{DB} = \frac{AX}{XB}$ (1)

In Δ ADC : $\because \overrightarrow{DY}$ bisects ∠ ADC

$\therefore \frac{AD}{DC} = \frac{AY}{YC}$ (2)

$\because \overline{AD}$ is a median $\quad$ ∴ DB = DC (3)

From (1) , (2) , (3) we get : $\frac{AX}{XB} = \frac{AY}{YC}$ (4)

In Δ ABC : from (4) we get $\overline{XY} \parallel \overline{BC}$

3

L.H.S. = sin 60° cos 30° − cos 60° sin 30°

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$

R.H.S. $= \sin^2 \frac{\pi}{4} = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

∵ L.H.S. = R.H.S.

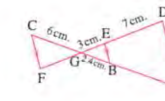
4

$\because m(\angle A) = \frac{1}{2}\left(m(\text{major } \widehat{BC}) - m(\widehat{BC})\right)$

$= \frac{1}{2}\left((360 - 140) - (140)\right) = \frac{1}{2}(80) = 40°$

5

$\therefore \overline{CF} \parallel \overline{EB} \parallel \overline{DA}$ and $\overleftrightarrow{ED}$



, $\overleftrightarrow{CA}$ are two transversal

$\therefore \frac{CG}{FG} = \frac{GB}{GE} = \frac{BA}{ED}$

$\therefore \frac{6}{FG} = \frac{2.4}{3} = \frac{BA}{7}$

$\therefore FG = \frac{6 \times 3}{2.4} = 7.5$ cm. $\quad AB = \frac{7 \times 2.4}{3} = 5.6$

∴ GA = 5.6 + 2.4 = 8 cm.

10 El-Fayoum

First Multiple choice questions

(1) (b)	(2) (a)	(3) (a)	(4) (a)
(5) (c)	(6) (c)	(7) (d)	(8) (c)
(9) (d)	(10) (b)	(11) (c)	(12) (b)
(13) (c)	(14) (c)	(15) (d)	(16) (a)
(17) (c)	(18) (b)	(19) (d)	(20) (d)
(21) (a)	(22) (c)	(23) (c)	(24) (c)
(25) (d)	(26) (a)	(27) (d)	(28) (c)

Second Essay questions

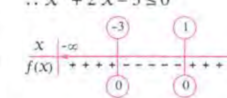
1

$x(x + 2) - 3 \le 0$ $\quad \therefore x^2 + 2x - 3 \le 0$

Let $x^2 + 2x - 3 = 0$

$\therefore (x + 3)(x - 1) = 0$

$\therefore x = -3$ or $x = 1$



∴ The solution set of the inequality $= [-3 , 1]$

2

$\cos 90° \csc 30° + \sec^2 45° \sin 30° - \cos 270° \sin 180°$

$= 0 \times 2 + \left(\sqrt{2}\right)^2 \times \frac{1}{2} - 0 \times 0 = 1$

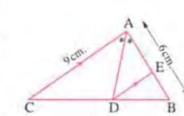
3

In Δ ABC : $\because \overrightarrow{AD}$ bisects ∠ A

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$ (1)

$\because \overline{DE} \parallel \overline{AC}$

$\therefore \frac{BE}{EA} = \frac{BD}{DC}$ (2)



From (1) , (2) : $\therefore \frac{BE}{EA} = \frac{BA}{AC}$ (The req.)

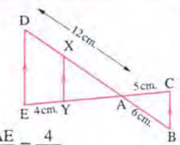
From (1) : $\therefore \frac{BD}{DC} = \frac{6}{9} = \frac{2}{3}$

$\therefore \frac{BE}{EA} = \frac{2}{3}$ $\therefore \frac{BE}{BA} = \frac{2}{5}$

$\therefore \frac{BE}{6} = \frac{2}{5}$

$\therefore$ BE = 2.4 cm. $\therefore$ AE = 6 – 2.4 = 3.6 cm.

4



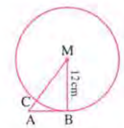
$\because \overleftrightarrow{BC} \,/\!/\, \overleftrightarrow{XY} \,/\!/\, \overleftrightarrow{DE}$ and $\overleftrightarrow{CE}$

, $\overleftrightarrow{BD}$ are two transversals

$\therefore \frac{AC}{AB} = \frac{AE}{AD} = \frac{EY}{DX}$ $\quad \frac{5}{6} = \frac{AE}{12} = \frac{4}{DX}$

$\therefore AE = \frac{5 \times 12}{6} = 10$ cm. , $DX = \frac{4 \times 6}{5} = 4.8$

5



$\because P_M(A) = 81$

$\therefore (AB)^2 = 81$

$\therefore$ AB = 9 cm.

, $(AM)^2 = 81 + 144 = 255$

$\therefore$ AM = 15 cm. $\therefore$ AC = 15 – 12 = 3 cm.

Answers of final models

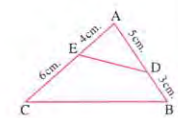
Model 1

First Multiple choice questions

1 (c)	2 (b)	3 (a)	4 (b)	5 (a)
6 (b)	7 (c)	8 (d)	9 (b)	10 (d)
11 (b)	12 (a)	13 (b)	14 (c)	15 (b)
16 (d)	17 (a)	18 (c)	19 (c)	20 (a)
21 (b)	22 (c)	23 (d)	24 (d)	25 (c)
26 (c)	27 (a)	28 (c)		

Second Essay questions

1



In Δ ADE , Δ ACB :

$\frac{AD}{AC} = \frac{5}{10} = \frac{1}{2}$

, $\frac{AE}{AB} = \frac{4}{8} = \frac{1}{2}$

$\therefore \frac{AD}{AC} = \frac{AE}{AB}$

, $\because$ ∠ A is a common angle.

$\therefore$ Δ ADE ~ Δ ACB (Q.E.D. 1)

From similarity : m (∠ ADE) = m (∠ ACB)

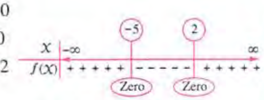
$\therefore$ DECB is a cyclic quadrilateral. (Q.E.D. 2)

2

Put $x^2 + 3x - 10 = 0$

$\therefore (x + 5)(x - 2) = 0$

$\therefore x = -5$ or $x = 2$



, $\because$ a > 0

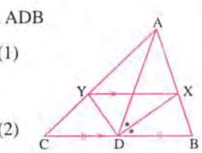
$\therefore f(x)$ is positive at $x \in \mathbb{R} - [-5 , 2]$

, $f(x)$ = zero at $x \in \{-5 , 2\}$

, $f(x)$ is negative at $x \in]-5 , 2[$

$\therefore$ The solution set of the inequality = $[-5 , 2]$

3



In Δ ADB : $\because \overrightarrow{DX}$ bisects ∠ ADB

$\therefore \frac{AD}{DB} = \frac{AX}{XB}$ (1)

In Δ ABC : $\because \overleftrightarrow{XY} \,/\!/\, \overline{BC}$

$\therefore \frac{AX}{XB} = \frac{AY}{YC}$ (2)

From (1) , (2) : $\therefore \frac{AD}{DB} = \frac{AY}{YC}$

, $\because$ DB = DC $\therefore \frac{AD}{DC} = \frac{AY}{YC}$

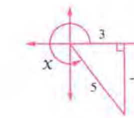
$\therefore \overrightarrow{DY}$ bisects ∠ ADC (Q.E.D. 1)

, $\because$ in Δ ABD : $\overrightarrow{DX}$ bisects ∠ ADB internally

, $\overrightarrow{DY}$ bisects it externally

$\therefore$ m (∠ XDY) = 90° (Q.E.D. 2)

4



$\therefore \sin(180° - x)$

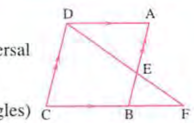
$+ \tan(90° - x)$

$+ \tan(270° - x)$

$= \sin x + \cot x + \cot x$

$= \frac{-4}{5} + \left(\frac{-3}{4}\right) + \left(\frac{-3}{4}\right) = \frac{-23}{10}$

5



$\because \overline{FC} \,/\!/\, \overline{AD}$, $\overleftrightarrow{DF}$ is a transversal

$\therefore$ m (∠ F) = m (∠ ADE)

(Alternate angles)

, $\because$ m (∠ C) = m (∠ A) (properties of a parallelogram)

$\therefore$ Δ DCF ~ Δ EAD (First req.)

$\therefore \frac{\text{Area of }(\Delta DCF)}{\text{Area of }(\Delta EAD)} = \left(\frac{DC}{EA}\right)^2 = \left(\frac{AB}{EA}\right)^2 = \frac{25}{9}$

(Second req.)

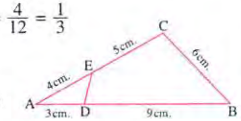
Model 2

First Multiple choice questions

1 (a)	2 (b)	3 (c)	4 (d)	5 (c)
6 (c)	7 (b)	8 (b)	9 (d)	10 (d)
11 (d)	12 (c)	13 (d)	14 (c)	15 (c)
16 (d)	17 (c)	18 (c)	19 (d)	20 (b)
21 (a)	22 (d)	23 (d)	24 (a)	25 (b)
26 (c)	27 (a)	28 (b)		

Second Essay questions

1



In Δ ADE , Δ ACB :

$\frac{AD}{AC} = \frac{3}{9} = \frac{1}{3}$, $\frac{AE}{AB} = \frac{4}{12} = \frac{1}{3}$

$\therefore \frac{AD}{AC} = \frac{AE}{AB}$

, ∠ A is common angle

$\therefore$ Δ ADE ~ Δ ACB

From similarity : $\therefore \frac{AD}{AC} = \frac{DE}{CB}$ $\quad \therefore \frac{DE}{6} = \frac{1}{3}$

$\therefore$ DE = 2 cm.

2

$\because \tan(\theta + 20°) = \cot(3\theta + 30°)$

$\therefore (\theta + 20°) + (3\theta + 30°) = 90° + 180° n$

$\therefore 4\theta + 50° = 90° + 180° n$

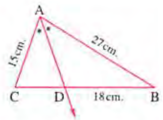
$\therefore 4\theta = 40° + 180° n$ $\quad \therefore \theta = 10° + 45° n$

, at $n = 0$ $\quad \therefore \theta = 10°$, at $n = 1$

$\therefore \theta = 55°$, at $n = 2$ $\quad \therefore \theta = 100°$ (refused)

$\therefore$ required values of θ are $10°$, $55°$

3



$\because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$

$\therefore \frac{27}{15} = \frac{18}{DC}$

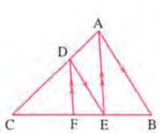
$\therefore DC = 10$ cm. , $AD = \sqrt{27 \times 15 - 18 \times 10} = 15$ cm.

4

$\frac{(4-3i)(4+3i)}{2+i} = \frac{25}{2+i} \times \frac{2-i}{2-i} = \frac{50-25i}{5} = 10 - 5i$

$\therefore x = 10$, $y = -5$

5



$\because \overline{DE} \,//\, \overline{AB}$

$\therefore \frac{CD}{CA} = \frac{CE}{CB}$ $\quad (1)$

, $\because \overline{AE} \,//\, \overline{DF}$

$\therefore \frac{CD}{CA} = \frac{CF}{CE}$ $\quad (2)$

From (1) , (2) :

$\therefore \frac{CE}{CB} = \frac{CF}{CE}$ $\quad \therefore (CE)^2 = CF \times CB$

Model 3

First Multiple choice questions

1 (c)	2 (d)	3 (b)	4 (a)	5 (d)
6 (a)	7 (a)	8 (b)	9 (b)	10 (c)
11 (a)	12 (a)	13 (c)	14 (c)	15 (b)
16 (d)	17 (c)	18 (c)	19 (c)	20 (c)
21 (b)	22 (d)	23 (a)	24 (a)	25 (c)
26 (a)	27 (b)	28 (a)		

Second Essay questions

1

$\because \frac{\text{a (The greater polygon)}}{\text{a (The smaller polygon)}} = \left(\frac{5}{3}\right)^2 = \frac{25}{9}$

$\therefore \frac{\text{a (The greater polygon)} - \text{a (The smaller polygon)}}{\text{a (The smaller polygon)}} = \frac{25-9}{9} = \frac{16}{9}$

$\therefore \frac{32}{\text{a (The smaller polygon)}} = \frac{16}{9}$

$\therefore$ The area of the smaller polygon is 18 cm^2.

, $\because \frac{\text{a (The greater polygon)}}{18} = \frac{25}{9}$

$\therefore$ The area of the greater polygon = 50 cm^2.

2

Write the quadratic function related to the inequality :

$f(x) = (x+3)^2 - 10 + 3(x+3)$

$= x^2 + 6x + 9 - 10 + 3x + 9 = x^2 + 9x + 8$

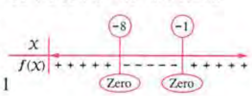
put $x^2 + 9x + 8 = 0$

$\therefore (x+8)(x+1) = 0$

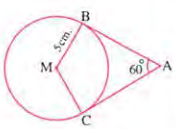
$\therefore x = -8$ or $x = -1$

$\because a > 0$

$\therefore$ The solution set = $[-8 , -1]$



3



In the quadrilateral ABCD :

$m(\angle BMC) = 360°$

$- (60° + 90° + 90°) = 120°$

In radians $= \frac{120° \times \pi}{180°} = \frac{2\pi}{3}$

$\therefore$ The length of the minor $\widehat{BC} = \frac{2\pi}{3} \times 5 = \frac{10\pi}{3}$ cm.

4

L.H.S. $= \sin 600° \cos(-30°) + \sin(150°)\cos(240°)$

$= \sin(360° + 240°)\cos(30°)$

$+ \sin(180° - 30°)\cos(180° + 60°)$

$= \sin(180° + 60°)\cos(30°)$

$+ \sin(180° - 30°)\cos(180° + 60°)$

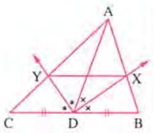
$= -\sin 60° \cos 30° + \sin 30° (-\cos 60°)$

$= -\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times -\frac{1}{2} = -1$

, R.H.S. $= \sin \frac{3\pi}{2} = -1$

$\therefore$ L.H.S. = R.H.S.

5



In Δ ABD :

$\because \overrightarrow{DX}$ bisects $\angle ADB$

$\therefore \frac{AX}{XB} = \frac{AD}{DB}$ $\quad (1)$

, in Δ ADC : $\because \overrightarrow{DY}$ bisects $\angle ADC$

$\therefore \frac{AY}{YC} = \frac{AD}{DC}$ $\quad (2)$

, $\because \overline{AD}$ is a median in Δ ABC $\quad \therefore BD = DC$ $\quad (3)$

From (1) , (2) , (3) : $\therefore \frac{AX}{XB} = \frac{AY}{YC}$ $\quad \therefore \overline{XY} \,//\, \overline{BC}$

Model 4

First Multiple choice questions

1 (b)	2 (a)	3 (c)	4 (a)	5 (b)
6 (b)	7 (d)	8 (b)	9 (c)	10 (c)
11 (d)	12 (c)	13 (b)	14 (a)	15 (a)
16 (c)	17 (c)	18 (c)	19 (b)	20 (c)
21 (c)	22 (a)	23 (c)	24 (c)	25 (a)
26 (d)	27 (d)	28 (c)		

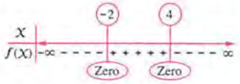
Second Essay questions

1

Put $8 + 2x - x^2 = 0$

$\therefore x^2 - 2x - 8 = 0$

$\therefore (x-4)(x+2) = 0$



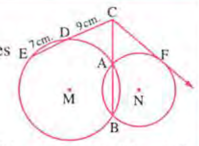
$\therefore x = 4$ or $x = -2$, $\because a < 0$

$\therefore f$ is positive at $x \in]-2 , 4[$, $f(x) = 0$ at $x \in \{-2 , 4\}$

, f is negative at $x \in \mathbb{R} - [-2 , 4]$

$\therefore$ The solution set of the inequality = $[-2 , 4]$

2



$\because$ A , B lies on the two circles

$\therefore P_M(A) = P_N(A) = 0$

, $P_M(B) = P_N(B) = 0$

$\therefore \overleftrightarrow{AB}$ is the principle axis of the two circles M and N

, $\because C \in \overleftrightarrow{AB}$ $\quad \therefore P_M(C) = P_N(C)$ $\quad$ (First req.)

, $P_M(C) = CD \times CE = 9 \times 16 = 144$

$\therefore CA \times (CA + 10) = 144$

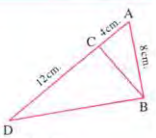
$\therefore (CA)^2 + 10(CA) - 144 = 0$

$((CA) + 18)((CA) - 8) = 0$

$\therefore CA = 8$ cm.

, $(CF)^2 = 144$ $\quad \therefore CF = 12$ cm. $\quad$ (Second req.)

3



$\because (AB)^2 = (8)^2 = 64$

, $AC \times AD = 4 \times 16 = 64$

$\therefore (AB)^2 = AC \times AD$

$\therefore \overleftrightarrow{AB}$ touches the circle passes through the points B , C , D

4

In Δ ABC : $\because \angle C$ is right

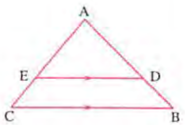
$\therefore \angle A$ complements $\angle B$ $\quad \therefore \cos B = \sin A$

$\therefore \sin A + \sin A = 1$ $\quad \therefore 2 \sin A = 1$

$\therefore \sin A = \frac{1}{2}$ $\quad \therefore m(\angle A) = 30°$

$\therefore \sin(5A) = \sin(150°) = \frac{1}{2}$

5



$\because \overline{DE} \,//\, \overline{BC}$

$\therefore \Delta ADE \sim \Delta ABC$

$\therefore \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \left(\frac{AD}{AB}\right)^2$

$= \left(\frac{2}{3}\right)^2$

$\therefore \frac{60}{\text{Area of } \Delta ABC} = \frac{4}{9}$

$\therefore$ Area of Δ ABC = 135 cm^2.

$\therefore$ Area of trapezium DBCE = 135 − 60 = 75 cm^2.

(The req.)

Model 5

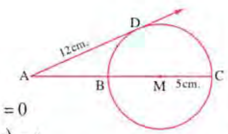
First Multiple choice questions

1 (b)	2 (d)	3 (d)	4 (c)	5 (c)
6 (b)	7 (b)	8 (c)	9 (c)	10 (c)
11 (b)	12 (b)	13 (b)	14 (c)	15 (a)
16 (b)	17 (c)	18 (d)	19 (c)	20 (a)
21 (b)	22 (b)	23 (c)	24 (d)	25 (d)
26 (b)	27 (a)	28 (d)		

Second Essay questions

1

∵ $\overleftrightarrow{AD}$ is a tangent



∴ $(AD)^2 = AB \times AC$

∴ $(12)^2 = AB\,(AB + 10)$

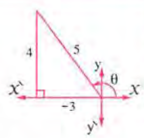
∴ $(AB)^2 + 10\,(AB) - 144 = 0$

∴ $\big((AB) + 18\big)\big((AB) - 8\big) = 0$

∴ AB = 8 cm. ∴ AC = 8 + 10 = 18 cm.

2

∵ $\sin\theta = \frac{4}{5}$, $90° < \theta < 180°$



∴ θ lies in the 2nd quadrant

∵ $\sin(180° - \theta) + \tan(360° - \theta) + 2\sin(270° - \theta)$

$= \sin\theta - \tan\theta - 2\cos\theta = \frac{4}{5} - \left(\frac{-4}{3}\right) - 2\left(\frac{-3}{5}\right) = \frac{10}{3}$

3

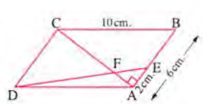
$x = \frac{13(1+i)}{5+i} \times \frac{5-i}{5-i} = \frac{13(5+4i-i^2)}{25+1} = \frac{13(6+4i)}{26} = 3 + 2i$

, $y = \frac{5+i}{1+i} \times \frac{1-i}{1-i} = \frac{5-5i+i-i^2}{1+1} = \frac{6-4i}{2} = 3 - 2i$

∴ $x + y = 3 + 2i + 3 - 2i = 6$

4

In △ABC : $AC = \sqrt{10^2 - 6^2}$ = 8 cm.



In △△AFE

, CFD : m (∠ AFE) = m (∠ CFD) (V.O.A)

, m (∠ EAF) = m (∠ ACD) (Alternate angles)

∴ △AFE ~ △CFD

From similarity : ∴ $\frac{AF}{FC} = \frac{AE}{CD}$ ∴ $\frac{AF}{8 - AF} = \frac{2}{6} = \frac{1}{3}$

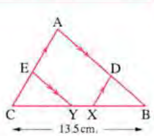
∴ 3 AF = 8 − AF ∴ 4 AF = 8

∴ AF = 2 cm. ∴ AE = AF = 2 cm.

∴ △AFE is an isosceles triangle

5

In △ABC : ∵ $\overline{DX} \parallel \overline{AC}$



∴ $\frac{BX}{BC} = \frac{BD}{BA}$ ∴ $\frac{BX}{13.5} = \frac{2}{5}$

∴ BX = 5.4 cm.

, ∵ $\overline{EY} \parallel \overline{AB}$ ∴ $\frac{CY}{CB} = \frac{CE}{CA}$ ∴ $\frac{CY}{13.5} = \frac{4}{9}$

∴ CY = 6 cm.

∴ XY = BC − (BX + CY) = 13.5 − (5.4 + 6) = 2.1 cm. (The req.)

Model 6

First Multiple choice questions

1 (c)	2 (c)	3 (a)	4 (a)	5 (d)
6 (c)	7 (b)	8 (d)	9 (d)	10 (c)
11 (c)	12 (b)	13 (c)	14 (c)	15 (b)
16 (d)	17 (b)	18 (c)	19 (d)	20 (c)
21 (c)	22 (d)	23 (d)	24 (d)	25 (a)
26 (b)	27 (c)	28 (d)		

Second Essay questions

1

θ + 20° + 3 θ + 30° = 90° + 180° n

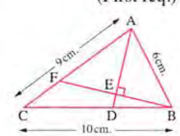
∴ 4 θ + 50 = 90° + 180° n ∴ 4 θ = 40° + 180° n

∴ θ = 10° + 45° n , at n = 0

∴ θ = 10° , at n = 1 ∴ θ = 55°

2

∵ BC = 10 cm. , BD = 4 cm. (First req.)



∴ DC = 6 cm.

In △ABC : $\frac{AB}{AC} = \frac{6}{9} = \frac{2}{3}$

, $\frac{BD}{DC} = \frac{4}{6} = \frac{2}{3}$

∴ $\frac{AB}{AC} = \frac{BD}{DC}$ ∴ $\overrightarrow{AD}$ bisects ∠ BAC

In △ABF : ∵ $\overrightarrow{AE}$ bisects ∠ BAF , $\overline{AE} \perp \overline{BF}$

∴ △ABF is an isosceles triangle

∴ AB = AF = 6 cm. , FC = 9 − 6 = 3 cm.

∴ a (△ABF) : a (△CBF) = AF : FC = 6 : 3 = 2 : 1 (Second req.)

3

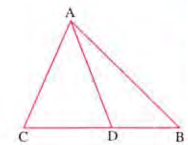
$\cos\theta = \frac{\sqrt{5}}{3}$, $\sin\theta = -\frac{2}{3}$

$\sin\left(\frac{\pi}{2} - \theta\right) + \cot(2\pi - \theta)$

$= \cos\theta - \cot\theta = \frac{\sqrt{5}}{3} - \left(\left(\frac{\sqrt{5}}{3}\right) \div \left(-\frac{2}{3}\right)\right) = \frac{5\sqrt{5}}{6}$

4

In △ACD , △BCA :



∵ $(AC)^2 = CD \times CB$

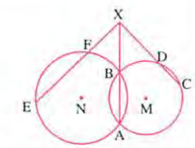
∴ $\frac{AC}{BC} = \frac{CD}{AC}$

, ∠ C is a common angle

∴ △ACD ~ △BCA

5

∵ A lies on the circle M



, A lies on the circle N

∴ $P_M(A) = P_N(A) = 0$

Similarly : $P_M(B) = P_N(B)$

∴ $\overleftrightarrow{AB}$ is the principle axis of the two circles M , N (First req.)

, ∵ X ∈ $\overleftrightarrow{AB}$, ∵ $P_M(X) = P_N(X)$

, ∵ $P_M(X) = XD \times XC$

, ∵ XD = 2 DC ∴ 144 = 2 DC × 3 DC

∴ $(DC)^2 = 24$ ∴ $DC = 2\sqrt{6}$ cm.

∴ $XC = 6\sqrt{6}$ cm.

, ∵ $P_N(X) = XF \times XE$ ∴ 144 = XF × (XF + 10)

∴ $144 = (XF)^2 + 10\,XF$

∴ $(XF)^2 + 10\,XF - 144 = 0$ ∴ (XF + 18) (XF − 8) = 0

∴ XF = 8 cm. (Second req.)

∵ $P_M(X) = P_N(X)$ ∴ XD × XC = XF × XE

∴ Figure CDFE is a cyclic quadrilateral. (Third req.)

Model 7

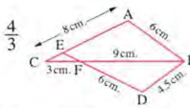
First Multiple choice questions

1 (c)	2 (c)	3 (c)	4 (a)	5 (d)
6 (b)	7 (c)	8 (b)	9 (d)	10 (b)
11 (c)	12 (b)	13 (b)	14 (b)	15 (c)
16 (b)	17 (c)	18 (a)	19 (a)	20 (b)
21 (a)	22 (d)	23 (a)	24 (b)	25 (c)
26 (b)	27 (b)	28 (d)		

Second Essay questions

1

In △ABC , △DBF



$\frac{AB}{DB} = \frac{6}{4.5} = \frac{4}{3}$, $\frac{AC}{DF} = \frac{8}{6} = \frac{4}{3}$

, $\frac{BC}{BF} = \frac{12}{9} = \frac{4}{3}$

∴ $\frac{AB}{DB} = \frac{AC}{DF} = \frac{BC}{BF}$ ∴ △ABC ~ △DBF (Q.E.D. 1)

From similarity : ∴ m (∠ C) = m (∠ DFB)

, ∵ m (∠ DFB) = m (∠ EFC) (V.O.A.)

∴ m (∠ C) = m (∠ EFC)

∴ △EFC is an isosceles triangle (Q.E.D. 2)

2

R.H.S. = sin 750° cos 300° + sin (− 60°) cot (120°)

= sin (720° + 30°) cos (360° − 60°) + sin (− 60°) cot (90° + 30°)

= sin 30° cos 60° − sin 60° (− tan 30°)

$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \frac{-1}{\sqrt{3}} = \frac{3}{4}$

∴ $\sin\theta = \frac{3}{4}$ (Positive)

∴ θ lies in the first or second quadrant

∴ θ ≃ 48° 35′ 25″ or θ = 180° − 48° 35′ 25″ ≃ 131° 24′ 35″

3

Put $x^2 - x + 12 = 0$

The discriminant = $b^2 - 4ac = (-1)^2 - 4(1)(12)$ = − 47 (< zero)

∴ The equation has no real roots , ∵ a > 0

∴ The sign of $f(x)$ is positive for all $x \in \mathbb{R}$

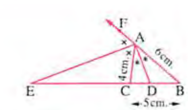
∴ $x^2 + 12 > x$

∴ $x^2 - x + 12 > 0$

∴ The solution set = ℝ

4

∵ $\overrightarrow{AD}$ bisects ∠ BAC



∴ $\frac{BA}{AC} = \frac{BD}{DC}$

∴ $\frac{6}{4} = \frac{BD}{5 - BD}$ ∴ 4 BD = 30 − 6 BD

∴ 10 BD = 30 ∴ BD = 3 cm. , DC = 2 cm.

∵ $\overrightarrow{AE}$ bisects ∠ BAC externally

$\therefore \frac{BA}{AC} = \frac{BE}{EC}$ $\therefore \frac{6}{4} = \frac{5 + EC}{EC}$

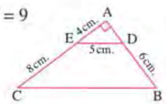
$\therefore 6\ EC = 20 + 4\ EC$ $\therefore 2\ EC = 20$

$\therefore EC = 10$ cm.

$\therefore ED = 2 + 10 = 12$ cm. (The req.)

5

In Δ DAE , which is right at A :



$(AD)^2 = (DE)^2 - (AE)^2 = 25 - 16 = 9$

$\therefore AD = 3$ cm.

In Δ ABC : $\because \frac{AD}{DB} = \frac{3}{6} = \frac{1}{2}$

, $\frac{AE}{EC} = \frac{4}{8} = \frac{1}{2}$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ $\therefore \overline{DE} \parallel \overline{BC}$ (First req.)

In Δ ABC which is right at A :

$\because (BC)^2 = (AB)^2 + (AC)^2 = 81 + 144 = 225$

$\therefore BC = 15$ cm. (Second req.)

Model 8

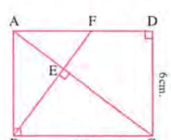
First Multiple choice questions

1 (c)	2 (d)	3 (c)	4 (b)	5 (d)
6 (c)	7 (b)	8 (c)	9 (c)	10 (c)
11 (c)	12 (d)	13 (d)	14 (d)	15 (b)
16 (c)	17 (d)	18 (c)	19 (b)	20 (d)
21 (b)	22 (a)	23 (b)	24 (b)	25 (b)
26 (c)	27 (b)	28 (c)		

Second Essay questions

1

In Δ ABC



$\because \angle$ B is right angle

, $\overline{BE} \perp \overline{CA}$

$\therefore (AB)^2 = AE \times AC$ (1)

$\therefore m(\angle D) + m(\angle FEC) = 180°$

$\therefore$ EFDC is a cyclic quadrilateral

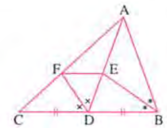
$\therefore AE \times AC = AF \times AD$ (2)

From (1) , (2) : $\therefore (AB)^2 = AF \times AD$ (First req.)

$\therefore (6)^2 = AF \times 8$ $\therefore AF = 4.5$ cm. (Second req.)

2

In Δ ABD :



$\because \overrightarrow{BE}$ bisects $\angle$ ABD

$\therefore \frac{AE}{ED} = \frac{AB}{BD}$ (1)

, in Δ ADC : $\because \overrightarrow{DF}$ bisects $\angle$ ADC

$\therefore \frac{AF}{FC} = \frac{AD}{DC}$ (2)

, $\because$ D is the midpoint of $\overline{BC}$ $\therefore BD = DC$ (3)

$\because AB = AD$ (4)

From (1) , (2) , (3) , (4) : $\therefore \frac{AE}{ED} = \frac{AF}{FC}$ $\therefore \overline{EF} \parallel \overline{BC}$

3

$\because \csc 6\theta = \sec 3\theta$ $\therefore 6\theta \pm 3\theta = \frac{\pi}{2} \pm 2\pi n$

$\therefore 6\theta + 3\theta = \frac{\pi}{2} + 2\pi n$ $\therefore 9\theta = \frac{\pi}{2} + 2\pi n$

$\therefore \theta = \frac{\pi}{18} + \frac{2\pi}{9} n$ where $n \in \mathbb{Z}$ or $6\theta - 3\theta = \frac{\pi}{2} + 2\pi n$

$\therefore 3\theta = \frac{\pi}{2} + 2\pi n$ $\therefore \theta = \frac{\pi}{6} + \frac{2\pi}{3} n$ where $n \in \mathbb{Z}$

4

The discriminant $= b^2 - 4ac = (-11)^2 - 4(7)(5)$

$= 121 - 140 = -19$

$\therefore$ The roots of the equation are non-real complex numbers.

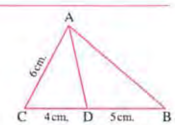
, $\because$ the coefficients and absolute term are real

$\therefore$ The two roots are conjugate

$\therefore x = \frac{11 \pm \sqrt{-19}}{2(7)} = \frac{11 \pm \sqrt{19}\, i}{14}$

5

$\because (AC)^2 = CD \times BC$



$\therefore \overleftrightarrow{AC}$ is a tangent

to the circle passing

through the points A , B , D (Q.E.D. 1)

, $\because \Delta\Delta$ ACD , BCA have :

$m(\angle DAC) = m(\angle B)$

(tangency and inscribed angles subtended by $\overset{\frown}{AD}$)

, $\angle$ C is a common angle.

$\therefore \Delta ACD \sim \Delta BCA$ (Q.E.D. 2)

$\because \frac{a(\Delta ACD)}{a(\Delta BCA)} = \left(\frac{CD}{CA}\right)^2 = \left(\frac{4}{6}\right)^2 = \frac{4}{9}$

$\therefore a(\Delta ACD) = 4k$, $a(\Delta BCA) = 9k$

$\therefore a(\Delta ABD) = 5k$

$\therefore \frac{a(\Delta ABD)}{a(\Delta ABC)} = \frac{5k}{9k} = \frac{5}{9}$ (Q.E.D. 3)

Model 9

First Multiple choice questions

1 (c)	2 (a)	3 (c)	4 (a)	5 (a)
6 (b)	7 (b)	8 (c)	9 (d)	10 (b)
11 (b)	12 (c)	13 (b)	14 (c)	15 (b)
16 (c)	17 (a)	18 (d)	19 (b)	20 (b)
21 (b)	22 (d)	23 (b)	24 (c)	25 (d)
26 (c)	27 (a)	28 (b)		

Second Essay questions

1

$\sin 420° \cos 330° + \frac{\sin 15°}{\sin 165°} + \tan^2 65° - \cot 25° \tan 65°$

$= \sin(360° + 60°) \cos(360° - 30°) + \frac{\sin(15°)}{\sin(180° - 15°)} + \tan(65°)\left(\tan(65°) - \cot(25°)\right)$

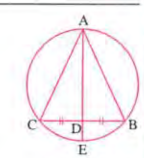
$= \sin(60°) \cos(30°) + \frac{\sin 15°}{\sin 15°} + \tan 65° \left(\tan 65° - \cot(90° - 65°)\right)$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + 1 + \tan 65° (\tan 65° - \tan 65°)$

$= \frac{3}{4} + 1 + \text{zero} = 1\frac{3}{4}$

2

(1) $\because \overline{AE} \cap \overline{BC} = \{D\}$



$\therefore DB \times DC = AD \times DE$

$\because$ D is the midpoint of $\overline{BC}$

, $\because BD = DC$

$\therefore (DB)^2 = AD \times DE$ (Q.E.D. 1)

(2) In Δ EBD , Δ CAD , $m(\angle EBD) = m(\angle EAC)$

two inscribed angles on the same arc $\overset{\frown}{EC}$

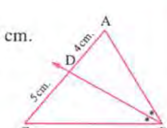
, $m(\angle AEB) = m(\angle ACB)$

two inscribed angles on the same arc $\overset{\frown}{AB}$

$\therefore \Delta EBD \sim \Delta CAD$ (Q.E.D. 2)

3

$\because$ The perimeter of Δ ABC = 27 cm.



$\therefore AB + BC = 27 - 9 = 18$ cm.

$\because \overrightarrow{BD}$ bisects $\angle$ ABC

$\therefore \frac{AB}{BC} = \frac{AD}{DC}$

$\therefore \frac{AB}{18 - AB} = \frac{4}{5}$ $\therefore 5\ AB = 72 - 4\ AB$

$\therefore 9\ AB = 72$ $\therefore AB = 8$ cm. , $BC = 18 - 8 = 10$ cm.

, $BD = \sqrt{8 \times 10 - 4 \times 5} = 2\sqrt{15}$ cm.

4

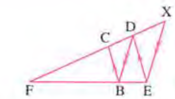
$y = \frac{3 + i}{i} \times \frac{i}{i} = \frac{3i + i^2}{i^2} = 1 - 3i$

the value of the expression $x^2 + 2xy + y^2$

$= (x + y)^2 = (2 + \cancel{3i} + 1 - \cancel{3i})^2 = (3)^2 = 9$

5

$\because \overline{BC} \parallel \overline{ED}$ and $\overleftrightarrow{FE}$



, $\overleftrightarrow{FD}$ are two transversals

$\therefore \frac{FB}{FE} = \frac{FC}{FD}$ (1)

, $\because \overline{BD} \parallel \overline{EX}$ and $\overleftrightarrow{FE}$, $\overleftrightarrow{FX}$ are two transversals

$\therefore \frac{FB}{FE} = \frac{FD}{FX}$ (2)

From (1) , (2) , by multiplying

$\therefore \left(\frac{FB}{FE}\right)^2 = \frac{FC}{FD} \times \frac{FD}{FX} = \frac{FC}{FX}$ (Q.E.D.)

Model 10

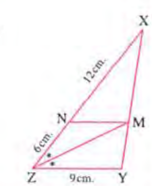
First Multiple choice questions

1 (b)	2 (d)	3 (b)	4 (b)	5 (c)
6 (b)	7 (c)	8 (d)	9 (b)	10 (b)
11 (a)	12 (c)	13 (a)	14 (d)	15 (a)
16 (b)	17 (a)	18 (d)	19 (b)	20 (d)
21 (c)	22 (b)	23 (b)	24 (a)	25 (b)
26 (a)	27 (b)	28 (b)		

Second Essay questions

1

In Δ XYZ : $\because \overrightarrow{ZM}$ bisects $\angle$ XZY



$\therefore \frac{XM}{MY} = \frac{XZ}{ZY}$

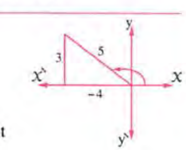
$\therefore \frac{XM}{MY} = \frac{18}{9} = \frac{2}{1}$

, $\because \frac{XN}{NZ} = \frac{12}{6} = \frac{2}{1}$

$\therefore \frac{XM}{MY} = \frac{XN}{NZ}$ $\therefore \overline{MN} \parallel \overline{YZ}$

2

$\therefore 5 \sin\theta - 3 = \text{zero}$



$\therefore \sin\theta = \frac{3}{5}$, $\frac{\pi}{2} < \theta < \pi$

$\therefore \theta$ lies in the second quadrant

$\therefore \cos\left(\frac{\pi}{2} - \theta\right) + \sin(2\pi - \theta) - \cos\left(\frac{3\pi}{2} - \theta\right) + \cos\theta$

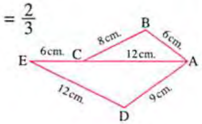
$= \cancel{\sin\theta} - \cancel{\sin\theta} + \sin\theta + \cos\theta$

$= \sin\theta + \cos\theta = \frac{3}{5} + \left(\frac{-4}{5}\right) = -\frac{1}{5}$

3

In Δ ABC , Δ ADE

$\therefore \frac{AB}{AD} = \frac{6}{9} = \frac{2}{3}$, $\frac{BC}{DE} = \frac{8}{12} = \frac{2}{3}$



, $\frac{AC}{AE} = \frac{12}{18} = \frac{2}{3}$

$\therefore \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$

$\therefore \Delta$ ABC $\sim \Delta$ ADE (Q.E.D. 1)

, from similarity : m ($\angle$ BAC) = m ($\angle$ DAE)

$\therefore \overrightarrow{AE}$ bisects $\angle$ BAD

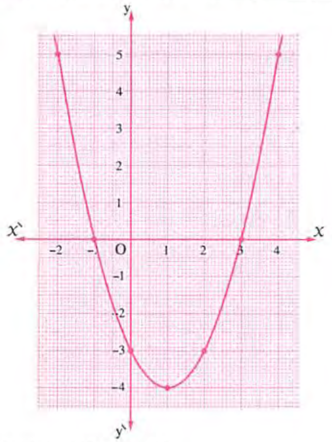
4

The x-coordinate of the vertex $= \frac{-b}{2a} = \frac{2}{2} = 1$

, $f(1) = (1)^2 - 2(1) - 3 = -4$

$\therefore$ The vertex of the curve is (1 , − 4)

x	−2	−1	0	1	2	3	4
y	5	0	−3	−4	−3	0	5



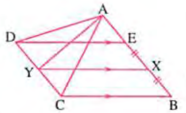
• $f(x) = 0$ at $x \in \{-1 , 3\}$

• f is negative at $x \in]-1 , 3[$

• f is positive at $x \in \mathbb{R} - [-1 , 3]$

5

$\because \overline{ED} \parallel \overline{XY} \parallel \overline{BC}$



$\therefore \frac{EX}{BX} = \frac{DY}{CY}$ (1)

, $\because$ AD × BX = AC × EX

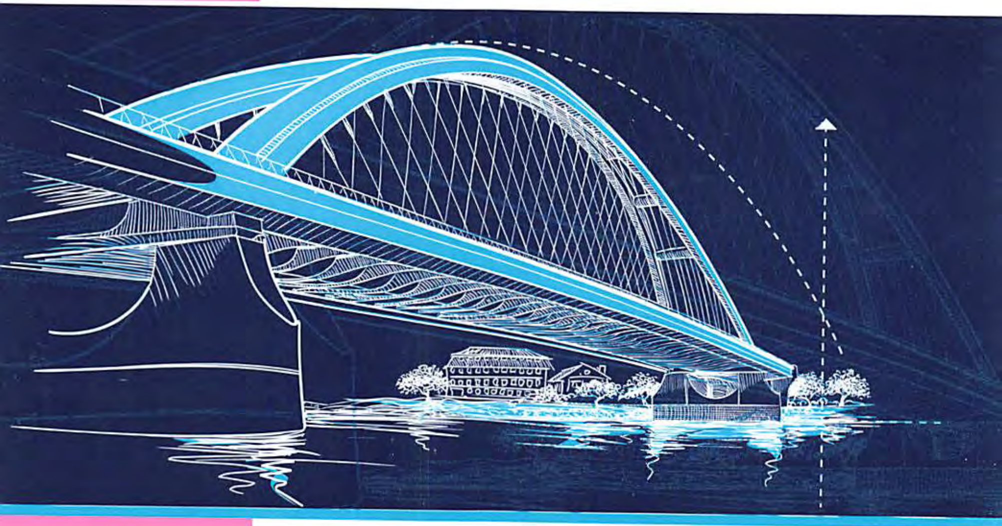
$\therefore \frac{EX}{BX} = \frac{AD}{AC}$ (2)

From (1) , (2) : $\therefore \frac{DY}{CY} = \frac{AD}{AC}$

$\therefore \overrightarrow{AY}$ bisects $\angle$ CAD (Q.E.D.)

Mathematics

By a group of supervisors



FIRST TERM

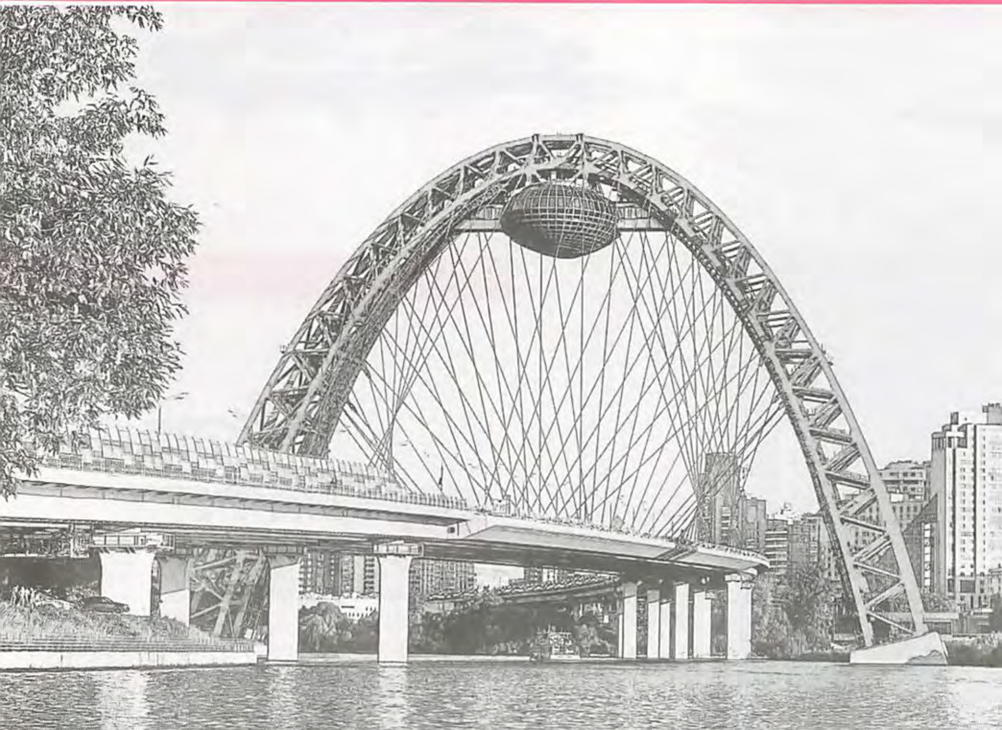
1 SEC.
2023

GUIDE ANSWERS

EL-MOASSER

First

Algebra and Trigonometry



Guide Answers of "Unit One"

Answers of pre-requirements

First Multiple choice questions

(1) d	(2) b	(3) c	(4) d
(5) d	(6) c	(7) a	(8) b
(9) b	(10) d	(11) a	(12) d
(13) b	(14) c	(15) a	(16) a
(17) c	(18) d	(19) a	(20) c
(21) d	(22) c		

Second Essay questions

1

(1) $\because a=1$, $b=-6$, $c=1$

$\therefore X=\frac{6\pm\sqrt{36-4\times1\times1}}{2\times1}=\frac{6\pm4\sqrt{2}}{2}=3\pm2\sqrt{2}$

$\therefore$ S.S. $=\{5.8 , 0.2\}$

(2) $\because a=1$, $b=3$, $c=5$

$\therefore X=\frac{-3\pm\sqrt{9-4\times1\times5}}{2\times1}=\frac{-3\pm\sqrt{-11}}{2}$

$\therefore$ S.S. $=\varnothing$

(3) $\because a=2$, $b=3$, $c=-4$

$\therefore X=\frac{-3\pm\sqrt{9-4\times2\times-4}}{2\times2}=\frac{-3\pm\sqrt{41}}{4}$

$\therefore$ S.S. $=\{0.9 , -2.4\}$

(4) $\because a=3$, $b=0$, $c=-65$

$\therefore X=\frac{\pm\sqrt{-4\times3\times-65}}{2\times3}=\frac{\pm\sqrt{780}}{6}$

$\therefore$ S.S. $=\{4.7 , -4.7\}$

(5) Multiplying by (X) $\quad\therefore X^2-3X-5=0$

$\therefore a=1$, $b=-3$, $c=-5$

$\therefore X=\frac{3\pm\sqrt{9-4\times1\times-5}}{2\times1}=\frac{3\pm\sqrt{29}}{2}$

$\therefore$ S.S. $=\{4.2 , -1.2\}$

(6) $\because \frac{3}{X-2}+\frac{2}{X+2}=2$

$\therefore \frac{3X+6+2X-4}{X^2-4}=2$

$\therefore 5X+2=2X^2-8$

$\therefore 2X^2-5X-10=0$

$\therefore a=2$, $b=-5$, $c=-10$

$\therefore X=\frac{5\pm\sqrt{25-4\times2\times-10}}{2\times2}=\frac{5\pm\sqrt{105}}{4}$

$\therefore$ S.S. $=\{3.8 , -1.3\}$

2

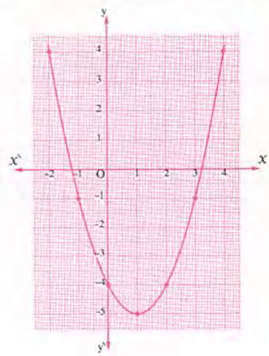
(1) $\because a=1$, $b=-2$, $c=-4$

$\therefore X=\frac{2\pm\sqrt{4-4\times1\times-4}}{2\times1}=\frac{2\pm\sqrt{20}}{2}=\frac{2\pm2\sqrt{5}}{2}$
$=1\pm\sqrt{5}$

$\therefore$ S.S. $=\{-1.2 , 3.2\}$

Let $f(X)=X^2-2X-4$

X	-2	-1	0	1	2	3	4
y	4	-1	-4	-5	-4	-1	4



From the graph :

S.S. $=\{-1.2 , 3.2\}$ approximately

(2) $\because a=-1$, $b=3$, $c=2$

$\therefore X=\frac{-3\pm\sqrt{9-4\times-1\times2}}{2\times-1}=\frac{-3\pm\sqrt{17}}{-2}$

$\therefore$ S.S. $=\{-0.6 , 3.6\}$

Let $f(X)=-X^2+3X+2$

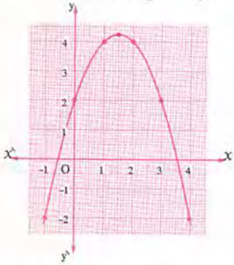
X	-1	0	1	2	3	4
y	-2	2	4	4	2	-2

$\because$ The X-coordinate of the curve vertex point

$= \frac{-b}{2a} = \frac{-3}{2 \times -1} = \frac{3}{2} = 1\frac{1}{2}$

$f\left(\frac{-b}{2a}\right) = f\left(1\frac{1}{2}\right) = -\left(1\frac{1}{2}\right)^2 + 3 \times 1\frac{1}{2} + 2 = 4\frac{1}{4}$

$\therefore$ The vertex point is $\left(1\frac{1}{2}, 4\frac{1}{4}\right)$



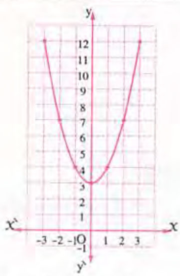
From the graph :
S.S. $= \{-0.6 , 3.6\}$ approximately

(3) $\because a = 1 , b = 0 , c = 3$

$\therefore X = \frac{\pm\sqrt{-4 \times 1 \times 3}}{2 \times 1} = \frac{\pm\sqrt{-12}}{2}$ $\quad \therefore$ S.S. $= \varnothing$

Let $f(X) = X^2 + 3$

X	-3	-2	-1	0	1	2	3
y	12	7	4	3	4	7	12



From the graph : S.S. $= \varnothing$

(4) $\because a = -2 , b = -4 , c = 1$

$\therefore X = \frac{4 \pm\sqrt{16 - 4 \times -2 \times 1}}{2 \times -2} = \frac{2 \pm \sqrt{6}}{-2}$

$\therefore$ S.S. $= \{0.2 , -2.2\}$

Let $f(X) = -2X^2 - 4X + 1$

$\therefore$ The vertex point is $(-1 , 3)$

X	-3	-2	-1	0	1
y	-5	1	3	1	-5

Draw the curve and from the graph :
S.S. $= \{0.2 , -2.2\}$ approximately.

3

(1) $\because 78 = \frac{n}{2}(1 + n)$

$\therefore \frac{n^2}{2} + \frac{n}{2} - 78 = 0$ (multiplying by 2)

$\therefore n^2 + n - 156 = 0$ $\quad \therefore (n - 12)(n + 13) = 0$

$\therefore n = 12$ or $n = -13$ (refused)

$\therefore$ no. of integers = 12 integers.

(2) $\because 171 = \frac{n}{2}(1 + n)$

$\therefore \frac{n^2}{2} + \frac{n}{2} - 171 = 0$ (multiplying by 2)

$\therefore n^2 + n - 342 = 0$ $\quad \therefore (n - 18)(n + 19) = 0$

$\therefore n = 18$ or $n = -19$ (refused)

$\therefore$ no. of integers = 18 integers.

(3) $\because 253 = \frac{n}{2}(1 + n)$

$\therefore \frac{n^2}{2} + \frac{n}{2} - 253 = 0$ (multiplying by 2)

$\therefore n^2 + n - 506 = 0$

$\therefore (n - 22)(n + 23) = 0$

$\therefore n = 22$ or $n = -23$ (refused)

$\therefore$ no. of integers = 22 integers.

(4) $\because 465 = \frac{n}{2}(1 + n)$

$\therefore \frac{n^2}{2} + \frac{n}{2} - 465 = 0$ (multiplying by 2)

$\therefore n^2 + n - 930 = 0$ $\quad \therefore (n - 30)(n + 31) = 0$

$\therefore n = 30$ or $n = -31$ (refused)

$\therefore$ no. of integers = 30 integers.

4

$\because X = 2$ is one root of the equation.

$\therefore 4 - 4a + 2a^2 - 12 = 0$ $\quad \therefore a^2 - 2a - 4 = 0$

$\therefore a = \frac{2 \pm\sqrt{4 - 4 \times 1 \times -4}}{2 \times 1} = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$

$\therefore a = 1 + \sqrt{5}$ or $a = 1 - \sqrt{5}$

5

$f(0) = -3$ $\qquad \therefore c = -3$

$\therefore f(x) = a x^2 + b x - 3$

$, \because 3$ is a root of the equation $f(x) = 0$

$\therefore 9a + 3b - 3 = 0$ $\qquad \therefore 3a + b = 1 \quad (1)$

$, \because -\frac{1}{2}$ is a root of the equation $f(x) = 0$

$\therefore \frac{1}{4}a - \frac{1}{2}b - 3 = 0$ $\qquad \therefore a - 2b = 12 \quad (2)$

Solving the two equations (1) and (2) :

$\therefore b = -5 \;, a = 2$

Answers of Exercise 1

First Multiple choice questions

(1) c	(2) d	(3) a	(4) b	(5) d
(6) c	(7) c	(8) b	(9) b	(10) c
(11) a	(12) c	(13) b	(14) b	(15) b
(16) a	(17) c	(18) d	(19) d	(20) c
(21) c	(22) d	(23) b	(24) a	(25) b
(26) d	(27) c	(28) d	(29) d	(30) a
(31) b	(32) d	(33) b	(34) c	(35) c
(36) c	(37) d	(38) d	(39) b	(40) c
(41) c	(42) a	(43) c	(44) a	(45) a
(46) b	(47) c			

Second Essay questions

1

(1) $6 + i - 12 i^2 = 18 + i$

(2) $4 - 20 i + 25 i^2 = -21 - 20 i$

(3) $9 - 12 i + 4 i^2 + 3 + 2 i = 8 - 10 i$

(4) $\left[(1+i)^2\right]^2 = \left[1 + 2i + i^2\right]^2 = (2i)^2 = 4i^2 = -4$

(5) $\left[(1+i)^2\right]^2 - \left[(1-i)^2\right]^2$

$= (1 + 2i + i^2)^2 - (1 - 2i + i^2)^2$

$= (2i)^2 - (-2i)^2 = 4i^2 - 4i^2 = \text{zero}$

(6) $\left[(1-i)^2\right]^5 = (1 - 2i + i^2)^5 = (-2i)^5$

$= -32 i^5 = -32 i$

(7) $\left(1 + (-2)\right)(2 + 3i + 4i^2)$

$= -1(2 + 3i - 4) = 2 - 3i$

2

(1) $\frac{4-5i}{7i} \times \frac{-7i}{-7i} = \frac{-28i + 35i^2}{-49i^2} = -\frac{5}{7} - \frac{4}{7} i$

(2) $\frac{26}{3-2i} \times \frac{3+2i}{3+2i} = \frac{78 + 52i}{9 - 4i^2} = \frac{78+52i}{13} = 6 + 4i$

(3) $\frac{2-3i}{3+i} \times \frac{3-i}{3-i} = \frac{6 - 11i + 3i^2}{9 - i^2} = \frac{3}{10} - \frac{11}{10} i$

(4) $\frac{3+4i}{5-2i} \times \frac{5+2i}{5+2i} = \frac{15 + 26i + 8i^2}{25 - 4i^2} = \frac{7}{29} + \frac{26}{29} i$

(5) $\frac{(3+2i)(2-i)}{3+i} = \frac{6 + i - 2i^2}{3+i} = \frac{8+i}{3+i}$

$\therefore \frac{8+i}{3+i} \times \frac{3-i}{3-1} = \frac{24 - 5i - i^2}{9 - i^2} = \frac{25 - 5i}{10} = \frac{5}{2} - \frac{1}{2} i$

(6) $\frac{(3+i)(3-i)}{3-4i} = \frac{9 - i^2}{3-4i} = \frac{10}{3-4i}$

$\therefore \frac{10}{3-4i} \times \frac{3+4i}{3+4i} = \frac{10(3+4i)}{9 - 16i^2} = \frac{10(3+4i)}{25}$

$= \frac{6}{5} + \frac{8}{5} i$

(7) $\frac{1}{(1+2i)^2} = \frac{1}{1 + 4i + 4i^2} = \frac{1}{-3+4i}$

$\therefore \frac{1}{-3+4i} \times \frac{-3-4i}{-3-4i} = \frac{-3-4i}{9 - 16i^2} = \frac{-3}{25} - \frac{4}{25} i$

(8) $\frac{1 + i + 2i^2 + 2i^3}{1 - 5i + 3i^2 - 3i^3} = \frac{-1-i}{-2-2i} = \frac{1}{2}$

(9) $\frac{2\sqrt{3} + 2\sqrt{2}\, i}{\sqrt{3} - 3\sqrt{2}\, i} \times \frac{\sqrt{3} + 3\sqrt{2}\, i}{\sqrt{3} + 3\sqrt{2}\, i} = \frac{6 + 8\sqrt{6}\, i + 12 i^2}{3 - 18 i^2}$

$= \frac{-2}{7} + \frac{8\sqrt{6}}{21} i$

3

(1) $\because 3x^2 + 12 = 0$ $\qquad \therefore 3x^2 = -12$

$\therefore x^2 = -4$ $\qquad \therefore x = \pm\sqrt{-4}$

$\therefore x = \pm\sqrt{4 i^2}$ $\qquad \therefore x = \pm 2i$

(2) $\because 4x^2 + 100 = 75$ $\qquad \therefore 4x^2 = -25$

$\therefore x^2 = -\frac{25}{4}$ $\qquad \therefore x = \pm\sqrt{-\frac{25}{4}}$

$\therefore x = \pm\sqrt{\frac{25}{4} i^2}$ $\qquad \therefore x = \pm \frac{5}{2} i$

(3) $x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times 5}}{2} = \frac{4 \pm \sqrt{-4}}{2}$

$= \frac{4 \pm \sqrt{4}\, i}{2} = 2 \pm i$

(4) $x = \frac{-6 \pm \sqrt{(6)^2 - 4 \times 2 \times 5}}{2 \times 2} = \frac{-6 \pm \sqrt{-4}}{4}$

$= \frac{-6 \pm \sqrt{4}\, i}{4}$

$= \frac{-3}{2} \pm \frac{1}{2} i$

4

(1) $\because (2X-3)+(3y+1)i = 7+10i$

$\therefore 2X-3=7$ $\qquad \therefore 2X=10$

$\therefore X=5 \quad , 3y+1=10$

$\therefore 3y=9$ $\qquad \therefore y=3$

(2) $\because (2X-y)+(X-2y)i = 5+i$

$\therefore 2X-y=5 \ (1) \quad , X-2y=1$ (2)

Multiply (1) by -2 : $\therefore -4X+2y=-10$ (3)

adding (2) and (3) : $\therefore -3X=-9$

$\therefore X=3$ $\qquad \therefore y=1$

(3) $\because 3X+Xi-2y+yi=5$

$\therefore (3X-2y)+(X+y)i=5$

$\therefore 3X-2y=5 \ (1) \quad , X+y=0$ (2)

Multiply (2) by 2 : $\therefore 2X+2y=0$ (3)

adding (1) and (3) : $\therefore 5X=5$

$\therefore X=1$ $\qquad \therefore y=-1$

(4) $\because X^2-y^2+(X+y)i=4i$

$\therefore X+y=4 \quad , X^2-y^2=0$

$\therefore (X+y)(X-y)=0$

$\therefore 4(X-y)=0$ $\qquad \therefore X=y=2$

(5) L.H.S. $= \frac{10}{2+i} \times \frac{2-i}{2-i} = \frac{10(2-i)}{4-i^2} = 4-2i$

$\therefore 4-2i = X+yi$ $\qquad \therefore X=4 \quad , y=-2$

(6) L.H.S. $= \frac{6-4i}{1-i} \times \frac{1+i}{1+i} = \frac{6+2i-4i^2}{1-i^2}$

$= \frac{10+2i}{2} = 5+i$

$\therefore 5+i = X+yi$ $\qquad \therefore X=5 \quad , y=1$

(7) L.H.S. $= \frac{(2+i)(2-i)}{3+4i} = \frac{4-i^2}{3+4i} = \frac{5}{3+4i}$

$= \frac{5}{3+4i} \times \frac{3-4i}{3-4i} = \frac{5(3-4i)}{9-16i^2}$

$= \frac{5(3-4i)}{25} = \frac{3}{5} - \frac{4}{5}i$

$\therefore \frac{3}{5} - \frac{4}{5}i = X+yi$ $\qquad \therefore X=\frac{3}{5} \quad , y=\frac{-4}{5}$

5

$\because X = \frac{13}{5-i} \times \frac{5+i}{5+i} = \frac{13(5+i)}{25-i^2} = \frac{13(5+i)}{26} = \frac{5}{2}+\frac{i}{2}$

$, y = \frac{3+2i}{1+i} \times \frac{1-i}{1-i} = \frac{3-i-2i^2}{1-i^2} = \frac{5}{2}-\frac{i}{2}$

$\therefore X , y$ are two conjugate numbers.

6

R.H.S. $= \frac{2+i}{2-i} \times \frac{2+i}{2+i} = \frac{4+4i+i^2}{4-i^2}$

$= \frac{3+4i}{5} = \frac{3}{5}+\frac{4}{5}i$

$\therefore a+bi = \frac{3}{5}+\frac{4}{5}i$ $\qquad \therefore a=\frac{3}{5} \ , b=\frac{4}{5}$

$\therefore a^2+b^2 = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = 1$

7 Ahmed's answer is the correct one because Karim expanded the expression $(2+3i)^2$ in a wrong way.

Third Higher skills

1

(1) c	(2) a	(3) d	(4) d
(5) d	(6) a	(7) b	(8) a
(9) c	(10) c	(11) c	(12) d
(13) d			

Instructions to solve 1 :

(1) $\because$ L is a root of the equation : $X^2+1=0$

$\therefore L^2+1=0$ $\qquad \therefore L^2=-1$

$\therefore L^{2018} = (L^2)^{1009} = (-1)^{1009} = -1$

Similarly $M^{2018} = -1$

$\therefore L^{2018} + M^{2018} = (-1)+(-1) = -2$

(2) $(1+i)^{2020} = [(1+i)^2]^{1010} = (2i)^{1010}$

$, (1-i)^{2020} = [(1-i)^2]^{1010} = (-2i)^{1010}$

$= (2i)^{1010}$

$\therefore (1+i)^{2020} = (1-i)^{2020}$

(3) $\left(\frac{1-i}{1+i}\right)^{100} = \left(\frac{(1-i)^2}{(1+i)^2}\right)^{50} = \left(\frac{-2i}{2i}\right)^{50} = 1$

(4) $(2+i)^{-1} = \frac{1}{2+i} \times \frac{2-i}{2-i} = \frac{2-i}{4+1} = \frac{2-i}{5}$

$\therefore$ Conjugate of the number $(2+i)^{-1}$ is $\frac{2+i}{5}$

(5) $X^2+4 = X^2-4i^2 = (X-2i)(X+2i)$

(6) $\because (X+2)+4yi = 3-4i$

$\therefore X+2=3$ $\qquad \therefore X=1$

$, 4y=-4$ $\qquad \therefore y=-1$

(7) $\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1+2i+i^2}{1-i^2} = \frac{1+2i-1}{1-(-1)} = i$

$\therefore \left(\frac{1+i}{1-i}\right)^n = (i)^n$

$\therefore$ The smallest positive integer which make $(i)^n = 1$ is 4

(8) $\because$ a , b , c , d are positive consecutive integer

$\therefore b = a + 1 \quad , \quad c = a + 2 \quad , d = a + 3$

$\therefore i^a + i^b + i^c + i^d = i^a + i^{a+1} + i^{a+2} + i^{a+3}$

$= i^a (1 + i + i^2 + i^3)$

$= i^a (1 + i - 1 - i)$

$= i^a \times \text{zero} = \text{zero}$

(9) $i + i^2 + i^3 + i^4 + ... + i^{100}$

$= i - 1 - i + 1 + ... + i^{100}$

(sum of each 4 consecutive terms = zero)

$\therefore$ The total sum = zero

(10) $(1 + i)(1 + i^2)(1 + i^3)(1 + i^4) ... (1 + i^{100})$

, $\because (1 + i^2) = (1 - 1) = \text{zero}$

$\therefore$ Product of

$(1 + i)(\text{zero})(1 + i^3)(1 + i^4) ... (1 + i^{100}) = \text{zero}$

(11) $\because i = i^5 \quad \therefore$ it is not necessary $m = n$

, $i^m = i^n$

$\therefore i^m = i^{n+4k}$ where k is an integer

$\therefore m = n + 4k \qquad \therefore m - n = 4k$

$\therefore m - n$ is a multiple of 4

, $\because i^m \times i^n = i^n \times i^n \qquad \therefore i^{m+n} = i^{2n+4k}$

$\therefore m + n = 2n + 4k$

$\therefore m + n = 2(n + 2k)$ is even number

(12) $\because a < b < 0$

$\therefore$ a , b are negative real numbers.

$\therefore \sqrt{ab}$ is a real number.

, $\because c > 0 \qquad \therefore bc < 0 \quad , \quad ba > 0$

$\therefore \sqrt{b(c-a)} = \sqrt{bc - ba}$ is an imaginary number.

$\therefore \sqrt{ab} = 2 \qquad \therefore ab = 4 , \sqrt{bc - ba} = 3i$

$\therefore bc - ba = -9 \qquad \therefore bc - 4 = -9$

$\therefore bc = -5$

(13) $\because$ There is no order in the set of non real complex number

$\therefore$ The correct answer is (d)

2

$\because 7i = (x + 3i)(y - i) - 9$

$= xy - xi + 3yi - 3i^2 - 9$

$= xy - xi + 3yi - 6$

$= (xy - 6) + (-x + 3y)i$

$\therefore xy - 6 = 0 \qquad \therefore xy = 6$

$, -x + 3y = 7 \qquad \therefore x = 3y - 7$

$\therefore y(3y - 7) = 6 \qquad \therefore 3y^2 - 7y = 6$

$\therefore 3y^2 - 7y - 6 = 0 \qquad \therefore (y - 3)(3y + 2) = 0$

$\therefore y = 3 \qquad \therefore x = 2$

or $y = -\frac{2}{3} \qquad \therefore x = -9$

3

$x = \frac{2+i}{2-i} \times \frac{2+i}{2+i} = \frac{4+4i+i^2}{4-i^2} = \frac{3+4i}{5} = \frac{3}{5} + \frac{4}{5}i$

$, y = \frac{2+3i}{2+i} \times \frac{2-i}{2-i} = \frac{4+4i-3i^2}{4-i^2} = \frac{7+4i}{5} = \frac{7}{5} + \frac{4}{5}i$

$\because 2x - y = a + bi$

$\therefore \frac{6}{5} + \frac{8}{5}i - \frac{7}{5} - \frac{4}{5}i = a + bi$

$\therefore -\frac{1}{5} + \frac{4}{5}i = a + bi \qquad \therefore a = -\frac{1}{5} , b = \frac{4}{5}$

$\therefore 9a^2 + b^2 = 9\left(-\frac{1}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = 1$

Answers of Exercise 2

First | Multiple choice questions

(1) a	(2) b	(3) b	(4) c	(5) c
(6) a	(7) c	(8) c	(9) b	(10) b
(11) b	(12) d	(13) b	(14) c	(15) d
(16) c	(17) d	(18) b	(19) d	(20) a
(21) d	(22) b	(23) d	(24) d	(25) d
(26) d	(27) d	(28) b	(29) d	(30) a
(31) b	(32) a	(33) c	(34) c	(35) a
(36) a	(37) d	(38) b	(39) c	(40) b
(41) c	(42) a	(43) d		

Second | Essay questions

1

(1) Discriminant $= (-2)^2 - 4 \times 1 \times 5 = -16 < 0$

$\therefore$ The two roots are complex and not real number.

(2) Discriminant $= (-10)^2 - 4 \times 1 \times 25 = 0$

$\therefore$ The two roots are real and equal.

(3) Discriminant $= (5)^2 - 4(-1)(-30) = -95 < 0$

$\therefore$ The two roots are complex and not real numbers.

(4) $\because x - 11 - x^2 + 6x = 0$

$\therefore x^2 - 7x + 11 = 0$

$\therefore$ The discriminant $= (-7)^2 - 4 \times 1 \times 11 = 5 > 0$

$\therefore$ The two roots are real and different.

(5) $\because x - \frac{2}{x-1} = 4$ multiplying by $(x-1)$

$\therefore x^2 - x - 2 = 4x - 4 \quad \therefore x^2 - 5x + 2 = 0$

$\therefore$ The discriminant $= (-5)^2 - 4 \times 1 \times 2 = 17 > 0$

$\therefore$ The two roots are real and different.

(6) $\because \frac{x}{x+1} + \frac{x}{x-1} = 3$

$\therefore \frac{x^2 - x + x^2 + x}{(x+1)(x-1)} = 3 \qquad \therefore \frac{2x^2}{x^2-1} = 3$

$\therefore 2x^2 = 3x^2 - 3 \qquad \therefore x^2 - 3 = 0$

$\therefore$ The discriminant $= (0)^2 - 4 \times 1 \times -3 = 12 > 0$

$\therefore$ The two roots are real and different.

(7) $\because (x-1)(x-7) = 2(x-3)(x-4)$

$\therefore x^2 - 8x + 7 = 2x^2 - 14x + 24$

$\therefore x^2 - 6x + 17 = 0$

$\therefore$ The discriminant $= (-6)^2 - 4 \times 1 \times 17 = -32 < 0$

$\therefore$ The two roots are complex and not real.

2

$\because$ The discriminant $= (-3)^2 - 4 \times 2 \times 2 = -7 < 0$

$\therefore$ The two roots are complex and not real.

$\because x = \frac{3 \pm \sqrt{-7}}{4} = \frac{3 \pm \sqrt{7}\, i}{4}$

$\therefore$ The two roots are : $\frac{3 + \sqrt{7}\, i}{4}$, $\frac{3 - \sqrt{7}\, i}{4}$

3

(1) $\because$ The two roots are equal

$\therefore$ The discriminant $= 0$

$\therefore (-3)^2 - 4 \times 1 \times \left(2 + \frac{1}{k}\right) = 0$

$\therefore 8 + \frac{4}{k} = 9 \qquad \therefore \frac{4}{k} = 1 \qquad \therefore k = 4$

(2) $\because$ The two roots are equal

$\therefore$ Discriminant = zero

$\therefore (2k+3)^2 - 4 \times 1 \times k^2 = 0$

$\therefore 4k^2 + 12k + 9 - 4k^2 = 0$

$\therefore k = -\frac{3}{4}$

(3) $\because$ The two roots are equal

$\therefore$ The discriminant $= 0$

$\therefore \left[2(k-1)\right]^2 - 4 \times 1 \times (2k+1) = 0$

$\therefore 4k^2 - 8k + 4 - 8k - 4 = 0$

$\therefore 4k^2 - 16k = 0$

$\therefore 4k(k-4) = 0 \qquad \therefore k = 0$ or $k = 4$

$\because$ The two roots are equal and each of them

$= \frac{-2k + 2 \pm \sqrt{0}}{2 \times 1} = \frac{-2k+2}{2} = -k + 1$

$\therefore$ At $k = 0$, then the two roots are equal and each of them $= 1$

At $k = 4$, then the two roots are equal and each of them $= -3$

(4) $\because$ The equation is : $x^2 - (2k+6)x + (7k+9) = 0$

$\because$ The two roots are equal

$\therefore$ The discriminant $= 0$

$\therefore (2k+6)^2 - 4 \times 1 \times (7k+9) = 0$

$\therefore 4k^2 + 24k + 36 - 28k - 36 = 0$

$\therefore 4k^2 - 4k = 0 \qquad \therefore 4k(k-1) = 0$

$\therefore k = 0$ or $k = 1$

$\because$ The two roots are equal and each of them

$= \frac{(2k+6) \pm \sqrt{0}}{2 \times 1} = k + 3$

$\therefore$ At $k = 0$

, then the two roots are equal and each of them $= 3$

, at $k = 1$

, then the two roots are equal and each of them $= 4$

4

The discriminant $= (-2m)^2 - 4 \times (m-1) \times m$

$= 4m^2 - 4m^2 + 4m = 4m$

, $\because$ the equation has no real roots

$\therefore 4m < 0 \qquad \therefore m < 0 \qquad \therefore m \in \left]-\infty , 0\right[$

5

(1) $\because$ The coefficients are rational numbers

, the discriminant $= (-3)^2 - 4 \times 2 \times -2$

$= 25$ (a perfect square)

$\therefore$ The two roots are rational.

• The algebraic check :

$\because 2x^2 - 3x - 2 = 0 \quad \therefore x = \frac{3 \pm \sqrt{25}}{4}$

$\therefore$ The two roots are 2 or $-\frac{1}{2}$ (rational)

(2) $\because$ One of the coefficients isn't a rational number

$\therefore$ The discriminant $= \left(\sqrt{5}\right)^2 - 4 \times 1 \times -5$

$= 25$ (a perfect square)

$\therefore$ The two roots are real and not rational number

• The algebraic check :

$\because x^2 + \sqrt{5}\, x - 5 = 0$

$\therefore x = \frac{-\sqrt{5} \pm \sqrt{25}}{2} = \frac{5-\sqrt{5}}{2}$ or $\frac{-5-\sqrt{5}}{2}$

$\therefore$ The two roots are : $\frac{5-\sqrt{5}}{2}$ or $\frac{-5-\sqrt{5}}{2}$

i.e. They are real and not rational numbers

(3) $\because 2x + 6 + x^2 - x = 9$

$\therefore x^2 + x - 3 = 0$

$\because$ The coefficients are rational.

$\therefore$ The discriminant $= (1)^2 - 4 \times 1 \times -3 = 13$ (not a perfect square)

$\therefore$ The two roots are real and not rational numbers

• The algebraic check :

$\because x^2 + x - 3 = 0 \qquad \therefore x = \frac{-1 \pm \sqrt{13}}{2}$

$\therefore$ The two roots are : $\frac{-1+\sqrt{13}}{2}$ or $\frac{-1-\sqrt{13}}{2}$

i.e. They are real and not rational.

6

$\because$ The coefficients are rational ,

discriminant $= b^2 - 4 \times a \times (b - a) = b^2 - 4\,ab + 4\,a^2$

$= (b - 2\,a)^2$ (a perfect square)

$\therefore$ The two roots are rational.

7

$\because$ The coefficients are rational numbers

$\therefore$ The discriminant $= (L - M)^2 - 4 \times L \times - M$

$= L^2 - 2\,L\,M + M^2 + 4\,L\,M$

$= L^2 + 2\,L\,M + M^2 = (L + M)^2$ (a perfect square)

$\therefore$ The two roots are rational.

8

$\because x^2 + k\,x + k - 1 = 0$

$\therefore$ The coefficients are rational

$\therefore$ The discriminant $= k^2 - 4 \times 1 \times (k - 1)$

$= k^2 - 4\,k + 4 = (k - 2)^2$

(a perfect square)

$\therefore$ The two roots are rational

9

$\because$ The coefficients are rational

, the discriminant $= (-2\,a^3)^2 - 4 \times 1 \times (a^6 - b^6)$

$= 4\,a^6 - 4\,a^6 + 4\,b^6 = 4\,b^6 = (2\,b^3)^2$ (a perfect square)

$\therefore$ The two roots are rational.

10

The discriminant $= (2\,a + 3)^2 - 4 \times (a + 2) \times (a - 1)$

$= 4\,a^2 + 12\,a + 9 - 4\,(a^2 + a - 2)$

$= 4\,a^2 + 12\,a + 9 - 4\,a^2 - 4\,a + 8 = 8\,a + 17$

$\because$ The two roots are real. $\qquad \therefore 8\,a + 17 \ge 0$

$\therefore a \ge -\frac{17}{8} \qquad \therefore a \in \left[-\frac{17}{8}, \infty\right[$

11

$\because$ The discriminant $= (-2\,a^3)^2 - 4\,(a^2 + 1) \times a^4$

$= 4\,a^6 - 4\,a^6 - 4\,a^4 = -4\,a^4$

$\because a^4$ is positive for all values of a except zero

$\therefore -4\,a^4$ is a negative number.

$\therefore$ The equation has no real roots for all the real values of a except zero.

12

$\because (x - a)(x - b) = 5$

$\therefore x^2 - (a + b)\,x + a\,b - 5 = 0$

$\therefore$ The discriminant $= (a + b)^2 - 4 \times 1 \times (a\,b - 5)$

$= a^2 + 2\,ab + b^2 - 4\,a\,b + 20$

$= a^2 - 2\,a\,b + b^2 + 20$

$= (a - b)^2 + 20$

is a positive quantity for all the real values of a , b

$\therefore$ The two roots are real and different.

13

The discriminant $= (-a)^2 - 4 \times (a - 1) \times 1$

$= a^2 - 4\,a + 4 = (a - 2)^2$

$\because a \ne 2 \qquad \therefore (a - 2)^2 > 0$ for every value of a

$\therefore$ The two roots are real and different.

Third **Higher skills**

1 (1) d (2) c (3) b
(4) d (5) d

Instructions to solve 1 :

(1) $\because$ The discriminant $= \left(-2\sqrt{5}\right)^2 - 4\,(1)\,(1)$

$= 20 - 4 = 16$

$\therefore$ The roots are real

, $\because$ the coefficient of "X" is irrational number.

$\therefore$ The roots are real but irrational

(2) $\because$ $(b^2 - 4\,a\,c)$ is not positive

$\therefore$ Either $(b^2 - 4\,a\,c)$ is negative and so the roots of the equation are complex and conjugate or $(b^2 - 4\,a\,c) =$ zero

$\therefore$ The roots are real and equal

, $\because a , b , c \in \mathbb{R}$

$\therefore$ The roots are complex and conjugate.

(3) $\because a + b + c = 0$

$\therefore$ The equation $(-2\,a)\,X^2 - 2\,b\,X - 2\,c = 0$ can be written as : $a\,X^2 + b\,X + c = 0$

, the discriminant $= b^2 - 4\,a\,c$

$= (-a - c)^2 - 4\,a\,c$

$= a^2 + 2\,a\,c + c^2 - 4\,a\,c$

$= a^2 - 2\,a\,c + c^2 = (a - c)^2$

, $\because a \neq c$ $\qquad \therefore (a - c)^2 > 0$

$\therefore$ The roots of the equation are real different

, $\because a , b , c \in \mathbb{Z}$

$\therefore$ The roots are rational different.

(4) • $X^2 - 4\,X - 5 = 0$

, discriminant : $(-4)^2 - 4 \times 1 \times -5 = 36 > 0$

$\therefore$ The two roots are real and different.

• $\sqrt{3}\,X^2 + \sqrt{5}\,X - 1 = 0$

, discriminant : $5 - 4\sqrt{3} \times -1 = 5 + 4\sqrt{3} > 0$

$\therefore$ The two roots are real and different.

• $X^2 - 3\sqrt{2}\,X + 4 = 0$

, discriminant : $\left(-3\sqrt{2}\right)^2 - 4 \times 1 \times 4 = 2 > 0$

$\therefore$ The two roots are real and different

• $3\,X^2 - \sqrt{7}\,X + 5 = 0$

, discriminant : $7 - 4 \times 3 \times 5 = -53 < 0$

, $\because$ the coefficients are real number , discriminant is negative.

$\therefore$ The two roots are non real conjugate complex numbers.

(5) $\because$ The two roots are conjugate complex numbers.

$\therefore$ Discriminant ≤ 0

$\therefore \left(-2\sqrt{2}\right)^2 - 4 \times 1 \times a \leq 0$

$\therefore -4\,a \leq -8$ $\qquad \therefore a \geq 2$

$\therefore a \in [2 , \infty[$

2

The discriminant $= (2\,a)^2 - 4 \times 1 \times (a^2 - b^2 - c^2)$

$= 4\,a^2 - 4\,a^2 + 4\,b^2 + 4\,c^2$

$= 4\,(b^2 + c^2) \geq 0$

(for every real value of b , c)

$\therefore$ The two roots are real.

3

$\because \dfrac{1}{X + a} = \dfrac{1}{X} + \dfrac{1}{a}$ $\qquad \therefore \dfrac{1}{X + a} = \dfrac{X + a}{a\,X}$

$\therefore (X + a)^2 - a\,X = 0$

$\therefore X^2 + 2\,a\,X + a^2 - a\,X = 0$

$\therefore X^2 + a\,X + a^2 = 0$

$\therefore$ The discriminant $= a^2 - 4 \times 1 \times a^2 = -3\,a^2 < 0$

for every $a \in \mathbb{R}^*$ $\qquad \therefore$ The two roots aren't real.

Answers of Exercise 3

First **Multiple choice questions**

(1) d	(2) a	(3) c	(4) d	(5) d
(6) c	(7) b	(8) c	(9) c	(10) b
(11) d	(12) d	(13) d	(14) a	(15) c
(16) a	(17) a	(18) b	(19) d	(20) c
(21) b	(22) c	(23) c	(24) b	(25) a
(26) c	(27) c	(28) a	(29) c	(30) a
(31) a	(32) a	(33) a	(34) a	(35) b
(36) b	(37) b	(38) b	(39) c	(40) b
(41) c	(42) b	(43) c	(44) a	(45) c
(46) c	(47) a	(48) b	(49) d	(50) b

Second Essay questions

1

(1) $\because 3x^2 - 23x + 30 = 0$

$\therefore$ The sum of the two roots $= \frac{23}{3}$

, their product = 10

(2) $\because 4x^2 + 25x + 6 = 3x^2 - 10x + 8$

$\therefore x^2 + 35x - 2 = 0$

$\therefore$ The sum of the two roots $= -35$

, their product $= -2$

(3) Multiplying both sides by L.C.M. of denominators which is $2x$

$\therefore x^2 + 2 = 3x \qquad \therefore x^2 - 3x + 2 = 0$

$\therefore$ The sum of the two roots = 3

, their product = 2

(4) $\because (3x+2)(x-1) = (x+1)(x+2)$

$\therefore 3x^2 - x - 2 = x^2 + 3x + 2$

$\therefore 2x^2 - 4x - 4 = 0$

$\therefore x^2 - 2x - 2 = 0$

$\therefore$ The sum of the two roots = 2

, their product $= -2$

(5) $\because (a-1)x^2 + (1-a^2)x + a - 1 = 0$

$\therefore$ The sum of the two roots

$= \frac{-(1-a^2)}{a-1} = \frac{a^2-1}{a-1} = \frac{(a-1)(a+1)}{a-1} = a+1$

, their product $= \frac{a-1}{a-1} = 1$

(6) The sum of the two roots

$= \frac{-(a^2-b^2)}{a+b} = \frac{-(a-b)(a+b)}{a+b} = -(a-b)$

, their product $= \frac{a^2+2ab+b^2}{a+b} = \frac{(a+b)^2}{a+b} = a+b$

2

$\because$ The product of the two roots $= \frac{c}{a}$

$\therefore \frac{-c}{3} = \frac{-8}{3} \qquad \therefore c = 8$

$\therefore 3x^2 + 10x - 8 = 0 \qquad \therefore (3x-2)(x+4) = 0$

$\therefore x = \frac{2}{3}$ or $x = -4$

3

$\because$ The sum of the two roots $= \frac{-b}{a}$

$\therefore \frac{-b}{2} = \frac{-3}{2} \qquad \therefore b = 3$

$\therefore 2x^2 + 3x - 5 = 0$

$\therefore (2x+5)(x-1) = 0 \qquad \therefore x = \frac{-5}{2}$ or $x = 1$

4

(1) $\because$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = 2$

$\therefore -1 +$ the other root $= 2$

$\therefore$ The other root = 3

$\because$ The product of the two roots

$= \frac{\text{the absolute term}}{\text{coefficient of } x^2} = a$

$\therefore -1 \times 3 = a \qquad \therefore a = -3$

(2) $\because$ The product of the two roots $= \frac{\text{the absolute term}}{\text{coefficient of } x^2}$

$= \frac{3}{2}$

$\therefore \frac{1}{2} \times$ the other root $= \frac{3}{2}$ $\therefore$ The other root = 3

$\because$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = \frac{a}{2}$

$\therefore \frac{1}{2} + 3 = \frac{a}{2} \qquad \therefore a = 7$

(3) $\because$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = 2$

$\therefore (1+i) +$ the other root $= 2$

$\therefore$ The other root $= 1 - i$

$\therefore$ The product of the two roots

$= \frac{\text{the absolute term}}{\text{coefficient of } x^2} = a$

$\therefore (1+i)(1-i) = a$

$\therefore 1 - i^2 = a \qquad \therefore a = 2$

(4) $\because$ The product of the two roots $= \frac{\text{the absolute term}}{\text{coefficient of } x^2} = 5$

$\therefore (2+i) \times$ the other root $= 5$

$\therefore$ The other root $= 2 - i$

$\therefore$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

$= -a$

$\therefore (2+i) + (2-i) = -a \qquad \therefore a = -4$

5

(1) $\because$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = -a$

$\therefore 2 + 5 = -a \qquad \therefore a = -7$

$\because$ The product of the two roots

$= \frac{\text{the absolute term}}{\text{coefficient of } x^2} = b$

$\therefore 2 \times 5 = b \qquad \therefore b = 10$

(2) $\because$ The product of the two roots $= \frac{\text{the absolute term}}{\text{coefficient of } x^2}$

$= \frac{-21}{a}$

$\therefore -3 \times 7 = \frac{-21}{a}$ $\therefore a = 1$

$\because$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = b$

$\therefore -3 + 7 = b$ $\therefore b = 4$

(3) $\because$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = \frac{1}{a}$

$\therefore -1 + \frac{3}{2} = \frac{1}{a}$ $\therefore a = 2$

$\because$ The product of the two roots

$= \frac{\text{the absolute term}}{\text{coefficient of } x^2} = \frac{b}{2}$

$\therefore -1 \times \frac{3}{2} = \frac{b}{2}$ $\therefore b = -3$

(4) $\because$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = -a$

$\therefore \sqrt{3}\,i + \left(-\sqrt{3}\,i\right) = -a$ $\therefore a = 0$

$\because$ The product of the two roots

$= \frac{\text{the absolute term}}{\text{coefficient of } x^2} = b$

$\therefore \sqrt{3}\,i \times \left(-\sqrt{3}\,i\right) = b$ $\therefore b = 3$

6

(1) $\because$ One of the two roots is the additive inverse of the other.

$\therefore k - 1 = 0$ $\therefore k = 1$

(2) $\because$ One of the two roots is the multiplicative inverse of the other.

$\therefore k - 2 = -4$ $\therefore k = -2$

(3) $\because$ One of the two roots is the multiplicative inverse of the other.

$\therefore 4k = k^2 + 4$ $\therefore k^2 - 4k + 4 = 0$

$\therefore (k-2)^2 = 0$ $\therefore k = 2$

(4) $\because 2x^2 - 5x + k^2 - 2 = 0$

One of the two roots is the multiplicative inverse of the other.

$\therefore k^2 - 2 = 2$ $\therefore k = \pm 2$

7

Let one of the two roots $= L$

$\therefore$ The other root $= 2L + 1$

$\therefore L(2L+1) = 21$ $\therefore 2L^2 + L - 21 = 0$

$\therefore (2L+7)(L-3) = 0$ $\therefore L = \frac{-7}{2}$ or $L = 3$

$\because L + (2L+1) = a$ $\therefore 3L + 1 = a$

$\therefore a = -9.5$ or $a = 10$

8

(1) $\because$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

$= \frac{-(a-3)}{a-2}$

$\therefore 3 = \frac{-a+3}{a-2}$ $\therefore 3a - 6 = -a + 3$

$\therefore a = \frac{9}{4}$

(2) $\because$ The product of the two roots

$= \frac{\text{the absolute term}}{\text{coefficient of } x^2} = \frac{-4}{a-2}$

$\therefore -4 = \frac{-4}{a-2}$ $\therefore a = 3$

9

(1) $\because$ The sum of the two roots $= \frac{3-k}{k-4} = 5$

$\therefore 3 - k = 5k - 20$

$\therefore 6k = 23$ $\therefore k = \frac{23}{6}$

(2) $\because$ The product of the two roots $= \frac{-3}{k-4} = -3$

$\therefore k - 4 = 1$ $\therefore k = 5$

(3) $\because$ The sum of the two roots $= 0$

$\therefore 3 - k = 0$ $\therefore k = 3$

(4) $\because$ One of the two roots is the multiplicative inverse of the other

$\therefore -3 = k - 4$ $\therefore k = 1$

10

Let the two roots be L , 2 L

$\therefore$ The sum of the two roots $= \frac{k-1}{2} = 3L$

$\therefore L = \frac{k-1}{6}$ (1)

$\therefore$ The product of the two roots $= \frac{k^2+2k-3}{2}$

$= 2L^2$ (2)

from (1) , (2) :

$\therefore \frac{k^2+2k-3}{2} = 2\left(\frac{k-1}{6}\right)^2$

$\therefore k^2 + 2k - 3 = 4\left(\frac{k^2-2k+1}{36}\right)$

$\therefore 9k^2 + 18k - 27 = k^2 - 2k + 1$

$\therefore 8k^2 + 20k - 28 = 0$ $\therefore 2k^2 + 5k - 7 = 0$

$\therefore (2k+7)(k-1) = 0$ $\therefore k = -3.5$ or $k = 1$

11

Let the two roots be L , 4 L

$\therefore$ The sum of the two roots $= a = 5L$

$\therefore L = \frac{1}{5} a$ (1)

, the product of the two roots $= 2a - 4 = 4L^2$ (2)

from (1) , (2) :

$\therefore 2a - 4 = 4\left(\frac{1}{5} a\right)^2$ $\therefore 2a - 4 = \frac{4}{25} a^2$

$\therefore 2a^2 - 25a + 50 = 0$

$\therefore (a - 10)(2a - 5) = 0$ $\therefore a = 10$ or $a = 2\frac{1}{2}$

12

$\because$ The sum of the two roots $= \frac{a}{a-2} = 3$

$\therefore a = 3a - 6$ $\therefore a = 3$

$\because$ The product of the two roots $= \frac{b^2}{a-2} = 5$

$\therefore b^2 = 5$ $\therefore b = \pm\sqrt{5}$

13

Let the two roots be L , L^2

$\because$ The sum of the two roots $= L + L^2 = 6$

$\therefore L^2 + L - 6 = 0$

$\therefore (L + 3)(L - 2) = 0$ $\therefore L = -3$ or $L = 2$

$\therefore$ The product of the two roots $= L \times L^2 = c$

$\therefore c = L^3$ At $L = -3$ $\therefore c = (-3)^3 = -27$

At $L = 2$ $\therefore c = 2^3 = 8$

14

Let the two roots be L , L^2

$\because$ The sum of the two roots $= L + L^2 = \frac{30}{8} = \frac{15}{4}$

$\therefore 4L^2 + 4L - 15 = 0$

$\therefore (2L - 3)(2L + 5) = 0$

$\therefore L = \frac{3}{2}$ or $L = \frac{-5}{2}$

$\because$ The product of the two roots $= L \times L^2 = \frac{c}{8}$

$\therefore c = 8L^3$

At $L = \frac{3}{2}$ $\therefore c = 8\left(\frac{3}{2}\right)^3 = 27$

At $L = \frac{-5}{2}$ $\therefore c = 8\left(\frac{-5}{2}\right)^3 = -125$

15

Let the two roots be L , $1 - L$

$\therefore$ The sum of the two roots $= \frac{a}{4} = 1$

$\therefore a = 4$

16

Let the two roots be L , $\frac{1}{L} + 1$

$\therefore$ The product of the two roots $= L\left(\frac{1}{L} + 1\right) = \frac{3}{2}$

$\therefore 1 + L = \frac{3}{2}$ $\therefore L = \frac{1}{2}$

$\because$ The sum of the two roots $= L + \frac{1}{L} + 1 = \frac{a}{2}$

$\therefore \frac{1}{2} + 2 + 1 = \frac{a}{2}$ $\therefore a = 7$

17

Let the two roots be L , $L^2 - 2$

$\therefore$ The sum of the two roots $= L^2 + L - 2 = 10$

$\therefore L^2 + L - 12 = 0$ $\therefore (L + 4)(L - 3) = 0$

$\therefore L = -4$ or $L = 3$

$\because$ The product of the two roots $= L^3 - 2L = c$

$\therefore c = -56$ or $c = 21$

18

Let the two roots be $2L$, $3L$

$\therefore$ The sum of the two roots $= \frac{-b}{a} = 5L$

$\therefore L = \frac{-b}{5a}$ (1)

$\because$ The product of the two roots $= \frac{c}{a} = 6L^2$ (2)

From (1) , (2) :

$\therefore \frac{c}{a} = 6\left(\frac{-b}{5a}\right)^2$ $\therefore \frac{c}{a} = \frac{6b^2}{25a^2}$

$\therefore 25ac = 6b^2$

19

Let the two roots be $2L$, $3L$

$\because$ The product of the two roots $= 6L^2 = \frac{3}{8}$

$\therefore L^2 = \frac{1}{16}$

$\therefore L = \frac{1}{4}$ (the negative solution is refused)

$\because$ The sum of the two roots $= 5L = \frac{b}{8}$

$\therefore L = \frac{b}{40}$ by substitution by $L = \frac{1}{4}$

$\therefore b = 10$

20

Let the two roots be L , M

$\therefore L + M = \frac{-(3a - 1)}{a + 1}$

, $LM = \frac{a^2 + 1}{a + 1}$ $\therefore \frac{1 - 3a}{a + 1} = \frac{a^2 + 1}{a + 1}$

$\therefore a^2 + 1 = 1 - 3a$ $\therefore a^2 + 3a = 0$

$\therefore a = 0$ or $a = -3$

21

(1) Let the two roots be L , $2L$

$\therefore$ The sum of the two roots $= \frac{-b}{a} = 3L$

$\therefore L = \frac{-b}{3a}$ (1)

, the product of the two roots $= \frac{c}{a} = 2L^2$ (2)

from (1) , (2) :

$\therefore \frac{c}{a} = 2\left(\frac{-b}{3a}\right)^2$ $\therefore \frac{c}{a} = \frac{2b^2}{9a^2}$

$\therefore 9ac = 2b^2$

$\therefore$ That is the satisfying condition.

(2) Let the two roots be L , L + 3

$\therefore$ The sum of the two roots $= \frac{-b}{a} = 2L + 3$

$\therefore L = \frac{1}{2}\left(\frac{-b}{a} - 3\right)$ (1)

, the product of the two roots $= \frac{c}{a} = L^2 + 3L$ (2)

From (1) , (2) :

$\therefore \frac{c}{a} = \frac{1}{4}\left(\frac{-b}{a} - 3\right)^2 + \frac{3}{2}\left(\frac{-b}{a} - 3\right)$

$= \frac{1}{4}\left(\frac{b^2}{a^2} + 6\frac{b}{a} + 9\right) - \frac{3b}{2a} - \frac{9}{2}$

$= \frac{b^2}{4a^2} + \frac{3b}{2a} + \frac{9}{4} - \frac{3b}{2a} - \frac{9}{2} = \frac{b^2}{4a^2} - \frac{9}{4}$

$\therefore \frac{c}{a} = \frac{b^2 - 9a^2}{4a^2}$ $\therefore 4ac = b^2 - 9a^2$

$\therefore$ That is the satisfying condition.

22

$\because$ The sum of the two roots of the first equation = a + 4

, the product of the two roots of the second equation $= \frac{a^2}{2}$

$\therefore a + 4 = \frac{a^2}{2}$ $\therefore a^2 - 2a - 8 = 0$

$\therefore (a-4)(a+2) = 0$ $\therefore a = 4$ or $a = -2$

23

Noura's answer is the correct because she put the equation on the form $a X^2 + b X + c = 0$

Third Higher skills

1 (1) c (2) b (3) a (4) c

Instructions to solve 1 :

(1) $\because$ The coefficients are real numbers and one of the two roots is 2 i , then the other root is – 2 i

, sum of the two roots = 2 i – 2 i = zero

, product of the two roots = 2 i × (– 2 i)

$= -4 \times -1 = 4$

and the discriminant < 0

(2) $\because$ b , c are real numbers.

$\therefore$ If one of the roots is (3 + i) , then the other root (3 – i) and that is sufficient to find b and c

(3) From the graph , the roots of the equation are 5 and 2

$\therefore$ The sum of the roots $= \frac{-b}{a} = 7$

, their product $\frac{c}{a} = 10$

$\therefore \frac{b+c}{a} = (-7) + (10) = 3$

(4) $\because X_1 < 0 < X_2$ $\therefore X_1 . X_2 < 0$

$\therefore \frac{c}{a} < 0$

$, \because |X_1| > |X_2|$ $\therefore X_1 + X_2 < 0$

$\therefore \frac{-b}{a} < 0$ $\therefore \frac{c}{a} . \frac{-b}{a} > 0$

$\therefore \frac{-bc}{a^2} > 0$ $\therefore -bc > 0$

$\therefore bc < 0$

2

$\because$ Discriminant $= (2a-1)^2 - 12(a-4)$

$= 4a^2 - 4a + 1 - 12a + 48$

$= 4a^2 - 16a + 49$

$= 4(a^2 - 4a + 4) + 33$

$= 4(a-2)^2 + 33 > 0$

whatever the value of a

$\therefore$ This equation has two different roots and these roots have different signs if the product of the roots < 0

$\therefore \frac{a-4}{3} < 0$ $\therefore a - 4 < 0$

$\therefore a < 4$ $\therefore a \in]-\infty , 4[$

Answers of Exercise 4

First Multiple choice questions

(1) d	(2) a	(3) c	(4) d
(5) c	(6) c	(7) b	(8) b
(9) a	(10) a	(11) b	(12) d
(13) d	(14) b	(15) c	(16) c
(17) c	(18) b	(19) a	(20) b
(21) d	(22) b	(23) d	(24) c
(25) c	(26) b	(27) c	(28) a
(29) c	(30) b	(31) b	(32) c
(33) d	(34) d	(35) a	(36) c

Second Essay questions

1

(1) $\because$ The sum of the two roots = 2

, their product = – 8

$\therefore$ The equation is : $x^2 - 2x - 8 = 0$

(2) $\because$ The sum of the two roots = 14

, their product = 49

$\therefore$ The equation is : $x^2 - 14x + 49 = 0$

(3) $\because$ The sum of the two roots = – 7

, their product = 0

$\therefore$ The equation is : $x^2 + 7x = 0$

(4) $\because$ The sum of the two roots = $\frac{13}{6}$

, their product = 1

$\therefore$ The equation is : $x^2 - \frac{13}{6}x + 1 = 0$

i.e. $6x^2 - 13x + 6 = 0$

(5) $\because$ The sum of the two roots = $\frac{-8}{5}$

, their product = $\frac{-33}{25}$

$\therefore$ The equation is : $x^2 - \frac{-8}{5}x + \frac{-33}{25} = 0$

i.e. $25x^2 + 40x - 33 = 0$

(6) $\because$ The sum of the two roots = $3\sqrt{3}$

, their product = – 30

$\therefore$ The equation is : $x^2 - 3\sqrt{3}x - 30 = 0$

(7) $\because$ The sum of the two roots = 14

, their product = 29

$\therefore$ The equation is : $x^2 - 14x + 29 = 0$

(8) $\because$ The sum of the two roots = 0

, their product = 25

$\therefore$ The equation is : $x^2 + 25 = 0$

(9) $\because$ The sum of the two roots = 2

, their product = 10

$\therefore$ The equation is : $x^2 - 2x + 10 = 0$

(10) $\because$ The sum of the two roots = 6

, their product = 17

$\therefore$ The equation is : $x^2 - 6x + 17 = 0$

(11) $\because$ The sum of the two roots $= \frac{3}{i} + \frac{3+3i}{1-i}$

$= \frac{3-3i+3i-3}{1+i} = 0$

, their product $= \frac{3}{i} \times \frac{3+3i}{1-i} = \frac{9+9i}{1+i} = 9$

$\therefore$ The equation is : $x^2 + 9 = 0$

(12) $\because$ The sum of the two roots

$= \frac{-2+2i}{1+i} + \frac{-2-4i}{2-i} = \frac{-2+6i-6i+2}{3+i} = 0$

, their product $= \frac{-2+2i}{1+i} \times \frac{-2-4i}{2-i}$

$= \frac{12+4i}{3+i} = 4$

$\therefore$ The equation is : $x^2 + 4 = 0$

(13) $\because$ The sum of the two roots = 2 a

, their product = $a^2 - b^2$

$\therefore$ The equation is : $x^2 - 2ax + a^2 - b^2 = 0$

(14) $\because$ The sum of the two roots

$= \frac{(a-b)(a+b)}{a-b} + \frac{(a-b)(a^2+ab+b^2)}{a^2+ab+b^2}$

$= a + b + a - b = 2a$

, their product $= (a+b)(a-b) = a^2 - b^2$

$\therefore$ The equation is : $x^2 - 2ax + a^2 - b^2 = 0$

2

$\boxed{L + M = 7}$, $\boxed{LM = 5}$

(1) $L^2M + M^2L = LM(L+M)$

$= 5 \times 7 = 35$

(2) $\frac{1}{M} + \frac{1}{L} = \frac{L+M}{LM} = \frac{7}{5}$

(3) $(L-2)(M-2) = LM - 2(L+M) + 4$

$= 5 - 14 + 4 = -5$

(4) $\left(L + \frac{1}{M}\right)\left(M + \frac{1}{L}\right) = LM + 2 + \frac{1}{LM}$

$= 5 + 2 + \frac{1}{5} = 7\frac{1}{5}$

3

$\boxed{L + M = 4}$, $\boxed{LM = 2}$

(1) $L^2 + M^2 = (L+M)^2 - 2LM$

$= 4^2 - 2 \times 2 = 12$

(2) $\because (L-M)^2 = (L+M)^2 - 4LM$

$= 4^2 - 4 \times 2 = 8$

$\therefore L - M = 2\sqrt{2}$, where $L > M$

(3) $L^3 + M^3 = (L+M)\left[(L+M)^2 - 3LM\right]$

$= 4(16 - 6) = 40$

(4) $\because$ L is a root for the equation : $x^2 - 4x + 2 = 0$

$\therefore L^2 - 4L + 2 = 0$ $\therefore L^2 - 4L + 7 = 5$

(5) $\because$ M is a root for the equation : $x^2 - 4x + 2 = 0$

$\therefore M^2 - 4M + 2 = 0$ $\therefore 2M^2 - 8M + 4 = 0$

$\therefore 2M^2 - 8M + 15 = 11$

4

$\because L + M = 3$, $LM = -5$

, let D , E be the two roots of the required equation

$\therefore D = L - 4$, $E = M - 4$

$\therefore D + E = L - 4 + M - 4 = (L + M) - 8$

$= 3 - 8 = -5$

, $D E = (L - 4)(M - 4) = L M - 4 (M + L) + 16$

$= -5 - 4 (3) + 16 = -1$

$\therefore$ The required equation is : $x^2 + 5 x - 1 = 0$

5

$\because L + M = \frac{5}{2}$, $L M = \frac{-7}{2}$

and let D , E be the two roots of the required equation

$\therefore D = 1 - L$, $E = 1 - M$

$\therefore D + E = 1 - L + 1 - M = 2 - (L + M)$

$= 2 - \frac{5}{2} = -\frac{1}{2}$

, $D E = (1 - L)(1 - M) = 1 - (L + M) + L M$

$= 1 - \frac{5}{2} + \frac{-7}{2} = -5$

$\therefore$ The required equation is : $x^2 + \frac{1}{2} x - 5 = 0$

i.e. $2 x^2 + x - 10 = 0$

6

$\because L + M = 3$, $L M = -4$

, let D , E be the two roots of the required equation.

$\therefore D = \frac{1}{L}$, $E = \frac{1}{M}$

$\therefore D + E = \frac{1}{L} + \frac{1}{M} = \frac{L + M}{L M} = \frac{-3}{4}$

, $D E = \frac{1}{L} \times \frac{1}{M} = \frac{1}{L M} = -\frac{1}{4}$

$\therefore$ The required equation is : $x^2 + \frac{3}{4} x - \frac{1}{4} = 0$

i.e. $4 x^2 + 3 x - 1 = 0$

7

$\because L + M = \frac{5}{2}$, $L M = \frac{1}{2}$

, let D , E be the two roots of the required equation

$\therefore D = 2 L^2$, $E = 2 M^2$

$\therefore D + E = 2 L^2 + 2 M^2 = 2 (L^2 + M^2)$

$= 2 [(L + M)^2 - 2 L M] = 2 \left[\frac{25}{4} - 1\right] = \frac{21}{2}$

, $D E = 2 L^2 \times 2 M^2$

$= 4 (L M)^2 = 4 \times \frac{1}{4} = 1$

$\therefore$ The required equation is : $x^2 - \frac{21}{2} x + 1 = 0$

i.e. $2 x^2 - 21 x + 2 = 0$

8

let the two roots of the given equation be : L , M

and the two roots of the required equation be : D , E

$\therefore D = L + 1$, $E = M + 1$

$\therefore L = D - 1$ (1)

, $\because$ L is one of the roots of the equation :

$x^2 - 7 x - 9 = 0$ $\therefore L^2 - 7 L - 9 = 0$

, from (1) : $\therefore (D - 1)^2 - 7 (D - 1) - 9 = 0$

$\therefore D^2 - 2 D + 1 - 7 D + 7 - 9 = 0$

$\therefore D^2 - 9 D - 1 = 0$

$\therefore$ D is a root of the equation :

$x^2 - 9 x - 1 = 0$ which is the required equation.

9

let the two roots of the given equation be : L , M

, the two roots of the required equation be : D , E

$\therefore D = \frac{1}{2} L$, $E = \frac{1}{2} M$ $\therefore L = 2 D$ (1)

, $\because$ L is one of the roots of the equation :

$4 x^2 - 12 x + 7 = 0$ $\therefore 4 L^2 - 12 L + 7 = 0$

, from (1) : $\therefore 4 (2 D)^2 - 12 (2 D) + 7 = 0$

$\therefore 16 D^2 - 24 D + 7 = 0$

$\therefore$ D is a root of the equation : $16 x^2 - 24 x + 7 = 0$

which is the required equation.

10

let the two roots of the given equation be : L , M

$\therefore L + M = -3$, $L M = -5$

, let the two roots of the required equation be : D , E

$\therefore D = L^2$, $E = M^2$

$\therefore D + E = L^2 + M^2 = (L + M)^2 - 2 L M$

$= 9 + 10 = 19$

, $D E = (L^2 M^2) = (L M)^2 = (-5)^2 = 25$

$\therefore$ The required equation is : $x^2 - 19 x + 25 = 0$

11

$\because L + M = \frac{3}{2}$, $L M = -\frac{1}{2}$

, let the two roots of the required equation be : D , E

$\therefore D = \frac{L}{M}$, $E = \frac{M}{L}$

$\therefore D+E=\frac{L}{M}+\frac{M}{L}=\frac{L^2+M^2}{LM}=\frac{(L+M)^2-2LM}{LM}$

$=\frac{\left(\frac{3}{2}\right)^2-2\times-\frac{1}{2}}{-\frac{1}{2}}=\frac{-13}{2}$

$, DE=\frac{L}{M}\times\frac{M}{L}=1$

$\therefore$ The required equation is : $x^2+\frac{13}{2}x+1=0$

i.e. $2x^2+13x+2=0$

12

$\because L+M=2 , LM=-4$

, let D , E be the two roots of the required equation

$\therefore D=\frac{1}{L^2} , E=\frac{1}{M^2}$

$\therefore D+E=\frac{1}{L^2}+\frac{1}{M^2}$

$=\frac{M^2+L^2}{(LM)^2}=\frac{(L+M)^2-2LM}{(LM)^2}=\frac{4+8}{16}=\frac{3}{4}$

$, DE=\frac{1}{L^2}\times\frac{1}{M^2}=\frac{1}{(LM)^2}=\frac{1}{16}$

$\therefore$ The required equation is : $x^2-\frac{3}{4}x+\frac{1}{16}=0$

i.e. $16x^2-12x+1=0$

13

$\because L+M=\frac{5}{3} , LM=\frac{2}{3}$

, let the two roots of the required equation be : D , E

$\therefore D=\frac{L^2}{M} , E=\frac{M^2}{L}$

$\therefore D+E=\frac{L^2}{M}+\frac{M^2}{L}=\frac{L^3+M^3}{LM}$

$=\frac{(L+M)\left[(L+M)^2-3LM\right]}{LM}$

$=\frac{\frac{5}{3}\left[\frac{25}{9}-2\right]}{\frac{2}{3}}=\frac{5}{2}\times\frac{7}{9}=\frac{35}{18}$

$, DE=\frac{L^2}{M}\times\frac{M^2}{L}=LM=\frac{2}{3}$

$\therefore$ The required equation is : $x^2-\frac{35}{18}x+\frac{2}{3}=0$

i.e. $18x^2-35x+12=0$

14

$\because L+M=\frac{-12}{10}=-\frac{6}{5}$

$, LM=\frac{-1}{10}$, let the two roots of the required equation be D , E

$\therefore D=2L+\frac{1}{M} , E=2M+\frac{1}{L}$

$\therefore D+E=2L+\frac{1}{M}+2M+\frac{1}{L}$

$=2(L+M)+\frac{L+M}{LM}$

$=2\left(\frac{-6}{5}\right)+\frac{-\frac{6}{5}}{-\frac{1}{10}}=-\frac{12}{5}+12=\frac{48}{5}$

$, DE=\left(2L+\frac{1}{M}\right)\left(2M+\frac{1}{L}\right)=4LM+4+\frac{1}{LM}$

$=4\left(-\frac{1}{10}\right)+4-10=\frac{-32}{5}$

$\therefore$ The required equation is : $x^2-\frac{48}{5}x-\frac{32}{5}=0$

i.e. $5x^2-48x-32=0$

15

$\because L+M=3 , LM=-5$

, let the two roots of the required equation be : D , E

$\therefore D=L^2M , E=M^2L$

$\therefore D+E=L^2M+M^2L=LM(L+M)$

$=-5\times 3=-15$

$, DE=L^2M\times M^2L$

$=(LM)^3=(-5)^3=-125$

$\therefore$ The required equation is : $x^2+15x-125=0$

16

$\because L+M=3 , LM=5$

, let D , E be the two roots of the required equation

$\therefore D=6 , E=L^2+M^2$

$\therefore D+E=6+L^2+M^2$

$=6+(L+M)^2-2LM$

$=6+9-2\times 5=5$

$, DE=6(L^2+M^2)$

$=6\left[(L+M)^2-2LM\right]$

$=6(9-10)=-6$

$\therefore$ The required equation is : $x^2-5x-6=0$

17

$\because L+M=3 , LM=-1$

, let the two roots of the required equation be : D , E

$\therefore D=3L-2M , E=2L-3M$

$\therefore D+E=5L-5M=5(L-M)$

$, \because (L-M)^2=(L+M)^2-4LM=9+4=13$

$\therefore L-M=\sqrt{13}$ (where $L>M$)

$\therefore D+E=5(L-M)=5\sqrt{13}$

$, D E = (3 L - 2 M)(2 L - 3 M)$

$= 6 L^2 - 9 L M - 4 L M + 6 M^2$

$= 6 (L^2 + M^2) - 13 L M$

$= 6 [(L + M)^2 - 2 L M] - 13 L M$

$= 6 (L + M)^2 - 25 L M$

$= 6 \times 9 + 25 = 79$

$\therefore$ The required equation is : $x^2 - 5\sqrt{13}\, x + 79 = 0$

18

$\because L + 2 + M + 2 = 11 \qquad \therefore L + M = 7$

$, \because (L + 2)(M + 2) = 3$

$\therefore L M + 2 (L + M) + 4 = 3$

$\therefore L M + 2 \times 7 + 4 = 3 \qquad \therefore L M = -15$

$\therefore$ The required equation is : $x^2 - 7x - 15 = 0$

19

$\because L + 3 , M + 3$ are the two roots of the given equation

$\therefore L + 3 + M + 3 = 5 \qquad \boxed{\therefore L + M = -1}$

$, \because (L + 3)(M + 3) = 11 \qquad \therefore L M + 3 (L + M) = 2$

$\therefore L M + 3 (-1) = 2 \qquad \boxed{\therefore L M = 5}$

and let the two roots of the required equation be : D , E

$\therefore D = L^2 M , E = M^2 L$

$\therefore D + E = L^2 M + M^2 L$

$= L M (L + M) = 5 (-1) = -5$

$, D E = L^2 M \times M^2 L = (L M)^3 = 5^3 = 125$

$\therefore$ The required equation is : $x^2 + 5x + 125 = 0$

20

$\because \frac{1}{L} , \frac{1}{M}$ are the two roots of the given equation.

$\therefore \frac{1}{L} + \frac{1}{M} = 3 \qquad \therefore \frac{L + M}{L M} = 3$

$\therefore L + M = 3 L M$ (1)

$, \frac{1}{L} \times \frac{1}{M} = 1 \qquad \therefore \frac{1}{L M} = 1$

$\boxed{\therefore L M = 1}$ (2)

From (1) , (2) : $\therefore L + M = 3$

, let the two roots of the required equation be : D , E

$\therefore D = L M - 7 = 1 - 7 = -6$

$, E = L + M + 3 = 3 + 3 = 6$

$\therefore D + E = 0 , D E = -36$

$\therefore$ The required equation is : $x^2 - 36 = 0$

21

$\because L + M = 2 , L M = -5$

, let the two roots of the required equation be : D , E

$\therefore D = L^2 + M , E = M^2 + L$

$\therefore D + E = L^2 + M^2 + M + L$

$= (L + M)^2 - 2 L M + (M + L)$

$= 4 + 10 + 2 = 16$

$, D E = (L^2 + M)(M^2 + L)$

$= (L M)^2 + L^3 + M^3 + L M$

$= 25 - 5 + (L + M) [(L + M)^2 - 3 L M]$

$= 20 + 2 [2^2 - 3 \times -5] = 58$

$\therefore$ The required equation is : $x^2 - 16x + 58 = 0$

22

$\because \frac{3}{L} + \frac{3}{M} = 12 \qquad \therefore \frac{3 L + 3 M}{L M} = 12$

$\therefore M + L = 4 L M$ (1)

$, \frac{3}{L} \times \frac{3}{M} = 9 \qquad \therefore L M = 1$ (2)

From (1) , (2) : $\therefore M + L = 4$

, let the two roots of the required equation be : D , E

$\therefore D = \frac{1}{L^3} , E = \frac{1}{M^3}$

$\therefore D + E = \frac{1}{L^3} + \frac{1}{M^3} = \frac{L^3 + M^3}{L^3 M^3}$

$= \frac{(L + M) [(L + M)^2 - 3 L M]}{(L M)^3}$

$= \frac{4 [(4)^2 - 3 \times 1]}{1^3} = 52$

$, D E = \frac{1}{L^3} \times \frac{1}{M^3} = \frac{1}{(L M)^3} = 1$

$\therefore$ The required equation is : $x^2 - 52x + 1 = 0$

23

let the two roots of the given equation be : L , M

$\therefore L + M = \frac{7}{6}$ (1)

$, L M = \frac{1 - c}{6}$ (2)

$, L - M = \frac{11}{6}$ (3)

, by adding (1) , (3) : $\therefore 2 L = \frac{18}{6} \qquad \therefore L = \frac{3}{2}$

, substituting in (1) :

$\therefore M = \frac{7}{6} - \frac{9}{6} = -\frac{1}{3} \qquad \therefore L M = -\frac{1}{2}$

, substituting in (2) $\therefore -\frac{1}{2} = \frac{1 - c}{6} \qquad \therefore c = 4$

24

let the two roots of the first equation be L and M

$\therefore L - M = \frac{\pm\sqrt{(-2)^2 - 4 \times 3 \times c}}{3} = \frac{\pm\sqrt{4 - 12c}}{3}$

and let the two roots of the second equation be D , E

$\therefore D - E = \frac{\pm\sqrt{(-c)^2 - 4 \times 2 \times 3}}{2} = \frac{\pm\sqrt{c^2 - 24}}{2}$

, $\because L - M = D - E$ $\quad \therefore \frac{\sqrt{4 - 12c}}{3} = \frac{\sqrt{c^2 - 24}}{2}$

, by squaring both sides $\quad \therefore \frac{4 - 12c}{9} = \frac{c^2 - 24}{4}$

$\therefore 9c^2 - 216 - 16 + 48c = 0$

$\therefore 9c^2 + 48c - 232 = 0$

25

let the two roots of the first equation be : L , M

$\therefore L - M = \pm\sqrt{k^2 - 8k}$

, let the two roots of the second equation be : D , E

$\therefore DE = k$

, $\because L - M = 2DE$ $\quad \therefore \pm\sqrt{k^2 - 8k} = 2k$

, by squaring both sides :

$\therefore k^2 - 8k = 4k^2$ $\quad \therefore 3k^2 + 8k = 0$

$\therefore k(3k + 8) = 0$ $\quad \therefore k = 0$ or $k = \frac{-8}{3}$

26

$\because$ L , M are the two roots of the given equation.

$\therefore L + M = \frac{6}{4} = \frac{3}{2}$, $LM = \frac{a}{4}$

, $\because L^2 + M^2 = 7LM$

$\therefore L^2 + M^2 + 2LM = 9LM$ $\quad \therefore (L + M)^2 = 9LM$

$\therefore \left(\frac{3}{2}\right)^2 = 9 \times \frac{a}{4}$ $\quad \therefore a = 1$

27

$\because L + M = 8$, $LM = c$

, $\because L^2 + M^2 = 40$ $\quad \therefore (L + M)^2 - 2LM = 40$

$\therefore 64 - 2c = 40$ $\quad \therefore c = 12$

$\therefore L + M = 8$, $LM = 12$

, let the two roots of the required equation be D , E

$\therefore D = L^2M + M^2L = LM(L + M)$, $E = LM$

$\therefore D + E = LM(L + M) + LM$

$= 12 \times 8 + 12 = 108$

, $DE = LM(L + M) \times LM = 12 \times 8 \times 12 = 1152$

$\therefore$ The required equation is : $x^2 - 108x + 1152 = 0$

28

$\because x^2 - 4x - 5 = 0$ $\quad \therefore (x - 5)(x + 1) = 0$

$\therefore x = 5$ or $x = -1$

, $\because L > M$ $\quad \therefore L = 5$, $M = -1$

$\therefore$ The two roots of the required equation are : -2 , 3

$\therefore$ The equation is : $(x + 2)(x - 3) = 0$

i.e. $x^2 - x - 6 = 0$

29

Yousef's answer is the correct because he used the two roots of the first equation to find the roots of the second equation , then he found the unknown equation.

Third Higher skills

1 (1) d (2) b (3) b (4) b (5) c
(6) d (7) d (8) d (9) c

Instructions to solve 1 :

(1) Let the roots of the equation (the rectangle dimensions) be L and M

$\therefore LM = 15$, $2(L + M) = 26$

$\therefore L + M = 13$

$\therefore$ The quadratic equation is : $x^2 - 13x + 15 = 0$

(2) $\because a^2 + 3a + 1 = 0$, $b^2 + 3b + 1 = 0$

$\therefore$ a and b are the roots of the equation :

$x^2 + 3x + 1 = 0$

$\therefore a + b = -3$, $ab = 1$

, $\because \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab} = \frac{(a + b)^2 - 2ab}{ab}$

$= \frac{(-3)^2 - 2(1)}{(1)} = 7$

(3) $\because (x - a)(x - b) = k$

$\therefore x^2 - (a + b)x + ab - k = 0$

$\therefore L + M = a + b$, $LM = ab - k$

and so $ab = LM + k$

$\therefore$ The quadratic equation whose roots are a , b is $x^2 - (L + M)x + LM + k = 0$

$\therefore (x - L)(x - M) + k = 0$

(4) $\because (L + M + 4)^2 + (LM - 3)^2 =$ zero

$\therefore L + M + 4 = 0$ and so $L + M = -4$

and $LM - 3 = 0$ and so $LM = 3$

$\therefore$ To form quadratic equation whose roots are $4L$, $4M$

The sum of the two roots $4L + 4M$

$= 4(L + M) = 4(-4) = -16$

and their product $4L \times 4M = 16LM = 16(3)$

$= 48$

$\therefore$ The sufficient condition to form the equation is (b)

(5) Omar made a mistake in the absolute term and the roots were 3 , 4

$\therefore$ The sum of the two roots is 7

, $\because$ Khaled made a mistake in the coefficient of x and the roots of the equation were 2 , 3

$\therefore$ The product of the two roots is 6

$\therefore$ The quadratic equation is : $x^2 - 7x + 6 = 0$ and its roots are 6 , 1

(6) Let the roots of the equation be L , L + 2

$\therefore$ The sum of the two roots $(-b) = (2L + 2)$

, their product $c = L^2 + 2L$

$\therefore b^2 - 4c = (2L + 2)^2 - 4(L^2 + 2L)$

$= 4L^2 + 8L + 4 - 4L^2 - 8L = 4$

Another solution :

$\frac{\pm\sqrt{b^2 - 4c}}{1} = 2$ $\qquad \therefore b^2 - 4c = 4$

(7) $\because$ The product of the roots = c and its prime

$\therefore$ The roots are c and 1

, $\because$ Their sum = b (where b is a prime)

$\therefore b = 1 + c$

$\therefore$ b , c are two consecutive primes

$\therefore b = 3$, $c = 2$

$\therefore b - c = 1$ (odd number)

, $b^2 - c = 9 - 2 = 7$ (prime number)

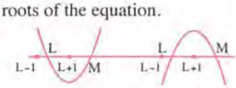
, $b + c = 3 + 2 = 5$ (prime number)

$\therefore$ The answer is (d)

(8) $\because$ L is one of the roots of the equation.

$\therefore f(L) = 0$

, $\because |L - M| > 1$



$\therefore f(L + 1)$, $f(L - 1)$ have always different signs.

$\therefore f(L + 1) \times f(L - 1) < 0$

(9) $\because$ L , M are the roots of the equation.

$\therefore L + M = \tan\theta$, $LM = -1$

$\therefore (L + M)^2 = (\tan\theta)^2$

$\therefore L^2 + 2LM + M^2 = \tan^2\theta$

, $\because L^2 + M^2 = 3$ $\qquad \therefore 3 + 2(-1) = \tan^2\theta$

$\therefore \tan^2\theta = 1$ $\qquad \therefore \tan\theta = \pm 1$

, $\because 0 < \theta < 90°$ $\qquad \therefore \tan\theta = 1$

$\therefore \theta = 45°$

2

$\because L + M = \frac{-2b}{a}$, $LM = \frac{c}{a}$

(1) $\because L - M = 2$ $\qquad \therefore (L - M)^2 = 4$

$\therefore (L + M)^2 - 4LM = 4$

$\therefore \frac{4b^2}{a^2} - \frac{4c}{a} = 4$ $\qquad \therefore \frac{4b^2 - 4ac}{a^2} = 4$

$\therefore 4b^2 - 4ac = 4a^2$ $\qquad \therefore b^2 - ac = a^2$

$\therefore b^2 = a^2 + ac = a(a + c)$

(2) $\because L + M = \frac{-2b}{a}$, $L - M = 2$

, by adding : $\therefore 2L = \frac{-2b}{a} + 2$

$\therefore L = 1 - \frac{b}{a}$

3

let the two roots of the given equation be : L , M

$\therefore L + M = \frac{-b}{a}$, $LM = \frac{c}{a}$

, $\because L - M = 2\left(\frac{1}{L} + \frac{1}{M}\right)$, by squaring

$\therefore (L - M)^2 = 4\left(\frac{1}{L} + \frac{1}{M}\right)^2$

$\therefore (L + M)^2 - 4LM = \frac{4(M + L)^2}{(LM)^2}$

$\therefore \left(\frac{-b}{a}\right)^2 - 4 \times \frac{c}{a} = \frac{4 \times \left(-\frac{b}{a}\right)^2}{\left(\frac{c}{a}\right)^2}$

$\therefore \frac{b^2}{a^2} - \frac{4c}{a} = \frac{4 \times \frac{b^2}{a^2}}{\frac{c^2}{a^2}}$ $\qquad \therefore \frac{b^2 - 4ac}{a^2} = \frac{4b^2}{c^2}$

$\therefore c^2(b^2 - 4ac) = 4a^2b^2$

Answers of Exercise 5

First Multiple choice questions

(1) c	(2) a	(3) b	(4) d	(5) a
(6) d	(7) a	(8) d	(9) b	(10) b

(11) c **(12)** b **(13)** d **(14)** c **(15)** c
(16) d **(17)** b **(18)** a **(19)** b **(20)** c
(21) b **(22)** b **(23)** d **(24)** b **(25)** c
(26) First : d **Second :** c
(27) First : d **Second :** c **Third:** a
(28) b **(29)** a **(30)** b **(31)** d **(32)** c
(33) c **(34)** b **(35)** d **(36)** b **(37)** b

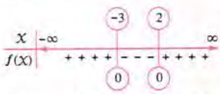
Second Essay questions

1

(1) $\because f(x) = (x-2)(x+3)$

$\therefore$ The roots of the equation :

$f(x) = 0$ are $x = 2$, $x = -3$

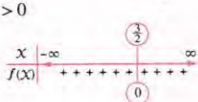


- f is positive at $x \in \mathbb{R} - [-3 , 2]$
- $f(x) = 0$ at $x \in \{-3 , 2\}$
- f is negative at $x \in \,]-3 , 2[$

(2) $\because f(x) = (2x-3)^2$ $\therefore$ when $f(x) = 0$

$\therefore (2x-3)^2 = 0$ $\therefore x = \frac{3}{2}$

$\because a = 4 > 0$

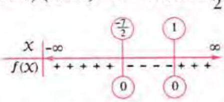


$\therefore f$ is positive $\forall x \in \mathbb{R} - \left\{\frac{3}{2}\right\}$

(3) $\because f(x) = 2x^2 + 5x - 7$

when $f(x) = 0$ $\therefore 2x^2 + 5x - 7 = 0$

$\therefore (2x+7)(x-1) = 0$ $\therefore x = -\frac{7}{2}$ or $x = 1$



- The sign of f is the same as a (where $a = 2 > 0$)

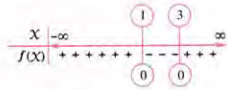
thus f is positive at $x \in \mathbb{R} - \left[-\frac{7}{2} , 1\right]$

- $f(x) = 0$ at $x \in \left\{-\frac{7}{2} , 1\right\}$
- The sign of f is negative at $x \in \left]-\frac{7}{2} , 1\right[$

(4) $\because f(x) = x^2 - 4x + 3$

when $f(x) = 0$ $\therefore x^2 - 4x + 3 = 0$

$\therefore (x-1)(x-3) = 0$ $\therefore x = 1$ or $x = 3$



- The sign of f is the same as a (where $a = 1 > 0$)

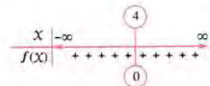
thus f is positive at $x \in \mathbb{R} - [1 , 3]$

- $f(x) = 0$ at $x \in \{1 , 3\}$
- The sign of f is negative at $x \in \,]1 , 3[$

(5) $\because f(x) = x^2 - 8x + 16$

when $f(x) = 0$ $\therefore x^2 - 8x + 16 = 0$

$\therefore (x-4)^2 = 0$ $\therefore x = 4$



- The sign of f is the same as a (where $a = 1 > 0$)

$\therefore f$ is positive at $x \in \mathbb{R} - \{4\}$

- $f(x) = 0$ at $x = 4$

(6) $\because f(x) = 2x^2 - 3x + 5$

$\therefore$ The discriminant $= b^2 - 4ac = (-3)^2 - 4 \times 2 \times 5$

$= 9 - 40 = -31 < 0$

$\therefore$ There's no real zeroes to the function.

$\therefore$ The equation has no real roots.

$\because$ a (coefficient of x^2) $= 2 > 0$

$\therefore f$ is positive $\forall x \in \mathbb{R}$

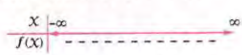
(7) $\because f(x) = -x^2 + 4x - 7$

$\therefore$ The discriminant $= -12 < 0$

$\therefore$ There's no real zeroes to the function.

$\therefore$ The equation has no real roots.

$\because$ a (coefficient of x^2) $= -1 < 0$



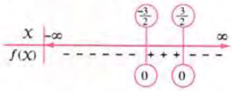
$\therefore f$ is negative $\forall x \in \mathbb{R}$

(8) $\because f(x) = 9 - 4x^2$

$\therefore f(x) = 0$ at $4x^2 - 9 = 0$

$\therefore (2x-3)(2x+3) = 0$

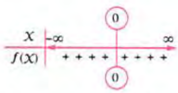
$\therefore x = \frac{3}{2}$ or $x = -\frac{3}{2}$



- f has sign as the same of a (where $a = -4 < 0$) thus f is negative at $x \in \mathbb{R} - \left[-\frac{3}{2} , \frac{3}{2}\right]$
- $f(x) = 0$ at $x \in \left\{-\frac{3}{2} , \frac{3}{2}\right\}$
- f is positive at $x \in \left]-\frac{3}{2} , \frac{3}{2}\right[$

(9) $\because f(x) = 2x^2$ $\quad\therefore f(x) = 0$ at $x = 0$

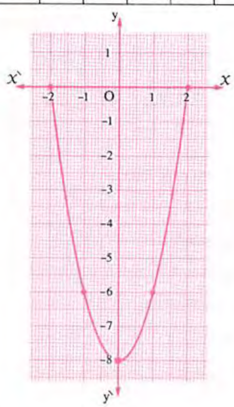
$\because$ a (coefficient of x^2) $= 2 > 0$



$\therefore f$ is positive $\forall x \in \mathbb{R} - \{0\}$

2 $f(x) = 2x^2 - 8$

x	-2	-1	0	1	2
$f(x)$	0	-6	-8	-6	0

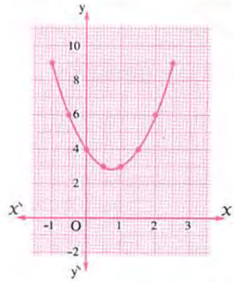


From the graph , we get :

- f is negative at $x \in]-2 , 2[$
- $f(x) = 0$ at $x \in \{-2 , 2\}$
- f is positive at $x \in \mathbb{R} - [-2 , 2]$

3 $f(x) = 2x^2 - 3x + 4$

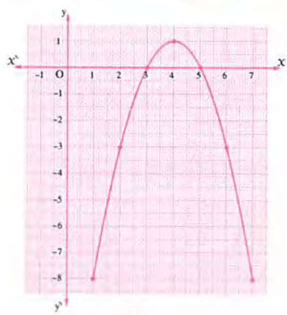
x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
$f(x)$	9	6	4	3	3	4	6	9



From the graph : f is positive $\forall x \in \mathbb{R}$

4 $f(x) = -x^2 + 8x - 15$

x	1	2	3	4	5	6	7
$f(x)$	-8	-3	0	1	0	-3	-8



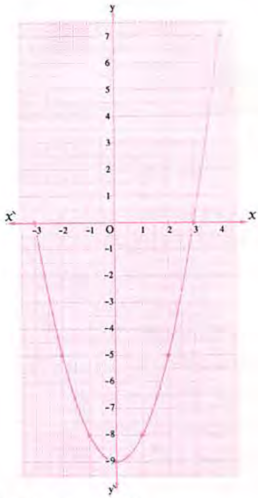
From the graph , we get :

- $f(x) = 0$ at $x \in \{3 , 5\}$
- f is negative at $x \in \mathbb{R} - [3 , 5]$
- f is positive at $x \in]3 , 5[$

$\therefore$ The S.S. of the equation : $f(x) = 0$ is $\{3 , 5\}$

5 $f(x) = x^2 - 9$

x	-3	-2	-1	0	1	2	3	4
$f(x)$	0	-5	-8	-9	-8	-5	0	7



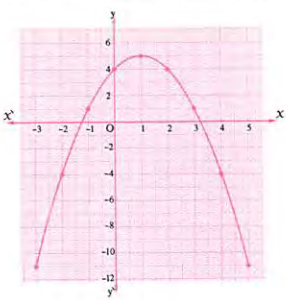
From the graph , we get :

- f is negative at $x \in]-3 , 3[$
- $f(x) = 0$ at $x \in \{-3 , 3\}$
- f is positive at $x \in]3 , 4[$

6

$f(x) = -x^2 + 2x + 4$

x	-3	-2	-1	0	1	2	3	4	5
$f(x)$	-11	-4	1	4	5	4	1	-4	-11



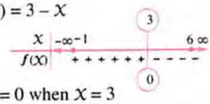
From the graph , we get :

- $f(x) = 0$ at $x \in \{-1.2 , 3.2\}$
- f is positive at $x \in [-1.2 , 3.2]$
- f is negative at $x \in [-3 , -1.2[\cup]3.2 , 5]$

Notice that : 3.2 , – 1.2 are approximated values for the roots of the equation related to the function.

7

(1) $\because f(x) = 3 - x$



- $f(x) = 0$ when $x = 3$
- f is positive when $3 - x > 0$ *i.e.* $x < 3$

$\therefore f$ is positive in the interval $[-1 , 3[$

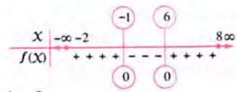
- f is negative when $3 - x < 0$ *i.e.* $x > 3$

$\therefore f$ is negative in the interval $]3 , 6]$

(2) $\because f(x) = x^2 - 5x - 6$

- The roots of the equation : $x^2 - 5x - 6 = 0$

$\therefore (x + 1)(x - 6) = 0 \quad \therefore x = -1$ or $x = 6$



$\because a = 1 > 0$

- f is positive when $x \in [-2 , 8] - [-1 , 6]$
- $f(x) = 0$ when $x \in \{-1 , 6\}$
- f is negative when $x \in]-1 , 6[$

8

(1) **From the graph , we get :**

- $f(x) = 0$ at $x \in \{-1 , 5\}$
- f is negative at $x \in \mathbb{R} - [-1 , 5]$
- f is positive at $x \in]-1 , 5[$

(2) **From the graph , we get :**

- $f(x) = 0$ at $x \in \{1 , 3\}$
- f is positive at $x \in \mathbb{R} - [1 , 3]$
- f is negative at $x \in]1 , 3[$

9

$f(x) = x - 3$ • $f(x) = 0$ at $x = 3$

- f is positive at $x > 3$ • f is negative at $x < 3$

$g(x) = x^2 - 5x - 6 = (x - 6)(x + 1)$

$x = 6$ or $x = -1$

- $g(x) = 0$ at $x \in \{-1 , 6\}$
- g is positive at $x \in \mathbb{R} - [-1 , 6]$
- g is negative at $x \in]-1 , 6[$

The two functions are positive together at $x > 6$

10

$f_1(x) = x - 3$, $f_1(x) = 0$ at $x = 3$

x	$-\infty$		3		∞
$f(x)$		$- - - - - -$	0	$+ + + +$	

- f_1 is positive at $x > 3$
- f_1 is negative at $x < 3$

$f_2(x) = 5 + 4x - x^2$

We find the two roots of the equation :

$-x^2 + 4x + 5 = 0$

$\therefore x^2 - 4x - 5 = 0$ $\qquad \therefore (x - 5)(x + 1) = 0$

$\therefore$ The two roots of the equation are 5 , -1

x	$-\infty$		-1		5		∞
$f(x)$		$- - - - - -$	0	$+ + +$	0	$- - -$	

- $f_2(x) = 0$ at $x \in \{-1, 5\}$
- f_2 is negative at $x \in \mathbb{R} - [-1, 5]$
- f_2 is positive at $x \in]-1, 5[$

f_1 , f_2 are negative together at $x \in]-\infty, -1[$

11

$f(x) = x^2 - 5x + 6$

We get the two roots of the equation : $x^2 - 5x + 6 = 0$

$\therefore (x - 2)(x - 3) = 0$ $\qquad \therefore x = 2$ or $x = 3$

x	$-\infty$		2		3		∞
$f(x)$		$+ + + + +$	0	$- - - -$	0	$+ + + +$	

- $f(x) = 0$ when $x \in \{2, 3\}$
- f is positive when $x \in \mathbb{R} - [2, 3]$
- f is negative when $x \in]2, 3[$

$g(x) = 2x^2 - 5x - 18$

We get the two roots of the equation :

$2x^2 - 5x - 18 = 0$ $\qquad \therefore (2x - 9)(x + 2) = 0$

$\therefore x = \frac{9}{2}$ or $x = -2$

x	$-\infty$		-2		$\frac{9}{2}$		∞
$g(x)$		$+ + + + +$	0	$- - - -$	0	$+ + + +$	

- $g(x) = 0$ when $x \in \left\{-2, \frac{9}{2}\right\}$
- g is positive when $x \in \mathbb{R} - \left[-2, \frac{9}{2}\right]$
- g is negative when $x \in \left]-2, \frac{9}{2}\right[$

$\therefore$ The two functions are both positive when :

$x \in]-\infty, -2[\cup \left]\frac{9}{2}, \infty\right[$

Thus $x \in \mathbb{R} - \left[-2, \frac{9}{2}\right]$

- The two functions are both negative when :

$x \in]2, 3[$

12

$\because 2x^2 - kx + k - 3 = 0$

$\therefore a = 2$, $b = -k$, $c = k - 3$

$\therefore$ The discriminant $= (-k)^2 - 4 \times 2 \times (k - 3)$

$= k^2 - 8k + 24$

$\therefore$ Investigate the sign of $f : f(k) = k^2 - 8k + 24$

$\because$ The discriminant $= (-8)^2 - 4 \times 1 \times 24 = -32 < 0$

$\therefore$ The equation : $k^2 - 8k + 24 = 0$

its two roots are not real numbers

, $\because$ the coefficient of $k^2 > 0$

$\therefore$ The sign of f is positive for all values of $k \in \mathbb{R}$

$\therefore$ The discriminant of the equation :

$2x^2 - kx + k - 3 = 0$ (positive for all values $x \in \mathbb{R}$)

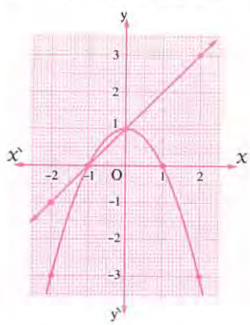
$\therefore$ The two roots are real and different for all $x \in \mathbb{R}$

13 The answer of Amira is correct.

$f(x) = x + 1$ $\qquad\qquad$ $g(x) = 1 - x^2$

x	-2	0	2
$f(x)$	-1	1	3

x	-2	-1	0	1	2
$g(x)$	-3	0	1	0	-3



From the graph , we get :

The two functions f and g are both positive in the interval $]-1, 1[$

Third Higher skills

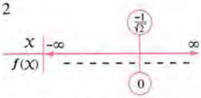
1

(1) $f(x) = -2x^2 - 2\sqrt{2}\,x - 1$

Let $f(x) = 0$ $\therefore -2x^2 - 2\sqrt{2}\,x - 1 = 0$

$\therefore 2x^2 + 2\sqrt{2}\,x + 1 = 0$ $\therefore \left(\sqrt{2}\,x + 1\right)^2 = 0$

$\therefore x = \frac{-1}{\sqrt{2}}$



- f is negative when $x \in \mathbb{R} - \left\{-\frac{1}{\sqrt{2}}\right\}$

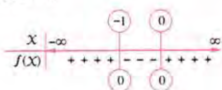
(2) $f(x) = x + (x+1)(2x+3) - 4(x+1) + 1$

$= x + 2x^2 + 5x + 3 - 4x - 4 + 1$

$= 2x^2 + 2x = 2x(x+1)$

$\therefore$ The two roots of the equation $f(x) = 0$ is

$x = 0 , x = -1$



- $f(x) = 0$ when $x \in \{-1 , 0\}$
- f is positive when $x \in \mathbb{R} -]-1 , 0[$

2

(1) From the graph :

- f is positive when $x \in \mathbb{R} - [-3 , 2]$
- $f(x) = 0$ when $x \in \{-3 , 2\}$
- f is negative when $x \in]-3 , 2[$

To find the rule of the function :

$\because f(x) = a(x-2)(x+3)$

$\because$ The curve passes through the point $(0 , -6)$

$\therefore -6 = a \times -2 \times 3$ $\therefore a = 1$

$\therefore f(x) = (x-2)(x+3) = x^2 + x - 6$

(2) From the graph :

- f is negative when $x \in \mathbb{R} - [-3 , 0]$
- $f(x) = 0$ when $x \in \{-3 , 0\}$
- f is positive when $x \in]-3 , 0[$

To find the rule of the function :

$\because f(x) = a\,x(x+3)$

$\because$ The curve passes through the point $(-1 , 2)$

$\therefore 2 = -a(-1+3)$ $\therefore a = -1$

$\therefore f(x) = -x(x+3) = -x^2 - 3x$

(3) From the graph :

- f is positive when $x \in \mathbb{R} - [1 , 5]$
- $f(x) = 0$ when $x \in \{1 , 5\}$
- f is negative when $x \in]1 , 5[$

$\because f(x) = a(x-1)(x-5)$

$\because$ The curve passes through the point $(3 , -4)$

$\therefore -4 = a(3-1)(3-5)$ $\therefore -4 = a \times 2 \times -2$

$\therefore a = 1$

$\therefore f(x) = (x-1)(x-5) = x^2 - 6x + 5$

Answers of Exercise 6

First Multiple choice questions

(1) b	(2) c	(3) d	(4) c
(5) d	(6) d	(7) c	(8) b
(9) d	(10) c	(11) c	(12) a
(13) a	(14) b	(15) b	(16) c
(17) c	(18) a	(19) d	(20) c
(21) c	(22) d	(23) c	(24) c
(25) c	(26) c	(27) a	(28) b

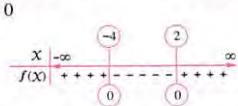
Second Essay questions

1

(1) $f(x) = x^2 + 2x - 8$ let $x^2 + 2x - 8 = 0$

$\therefore (x+4)(x-2) = 0$ $\therefore x = -4$ or $x = 2$

$\because a > 0$



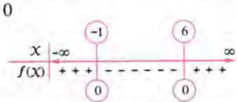
$\therefore f$ is positive when $x \in \mathbb{R} - [-4 , 2]$

$\therefore$ S.S. $= \mathbb{R} - [-4 , 2]$

(2) $f(x) = x^2 - 5x - 6$ let $x^2 - 5x - 6 = 0$

$\therefore (x+1)(x-6) = 0$ $\therefore x = -1$ or $x = 6$

$\because a > 0$



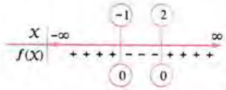
$\therefore f$ is negative when $x \in]-1 , 6[$

$\therefore$ S.S. $=]-1 , 6[$

(3) $f(x) = x^2 - x - 2$ let $x^2 - x - 2 = 0$

$\therefore (x+1)(x-2) = 0$ $\therefore x = -1$ or $x = 2$

$\because a > 0$



$\therefore f$ is negative when $x \in \mathbb{R} -]-1 , 2[$

, $f(x) = 0$ when $x \in \{-1 , 2\}$

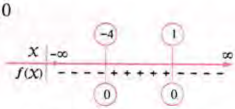
$\therefore$ S.S. $= [-1 , 2]$

(4) $f(x) = 4 - 3x - x^2$ let $4 - 3x - x^2 = 0$

$\therefore x^2 + 3x - 4 = 0$ $\therefore (x+4)(x-1) = 0$

$\therefore x = -4$ or $x = 1$

$\because a < 0$



$\therefore f$ is positive when $x \in]-4 , 1[$

, $f(x) = 0$ when $x \in \{-4 , 1\}$

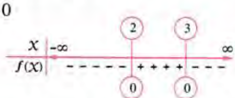
$\therefore$ S.S. $= [-4 , 1]$

(5) $f(x) = 5x - x^2 - 6$ let $5x - x^2 - 6 = 0$

$\therefore x^2 - 5x + 6 = 0$ $\therefore (x-2)(x-3) = 0$

$\therefore x = 2$ or $x = 3$

$\because a < 0$



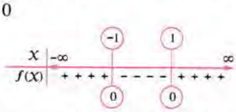
$\therefore f$ is negative when $x \in \mathbb{R} - [2 , 3]$

$\therefore$ S.S. $= \mathbb{R} - [2 , 3]$

(6) $f(x) = x^2 - 1$ let $x^2 - 1 = 0$

$\therefore (x+1)(x-1) = 0$ $\therefore x = -1$ or $x = 1$

$\because a > 0$



$\therefore f$ is negative when $x \in]-1 , 1[$

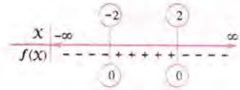
, $f(x) = 0$ when $x \in \{-1 , 1\}$

$\therefore$ S.S. $= [-1 , 1]$

(7) $f(x) = 4 - x^2$ let $4 - x^2 = 0$

$\therefore (2+x)(2-x) = 0$ $\therefore x = -2$ or $x = 2$

$\because a < 0$



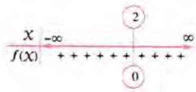
$\therefore f$ is negative when $x \in \mathbb{R} - [-2 , 2]$

$\therefore$ S.S. $= \mathbb{R} - [-2 , 2]$

(8) $f(x) = x^2 - 4x + 4$ let $x^2 - 4x + 4 = 0$

$\therefore (x-2)^2 = 0$ $\therefore x = 2$

$\because a > 0$



$\therefore f$ is positive when $x \in \mathbb{R} - \{2\}$

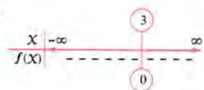
, $f(x) = 0$ when $x = 2$ $\therefore$ S.S. $= \mathbb{R}$

(9) $f(x) = 6x - x^2 - 9$ let $6x - x^2 - 9 = 0$

$\therefore x^2 - 6x + 9 = 0$ $\therefore (x-3)^2 = 0$

$\therefore x = 3$

$\because a < 0$



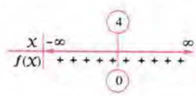
$\therefore f$ is negative when $x \in \mathbb{R} - \{3\}$

$\therefore$ S.S. $= \mathbb{R} - \{3\}$

(10) $f(x) = x^2 - 8x + 16$ let $x^2 - 8x + 16 = 0$

$\therefore (x-4)^2 = 0$ $\therefore x = 4$

$\because a > 0$



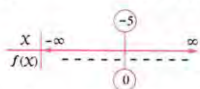
$\therefore f$ is positive when $x \in \mathbb{R} - \{4\}$

$\therefore$ S.S. $= \varnothing$

(11) $f(x) = -x^2 - 10x - 25$ let $-x^2 - 10x - 25 = 0$

$\therefore x^2 + 10x + 25 = 0$ $\therefore (x+5)^2 = 0$

$\therefore x = -5$ $\because a < 0$



$\therefore f$ is negative when $x \in \mathbb{R} - \{-5\}$

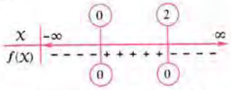
, $f(x) = 0$ when $x = -5$

$\therefore$ S.S. $= \{-5\}$

(12) $f(x) = 2x - x^2$ let $2x - x^2 = 0$

$\therefore x(2 - x) = 0$ $\therefore x = 0$ or $x = 2$

$\because a < 0$



$\therefore f$ is negative when $x \in \mathbb{R} - [0, 2]$

$\therefore$ S.S. $= \mathbb{R} - [0, 2]$

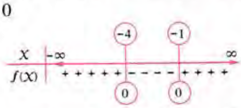
2

(1) $\because x^2 + 5x < -4$ $\therefore x^2 + 5x + 4 < 0$

$\therefore f(x) = x^2 + 5x + 4$ let $x^2 + 5x + 4 = 0$

$\therefore (x + 4)(x + 1) = 0$ $\therefore x = -4$ or $x = -1$

$\because a > 0$



$\therefore f$ is negative when $x \in]-4, -1[$

$\therefore$ S.S. $=]-4, -1[$

(2) $\because 5x^2 + 12x \geq 44$

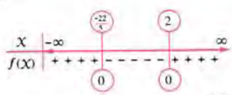
$\therefore 5x^2 + 12x - 44 \geq 0$

$\therefore f(x) = 5x^2 + 12x - 44$

let $5x^2 + 12x - 44 = 0$

$\therefore (5x + 22)(x - 2) = 0$ $\therefore x = \frac{-22}{5}$ or $x = 2$

$\because a > 0$



$\therefore f$ is positive when $x \in \mathbb{R} - \left[\frac{-22}{5}, 2\right]$

, $f(x) = 0$ when $x \in \left\{\frac{-22}{5}, 2\right\}$

$\therefore$ S.S. $= \mathbb{R} - \left]\frac{-22}{5}, 2\right[$

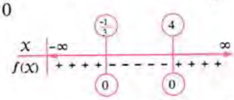
(3) $\because 3x^2 \leq 11x + 4$ $\therefore 3x^2 - 11x - 4 \leq 0$

$\therefore f(x) = 3x^2 - 11x - 4$

let $3x^2 - 11x - 4 = 0$

$\therefore (3x + 1)(x - 4) = 0$ $\therefore x = \frac{-1}{3}$ or $x = 4$

$\because a > 0$



$\therefore f$ is negative when $x \in \left]\frac{-1}{3}, 4\right[$

, $f(x) = 0$ when $x \in \left\{\frac{-1}{3}, 4\right\}$

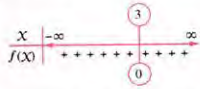
$\therefore$ S.S. $= \left[\frac{-1}{3}, 4\right]$

(4) $\because x^2 \geq 6x - 9$ $\therefore x^2 - 6x + 9 \geq 0$

$\therefore f(x) = x^2 - 6x + 9$ let $x^2 - 6x + 9 = 0$

$\therefore (x - 3)^2 = 0$ $\therefore x = 3$

$\because a > 0$



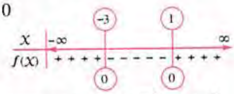
$\therefore f$ is positive when $x \in \mathbb{R} - \{3\}$ $\therefore$ S.S. $= \mathbb{R}$

(5) $\because 3 - 2x \geq x^2$ $\therefore x^2 + 2x - 3 \leq 0$

$\therefore f(x) = x^2 + 2x - 3$ let $x^2 + 2x - 3 = 0$

$\therefore (x + 3)(x - 1) = 0$ $\therefore x = -3$ or $x = 1$

$\because a > 0$



$\therefore f$ is negative when $x \in]-3, 1[$

, $f(x) = 0$ when $x \in \{-3, 1\}$

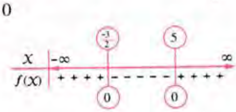
$\therefore$ S.S. $= [-3, 1]$

(6) $\because 7x + 15 \leq 2x^2$ $\therefore 2x^2 - 7x - 15 \geq 0$

$\therefore f(x) = 2x^2 - 7x - 15$ let $2x^2 - 7x - 15 = 0$

$\therefore (2x + 3)(x - 5) = 0$ $\therefore x = \frac{-3}{2}$ or $x = 5$

$\because a > 0$



$\therefore f$ is positive when $x \in \mathbb{R} - \left[\frac{-3}{2}, 5\right]$

, $f(x) = 0$ when $x \in \left\{\frac{-3}{2}, 5\right\}$

$\therefore$ S.S. $= \mathbb{R} - \left]\frac{-3}{2}, 5\right[$

(7) $\because x^2 + 5 \leq 1$ $\therefore x^2 + 4 \leq 0$

$\therefore f(x) = x^2 + 4$ let $x^2 + 4 = 0$

$\therefore$ The discriminant $= b^2 - 4ac$

$= 0 - 4 \times 1 \times 4 = -16 < 0$

$\therefore$ The equation has no real roots.

$\because a > 0$ $\therefore f$ is positive $\forall x \in \mathbb{R}$

$\therefore$ S.S. $= \varnothing$

(8) $\because -x^2-7<2$ $\quad\therefore -x^2-9<0$

$\therefore x^2+9>0$ $\quad\therefore f(x)=x^2+9$

let $x^2+9=0$

$\therefore$ The discriminant $= b^2-4ac$

$= 0-4\times 1\times 9=-36<0$

$\therefore$ The equation has no real roots.

$\because a>0$ $\quad\therefore f$ is positive $\forall\, x\in\mathbb{R}$

$\therefore$ S.S. $=\mathbb{R}$

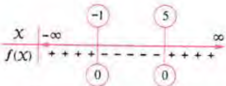
(9) $\because (x-2)^2\geq 9$ $\quad\therefore x^2-4x+4\geq 9$

$\therefore x^2-4x-5\geq 0$ $\quad\therefore f(x)=x^2-4x-5$

let $x^2-4x-5=0$

$\therefore (x-5)(x+1)=0$ $\quad\therefore x=5$ or $x=-1$

$\because a>0$



$\therefore f$ is positive when $x\in\mathbb{R}-[-1\,,5]$

, $f(x)=0$ when $x\in\{-1\,,5\}$

$\therefore$ S.S. $=\mathbb{R}-]-1\,,5[$

(10) $\because (x-2)^2\leq -5$ $\quad\therefore x^2-4x+4\leq -5$

$\therefore x^2-4x+9\leq 0$

$\therefore f(x)=x^2-4x+9$ $\quad$ let $x^2-4x+9=0$

$\therefore$ The discriminant $= b^2-4ac$

$=(-4)^2-4\times 1\times 9=-20<0$

$\therefore$ The equation has no real roots.

$\because a>0$ $\quad\therefore f$ is positive $\forall\, x\in\mathbb{R}$

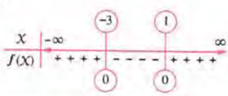
$\therefore$ S.S. $=\varnothing$

(11) $\because x(x+2)-3\leq 0$ $\quad\therefore x^2+2x-3\leq 0$

$\therefore f(x)=x^2+2x-3$ $\quad$ let $x^2+2x-3=0$

$\therefore (x+3)(x-1)=0$ $\quad\therefore x=-3$ or $x=1$

$\because a>0$



$\therefore f$ is negative at $x\in\,]-3\,,1[$

, $f(x)=0$ when $x\in\{-3\,,1\}$

$\therefore$ S.S. $=[-3\,,1]$

(12) $\because (x+2)^2+(x+1)(x-4)<0$

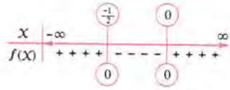
$\therefore x^2+4x+4+x^2-3x-4<0$

$\therefore 2x^2+x<0$

$\therefore f(x)=2x^2+x$ $\quad$ let $2x^2+x=0$

$\therefore x(2x+1)=0$ $\quad\therefore x=0$ or $x=\frac{-1}{2}$

$\because a>0$



$\therefore f$ is negative at $x\in\left]\frac{-1}{2}\,,0\right[$

$\therefore$ S.S. $=\left]\frac{-1}{2}\,,0\right[$

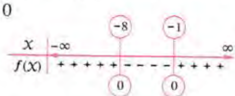
(13) $\because (x+3)^2<10-3(x+3)$

$\therefore x^2+6x+9<1-3x$ $\quad\therefore x^2+9x+8<0$

$\therefore f(x)=x^2+9x+8$ $\quad$ Let $x^2+9x+8=0$

$\therefore (x+8)(x+1)=0$ $\quad\therefore x=-8$ or $x=-1$

$\because a>0$



$\therefore f$ is negative at $x\in\,]-8\,,-1[$

$\therefore$ S.S. $=]-8\,,-1[$

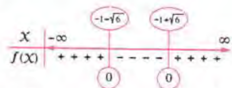
(14) $\because 5-2x\leq x^2$ $\quad\therefore x^2+2x-5\geq 0$

$\therefore f(x)=x^2+2x-5$ $\quad$ let $x^2+2x-5=0$

$\therefore x=\frac{-2\pm\sqrt{(2)^2-4\times 1\times -5}}{2\times 1}=\frac{-2\pm\sqrt{24}}{2}=-1\pm\sqrt{6}$

$\therefore x=-1+\sqrt{6}$ or $x=-1-\sqrt{6}$

$\because a>0$



$\therefore f$ is positive when $x\in\mathbb{R}-\left[-1-\sqrt{6}\,,-1+\sqrt{6}\right]$

, $f(x)=0$ at $x\in\left\{-1-\sqrt{6}\,,-1+\sqrt{6}\right\}$

$\therefore$ S.S. $=\mathbb{R}-\left]-1-\sqrt{6}\,,-1+\sqrt{6}\right[$

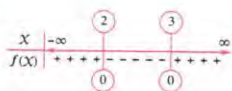
3

$f(x)=x^2-5x+6$

Let $x^2-5x+6=0$

$\therefore (x-2)(x-3)=0$ $\quad\therefore x=2$ or $x=3$

$\because a>0$



$\therefore f$ is positive when $x\in\mathbb{R}-[2\,,3]$

, $f(x) = 0$ at $x \in \{2 , 3\}$

, f is negative when $x \in]2 , 3[$

$\therefore$ S.S. = $]2 , 3[$

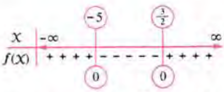
4

$f(x) = 2x^2 + 7x - 15$

Let $2x^2 + 7x - 15 = 0$

$\therefore (2x - 3)(x + 5) = 0$ $\therefore x = \frac{3}{2}$ or $x = -5$

$\because a > 0$



$\therefore f$ is positive when $x \in \mathbb{R} - \left[-5 , \frac{3}{2}\right]$

, $f(x) = 0$ at $x \in \left\{-5 , \frac{3}{2}\right\}$

, f is negative when $x \in \left]-5 , \frac{3}{2}\right[$

$\because 2x^2 + 7x \le 15$ $\therefore 2x^2 + 7x - 15 \le 0$

$\therefore$ S.S. = $\left[-5 , \frac{3}{2}\right]$

5

$f(x) = x^2 + 4$

, Let $x^2 + 4 = 0$

$\therefore$ discriminant $= -4 \times 1 \times 4 = -16 < 0$

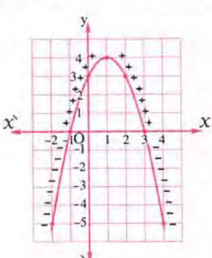
$\therefore$ The equation has no real roots

, $\because a > 0$ $\therefore f$ is positive for all $x \in \mathbb{R}$

$\therefore$ S.S. of the inequality = $\varnothing$

6 $f(x) = -x^2 + 2x + 3$

x	-2	-1	0	1	2	3	4
$f(x)$	-5	0	3	4	3	0	-5



From the graph :

(1) The S.S. of the equality $f(x) = 0$ is $\{-1 , 3\}$

(2) The S.S. of the inequality $f(x) \le 0$ is $\mathbb{R} -]-1 , 3[$

(3) The S.S. of the inequality $f(x) > 0$ is $]-1 , 3[$

7 Nour's answer is the correct.

8 Eslam's answer is the correct because S.S. = $\mathbb{R}$

Third Higher skills

1

(1) d	(2) d	(3) d	(4) b
(5) a	(6) c	(7) c	(8) b
(9) c	(10) c	(11) c	(12) c
(13) c	(14) c		

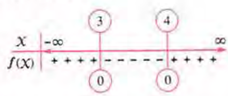
Instructions to solve 1 :

(1) $f(x) = x^2 - 7x + 12$

, Let $x^2 - 7x + 12 = 0$

$\therefore (x - 3)(x - 4) = 0$

$\therefore x = 3$ or $x = 4$



$\therefore$ S.S. of the equation

$f(x) = 0$ is $\{3 , 4\}$

, S.S. of the inequality $f(x) > 0$ is $\mathbb{R} - [3 , 4]$

, S.S. of the inequality $f(x) < 0$ is $]3 , 4[$

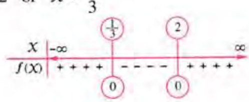
$\therefore$ The wrong choise is (d)

(2) The function related to the inequality is

$f : f(x) = (x - 2)(3x - 1)$

, Put $(x - 2)(3x - 1) = 0$

$\therefore x = 2$ or $x = \frac{1}{3}$



$\therefore$ The solution set = $\left[\frac{1}{3} , 2\right]$

$\therefore$ The sum of integers belong to the solution set is $1 + 2 = 3$

(3) $\because (x+3)^2 < 4(x+1)^2$

$\therefore x^2 + 6x + 9 < 4x^2 + 8x + 4$

$\therefore 3x^2 + 2x - 5 >$ zero

The function related to the inequality is

$f : f(x) = 3x^2 + 2x - 5$

Put $3x^2 + 2x - 5 = 0$

$\therefore (3x + 5)(x - 1) =$ zero

$\therefore x = \frac{-5}{3}$ or $x = 1$

$\therefore$ The solution set of the inequality $= \mathbb{R} - \left[\frac{-5}{3}, 1\right]$

x	$-\infty$		$-\frac{5}{3}$		1		∞
$f(x)$		+ + + +	0	− − − − −	0	+ + + +	

(4) $\because$ L , M are the roots of the equation

$a x^2 + b x + c = 0$, $a > 0$

Let $f(x) = a x^2 + b x + c$

x	$-\infty$		L		M		∞
$f(x)$		+ + + +	0	− − − − −	0	+ + + +	

$\therefore$ The solution set of the inequality $=]L , M[$

(5) $\because$ The discriminant is negative , $a < 0$

$\therefore$ The function related to the inequality lies below x-axis (negative)

$\therefore$ The solution set of the inequality $= \mathbb{R}$

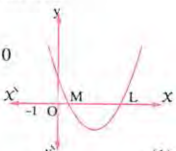
(6) $\because$ The equation has two real roots

$\therefore$ Discriminant ≥ 0

$\therefore (k-2)^2 - 4(2)(-5) \geq 0$

$\therefore (k-2)^2 \geq -40$

is satisfied for all values of $k \in \mathbb{R}$ (1)



, $\because$ each of the two roots is greater than -1

$\therefore$ (coefficient of x^2) $\times f(-1) > 0$

$\therefore 2(2 - (k-2) - 5) > 0$ $\quad\therefore 2(-k-1) > 0$

$\therefore -k - 1 > 0$ $\quad\therefore k < -1$ (2)

From (1) , (2) :

$\therefore k < -1$

(7) $\because$ The roots of the equation are real

$\therefore$ The discriminant ≥ 0

$\therefore (-2k)^2 - 4(k^2 + k - 5) \geq 0$

$\therefore 4k^2 - 4k^2 - 4k + 20 \geq 0$

$\therefore 4k \leq 20$ $\quad\therefore k \leq 5$ (1)

, $\because$ the two roots less than 5

$\therefore f(5) > 0$ $\quad\therefore 25 - 10k + k^2 + k - 5 > 0$

k	$-\infty$		4		5		∞
$f(k)$		+ + + +	0	− − − − −	0	+ + + +	

$\therefore k^2 - 9k + 20 > 0$

$\therefore (k-5)(k-4) > 0$ $\quad\therefore k \in \mathbb{R} - [4 , 5]$ (2)

From (1) , (2) : $\therefore k \in]-\infty , 4[$

(8) $\because$ The roots of the equation are not real

$\therefore$ The discriminant < zero

$\therefore (-k)^2 - (4)(1)(1) < 0$ $\quad\therefore k^2 - 4 < 0$

, the equation related to the inequality is

$k^2 - 4 = 0$

$\therefore k^2 = 4$ $\quad\therefore k = 2$ or $k = -2$

, $\because a >$ zero

k	$-\infty$		−2		2		∞
$f(k)$		+ + + +	0	− − − − −	0	+ + + +	

$\therefore$ The solution of the inequality is : $-2 < k < 2$

(9) $\because x^2 - 4 \leq x + k$ $\quad\therefore x^2 - x - 4 - k \leq 0$

, $\because$ the solution set of the inequality is $[-2 , 3]$

$\therefore$ The roots of the related equation are : -2 , 3

$\therefore (-2)^2 - (-2) - 4 - k = 0$

$\therefore k = 2$

(10) $\because x^2 - 10 < b x$ $\quad\therefore x^2 - b x - 10 < 0$

, $\because$ the solution set of the inequality is $]-2 , 5[$

$\therefore$ The roots of the equation related to the inequality are -2 , 5

$\therefore b = -2 + 5$ $\quad\therefore b = 3$

(11) $\because$ Only one of the two roots of the equation lies in the interval $]1 , 2[$

$\therefore f(1) \times f(2) < 0$

$\therefore (1 - b + 3)(4 - 2b + 3) < 0$

$\therefore (4 - b)(7 - 2b) < 0$

$\therefore b \in]3\frac{1}{2} , 4[$

(12) The function related to the inequality is

$f : f(x) = x^2 - x - 2$

, put $x^2 - x - 2 = 0$

$\therefore (x-2)(x+1) = 0 \quad \therefore x = 2$ or $x = -1$

, $\because a > 0$

x	$-\infty$		-1		2		∞
$f(x)$		+ + + +	0	− − − − −	0	+ + + +	

$\therefore D_1 = [-1 , 2]$

, the function related to the inequality is

$f : f(x) = x^2 + x - 2$

, put $x^2 + x - 2 = 0$

$\therefore (x+2)(x-1) = 0 \quad \therefore x = -2$ or $x = 1$

, $\because a > 0$

x	$-\infty$		-2		1		∞
$f(x)$		+ + + +	0	− − − − −	0	+ + + +	

$\therefore D_2 = [-2 , 1]$

$\therefore D_1 \cap D_2 = [-1 , 1]$

(13) $\because$ L , M are the roots of the equation :

$a x^2 + a x + a + 2 = 0$

, the function related to the equation is

$f(x) = a x^2 + a x + a + 2$

$\therefore f(L) = f(M) = 0$

If $a > 0$, $2 \in]L , M[$

$\therefore f(2) < 0 \qquad \therefore (2)^2 a + 2a + a + 2 < 0$

$\therefore 7a + 2 < 0 \qquad \therefore a < \frac{-2}{7}$ (refused)

and if $a < 0$, $2 \in]L , M[$

$\therefore f(2) > 0 \qquad \therefore 7a + 2 > 0$

$\therefore a > \frac{-2}{7} \qquad \therefore \frac{-2}{7} < a <$ zero

(14) $\because$ The two roots of the equation belong to the interval $]-1 , 1[$

$$\therefore \frac{2 + \sqrt{(-2)^2 - 4(4)(m)}}{2(4)} < 1$$

$\therefore 2 + \sqrt{4 - 16m} < 8 \qquad \therefore \sqrt{4 - 16m} < 6$

$\therefore 0 \le 4 - 16m < 36 \qquad \therefore -4 \le -16m < 32$

$\therefore \frac{-4}{-16} \ge m > \frac{32}{-16} \qquad \therefore -2 < m \le \frac{1}{4}$

2

$\because 10 > x^2 + 2x - 5 \ge 3$

$\therefore x^2 + 2x - 5 < 10 \qquad \therefore x^2 + 2x - 15 < 0$

$\therefore f(x) = x^2 + 2x - 15 \quad$ let $x^2 + 2x - 15 = 0$

$\therefore (x+5)(x-3) = 0 \qquad \therefore x = -5$ or $x = 3$

$\because a > 0$

x	$-\infty$		-5		3		∞
$f(x)$		+ + + +	0	− − − − −	0	+ + + +	

$\therefore f$ is negative at $x \in]-5 , 3[$

$\therefore$ S.S. $=]-5 , 3[$ (1)

$\because x^2 + 2x - 5 \ge 3 \qquad \therefore x^2 + 2x - 8 \ge 0$

$\therefore f(x) = x^2 + 2x - 8 \qquad$ let $x^2 + 2x - 8 = 0$

$\therefore (x+4)(x-2) = 0 \qquad \therefore x = -4$ or $x = 2$

$\because a > 0$

x	$-\infty$		-4		2		∞
$f(x)$		+ + + +	0	− − − − −	0	+ + + +	

$\therefore f$ is positive at $x \in \mathbb{R} - [-4 , 2]$

, $f(x) = 0$ at $x \in \{-4 , 2\}$

$\therefore$ S.S. $= \mathbb{R} -]-4 , 2[$ (2)

From (1) , (2) :

$\therefore$ The S.S. of the inequality :

$10 > x^2 + 2x - 5 \ge 3 =]-5 , -4] \cup [2 , 3[$

Answers of Life Applications on Unit One

1

By substituting in the relation :

$S = u t - 4.9 t^2$, where $S = 29.4$ m

and $u = 24.5$ m./sec.

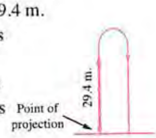
$\therefore 29.4 = 24.5 t - 4.9 t^2 \qquad \therefore 6 = 5t - t^2$

$\therefore t^2 - 5t + 6 = 0 \qquad \therefore (t-2)(t-3) = 0$

$\therefore t = 2$ sec. or $t = 3$ sec.

Explanation of getting of two answers :

The missile reaches a height of 29.4 m. after 2 seconds , then it continues moving up untill it reaches the maximum height , then it returns to the same height after 3 seconds from the moment of projection.



2

By substituting in the relation :

$S = -4.9\,t^2 + 3.5\,t + 10$

where S = 10 m. $\therefore 10 = -4.9\,t^2 + 3.5\,t + 10$

$\therefore -4.9\,t^2 + 3.5\,t = 0$ $\therefore 4.9\,t^2 = 3.5\,t$

$\therefore 4.9\,t = 3.5$ where $t \neq 0$ $\therefore t = \frac{5}{7}$ sec

3

$\because$ The present area of land $= 9 \times 6 = 54\ m^2$.

$\therefore$ The area of the land after doubling the area

$= 2 \times 54 = 108\ m^2$.

Let the increase in the land $= x$ m.

$\therefore (x + 6)(x + 9) = 108$ $\therefore x^2 + 15\,x + 54 = 108$

$\therefore x^2 + 15\,x - 54 = 0$ $\therefore (x - 3)(x + 18) = 0$

$\therefore x = 3$ or $x = -18$ "refused"

$\therefore$ The increase magnitude = 3 m.

4

(1) $\because -16\,t^2 + 80\,t + 20 = 0$

$\therefore a = -16$, $b = 80$, $c = 20$

$\therefore t = \frac{-80 \pm \sqrt{6400 + 1280}}{-32} = \frac{-80 \pm \sqrt{7680}}{-32}$

$\therefore t \simeq 5.24$ sec. or $t \simeq -0.24$ (refused)

$\therefore$ The ball will reach the ground after 5.24 sec. approximately

(2) Calculate the coordinates of the vertex of the curve to know the maximum height that the ball can reach.

$\because$ Vertex point $= \left(\frac{-b}{2a} , f\left(\frac{-b}{2a}\right)\right) = (2.5 , 120)$

i.e. Maximum height the ball reached to is 120 ft. , during 2.5 sec.

So , it will not reach the height 130 ft.

5

(1) At n = 0 $\therefore Z = 91$ million.

(2) At n = 10

$\therefore Z = (10)^2 + 1.2 \times 10 + 91 = 203$ million.

(3) At Z = 334 $\therefore 334 = n^2 + 1.2\,n + 91$

$\therefore n^2 + 1.2\,n - 243 = 0$

$\therefore n = \frac{-1.2 \pm \sqrt{1.44 - 4 \times 1 \times -243}}{2}$

$\therefore n = 15$ or $n = -16.2$ (refused)

$\therefore$ Population reaches 334 millions after 15 years

i.e. In year 2028

6

Total current intensity $= 4 - 2\,i + \frac{6 + 3\,i}{2 + i}$

$= \frac{(4 - 2\,i)(2 + i) + 6 + 3\,i}{2 + i}$

$= \frac{8 - 2\,i^2 + 6 + 3\,i}{2 + i} = \frac{16 + 3\,i}{2 + i}$

$= \frac{16 + 3\,i}{2 + i} \times \frac{2 - i}{2 - i}$

$= \frac{32 - 10\,i - 3\,i^2}{4 - i^2}$

$= \frac{35 - 10\,i}{5} = (7 - 2\,i)$ Ampere.

7

The intensity of the current passing through the other resistance

$= 6 + 4\,i - \frac{17}{4 - i} = \frac{(6 + 4\,i)(4 - i) - 17}{4 - i}$

$= \frac{24 + 10\,i - 4\,i^2 - 17}{4 - i} = \frac{11 + 10\,i}{4 - i}$

$= \frac{11 + 10\,i}{4 - i} \times \frac{4 + i}{4 + i} = \frac{44 + 51\,i + 10\,i^2}{16 - i^2}$

$= \frac{34 + 51\,i}{17} = (2 + 3\,i)$ Ampere.

8

(1) $\because f(n) = 12\,n^2 - 96\,n + 480$

$\therefore$ The discriminant $= b^2 - 4\,ac$

$= (-96)^2 - 4 \times 12 \times 480$

$= -13824 < 0$

$\therefore$ The two roots are not real.

, $\because a = 12 > 0$

$\therefore f$ is positive for all values $n \in \mathbb{R}$

(2) In the year 1990 : n = 0 $\therefore f(0) = 480$

$\therefore$ The mine production = 480 thousands ounces.

• In the year 2005 : n = 15

$f(15) = 12 \times (15)^2 - 96 \times 15 + 480 = 1470$

$\therefore$ The mine production = 1740 thousands ounces.

(3) $\because f(n) = 2016$ $\therefore 12\,n^2 - 96\,n + 480 = 2016$

$\therefore 12\,n^2 - 96\,n - 1536 = 0$

$\therefore n^2 - 8\,n - 128 = 0$ $\therefore (n - 16)(n + 8) = 0$

$\therefore n = 16$ or $n = -8$ (refused)

$\therefore$ The required year is 2006

Guide Answers of "Unit Two"

Answers of Exercise 7

First — Multiple choice questions

(1) b	(2) d	(3) c	(4) c
(5) c	(6) d	(7) b	(8) b
(9) a	(10) b	(11) c	(12) d
(13) c	(14) b	(15) c	(16) c
(17) c	(18) c	(19) c	(20) b
(21) c	(22) c		

Second — Essay questions

1

(1) The directed angle isn't in standard position , because the vertex angle isn't the origin point.

(2) The directed angle isn't in standard position , because its initial side doesn't lie on $\overrightarrow{OX}$

(3) The directed angle is in standard position.

(4) The directed angle is in standard position.

(5) The directed angle isn't in standard position , because the vertex angle isn't the origin point.

(6) The directed angle isn't in standard position , because its initial side doesn't lie on $\overrightarrow{OX}$

(7) The directed angle is in standard position.

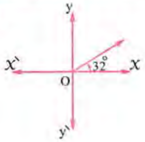
(8) The directed angle isn't in standard position because its initial side doesn't lie on $\overrightarrow{OX}$

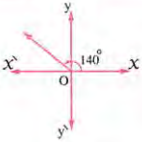
(9) The directed angle is in standard position.

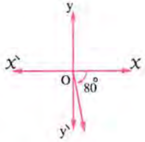
2

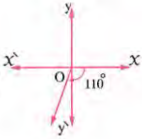
(1) -306°	(2) 270°	(3) 225°
(4) $300^\circ\ 28'$	(5) 245°	(6) $-69^\circ\ 20'$

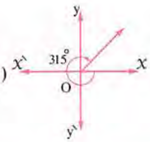
3

(1) 

(2) 

(3) 

(4) 

(5) 

4

(1) first	(2) third	(3) fourth
(4) second	(5) second	(6) first
(7) quadrantal	(8) quadrantal	

5

(1) 304° , fourth	(2) 240° , third
(3) 145° , second	(4) 220° , third
(5) 55° , first	(6) 210° , third
(7) $40^\circ\ 15'$, first	(8) $129^\circ\ 42'$, second

6

(1) -277°	(2) -224°	(3) -270°
(4) -96°	(5) -116°	(6) -10°

7

(1) 400° , -320°	(2) 510° , -210°
(3) 235° , -485°	(4) 120° , -600°
(5) 180° , -540°	

8 Ziad's answer is the correct answer.

Third — Higher skills

(1) d	(2) c	(3) c	(4) d
(5) d			

Instructions to solve :

(1) $\because$ A and B are equivalent angles.

$\therefore B = A \pm 360^\circ n \qquad \therefore B + C = A + C \pm 360^\circ n$

$\therefore (B + C)$, $(A + C)$ are measures of two equivalent angles.

, $B - C = A - C \pm 360^\circ n$

$\therefore (B - C)$, $(A - C)$ are measures of two equivalent angles.

, $CB = CA \pm 360^\circ Cn$, $C \in \mathbb{Z}$

$\therefore$ (CB) , (CA) are also measures of two equivalent angles.

$\therefore$ The answer is (d)

(2) $\because A = -A \pm 360^\circ n$

Put $n = 1$: $A = -A + 360^\circ$

$\therefore 2A = 360^\circ$ $\qquad \therefore A = 180^\circ$

(3) $\because (3x - 5)^\circ = (3y - 5)^\circ + 360^\circ$

$\therefore 3x - 3y = 360^\circ$ $\qquad \therefore x - y = 120^\circ$

(4) $(\theta + 20^\circ) = (20 - 8\theta)^\circ + 360^\circ$

$\therefore 9\theta = 360^\circ$ $\qquad \therefore \theta = 40^\circ$

(5) The terminal side passes through the point $(-1, 0)$

$\therefore$ The given directed angle is a quadrantal

$\therefore$ The answer is (d)

Answers of Exercise 8

First Multiple choice questions

(1) b	(2) c	(3) a	(4) d
(5) d	(6) b	(7) d	(8) a
(9) c	(10) d	(11) b	(12) b
(13) b	(14) b	(15) b	(16) b
(17) c	(18) a	(19) b	(20) c
(21) c	(22) c	(23) c	(24) d
(25) d			

Second Essay questions

1

$\theta^{rad} = x^\circ \times \frac{\pi}{180^\circ}$

(1) $\theta^{rad} = \frac{135^\circ}{180^\circ}\pi = \frac{3}{4}\pi$

(2) $\theta^{rad} = \frac{90^\circ}{180^\circ}\pi = \frac{1}{2}\pi$

(3) $\theta^{rad} = \frac{300^\circ}{180^\circ}\pi = \frac{5}{3}\pi$

(4) $\theta^{rad} = \frac{-235^\circ}{180^\circ}\pi = -\frac{47}{36}\pi$

(5) $\theta^{rad} = \frac{-210^\circ}{180^\circ}\pi = \frac{-7}{6}\pi$

(6) $\theta^{rad} = \frac{112.5^\circ}{180^\circ}\pi = \frac{5}{8}\pi$

(7) $\theta^{rad} = \frac{390^\circ}{180^\circ}\pi = \frac{13}{6}\pi$

(8) $\theta^{rad} = \frac{780^\circ}{180^\circ}\pi = \frac{13}{3}\pi$

2

$\theta^{rad} = x^\circ \times \frac{\pi}{180^\circ}$

(1) $\theta^{rad} = 58^\circ \times \frac{\pi}{180^\circ} \simeq 1.012^{rad}$

(2) $\theta^{rad} = 56.6^\circ \times \frac{\pi}{180^\circ} \simeq 0.988^{rad}$

(3) $\theta^{rad} = 37^\circ\, 15' \times \frac{\pi}{180^\circ} \simeq 0.650^{rad}$

(4) $\theta^{rad} = 115^\circ\, 38'\, 6'' \times \frac{\pi}{180^\circ} \simeq 2.018^{rad}$

(5) $\theta^{rad} = 257^\circ\, 54'' \times \frac{\pi}{180^\circ} \simeq 4.486^{rad}$

(6) $\theta^{rad} = 160^\circ\, 50'\, 48'' \times \frac{\pi}{180^\circ} \simeq 2.807^{rad}$

3

$x^\circ = \theta^{rad} \times \frac{180^\circ}{\pi}$

(1) $x^\circ = \frac{11}{15} \times 180^\circ = 132^\circ$

(2) $x^\circ = 0.72 \times 180^\circ = 129^\circ\, 36'$

(3) $x^\circ = 0.49 \times \frac{180^\circ}{\pi} = 28^\circ\, 4'\, 30''$

(4) $x^\circ = -1.67 \times \frac{180^\circ}{\pi} = -95^\circ\, 41'\, 2''$

(5) $x^\circ = 2.27 \times \frac{180^\circ}{\pi} = 130^\circ\, 3'\, 41''$

(6) $x^\circ = -3\frac{1}{2} \times \frac{180^\circ}{\pi} = -200^\circ\, 32'\, 7''$

4

5

$\theta^{rad} = \frac{L}{r}$

(1) $\theta^{rad} = \frac{12}{10} = 1.2^{rad}$

$\therefore x^\circ = 1.2 \times \frac{180^\circ}{\pi} = 68^\circ\, 45'\, 18''$

(2) $\theta^{rad} = \frac{14}{7} = 2^{rad}$

$\therefore x^\circ = 2 \times \frac{180^\circ}{\pi} = 114^\circ\, 35'\, 30''$

(3) $\theta^{rad} = \frac{2\pi}{6} = \frac{\pi}{3}$

$\therefore x^\circ = \frac{1}{3} \times 180^\circ = 60^\circ$

(4) $\theta^{rad} = \frac{15.72}{9.17} = 1\frac{5}{7}^{rad}$

$\therefore x^\circ = 1\frac{5}{7} \times \frac{180^\circ}{\pi} \simeq 98^\circ\, 13'\, 17''$

6

$r = \frac{l}{\theta^{rad}}$

(1) $\theta^{rad} = \frac{9}{8}\pi \simeq 3.534^{rad}$

$\therefore r = \frac{22.5}{3.534} \simeq 6.37$ cm.

(2) $r = \frac{38.35}{0.767} = 50$ cm.

(3) $\theta^{rad} = 139^\circ \times \frac{\pi}{180^\circ} \simeq 2.426^{rad}$

$\therefore r = \frac{24.325}{2.426} \simeq 10$ cm.

(4) $\theta^{rad} = 78^\circ\, 36'\, 26'' \times \frac{\pi}{180^\circ} \simeq 1.37^{rad}$

$\therefore r = \frac{43.92}{1.37} \simeq 32$ cm.

7

(1) $l = \theta^{rad} \times r = 1.6 \times 12.5 = 20$ cm.

(2) $l = \theta^{rad} \times r = 2.43 \times 20 = 48.6$ cm.

(3) $l = \theta^{rad} \times r = 67^\circ\, 40' \times \frac{\pi}{180^\circ} \times 7.5 \simeq 8.9$ cm.

(4) $l = \theta^{rad} \times r = 104^\circ\, 58'\, 6'' \times \frac{\pi}{180^\circ} \times 15 \simeq 27.5$ cm.

8

$\because$ The measure of the inscribed angle = 45°

$\therefore$ The measure of the central angle subtended by the same arc = $2 \times 45^\circ = 90^\circ$

$\therefore \theta^{rad} = 90^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{2}$

$\therefore r = \frac{L}{\theta^{rad}} = 12 \div \frac{\pi}{2} = \frac{24}{\pi}$ cm.

$\therefore$ The circumference = $2\pi r = 2\pi \times \frac{24}{\pi} = 48$ cm.

9

$\because l = 3r \qquad \therefore \theta^{rad} = \frac{3r}{r} = 3^{rad}$

$\therefore x^\circ = 3 \times \frac{180^\circ}{\pi} = 171^\circ\, 53'\, 14''$

10

$\because \theta^{rad} = 105^\circ \times \frac{\pi}{180^\circ} = \frac{7\pi}{12}$

$\therefore r = \frac{l}{\theta^{rad}} = \frac{7}{3}\pi \div \frac{7\pi}{12} = \frac{7}{3}\pi \times \frac{12}{7\pi} = 4$ cm.

$\therefore$ The diameter length = 8 cm.

11

The degree measure to the other angle = $\frac{1}{4} \times 180^\circ = 45^\circ$

$\therefore$ The measure of the third angle

$= 180^\circ - (45^\circ + 60^\circ) = 75^\circ$

$\therefore$ The radian measure = $75^\circ \times \frac{\pi}{180^\circ} = \frac{5}{12}\pi$

12

$\frac{11}{6}^{rad}$ equivalent $\frac{11}{6} \times \frac{180^\circ}{\frac{22}{7}} = 105^\circ$

$2\frac{4}{9}^{rad}$ equivalent $\frac{22}{9} \times \frac{180^\circ}{\frac{22}{7}} = 140^\circ$

$\therefore$ The degree measure to the fourth angle

$= 360^\circ - (105^\circ + 140^\circ + 45^\circ) = 70^\circ$

$\therefore$ The radian measure = $70^\circ \times \frac{\frac{22}{7}}{180^\circ} = \left(\frac{11}{9}\right)^{rad}$

13

Let the measures of the two angles be x , y , $x^\circ > y^\circ$

$\therefore x^\circ + y^\circ = 70^\circ$ (1)

$\therefore x^\circ - y^\circ = \frac{1}{5} \times 180^\circ = 36^\circ$ (2)

by adding (1) , (2) : $\therefore 2x = 106^\circ$

$\therefore x^\circ = 53^\circ$

$\therefore x^{rad} = 53^\circ \times \frac{\pi}{180^\circ} = \frac{53}{180}\pi$

$y^\circ = 70^\circ - 53^\circ = 17^\circ$

$\therefore y^{rad} = 17^\circ \times \frac{\pi}{180^\circ} = \frac{17}{180}\pi$

14

Let the measures of the two angles be :

x , y , $x^\circ > y^\circ$

$\therefore x^{rad} + y^{rad} = \pi$, $x^{rad} - y^{rad} = \frac{\pi}{3}$

$\therefore 2x^{rad} = \frac{4}{3}\pi \qquad \therefore x^{rad} = \frac{2}{3}\pi$

, $y^{rad} = \frac{1}{3}\pi$

, $x^\circ = \frac{2}{3} \times 180^\circ = 120^\circ$, $y^\circ = \frac{1}{3} \times 180^\circ = 60^\circ$

15

The area of Δ AMB $= \frac{1}{2} \times$ AM $\times$ BM

, $\because$ AM = BM = r $\qquad \therefore \frac{1}{2} r^2 = 32$

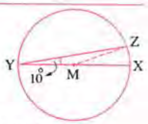
$\therefore r^2 = 64 \qquad \therefore r = 8$ cm.

$\therefore$ Length of $\widehat{AB} = 90° \times \frac{\pi}{180°} \times 8 = 12.57$ cm.

$\therefore$ The perimeter of the shaded part

$= 8 + 8 + 12.57 = 28.57$ cm.

16

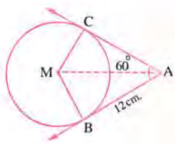


Const. : Draw $\overline{MZ}$

m ($\angle$ ZMX) = 20°

$\therefore$ length of $\widehat{XZ} = 20° \times \frac{\pi}{180°} \times 9 = 3.14$ cm.

17



Const. : Draw $\overline{AM}$

Proof : $\because \overrightarrow{AB}$, $\overrightarrow{AC}$

are two tangents

to the circle M.

$\therefore \overline{MB} \perp \overline{AB}$, $\overline{MC} \perp \overline{AC}$

$\therefore$ m ($\angle$ M) = 360° – (90° + 90° + 60°) = 120°

$\therefore$ m (reflex M) = 360° – 120° = 240°

$\because \overrightarrow{AM}$ bisects $\angle$ A $\qquad \therefore$ m ($\angle$ BAM) = 30°

$\therefore$ MB $= \frac{1}{2}$ AM

$\because$ MB = r $\qquad \therefore$ AM = 2r

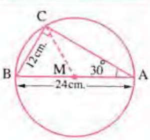
In Δ ABM is right-angled at B

$\therefore (2r)^2 = r^2 + (12)^2 \qquad \therefore 3r^2 = 144$

$\therefore r^2 = 48 \qquad \therefore r = 4\sqrt{3}$ cm.

$\therefore$ Length of greater $\widehat{BC} = 240° \times \frac{\pi}{180°} \times 4\sqrt{3} = 29$ cm.

18



$\because \angle$ C is right.

$\therefore \overline{AB}$ is a diameter.

$\therefore r = \frac{24}{2} = 12$ cm.

$\because \angle$ C is right , BC $= \frac{1}{2}$ AB

, m ($\angle$ A) = 30° , m ($\angle$ B) = 60°

Draw $\overline{MC}$, where M is the centre of the circle and the midpoint of $\overline{AB}$

$\therefore$ m ($\angle$ BMC) = 2 m ($\angle$ A) = 60°

m ($\angle$ AMC) = 2 m ($\angle$ B) = 120°

$\therefore \widehat{BC}$ is opposite to the central angle of measure 60°

$\therefore$ Length of $\widehat{BC} = 60° \times \frac{\pi}{180°} \times 12 = 12.6$ cm.

$\because \widehat{AC}$ is opposite to the central angle of measure 120°

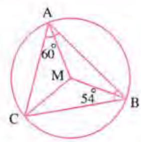
$\therefore$ The length of $\widehat{AC} = 120° \times \frac{\pi}{180°} \times 12 = 25.1$ cm.

$\because \widehat{AB}$ is opposite to central angle of measure 180°

$\therefore$ Length of $\widehat{AB}$ (half the circumference)

$= 180° \times \frac{\pi}{180°} \times 12 = 37.7$ cm.

19



m ($\angle$ BMC) = 2 m ($\angle$ A) = 120°

$\therefore$ Length of $\widehat{BC}$

$= 120° \times \frac{\pi}{180°} \times 7.5 = 15.7$ cm.

m ($\angle$ AMC) = 2 m ($\angle$ B) = 108°

$\therefore$ Length of $\widehat{AC} = 108° \times \frac{\pi}{180°} \times 7.5 = 14.1$ cm.

$\because$ m ($\angle$ AMB) = 360° – (120° + 108°) = 132°

$\therefore$ Length of $\widehat{AB} = 132° \times \frac{\pi}{180°} \times 7.5 = 17.3$ cm.

Third Higher skills

1

(1) b	(2) d	(3) b	(4) c
(5) b	(6) c	(7) c	(8) b
(9) b	(10) b	(11) b	

Instructions to solve 1 :

(1) The length of the arc $= \theta^{rad}\, r = \frac{72°}{180°} \times \pi \times 14$

$= \frac{28}{5} \pi$ cm.

$\therefore$ The circumference of the circle $= \frac{28}{5} \pi$

$\therefore 2 \pi r = \frac{28}{5} \pi$

$\therefore r = \frac{14}{5} = 2.8$ cm.

(2) $\because 5 <$ the length of arc $\widehat{AB} < 6$

$\therefore 5 < \frac{x}{180°} \times \pi \times 10 < 6$

$\therefore 5 < \frac{\pi}{180°} x < 6 \qquad \therefore 28.6° < x < 34.4°$

(3) $\because$ The ratio between measures of angles of the quadrilateral = 5 : 4 : 9 : 6

$\therefore 5X + 4X + 9X + 6X = 360°$

$\therefore 24X = 360°$ $\therefore X = 15°$

$\therefore$ Measure of the smallest angle in the quadrilateral $= 4 \times 15° = 60°$

, in radian measure $= 60° \times \frac{\pi}{180°} = \frac{\pi}{3}$

(4) Number of hours between the minute hand and hour hand at half past two = 3.5 hours.

$\therefore$ The angle between the minute hand and hour hand $= \frac{3.5}{12} \times 2\pi = \frac{7}{12}\pi$

(5) The radian measure of $60° = 60° \times \frac{\pi}{180°} = \frac{\pi}{3}$

Let the radius length of its circle be r_1

$\therefore$ The arc length $= r_1 \times \frac{\pi}{3}$

, the radian measure of $80° = 80° \times \frac{\pi}{180°} = \frac{4}{9}\pi$

Let the radius length of its circle be r_2

$\therefore$ The arc length $= r_2 \times \frac{4}{9}\pi$

$\therefore r_1 \times \frac{\pi}{3} = r_2 \times \frac{4}{9}\pi$ $\therefore \frac{r_1}{r_2} = \frac{4}{3}$

(6) Number of revolutions in one second $= \frac{45}{60} = \frac{3}{4}$

$\therefore$ The angle of rotation of a point on its lateral surface in a second $= \frac{3}{4} \times 2\pi = \frac{3}{2}\pi$

(7) (The measure of a circle)$^{rad} = 2\pi \simeq 6.28$

, $\because 6.28 > n$ where n is the greatest possible value

$\therefore n = 6$

(8) Number of rotations covered by the minute hand between 6 am and quarter past two pm

$= 9\frac{1}{4}$ revolutions.

$\therefore$ The covered distance by the tip of the minute hand $= 9\frac{1}{4} \times 2\pi \times 8 = 148\pi$ cm.

(9) When the smaller gear revolves one revolution anti clockwise , the greater gear revolves $\frac{1}{3}$ revolution clockwise.

$\therefore$ The central angle of revolution of the greater gear $= \frac{-1}{3} \times 2\pi = \frac{-2\pi}{3}$

(10) The circumference of circle N $= 2\pi \times 7$

$= 14\pi$ cm.

$\therefore$ The length $(\widehat{AB}) = 14\pi$

$\therefore m(\angle AMB) = \frac{\text{The arc length}}{r} = \frac{14\pi}{21} = \frac{2\pi}{3}$

(11) $\because$ ABCDEF is a regular hexagon.

$\therefore m(\angle AMB) = \frac{2\pi}{6} = \frac{\pi}{3}$

$\therefore \Delta AMB$ is an equilateral triangle.

$\therefore r = 4$ cm.

$\therefore$ The length of $(\widehat{AB}) = \frac{\pi}{3} \times 4 = \frac{4\pi}{3}$ cm.

2

The degree measure of the angle which the straight line makes with the X-axis $= \frac{180°}{3} = 60°$

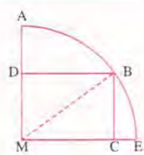
$\therefore$ The slope of the straight line $= \tan 60° = \sqrt{3}$

$\therefore$ The equation of the straight line : $y = \sqrt{3}X + c$

$\therefore$ The angle in the standard position.

$\therefore c = 0$ $\therefore y = \sqrt{3}X$

3



Const. : Draw $\overline{BM}$

Proof : BM = CD

(two diagonals of rectangle)

$\therefore$ BM = 10 cm.

$\therefore$ r = 10 cm.

$\because$ Measure of the central angle $= \frac{\pi}{2}$

$\therefore \ell$ (length of $\widehat{ABE}$) $= \theta^{rad} \times r = \frac{\pi}{2} \times 10 = 5\pi$ cm.

Answers of Exercise 9

First Multiple choice questions

(1) a	(2) d	(3) d	(4) b
(5) d	(6) c	(7) c	(8) b
(9) c	(10) a	(11) d	(12) c
(13) a	(14) d	(15) c	(16) d
(17) d	(18) c	(19) a	(20) c
(21) b	(22) d	(23) c	(24) d
(25) c	(26) c	(27) b	(28) d
(29) c	(30) d	(31) b	(32) d
(33) a	(34) a	(35) b	(36) a
(37) d	(38) d	(39) d	(40) c
(41) c	(42) d	(43) c	(44) c
(45) b	(46) d	(47) b	(48) a

Second Essay questions

1

(1) $\because 270° < 350° < 360°$

$\therefore$ 350° lies in the fourth quad.

$\therefore$ cos 350° is positive.

(2) $\because 90° < 100° < 180°$ $\therefore$ 100° lies in 2^{nd} quad.

$\therefore$ tan 100° is negative.

(3) $\because 180° < 265° < 270°$ $\therefore$ 265° lies in 3^{rd} quad.

$\therefore$ sec 265° is negative.

(4) $\because \frac{5\pi}{4} = \frac{5 \times 180°}{4} = 225°$

and it lies in 3^{rd} quad. $\therefore \sin \frac{5\pi}{4}$ is negative.

(5) $\because \frac{3\pi}{7} = \frac{3 \times 180°}{7} = 77° \frac{1}{7}$

and it lies in 1^{st} quad.

$\therefore \csc \frac{3\pi}{7}$ is positive.

(6) $\because \frac{3\pi}{4} = \frac{3 \times 180°}{4} = 135°$

and it lies in 2^{nd} quad. $\therefore \cot \frac{3\pi}{4}$ is negative.

(7) $\because \tan 410° = \tan (50° + 360°) = \tan 50°$

$\because$ 50° lies in 1^{st} quad.

$\therefore$ tan 410° is positive.

(8) $\because \csc 1200° = \csc (120° + 3 \times 360°) = \csc 120°$

$\because$ 120° lies in 2^{nd} quad.

$\therefore$ csc 1200° is positive.

(9) $\because \cos (-165°) = \cos (-165° + 360°)$

$= \cos 195°$

$\because$ 195° lies in 3^{rd} quad.

$\therefore$ cos (– 165°) is negative.

(10) $\because \frac{32\pi}{3} = \frac{32 \times 180°}{3} = 1920°$

$= (120° + 5 \times 360°)$

$\therefore \cot \frac{32\pi}{3} = \cot 120°$

$\because$ 120° lies in 2^{nd} quad.

$\therefore \cot \frac{32\pi}{3}$ is negative.

(11) $\because \frac{-3\pi}{4} = \frac{-3 \times 180°}{4} = -135°$

$= (-135° + 360°) = 225°$

and it lies in 3^{rd} quad.

$\therefore \cot \frac{-3\pi}{4}$ is positive.

(12) $\because \frac{-25\pi}{6} = \frac{-25 \times 180°}{6} = -750°$

$= (-750° + 3 \times 360°)$

$= 330°$

$\because$ 330° lies in 4^{th} quad.

$\therefore \sec \left(\frac{-25\pi}{6}\right)$ is positive.

2

(1) $\because x = \frac{2}{3}$, $y = \frac{\sqrt{5}}{3}$

$\therefore \sin\theta = \frac{\sqrt{5}}{3}$, $\cos\theta = \frac{2}{3}$

$\therefore \tan\theta = \frac{\sqrt{5}}{2}$, $\cot\theta = \frac{2}{\sqrt{5}}$

, $\csc\theta = \frac{3}{\sqrt{5}}$, $\sec\theta = \frac{3}{2}$

(2) $\because x = \frac{-3}{5}$, $y = \frac{-4}{5}$

$\therefore \sin\theta = \frac{-4}{5}$, $\cos\theta = \frac{-3}{5}$

$\therefore \tan\theta = \frac{4}{3}$, $\cot\theta = \frac{3}{4}$

, $\csc\theta = \frac{-5}{4}$, $\sec\theta = \frac{-5}{3}$

(3) $\because x = 0$, $y = -1$

$\therefore \sin\theta = -1$, $\cos\theta = 0$

, $\tan\theta$ is undefind , $\cot\theta = 0$

, $\csc\theta = -1$, $\sec\theta$ is undefind.

3

(1) $\because x^2 + y^2 = 1$ $\therefore (0.6)^2 + y^2 = 1$

$\therefore y^2 = 0.64$ $\therefore y = 0.8$ such that $y > 0$

$\therefore$ B (0.6 , 0.8)

$\therefore \cos\theta = 0.6$, $\sin\theta = 0.8$,

$\tan\theta = \frac{4}{3}$, $\sec\theta = \frac{5}{3}$, $\csc\theta = \frac{5}{4}$, $\cot\theta = \frac{3}{4}$

(2) $\because x^2 + y^2 = 1$

$\therefore x^2 + (-0.6)^2 = 1$ $\therefore x^2 = 0.64$

$\therefore x = 0.8$ such that $x > 0$ $\therefore$ B (0.8 , – 0.6)

$\therefore \cos\theta = 0.8$, $\sin\theta = -0.6$

, $\tan\theta = \frac{-3}{4}$, $\sec\theta = \frac{5}{4}$

, $\csc\theta = \frac{-5}{3}$, $\cot\theta = \frac{-4}{3}$

(3) $\because x^2 + y^2 = 1$ $\therefore \frac{3}{4} + y^2 = 1$

$\therefore y^2 = \frac{1}{4}$

$\therefore y = \frac{1}{2}$ such that $90° < \theta < 180°$

$\therefore B\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\therefore \cos\theta = -\frac{\sqrt{3}}{2}$, $\sin\theta = \frac{1}{2}$, $\tan\theta = \frac{-1}{\sqrt{3}}$

, $\sec\theta = \frac{-2}{\sqrt{3}}$, $\csc\theta = 2$, $\cot\theta = -\sqrt{3}$

(4) $\because x^2 + y^2 = 1$ $\quad \therefore x^2 + \frac{5}{9} = 1$

$\therefore x^2 = \frac{4}{9}$ $\quad \therefore x = \frac{-2}{3} : x < 0$

$\therefore B\left(\frac{-2}{3}, \frac{\sqrt{5}}{3}\right)$

$\therefore \cos\theta = \frac{-2}{3}$, $\sin\theta = \frac{\sqrt{5}}{3}$, $\tan\theta = \frac{-\sqrt{5}}{2}$

, $\sec\theta = \frac{-3}{2}$, $\csc\theta = \frac{3}{\sqrt{5}}$, $\cot\theta = \frac{-2}{\sqrt{5}}$

(5) $\because x^2 + y^2 = 1$ $\quad \therefore 1 + y^2 = 1$

$\therefore y^2 = 0$ $\quad \therefore y = 0$

$\therefore B(-1, 0)$

$\therefore \cos\theta = -1$, $\sin\theta = 0$, $\tan\theta = 0$

, $\sec\theta = -1$, $\csc\theta$ is undefind, $\cot\theta$ is undefind.

(6) $\because x^2 + y^2 = 1$ $\quad \therefore (-x^2) + x^2 = 1$

$\therefore 2x^2 = 1$ $\quad \therefore x = \frac{1}{\sqrt{2}} : x > 0$

$\therefore B\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\therefore \cos\theta = \frac{-1}{\sqrt{2}}$, $\sin\theta = \frac{1}{\sqrt{2}}$, $\tan\theta = -1$

, $\sec\theta = -\sqrt{2}$, $\csc\theta = \sqrt{2}$, $\cot\theta = -1$

(7) $\because x^2 + y^2 = 1$

$\therefore x^2 + x^2 = 1$ $\quad \therefore x^2 = \frac{1}{2}$

$\therefore x = \frac{1}{\sqrt{2}} : x > 0$ $\quad \therefore B\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

$\therefore \sin\theta = \cos\theta = -\frac{1}{\sqrt{2}}$

, $\tan\theta = \cot\theta = 1$, $\sec\theta = \csc\theta = -\sqrt{2}$

(8) $\because \theta$ lies in the 3rd quad.

$\therefore 9a$, $12a$ is negative. $\quad \therefore a < 0$

, $\because x^2 + y^2 = 1$ $\quad \therefore 81a^2 + 144a^2 = 1$

$\therefore 225a^2 = 1$ $\quad \therefore a^2 = \frac{1}{225}$

$\therefore a = -\frac{1}{15}$

$\therefore B\left(\frac{-9}{15}, \frac{-12}{15}\right) = \left(\frac{-3}{5}, \frac{-4}{5}\right)$

$\therefore \cos\theta = \frac{-3}{5}$, $\sin\theta = \frac{-4}{5}$, $\tan\theta = \frac{4}{3}$

, $\sec\theta = \frac{-5}{3}$, $\csc\theta = \frac{-5}{4}$, $\cot\theta = \frac{3}{4}$

(9) $\because \theta$ lies in the 4th quad.

$\therefore \frac{3}{2}a > 0$, $-2a < 0$ $\quad \therefore a > 0$

, $\because x^2 + y^2 = 1$ $\quad \therefore \frac{9}{4}a^2 + 4a^2 = 1$

$\therefore \frac{25}{4}a^2 = 1$ $\quad \therefore a^2 = \frac{4}{25}$

$\therefore a = \frac{2}{5}$ $\quad \therefore B\left(\frac{3}{5}, \frac{-4}{5}\right)$

$\therefore \cos\theta = \frac{3}{5}$, $\sin\theta = \frac{-4}{5}$, $\tan\theta = \frac{-4}{3}$

, $\sec\theta = \frac{5}{3}$, $\csc\theta = \frac{-5}{4}$, $\cot\theta = \frac{-3}{4}$

4

(1) $\tan 0° + \tan 45° + \tan 180° = 0 + 1 + 0 = 1$

(2) $\sin 180° \cos 45° - \cos 180° \sin 45°$

$= 0 \times \frac{1}{\sqrt{2}} - (-1) \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

(3) $\sec\frac{\pi}{6}\tan\frac{\pi}{3} - \cot\frac{\pi}{3}\cos\frac{\pi}{6}$

$= \sec 30° \tan 60° - \cot 60° \cos 30°$

$= \frac{2}{\sqrt{3}} \times \sqrt{3} - \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 2 - \frac{1}{2} = \frac{3}{2}$

(4) $\frac{4\sin^2 30° - 3\tan 45° \cos 0°}{2\cos 60° + 2\sin 45° \cos 45°}$

$= \frac{4 \times \left(\frac{1}{2}\right)^2 - 3 \times 1 \times 1}{2 \times \frac{1}{2} + 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = \frac{1-3}{1+1} = -1$

(5) $3\sin 30° \sin^2 60° - \cos 0° \sec 60°$

$+ \sin 270° \cos^2 45°$

$= 3 \times \frac{1}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1 \times 2 + (-1) \times \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{-11}{8}$

5

(1) L.H.S $= 2 \times (1)^2 = 2$,

R.H.S $= -2 \times (-1) = 2$ $\quad \therefore$ L.H.S = R.H.S

(2) L.H.S $= 3 \times \frac{\sqrt{3}}{2} \times \sqrt{3} - 2 \times \sqrt{2} \times \sqrt{2}$

$= \frac{9}{2} - 4 = \frac{1}{2} =$ R.H.S

(3) L.H.S $= 3\,(1)^2 - 2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= 3 - \frac{3}{2} = \frac{3}{2}$

, R.H.S $= \frac{3}{2} \times 1 = \frac{3}{2}$ $\therefore$ L.H.S. = R.H.S.

(4) L.H.S $= \frac{2}{\sqrt{3}} \times \sqrt{3} + \left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2$

$= 2 + \frac{4}{3} - 1 = \frac{7}{3} =$ R.H.S

(5) L.H.S $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{1}{2}$,

R.H.S $= \sin^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

$\therefore$ L.H.S = R.H.S

(6) L.H.S $= 3\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{4}{3}\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{1}{4}\,(1)^2\,(2)^2$

$= 3 \times \frac{1}{3} + \frac{4}{3} \times \frac{3}{4} - \frac{1}{4} \times 1 \times 4$

$= 1 + 1 - 1 = 1 =$ R.H.S

(7) L.H.S $= 2\cos^2 60° + 3\sin^2 45° + 4\tan^2 60° - 4\sin 90°$

$= 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\sqrt{3}\right)^2 - 4 \times 1$

$= \frac{1}{2} + \frac{3}{2} + 12 - 4 = 10 =$ R.H.S

(8) L.H.S $= \dfrac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}} = \dfrac{\sqrt{3} - \frac{\sqrt{3}}{3}}{1+1}$

$= \frac{1}{2} \times \frac{2\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = \cot 60° =$ R.H.S

(9) L.H.S $= \dfrac{\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}}$

$= \dfrac{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}}{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}} = 1 = \sin 90° =$ R.H.S

6

(1) $\because x \sin^2 45° \cos 180° = \tan^2 60° \sin 270°$

$\therefore x\left(\frac{1}{\sqrt{2}}\right)^2 \times (-1) = \left(\sqrt{3}\right)^2 \times (-1)$

$\therefore -\frac{1}{2}\,x = -3$ $\therefore x = 6$

(2) $\because x \sin 45° \cos 45° \cot 30° = \tan^2 45° - \cos^2 60°$

$\therefore x\left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{\sqrt{2}}\right) \times \left(\sqrt{3}\right) = (1)^2 - \left(\frac{1}{2}\right)^2$

$\therefore \frac{\sqrt{3}}{2}\,x = \frac{3}{4}$ $\therefore x = \frac{\sqrt{3}}{2}$

7

(1) $\because \cos x = \left(\frac{\sqrt{3}}{2} \div 1\right) - 0$

$\therefore \cos x = \frac{\sqrt{3}}{2}$ $\therefore x = 30°$

(2) $\because \sin x = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\therefore \sin x = \frac{1}{4} + \frac{3}{4} = 1$ $\therefore x = 90°$

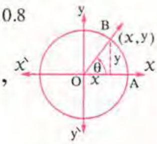
8

(1) $x = \cos\theta = 0.6$, $y = \sin\theta : y > 0$

$\therefore x^2 + y^2 = 1$ $\therefore 0.36 + y^2 = 1$

$\therefore y^2 = 0.64$ $\therefore y = 0.8$

$\therefore$ B (0.6 , 0.8)



$\therefore \cos\theta = 0.6$, $\sin\theta = 0.8$,

$\tan\theta = \frac{4}{3}$, $\sec\theta = \frac{5}{3}$

, $\csc\theta\ \frac{5}{4}$, $\cot\theta = \frac{3}{4}$

(2) $x = \cos\theta : x < 0$, $y = \sin\theta = \frac{12}{13}$

$\because x^2 + y^2 = 1$

$\therefore x^2 + \frac{144}{169} = 1$

$\therefore x^2 = \frac{25}{169}$

$\therefore x = \frac{-5}{13}$ $\therefore B\left(\frac{-5}{13}, \frac{12}{13}\right)$

$\therefore \cos\theta = \frac{-5}{13}$, $\sin\theta = \frac{12}{13}$, $\tan\theta = \frac{-12}{5}$

, $\sec\theta = \frac{-13}{5}$, $\csc\theta = \frac{13}{12}$, $\cot\theta = \frac{-5}{12}$

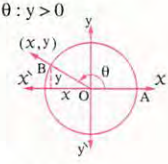
(3) $x = \cos\theta : x < 0$, $y = \sin\theta : y > 0$

$\tan\theta = \frac{y}{x} = \frac{-3}{4}$

$\therefore y = \frac{-3}{4}\,x$,

$\because x^2 + y^2 = 1$

$\therefore x^2 + \frac{9}{16}\,x^2 = 1$



$\therefore \frac{25}{16}\,x^2 = 1$ $\therefore x^2 = \frac{16}{25}$

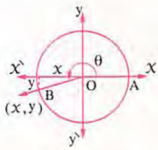
$\therefore x = \frac{-4}{5}$

$\therefore y = \frac{-3}{4} \times \frac{-4}{5} = \frac{3}{5}$ $\quad \therefore B\left(\frac{-4}{5}, \frac{3}{5}\right)$

$\therefore \cos\theta = \frac{-4}{5}$, $\sin\theta = \frac{3}{5}$, $\tan\theta = \frac{-3}{4}$

, $\sec\theta = \frac{-5}{4}$, $\csc\theta = \frac{5}{3}$, $\cot\theta = \frac{-4}{3}$

(4) $\csc\theta = \frac{1}{y} = \frac{-25}{7}$



$\therefore y = \frac{-7}{25}$, $x < 0$

$\because x^2 + y^2 = 1$

$\therefore x^2 + \frac{49}{625} = 1$

$\therefore x^2 = \frac{576}{625}$ $\quad \therefore x = \frac{-24}{25}$

$\therefore B\left(\frac{-24}{25}, \frac{-7}{25}\right)$

$\therefore \cos\theta = \frac{-24}{25}$, $\sin\theta = \frac{-7}{25}$, $\tan\theta = \frac{7}{24}$

, $\sec\theta = \frac{-25}{24}$, $\csc\theta = \frac{-25}{7}$, $\cot\theta = \frac{24}{7}$

(5) $\sec\theta = \frac{1}{x} = 2$ $\quad \therefore x = \frac{1}{2}$, $y < 0$

$\because x^2 + y^2 = 1$

$\therefore \frac{1}{4} + y^2 = 1$

$\therefore y^2 = \frac{3}{4}$ $\quad \therefore y = \frac{-\sqrt{3}}{2}$

$\therefore B\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$

$\therefore \cos\theta = \frac{1}{2}$, $\sin\theta = \frac{-\sqrt{3}}{2}$, $\tan\theta = -\sqrt{3}$

, $\sec\theta = 2$, $\csc\theta = \frac{-2}{\sqrt{3}}$, $\cot\theta = -\frac{1}{\sqrt{3}}$

9

$\because 0 < \theta < \frac{\pi}{2}$ $\quad \therefore$ Each of 2 a , 3 a is positive.

, $a > 0$ $\quad \because x^2 + y^2 = 1$

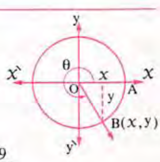
$\therefore (2a)^2 + (3a)^2 = 1$ $\quad \therefore 13a^2 = 1$

$\therefore a = \frac{1}{\sqrt{13}}$ $\quad \therefore$ The point is $\left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right)$

$\therefore \sec\theta = \frac{\sqrt{13}}{2}$, $\tan\theta = \frac{3}{2}$

$\therefore \sec^2\theta - \tan^2\theta = \frac{13}{4} - \frac{9}{4} = 1$

10



$y = \sin\theta = \frac{-24}{25}$

, $x = \cos\theta$, $x > 0$

$\because x^2 + y^2 = 1$

$\therefore x^2 + \frac{576}{625} = 1$ $\quad \therefore x^2 = \frac{49}{625}$

$\therefore x = \cos\theta = \frac{7}{25}$ $\quad \therefore B\left(\frac{7}{25}, \frac{-24}{25}\right)$

(1) $\frac{\cot\theta - \csc\theta}{\tan\theta - \sec\theta} = \frac{\frac{-7}{24} - \left(-\frac{25}{24}\right)}{\frac{-24}{7} - \frac{25}{7}} = \frac{-3}{28}$

(2) $\cos\theta - \csc\theta \tan\theta$

$= \frac{7}{25} - \left(-\frac{25}{24}\right) \times \frac{-24}{7} = \frac{-576}{175}$

11 Ahmed's answer is the correct because he uses direct substitution.

Third | Higher skills

(1) d	(2) c	(3) c
(4) b	(5) b	(6) c
(7) **First :** d	**Second :** b	**Third :** b
(8) d	(9) a	(10) b
(11) c	(12) c	

Instructions to solve :

(1) $\because$ The length of $(\widehat{BC}) = \frac{1}{3}\pi$

$\therefore m(\widehat{BC}) = \frac{\left(\frac{1}{3}\pi\right)}{2\pi} \times 360° = 60°$

$\therefore \sec(\angle BOC) = \frac{1}{\cos 60°} = \frac{1}{\left(\frac{1}{2}\right)} = 2$

(2) $\because$ A is the greatest acute angle in the triangle whose side lengths 5 , 12 , 13

, $\because (13)^2 = (5)^2 + (12)^2$

$\therefore$ The triangle is right angled

$\therefore \cot A = \frac{5}{12}$

(3) $\because (x+1)$ is the longest side

$\therefore (x+1)$ is the hypotenuse

$\therefore (x+1)^2 = (x)^2 + (x-7)^2$

$\therefore x^2 + 2x + 1 = x^2 + x^2 - 14x + 49$

$\therefore x^2 - 16x + 48 = 0$

$\therefore (x-12)(x-4) = 0$

$\therefore x = 12$ or $x = 4$ (refused)

for $x - 7 = -3$

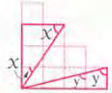
$\therefore$ The side lengths are 5 , 12 , 13

, $\because \overline{BC}$ is the smallest side $\quad \therefore$ BC = 5 cm.

$\therefore \sec A = \frac{1}{\cos A} = \frac{1}{\left(\frac{12}{13}\right)} = \frac{13}{12}$

(4) $\cot x + \cot y + \cot z = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$

(5) $\tan X = \frac{3}{2}$, $\cot y = \frac{1}{4}$



$\therefore \tan X + \cot y = \frac{3}{2} + \frac{1}{4} = \frac{7}{4}$

(6) $AO = \sqrt{(1)^2 + \left(\sqrt{3}\right)^2} = 2$

$, OB = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} = 2$

$, AB = \sqrt{(1+1)^2 + \left(\sqrt{3} - \sqrt{3}\right)^2} = 2$

$\therefore \Delta$ AOB is an equilateral triangle.

$\therefore \cot(\angle AOB) = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}}$

(7) **First :** $\because$ The circle is a unit circle

$\therefore AO = 1$ $\qquad \therefore \cos\theta = \frac{1}{OB}$

$\therefore OB = \sec\theta$

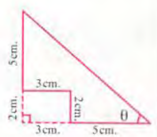
Second : $BC = BO - OC = \sec\theta - 1$

Third : The area of $\Delta ABO = \frac{1}{2} AO \times AB$

$= \frac{1}{2} \times 1 \times \tan\theta$

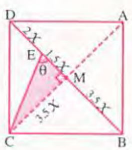
$= \frac{1}{2} \tan\theta$

(8) $\cot\theta = \frac{5+3}{5+2} = \frac{8}{7}$



(9) Draw $\overline{AC}$, $\overline{AC} \cap \overline{BD} = \{M\}$

$, \because \frac{DE}{EB} = \frac{2}{5}$



$\therefore DE = 2X$, $EB = 5X$

$\therefore BD = 7X$

$, \because$ ABCD is a square.

$\therefore AC = BD = 7X$

$, CM = 3.5X$, $ME = 1.5X$

In Δ CME , $\angle$ M is right

$\therefore \tan\theta = \frac{3.5X}{1.5X} = \frac{7}{3}$

(10) $\because \angle ADB$ is an exterior angle of Δ ADC

$\therefore m(\angle DAC) + m(\angle DCA) = \theta$

$, \because DA = DC$ $\qquad \therefore m(\angle C) = \frac{\theta}{2}$

In $\Delta ABD : m(\angle B) = 90^\circ$, $\tan\theta = \frac{4}{3}$

$\therefore AB = 4X$, $BD = 3X$

$\therefore AD = \sqrt{(4X)^2 + (3X)^2} = 5X$

$\therefore DA = DC = 5X$

In $\Delta ABC : \cot\frac{\theta}{2} = \frac{3X + 5X}{4X} = 2$

(11) In Δ ABD : $\angle$ D is right.

$\tan B = \frac{AD}{BD} = \frac{4}{BD}$

, in Δ ADC : $\angle$ D is right.

$\tan C = \frac{AD}{DC} = \frac{4}{DC}$

$\therefore \tan B + \tan C = \frac{4}{BD} + \frac{4}{DC}$

$= \frac{4(DC + BD)}{BD \cdot DC} = \frac{4\,BC}{BD \cdot DC}$

$, \because (AD)^2 = BD \times DC$ $\qquad \therefore BD \times DC = 16$

$\therefore \tan B + \tan C = \frac{4\,BC}{16} = \frac{BC}{4}$

$\therefore \frac{BC}{4} = \frac{5}{2}$ $\qquad \therefore BC = 10$ cm.

(12) $\because$ The slope of the straight line = 2

$\therefore \tan\theta = 2$

In Δ ABC : $\angle$ B is right.

$\therefore AC = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$

$\therefore \sin\theta = \frac{2}{\sqrt{5}}$

Answers of Exercise 10

First Multiple choice questions

(1) c	(2) b	(3) b	(4) b
(5) b	(6) c	(7) b	(8) b
(9) b	(10) c	(11) d	(12) a
(13) b	(14) d	(15) d	(16) a
(17) a	(18) c	(19) a	(20) c
(21) d	(22) b	(23) c	(24) b
(25) c	(26) c	(27) c	(28) a
(29) c	(30) c	(31) b	(32) c
(33) d	(34) d	(35) c	(36) d
(37) d	(38) d	(39) d	(40) a
(41) b	(42) a	(43) d	(44) a
(45) a	(46) a	(47) c	(48) b
(49) c	(50) a	(51) c	(52) d

(53) b (54) c (55) d (56) a

(57) c (58) d (59) c (60) b

(61) d (62) b (63) b (64) c

(65) d (66) a (67) d (68) c

(69) c (70) b (71) b (72) b

(73) d

Second Essay questions

1

(1) $\sin 150° = \sin (180° - 30°) = \sin 30° = \frac{1}{2}$

(2) $\sec 210° = \sec (180° + 30°) = -\sec 30° = \frac{-2}{\sqrt{3}}$

(3) $\tan 240° = \tan (180° + 60°) = \tan 60° = \sqrt{3}$

(4) $\cos (-150°) = \cos 150°$

$= \cos (180° - 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

(5) $\tan 225° = \tan (180° + 45°) = \tan 45° = 1$

(6) $\csc \frac{11\pi}{6} = \csc \left(\frac{12\pi}{6} - \frac{\pi}{6} \right) = -\csc \frac{\pi}{6} = -2$

(7) $\cot 780° = \cot (720° + 60°) = \cot 60° = \frac{1}{\sqrt{3}}$

(8) $\cos (-900°) = \cos (-900° + 3 \times 360°)$

$= \cos 180° = -1$

(9) $\sin \left(\frac{-4\pi}{3} \right) = -\sin \left(\frac{4\pi}{3} \right)$

$= -\sin \left(\pi + \frac{\pi}{3} \right)$

$= \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

(10) $\sec \left(\frac{-2\pi}{3} \right) = \sec \left(\frac{-2}{3} \times 180° \right)$

$= \sec (-120°) = \sec 120°$

$= \sec (180° - 60°)$

$= -\sec 60° = -2$

(11) $\sec (-480°) = \sec 480° = \sec (360° + 120°)$

$= \sec 120° = \sec (180° - 60°)$

$= -\sec 60° = -2$

(12) $\sin \left(\frac{-7\pi}{4} \right) = \sin \left(\frac{-7 \times 180°}{4} \right) = \sin (-315°)$

$= \sin (-315° + 360°)$

$= \sin 45° = \frac{1}{\sqrt{2}}$

2

(1) $\cos 120° + \tan 225° + \csc 330° + \cos 420°$

$= \cos (180° - 60°) + \tan (180° + 45°)$

$+ \csc (360° - 30°) + \cos (360° + 60°)$

$= -\cos 60° + \tan 45° - \csc 30° + \cos 60°$

$= -\frac{1}{2} + 1 - 2 + \frac{1}{2} = -1$

(2) $\sin 390° \cos (-60°) + \cos 30° \sin 120°$

$= \sin (360° + 30°) \cos 60° + \cos 30° \sin (180° - 60°)$

$= \sin 30° \cos 60° + \cos 30° \sin 60°$

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$

(3) $\because \cos 930° = \cos (2 \times 360° + 210°)$

$= \cos 210° = \cos (180° + 30°) = -\cos 30°$

$\therefore \sin 150° \cos (-300°) - \cos 30° \cot 240°$

$= \sin (180° - 30°) \cos (360° - 60°)$

$- \cos 30° \cot (180° + 60°)$

$= \sin 30° \cos 60° - \cos 30° \cot 60°$

$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = -\frac{1}{4}$

(4) $\tan \frac{2\pi}{3} \sec \frac{11\pi}{3} + \cot \frac{11\pi}{6} \csc \frac{19\pi}{6}$

$+ \tan \frac{25\pi}{6} \csc \left(\frac{-19\pi}{3} \right)$

$= \tan \left(\pi - \frac{\pi}{3} \right) \sec \left(\frac{12\pi}{3} - \frac{\pi}{3} \right)$

$+ \cot \left(\frac{12\pi}{6} - \frac{\pi}{6} \right) \csc \left(\frac{12\pi}{6} + \frac{7\pi}{6} \right)$

$+ \tan \left(\frac{24\pi}{6} + \frac{\pi}{6} \right) \csc \left(\frac{-18\pi}{3} - \frac{\pi}{3} \right)$

$= -\tan \frac{\pi}{3} \sec \frac{\pi}{3} - \cot \left(\frac{\pi}{6} \right) \csc \left(\pi + \frac{1}{6}\pi \right)$

$- \tan \frac{\pi}{6} \csc \left(\frac{\pi}{3} \right)$

$= -\tan \frac{\pi}{3} \sec \frac{\pi}{3} + \cot \frac{\pi}{6} \csc \frac{\pi}{6} - \tan \frac{\pi}{6} \csc \frac{\pi}{3}$

$= -\sqrt{3} \times 2 + \sqrt{3} \times 2 - \frac{1}{\sqrt{3}} \times \frac{2}{\sqrt{3}} = -\frac{2}{3}$

3

(1) $\cos (-300°) = \cos 300° = \cos (360° - 60°)$

$= \cos 60° = \frac{1}{2}$

, $\sin 420° = \sin (60° + 360°) = \sin 60° = \frac{\sqrt{3}}{2}$

$$, \cos 750° = \cos (30° + 360° \times 2)$$
$$= \cos 30° = \frac{\sqrt{3}}{2} ,$$
$$\cos 660° = \cos (300° + 360°) = \cos 300°$$
$$= \cos (360° - 60°) = \cos 60° = \frac{1}{2}$$
$$\therefore \text{L.H.S} = \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = 0 = \text{R.H.S}$$

(2) $\sin 600° = \sin (360° + 240°) = \sin 240°$
$$= \sin (180° + 60°) = -\sin 60° = -\frac{\sqrt{3}}{2}$$
$$, \cos (-30°) = \cos 30° = \frac{\sqrt{3}}{2}$$
$$, \sin 150° = \sin (180° - 30°) = \sin 30° = \frac{1}{2}$$
$$, \cos (-240°) = \cos 240° = \cos (180° + 60°)$$
$$= -\cos 60° = -\frac{1}{2}$$
$$\therefore \text{L.H.S} = -\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \left(-\frac{1}{2}\right)$$
$$= -1 = \text{R.H.S}$$

(3) $\sin 480° = \sin (360° + 120°) = \sin 120°$
$$= \sin (180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}$$
$$, \cos (-60°) = \cos 60° = \frac{1}{2}$$
$$, \cos 300° = \cos (360° - 60°) = \cos 60° = \frac{1}{2}$$
$$, \sin (-120°) = -\sin 120°$$
$$= -\sin (180° - 60°)$$
$$= -\sin 60° = -\frac{\sqrt{3}}{2}$$
$$\therefore \text{L.H.S} = \frac{\sqrt{3}}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{-\sqrt{3}}{2}$$
$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0 = \text{R.H.S}$$

(4) $\sin 150° = \sin (180° - 30°) = \sin 30° = \frac{1}{2}$
$$, \tan 225° = \tan (180° + 45°) = \tan 45° = 1$$
$$, \cos 315° = \cos (360° - 45°) = \cos 45° = \frac{1}{\sqrt{2}}$$
$$, \sec (-120°) = \sec 120° = \sec (180° - 60°)$$
$$= -\sec 60° = -2$$
$$, \sin (-135°) = -\sin 135° = -\sin (180° - 45°)$$
$$= -\sin 45° = -\frac{1}{\sqrt{2}}$$
$$, \csc 210° = \csc (180° + 30°)$$
$$= -\csc 30° = -2$$
$$\therefore \text{L.H.S} = \frac{1}{2} \times 1 + \frac{1}{\sqrt{2}} \times (-2) + \left(-\frac{1}{\sqrt{2}}\right) \times (-2)$$
$$= \frac{1}{2} - \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{1}{2} = \text{R.H.S}$$

4

$\sin \theta = \frac{4}{5}$, $\cos \theta = \frac{-3}{5}$

(1) $\sin (180° + \theta) = -\sin \theta = \frac{-4}{5}$

(2) $\cos \left(\frac{\pi}{2} - \theta\right) = \cos (90° - \theta) = \sin \theta = \frac{4}{5}$

(3) $\tan (360° - \theta) = -\tan \theta = -\frac{\frac{4}{5}}{\frac{-3}{5}} = \frac{4}{3}$

(4) $\csc \left(\frac{3\pi}{2} - \theta\right) = \csc (270° - \theta) = -\sec \theta = \frac{5}{3}$

(5) $\sec (\theta + \pi) = \sec (\theta + 180°) = -\sec \theta = \frac{5}{3}$

(6) $\sin (\theta - \pi) = \sin (\theta - 180°) = \sin (180° + \theta)$
$$= -\sin \theta = \frac{-4}{5}$$

5

$\sin \theta = \frac{2}{3}$, $\cos \theta = \frac{\sqrt{5}}{3}$

(1) $\sin (270° + \theta) = -\cos \theta = -\frac{\sqrt{5}}{3}$

(2) $\sec (270° + \theta) = \csc \theta = \frac{3}{2}$

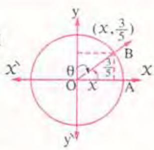
(3) $\csc \left(\theta + \frac{\pi}{2}\right) = \csc (\theta + 90°) = \sec \theta = \frac{3}{\sqrt{5}}$

(4) $\tan \left(\frac{\pi}{2} - \theta\right) = \tan (90° - \theta) = \cot \theta$
$$= \frac{\frac{\sqrt{5}}{3}}{\frac{2}{3}} = \frac{\sqrt{5}}{2}$$

(5) $\cot (\theta - 180°) = \cot \theta = \frac{\sqrt{5}}{2}$

(6) $\sec (-\theta) = \sec \theta = \frac{3}{\sqrt{5}}$

6



$\because x^2 + y^2 = 1 \quad \therefore x^2 + \frac{9}{25} = 1$

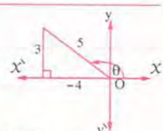
$\therefore x^2 = \frac{16}{25}$

$\therefore x = \frac{4}{5} \; ; x > 0$

$\therefore B \left(\frac{4}{5} , \frac{3}{5}\right)$

$\therefore \sin (90° - \theta) + \tan (90° - \theta) \cos (90° + \theta)$
$$= \cos \theta + \cot \theta (-\sin \theta)$$
$$= \frac{4}{5} + \frac{4}{3} \times \frac{-3}{5} = 0$$

7



$\sin \theta = \frac{3}{5}$

$\because 90° < \theta < 180°$

$\therefore \theta$ lies in 2^{nd} quad.

(1) $\cos (180° - \theta) = -\cos \theta = -\left(\frac{-4}{5}\right) = \frac{4}{5}$

(2) $\tan(180^\circ + \theta) = \tan\theta = \frac{-3}{4}$

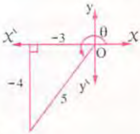
(3) $\csc(-\theta) = -\csc\theta = -\left(\frac{5}{3}\right) = \frac{-5}{3}$

(4) $\cot(360^\circ - \theta) = -\cot\theta = -\left(\frac{-4}{3}\right) = \frac{4}{3}$

(5) $\sin(90^\circ - \theta) = \cos\theta = \frac{-4}{5}$

(6) $\sin(270^\circ - \theta) = -\cos\theta = -\left(\frac{-4}{5}\right) = \frac{4}{5}$

8



$\cos\theta = -\frac{3}{5}$

, $\because 180^\circ < \theta < 270^\circ$

$\therefore$ θ lies in the 3rd quad.

(1) $\csc(180^\circ + \theta) = -\csc\theta = \frac{5}{4}$

(2) $\sec(-\theta) = \sec\theta = -\frac{5}{3}$

(3) $\tan(360^\circ - \theta) = -\tan\theta = \frac{-4}{3}$

(4) $\cot(\theta - 90^\circ) = -\cot(90^\circ - \theta) = -\tan\theta = -\frac{4}{3}$

(5) $\sec(90^\circ + \theta) = -\csc\theta = \frac{5}{4}$

(6) $\tan(270^\circ - \theta) = \cot\theta = \frac{3}{4}$

9

(1) $\because \sin(3\theta + 15^\circ) = \cos(2\theta - 5^\circ)$

$\therefore 3\theta + 15^\circ + 2\theta - 5^\circ = 90^\circ$

$\therefore 5\theta + 10^\circ = 90^\circ$

$\therefore 5\theta = 80^\circ \qquad \therefore \theta = 16^\circ$

(2) $\because \sec(\theta + 25^\circ) = \csc(\theta + 15^\circ)$

$\therefore \theta + 25^\circ + \theta + 15^\circ = 90^\circ$

$\therefore 2\theta + 40^\circ = 90^\circ \qquad \therefore 2\theta = 50^\circ \qquad \therefore \theta = 25^\circ$

(3) $\because \tan(\theta + 20^\circ) = \cot(3\theta + 30^\circ)$

$\therefore \theta + 20^\circ + 3\theta + 30^\circ = 90^\circ$

$\therefore 4\theta + 50^\circ = 90^\circ \qquad \therefore 4\theta = 40^\circ \qquad \therefore \theta = 10^\circ$

(4) $\because \cos\left(\frac{\theta + 20^\circ}{2}\right) = \sin\left(\frac{\theta + 40^\circ}{2}\right)$

$\therefore \frac{\theta + 20^\circ}{2} + \frac{\theta + 40^\circ}{2} = 90^\circ$

$\therefore \theta + 20^\circ + \theta + 40^\circ = 180^\circ$

$\therefore 2\theta + 60^\circ = 180^\circ$

$\therefore 2\theta = 120^\circ \qquad \therefore \theta = 60^\circ$

(5) $\because \tan(\theta + 18^\circ\, 24') = \cot(\theta + 52^\circ\, 10')$

$\therefore \theta + 18^\circ\, 24' + \theta^\circ + 52^\circ\, 10' = 90^\circ$

$\therefore 2\theta + 70^\circ\, 34' = 90^\circ$

$\therefore 2\theta = 19^\circ\, 26' \qquad \therefore \theta = 9^\circ\, 43'$

10

(1) $\because \sin 2\theta = \cos\theta$

$\therefore 2\theta \pm \theta = \frac{\pi}{2} + 2\pi n$ where $n \in \mathbb{Z}$

either $2\theta + \theta = \frac{\pi}{2} + 2\pi n$

$\therefore 3\theta = \frac{\pi}{2} + 2\pi n \qquad \therefore \theta = \frac{\pi}{6} + \frac{2\pi}{3} n$

or $2\theta - \theta = \frac{\pi}{2} + 2\pi n$

$\therefore \theta = \frac{\pi}{2} + 2\pi n$

$\therefore$ The solution is : $\frac{\pi}{6} + \frac{2\pi}{3} n$ or $\frac{\pi}{2} + 2\pi n$

(2) $\because \cos 5\theta = \sin\theta \qquad \therefore \sin\theta = \cos 5\theta$

$\therefore \theta \pm 5\theta = \frac{\pi}{2} + 2\pi n$

either $\theta + 5\theta = \frac{\pi}{2} + 2\pi n$

$\therefore 6\theta = \frac{\pi}{2} + 2\pi n \qquad \therefore \theta = \frac{\pi}{12} + \frac{\pi}{3} n$

or $\theta - 5\theta = \frac{\pi}{2} + 2\pi n$

$\therefore -4\theta = \frac{\pi}{2} + 2\pi n \qquad \therefore \theta = -\frac{\pi}{8} - \frac{\pi}{2} n$

$\therefore$ The solution is : $\frac{\pi}{12} + \frac{\pi}{3} n$ or $\frac{-\pi}{8} - \frac{\pi}{2} n$

11

(1) $\because \csc(\theta + 15^\circ) = \sec 42^\circ$

$\therefore (\theta + 15^\circ) \pm (42^\circ) = 90^\circ + 360^\circ n$

$\therefore \theta + 15^\circ + 42^\circ = 90^\circ \qquad \therefore \theta = 33^\circ$

(2) $\because \sin(\theta + 30^\circ) = \cos\theta$

$\therefore (\theta + 30^\circ) \pm \theta = 90^\circ + 360^\circ n$

$\therefore \theta + 30^\circ + \theta = 90^\circ$

$\therefore 2\theta = 60^\circ \qquad \therefore \theta = 30^\circ$

(3) $\because \sin\theta = \cos\theta$

$\therefore \theta \pm \theta = 90^\circ + 360^\circ n$

$\therefore 2\theta = 90^\circ \qquad \therefore \theta = 45^\circ$

(4) $\because \csc\left(\theta - \frac{\pi}{6}\right) = \sec\theta$

$\therefore \left(\theta - \frac{\pi}{6}\right) \pm \theta = 90^\circ + 360^\circ n$

$\therefore 2\theta - 30^\circ = 90^\circ$

$\therefore 2\theta = 120^\circ \qquad \therefore \theta = 60^\circ$

(5) $\because \tan(\theta + 27^\circ) = \cot 2\theta$

$\therefore \theta + 27 + 2\theta = 90^\circ + 180^\circ n$

$\therefore 3\theta + 27 = 90^\circ$

$\therefore 3\,\theta = 63°$ $\quad\quad \therefore \theta° = 21°$

or $\theta + 27 + 2\,\theta = 270°$

$\therefore 3\,\theta = 243°$ $\quad\quad \therefore \theta = 81°$

(6) $\because \tan(\theta + 10°) = \cot(4\,\theta - 10°)$

$\therefore (\theta + 10°) + (4\,\theta - 10) = 90° + 180°\, n$

$\therefore 5\,\theta = 90°$ $\quad\quad \therefore \theta = 18°$

or $5\,\theta = 270°$ $\quad\quad \therefore \theta = 54°$

or $5\,\theta = 450°$ $\quad\quad \therefore \theta = 90°$

(7) $\because \sec(2\,\theta + 35°) = \csc(3\,\theta - 10°)$

$\therefore \csc(3\,\theta - 10°) = \sec(2\,\theta + 35°)$

$\therefore (3\,\theta - 10°) \pm (2\,\theta + 35°) = 90° + 360°\, n$

$\therefore 3\,\theta - 10° + 2\,\theta + 35° = 90°$

$\therefore 5\,\theta = 65°$ $\quad\quad \therefore \theta = 13°$

or $3\,\theta - 10° + 2\,\theta + 35° = 90° + 360° = 450°$

$\therefore 5\,\theta = 425°$ $\quad\quad \therefore \theta = 85°$

(8) $\because \sec\theta = \csc(3\,\theta - 90°)$

$\therefore \csc(3\,\theta - 90°) = \sec\theta$

$\therefore (3\,\theta - 90°) \pm \theta = 90° + 360°\, n$

$\therefore 3\,\theta - 90° + \theta = 90°$

$\therefore 4\,\theta = 180°$ $\quad\quad \therefore \theta = 45°$

or $3\,\theta - 90° - \theta = 90°$ $\quad\quad \therefore 2\,\theta = 180°$

$\therefore \theta = 90°$ (refused)

(9) $\because \sin(4\,\theta + 48°) = \cos(\theta - 33°)$

$\therefore (4\,\theta + 48°) \pm (\theta - 33°) = 90° + 360°\, n$

$\therefore 4\,\theta + 48° + \theta - 33° = 90°$

$\therefore 5\,\theta + 15 = 90°$ $\quad\quad \therefore \theta = 15°$

or $5\,\theta + 15° = 450°$ $\quad\quad \therefore \theta = 87°$

or $4\,\theta + 48° - \theta + 33° = 90°$

$\therefore 3\,\theta + 81° = 90°$ $\quad\quad \therefore \theta = 3°$

(10) $\because \csc 8\,\theta = \sec 2\,\theta$

$\therefore (8\,\theta) \pm (2\,\theta) = 90° + 360°\, n$

$\therefore 8\,\theta + 2\,\theta = 90°$

$\therefore 10\,\theta = 90°$ $\quad\quad \therefore \theta = 9°$

or $10\,\theta = 450°$ $\quad\quad \therefore \theta = 45°$

or $10\,\theta = 810°$ $\quad\quad \therefore \theta = 81°$

or $8\,\theta - 2\,\theta = 90°$

$\therefore 6\,\theta = 90°$ $\quad\quad \therefore \theta = 15°$

or $6\,\theta = 450°$ $\quad\quad \therefore \theta = 75°$

12

(1) $\because \tan\theta - 1 = 0$ $\quad\quad \therefore \tan\theta = 1$

, $\because$ tan is positive in the first and third quad.

$\therefore \theta = 45°$ or $\theta = 180° + 45° = 225°$,

$\because \theta \in \left]0 , \frac{\pi}{2}\right[$ $\quad\quad \therefore \theta = 45°$

(2) $\because 2\cos\theta - 1 = 0$ $\quad\quad \therefore \cos\theta = \frac{1}{2}$

$\because$ cos is positive in the first and fourth quad.

$\therefore \theta = 60°$ or $\theta = 360° - 60° = 300°$,

$\because \theta \in \left]0 , \frac{\pi}{2}\right[$ $\quad\quad \therefore \theta = 60°$

(3) $\because 2\cos\left(\frac{\pi}{2} - \theta\right) = 1$ $\quad\quad \therefore \cos\left(\frac{\pi}{2} - \theta\right) = \frac{1}{2}$

$\therefore \sin\theta = \frac{1}{2}$

$\because$ sin is positive in the first and the second quad.

$\therefore \theta = 30°$ or $\theta = 180° - 30° = 150°$

$\because \theta \in \left]0 , \frac{\pi}{2}\right[$ $\quad\quad \therefore \theta = 30°$

(4) $\because 2\sin\left(\frac{\pi}{2} - \theta\right) = \sqrt{3}$

$\therefore \sin(90° - \theta) = \frac{\sqrt{3}}{2}$ $\quad\quad \therefore \cos\theta = \frac{\sqrt{3}}{2}$

$\because$ cos is positive in the first and fourth quad.

$\therefore \theta = 30°$ or $360° - 30° = 330°$,

$\because \theta \in \left]0 , \frac{\pi}{2}\right[$ $\quad\quad \therefore \theta = 30°$

13

(1) $\because \cos\theta = -\frac{1}{2}$ (negative)

$\therefore \theta$ lies in the 2^{nd} or the 3^{rd} quad.

$\because$ The acute angle whose cosine $\frac{1}{2}$ is $60°$

$\therefore \theta = 180° - 60° = 120°$

or $\theta = 180° + 60° = 240°$

$\therefore$ The S.S. $= \{120° , 240°\}$

(2) $\because \sec\theta = \sqrt{2}$

$\therefore \cos\theta = \frac{1}{\sqrt{2}}$ (positive)

$\therefore \theta$ lies in the 1^{st} or the 4^{th} quad.

$\because$ The acute angle whose cosine $= \frac{1}{\sqrt{2}}$ is $45°$

$\therefore \theta = 45°$ or $\theta = 360° - 45° = 315°$

$\therefore$ The S.S. $= \{45° , 315°\}$

(3) $\because \sin\theta = \frac{\sqrt{3}}{2}$ (positive)

$\therefore \theta$ lies in the 1^{st} or the 2^{nd} quad.

$\because$ The acute angle whose sin $\frac{\sqrt{3}}{2}$ is $60°$

$\therefore \theta° = 60°$ or $\theta = 180° - 60° = 120°$

$\therefore$ The S.S. $= \{60°, 120°\}$

(4) $\because \cos\theta = -1$ $\quad\therefore$ The S.S. $= \{180°\}$

(5) $\because \sin\theta = -\frac{\sqrt{3}}{2}$ (negative)

$\therefore \theta$ lies in the 3^{rd} or the 4^{th} quad.

$\because$ The acute angle whose sin $= \frac{\sqrt{3}}{2}$ is $60°$

$\therefore \theta = 180° + 60° = 240°$

or $\theta = 360° - 60° = 300°$

$\therefore$ The S.S. $= \{240°, 300°\}$

(6) $\because \tan\theta = -1$ (negative)

$\therefore \theta$ lies in the 2^{nd} or the 4^{th} quad.

$\because$ The acute angle whose tan $= 1$ is $45°$

$\therefore \theta = 180° - 45° = 135°$

or $\theta = 360° - 45° = 315°$

$\therefore$ The S.S. $= \{135°, 315°\}$

(7) $\because \csc\theta = \frac{-2}{\sqrt{3}}$ $\quad\therefore \sin\theta = \frac{-\sqrt{3}}{2}$ (negative)

$\therefore \theta$ lies in the 3^{rd} or the 4^{th} quad.

$\because$ The acute angle whose $\sin\theta = \frac{\sqrt{3}}{2}$ is $60°$

$\therefore \theta = 180° + 60° = 240°$

or $\theta = 360° - 60° = 300°$

$\therefore$ The S.S. $= \{240°, 300°\}$

(8) $\because \sin^2\theta = \frac{1}{4}$ $\quad\therefore \sin\theta = \pm\frac{1}{2}$

$\therefore \sin\theta = \frac{1}{2}$ (positive)

$\therefore \theta$ lies in the 1^{st} or the 2^{nd} quad. ,

$\because$ The acute angle whose sin $= \frac{1}{2}$ is $30°$

$\therefore \theta = 30°$ or $\theta = 180° - 30° = 150°$

or $\sin\theta = \left(-\frac{1}{2}\right)$ negative.

$\therefore \theta$ lies in the 3^{rd} or the 4^{th} quad.

$\therefore \theta = 180° + 30° = 210°$

or $\theta = 360° - 30° = 330°$

$\therefore$ The S.S. $= \{30°, 150°, 210°, 330°\}$

14

$\because \cos\left(\frac{3\pi}{2} - \theta\right) = \frac{\sqrt{3}}{2}$ $\quad\therefore \cos(270° - \theta) = \frac{\sqrt{3}}{2}$

$\therefore -\sin\theta = \frac{\sqrt{3}}{2}$ $\quad\therefore \sin\theta = \frac{-\sqrt{3}}{2}$

$\because \sin\left(\frac{\pi}{2} + \theta\right) = \frac{1}{2}$ $\quad\therefore \sin(90° + \theta) = \frac{1}{2}$

$\therefore \cos\theta = \frac{1}{2}$

$\because$ sin (negative) and cos (positive)

$\therefore \theta$ lies in 4^{th} quad.

$\because$ The acute angle whose sin $= \frac{\sqrt{3}}{2}$ is $60°$

$\therefore \theta = 360° - 60° = 300°$

15

$\because \sin(2\theta + 15°) = \cos(\theta + 30°)$

$\therefore (2\theta + 15°) \pm (\theta + 30°) = 90° + 360°\, n$

$\therefore 2\theta + 15° + \theta + 30° = 90°$

$\therefore 3\theta + 45° = 90°$ $\quad\therefore 3\theta = 45°$

$\therefore \theta = 15°$

$\therefore \csc^2 2\theta + \cot^2 3\theta + \sec^2 4\theta$

$= \csc^2 30° + \cot^2 45° + \sec^2 60° = 4 + 1 + 4 = 9$

16

$\because \frac{\sin(3\theta - 25°)}{\cos(2\theta - 35°)} = 1$

$\therefore \sin(3\theta - 25°) = \cos(2\theta - 35°)$

$\therefore (3\theta - 25°) \pm (2\theta - 35°) = 90° + 360°\, n$

$\therefore 3\theta - 25° + 2\theta - 35° = 90°$

$\therefore 5\theta = 150°$ $\quad\therefore \theta = 30°$

$\therefore \frac{\sin 18°}{\cos 72°} + \sin(180° - \theta) = \frac{\cos 72°}{\cos 72°} + \sin\theta$

$= 1 + \sin 30° = 1\frac{1}{2}$

17

$\because \frac{\tan\theta}{\cot 2\theta} = 1$ $\quad\therefore \tan\theta = \cot 2\theta$

$\therefore \theta + 2\theta = 90° + 180°\, n$

$\therefore \theta + 2\theta = 90°$ $\quad\therefore 3\theta = 90°$

$\therefore \theta = 30°$

$\therefore \sin(180° - 3\theta)\cos(360° - 2\theta)$

$+ \tan 2\theta \cot(\theta - 180°)$

$= \sin 90° \cos 60° + \tan 60° \cot(-150°)$

$= \cos 60° - \tan 60° \cot 150°$

$= \cos 60° - \tan 60° \cot(180° - 30°)$

$= \cos 60° + \tan 60° \cot 30°$

$= \frac{1}{2} + \sqrt{3} \times \sqrt{3} = 3\frac{1}{2}$

18

$\because \tan(\theta - 15°) = \cot(2\theta + 15°)$

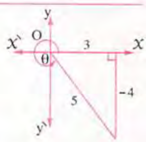
$\therefore (\theta - 15°) + (2\theta + 15°) = 90° + 180°\, n$

$\therefore \theta - 15° + 2\theta + 15° = 90°$

$\therefore 3\theta = 90° \qquad \therefore \theta = 30°$

$$\therefore \frac{1+\sin(270°+2\theta)}{1+\sin(90°+2\theta)} = \frac{1+\sin(270°+60°)}{1+\sin(90°+60°)} = \frac{1-\cos 60°}{1+\cos 60°} = \frac{1-\frac{1}{2}}{1+\frac{1}{2}} = \frac{1}{3}$$

19



$\cos\theta = \frac{3}{5}$

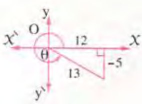
$\because 270° < \theta < 360°$

$\therefore \theta$ lies in 4^{th} quad.

$\therefore \sin(180° - \theta) + \tan(90° - \theta)$

$- \tan(270° - \theta) = \sin\theta + \cot\theta - \cot\theta = \sin\theta = \frac{-4}{5}$

20



$\cos\theta = \frac{12}{13}$ (positive)

$\because \theta$ lies in 1^{st} or 4^{th} quad.

$\because 90° < \theta < 360°$

$\therefore \theta$ lies in 4^{th} quad.

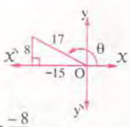
$\therefore$ The expression $= 13\sin\theta - 10\left(\frac{1}{\sqrt{2}}\right)^2\left(\sqrt{3}\right)^2$

$+ 50\sin(180° - 30°)$

$= 13 \times \frac{-5}{13} - 10 \times \frac{1}{2} \times 3 + 50\sin 30°$

$= -5 - 15 + 50 \times \frac{1}{2} = 5$

21



$\tan\theta = \frac{-8}{15}$

$\because 90° < \theta < 180°$

$\therefore \theta$ lies in 2^{nd} quad.

$\therefore \sin\theta = \frac{8}{17}, \cos\theta = \frac{-15}{17}, \tan\theta = \frac{-8}{15}$

$, \csc\theta = \frac{17}{8}, \sec\theta = \frac{-17}{15}, \cot\theta = \frac{-15}{8}$

$, 2\sin\theta\cos\theta = 2 \times \frac{8}{17} \times \frac{-15}{17} = \frac{-240}{289}$

$\sec(1080° + \theta) = \sec(\theta + 3 \times 360°) = \sec\theta = \frac{-17}{15}$

22

$\because \sin\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\because \theta \in \left]0, \frac{\pi}{2}\right[\qquad \therefore \theta = 45°$

(1)
$$\frac{1-2\cot(270°-\theta)}{1+\cos^2(270°+\theta)} = \frac{1-2\cot(270°-45°)}{1+\cos^2(270°+45°)} = \frac{1-2\tan 45°}{1+\sin^2 45°} = \frac{1-2}{1+\frac{1}{2}} = \frac{-1}{\frac{3}{2}} = \frac{-2}{3}$$

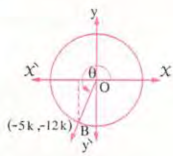
(2) L.H.S. $= \cos 2\theta = \cos 90° =$ zero

, R.H.S. $= \frac{1-\tan^2(270°-\theta)}{\csc^2(90°+\theta)} = \frac{1-\cot^2\theta}{\sec^2\theta}$

$= \frac{1-\cot^2 45°}{\sec^2 45°} = \frac{1-1}{2} =$ zero

$\therefore$ The two sides are equal.

23



$\because x^2 + y^2 = 1$

$\therefore 25k^2 + 144k^2 = 1$

$\therefore k^2 = \frac{1}{169}$

$\therefore k = \frac{1}{13}$ where $k > 0$

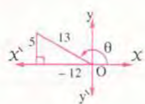
$\therefore B\left(\frac{-5}{13}, \frac{-12}{13}\right)$

$\therefore \csc(90° - \theta)\sin(90° + \theta) + 12\tan(270° + \theta)$

$= \sec\theta\cos\theta + 12(-\cot\theta)$

$= -\frac{13}{5} \times \frac{-5}{13} - 12 \times \frac{5}{12} = 1 - 5 = -4$

24



$\because 13\sin\theta - 5 = 0$

$\therefore \sin\theta = \frac{5}{13}$

$, \because \theta \in \left]\frac{\pi}{2}, \pi\right[$

$\therefore \csc(270° + \theta) = -\sec\theta = -\left(\frac{-13}{12}\right) = \frac{13}{12}$

$, \cos(\theta - 270°) = \cos(\theta - 270° + 360°)$

$= \cos(90° + \theta) = -\sin\theta = -\frac{5}{13}$

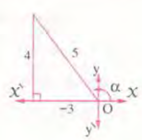
$, \tan(270° + \theta) = -\cot\theta = -\left(\frac{-12}{5}\right) = \frac{12}{5}$

$, \sin(270° - \theta) \times \sec(270° + \theta) \times \cot(270° + \theta)$

$= -\cos\theta \times \csc\theta \times -\tan\theta$

$= \frac{12}{13} \times \frac{13}{5} \times \frac{5}{12} = 1 = \sin 90°$

25



$\because \cos^2\alpha = \frac{9}{25}$

$, 90° < \alpha < 180°$

$\therefore \alpha$ lies in 2^{nd} quad.

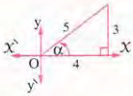
$\therefore \cos \propto$ (negative)

$\therefore \cos \propto = \frac{-3}{5}$

$\therefore 25 \sin \propto - 4 \cot \propto = 25 \times \frac{4}{5} - 4 \times \frac{-3}{4}$

$= 20 + 3 = 23$

26



$\tan \alpha = \frac{3}{4}$ (positive)

$\therefore \alpha$ lies in the 1^{st} or 3^{rd} quad.

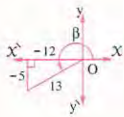
$\because \alpha$ is the smallest positive angle

$\therefore \alpha$ lies in the 1^{st} quad.

$\therefore \sin \alpha = \frac{3}{5}$, $\cos \alpha = \frac{4}{5}$

$\therefore \tan \alpha = \frac{3}{4}$, $\csc \alpha = \frac{5}{3}$

, $\sec \alpha = \frac{5}{4}$, $\cot \alpha = \frac{4}{3}$



$\because \tan \beta = \frac{5}{12}$, $180° < \beta < 270°$

$\therefore \beta$ lies in the 3^{rd} quad.

$\therefore \sin \beta = \frac{-5}{13}$, $\cos \beta = \frac{-12}{13}$

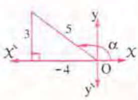
, $\tan \beta = \frac{5}{12}$, $\csc \beta = \frac{-13}{5}$

, $\sec \beta = \frac{-13}{12}$, $\cot \beta = \frac{12}{5}$

$\therefore \sin \alpha \cos \beta - \cos \alpha \sin \beta$

$= \frac{3}{5} \times \frac{-12}{13} - \frac{4}{5} \times \frac{-5}{13} = \frac{-36}{65} + \frac{20}{65} = \frac{-16}{65}$

27



$\sin \alpha = \frac{3}{5}$,

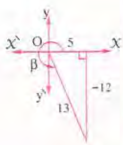
$\because \alpha \in \left]\frac{\pi}{2}, \pi\right[$

$\therefore \alpha$ lies in 2^{nd} quad.

$\cos \beta = \frac{5}{13}$

$\because \beta \in \left]\frac{3\pi}{2}, 2\pi\right[$

$\therefore \beta$ lies in the 4^{th} quad.

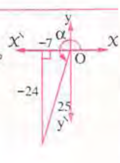


$\therefore \cos \alpha \cos \beta + \sin \alpha \sin \beta$

$= \frac{-4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{-12}{13}$

$= \frac{-20}{65} - \frac{36}{65} = -\frac{56}{65}$

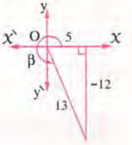
28



$\sin \alpha = \frac{-24}{25}$ $\quad \because 180° < \alpha < 270°$

$\therefore \alpha$ lies in 3^{rd} quad.

, $\tan \beta = \frac{-12}{5}$ (negative)



$\therefore \beta$ lies in the 2^{nd} or 4^{th} quad.

$\because \beta$ is the greatest positive angle ,

$\beta \in]0° , 360°[$

$\therefore \beta$ lies in the 4^{th} quad.

(1) The expression

$= -\sin \alpha - \cos \beta$

$= \frac{24}{25} - \frac{5}{13} = \frac{187}{325}$

(2) The expression $= -\csc \alpha \tan \beta - \sec \alpha (-\tan \beta)$

$= -\left(\frac{-25}{24}\right)\left(\frac{-12}{5}\right) - \left(\frac{-25}{7}\right)\left(\frac{12}{5}\right)$

$= \frac{-5}{2} + \frac{60}{7} = \frac{85}{14}$

(3) The expression $= \sec \alpha (-\tan \beta)(\cot \alpha)(-\sec \beta)$

$= -\frac{25}{7}\left(\frac{12}{5}\right)\left(\frac{7}{24}\right)\left(\frac{-13}{5}\right) = 6\frac{1}{2}$

29

$\because \theta$ is complementary of $(90° - \theta)$

$\therefore$ The terminal side of the angle whose measure is θ intersects the unit circle at the point $\left(y , \frac{5}{13}\right)$

$\because x^2 + y^2 = 1 \qquad \therefore y^2 + \frac{25}{169} = 1$

$\therefore y^2 = \frac{144}{169} \qquad \therefore y = \frac{12}{13}$

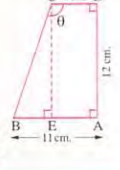
$\therefore \theta$ makes the point $\left(\frac{12}{13} , \frac{5}{13}\right)$ on the unit circle

$\therefore \cos \theta = \frac{12}{13}$, $\sin \theta = \frac{5}{13}$

, $\tan \theta = \frac{5}{12}$, $\sec \theta = \frac{13}{12}$

, $\csc \theta = \frac{13}{5}$, $\cot \theta = \frac{12}{5}$

30



Const. : Draw $\overline{CE} \perp \overline{AB}$

Proof : In the quadrilateral ABCD

$m(\angle B) = 180° - \theta$

In the right-angled triangle

BEC at E : $BC = \sqrt{144 + 25} = 13$ cm.

$\therefore \sin B = \sin(180° - \theta) = \sin \theta = \frac{12}{13}$

31

$\because m(\angle ABE) = m(\angle BFC)$ (alternate angles)

, $\because m(\angle ABE) = 180° - \theta$

$\therefore m(\angle BFC) = 180° - \theta$

in the right-angled triangle BCF at C :

$BF = \sqrt{9 + 4} = \sqrt{13}$ length unit.

$\therefore \csc(\angle BFC) = \csc(180° - \theta) = \csc \theta = \frac{\sqrt{13}}{3}$

32

Karim's answer is correct because

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$

Third **Higher skills**

1

(1) a	(2) d	(3) c	(4) c
(5) d	(6) d	(7) **First :** b	**Second :** a
(8) b	(9) a	**(10)** c	**(11)** b
(12) a	**(13)** b	**(14)** c	**(15)** b

Instructions to solve 1 :

(1) $\because \cos 90° = \text{zero}$

$\therefore \cos 45° \times \cos 46° \times \cos 47° \times ... \times \cos 90° \times ... \times \cos 135° = 0$

(2) $\because \sin 75° = \sin(90° - 15°) = \cos 15°$

$, \cos 12° = \cos(90° - 78°) = \sin 78°$

$\therefore \sin 75° \times \cos 12° \times \sec 15° \times \csc 78°$

$= \cos 15° \times \sin 78° \times \sec 15° \times \csc 78° = 1$

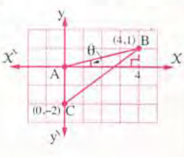
(3) $AB = \sqrt{(4)^2+(1)^2} = \sqrt{17}$

$, \because \sin(\angle BAC)$

$= \sin(90+\theta)$

$= \cos\theta$

$= \frac{4}{\sqrt{17}}$



(4) $\frac{\sec 1° \times \sec 2° \times ... \times \sec 88° \times \sec 89°}{\csc 1° \times \csc 2° \times ... \times \csc 88° \times \csc 89°}$

$= \frac{\sec 1° \times \sec 2° \times ... \times \sec 88° \times \sec 89°}{\csc(90°-1°) \times \csc(90°-88°) \times ... \times \csc(90°-2°) \times \csc(90°-1°)}$

$= \frac{\sec 1° \times \sec 2° \times ... \times \sec 88° \times \sec 89°}{\sec 89° \times \sec 88° \times ... \times \sec 2° \times \sec 1°} = 1$

(5) $\frac{\sin(60\pi+\theta)+\cos(90\pi+\theta)}{\cos\left(\frac{5\pi}{2}+\theta\right)-\sin\left(\frac{9\pi}{2}+\theta\right)}$

$= \frac{\sin\left(30(2\pi)+\theta\right)+\cos\left(45(2\pi)+\theta\right)}{\cos\left(2\pi+\frac{1}{2}\pi+\theta\right)-\sin\left(4\pi+\frac{1}{2}\pi+\theta\right)}$

$= \frac{\sin\theta+\cos\theta}{\cos\left(\frac{1}{2}\pi+\theta\right)-\sin\left(\frac{1}{2}\pi+\theta\right)}$

$= \frac{\sin\theta+\cos\theta}{-\sin\theta-\cos\theta} = \frac{\sin\theta+\cos\theta}{-(\sin\theta+\cos\theta)} = -1$

(6) $\frac{\sin 3X}{\cos 4X} + \frac{\tan 2X}{\cot 5X}$

$= \frac{\sin 3X}{\cos(7X-3X)} + \frac{\tan 2X}{\cot(7X-2X)}$

$= \frac{\sin 3X}{\cos\left(\frac{\pi}{2}-3X\right)} + \frac{\tan 3X}{\cot\left(\frac{\pi}{2}-2X\right)}$

$= \frac{\sin 3X}{\sin 3X} + \frac{\tan 2X}{\tan 2X} = 1+1 = 2$

(7) **First :** $\because X+y=30° \quad \therefore 3X+3y=90°$

$, \because X+2y=(3X+3y)-(2X+y)$

$= 90°-(2X+y)$

$\therefore \tan(X+2y)\tan(2X+y)$

$= \tan\left(90°-(2X+y)\right)\tan(2X+y)$

$= \cot(2X+y)\tan(2X+y) = 1$

Second : $\sin(3X+2y)+\sin(9X+8y)$

$= \sin(3X+3y-y)+\sin(9X+9y-y)$

$= \sin(90°-y)+\sin(270°-y)$

$= \cos y - \cos y = \text{zero}$

(8) $f(\theta)+f\left(\theta+\frac{\pi}{2}\right)+f(\theta+\pi)+f\left(\theta+\frac{3\pi}{2}\right)+...$

$+f(\theta+99\pi)+f\left(\theta+\frac{199}{2}\pi\right)$

$= \sin(2\theta)+\sin(2\theta+\pi)+\sin(2\theta+2\pi)$

$+\sin(2\theta+3\pi)+...+\sin(2\theta+198\pi)$

$+\sin(2\theta+199\pi) = \sin(2\theta)-\sin(2\theta)$

$+\sin(2\theta)-\sin(2\theta)+...+\sin(2\theta)$

$-\sin(2\theta) = \text{zero}$

(9) $\because \cos^2\theta = 1 \qquad \therefore \cos\theta = \pm 1$

$\therefore \cos\theta = 1$

$\therefore \theta = \text{zero or } \pm 2\pi \text{ or } \pm 4\pi \text{ or } ...$

or $\cos\theta = -1$

$\therefore \theta = \pm\pi \text{ or } \pm 3\pi \text{ or } \pm 5\pi \text{ or } ...$

$\therefore \theta = \text{zero or } \pm\pi \text{ or } \pm 2\pi \text{ or } \pm 3\pi \text{ or } ...$

$= n\pi$ where $n \in \mathbb{Z}$

(10) $\tan X = -\sqrt{3}$

$\therefore X$ belongs to the second quadrant or fourth quadrant.

$\therefore$ There is a solution to the equation every half revolution.

$, \because 0 \le X \le 15\pi$ includes 15 half revolution.

$\therefore$ Number of solutions = 15 solutions.

(11) $\because 2\theta = 180° - 2\alpha$

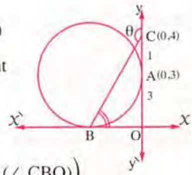
$\tan\theta = \tan\left(\frac{180° - 2\alpha}{2}\right)$

$= \tan(90 - \alpha)$

$= \cot\alpha$

(12) $\because$ OA = OB = 3 cm.

(two tangent segments)



In Δ COB : $\angle$ O is right

$\therefore BC = \sqrt{(3)^2 + (4)^2}$

$= 5$ length units.

, $\cos\theta = \cos\left(90° + m(\angle CBO)\right)$

$= -\sin(\angle CBO) = -\frac{4}{5}$

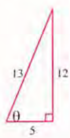
(13) $\because$ ADCB is a cyclic quadrilateral

$\therefore m(\angle ADC) = 180° - \theta$

$\therefore \cos(\angle ADC) = \cos(180° - \theta) = -\cos\theta$

, $\because \overline{AB}$ is a diameter in the semi-circle M

$\therefore \angle\theta$ is an acute angle



$\therefore \sin\theta = \frac{12}{13}$

$\therefore \cos\theta = \frac{5}{13}$

$\therefore \cos(\angle ADC) = -\frac{5}{13}$

(14) $\because$ The equation of the straight line is :

$y = \frac{-3}{4}x + 5$

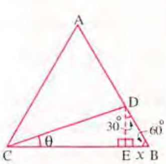
$\therefore \tan(90° + \theta) = \frac{-3}{4}$

$\therefore -\cot\theta = \frac{-3}{4}$

$\therefore \cot\theta = \frac{3}{4}$

$\therefore \tan\theta = \frac{4}{3}$

(15) In Δ DBE :



$m(\angle BED) = 90°$

$\therefore DE = \sqrt{(2x)^2 - x^2}$

$= \sqrt{3}\,x$

$\therefore \tan\theta = \frac{DE}{EC} = \frac{\sqrt{3}\,x}{4x} = \frac{\sqrt{3}}{4}$

2

(1) $\because \cos(180° - x) = -\cos x$

$\therefore \cos 160° = -\cos 20°$

, $\cos 140° = -\cos 40°$ ... etc

$\therefore$ The expression $= (\cos 20° + \cos 160°)$
$+ (\cos 40° + \cos 140°) + (\cos 60° + \cos 120°)$
$+ (\cos 80° + \cos 100°) + \cos 180°$
$= (\cos 20° - \cos 20°) + (\cos 40° - \cos 40°)$
$+ (\cos 60° - \cos 60°) + (\cos 80° - \cos 80°)$
$+ \cos 180° = 0 + 0 + 0 + 0 + (-1) = -1$

(2) $\because \sin(360° - x) = -\sin x$

$\therefore \sin 359° = -\sin 1°$

, $\sin 358° = -\sin 2°$ ... etc.

$\therefore$ The expression $= (\sin 1° + \sin 359°)$
$+ (\sin 2° + \sin 358°) + \ldots\ldots + \sin 180°$
$= 0 + 0 + \ldots + 0 = 0$

Answers of Exercise 11

First Multiple choice questions

(1) b	(2) b	(3) a	(4) b
(5) a	(6) d	(7) c	(8) c
(9) a	(10) a	(11) c	(12) d
(13) c	(14) b	(15) b	(16) b
(17) c	(18) d	(19) c	

Second Essay questions

1

	max. value	min. value	the range
(1)	$\frac{1}{2}$	$-\frac{1}{2}$	$\left[-\frac{1}{2}, \frac{1}{2}\right]$
(2)	$\frac{1}{3}$	$\frac{-1}{3}$	$\left[\frac{-1}{3}, \frac{1}{3}\right]$
(3)	2	-2	$[-2, 2]$

2 Form the table and draw by yourself , from the graph we get :

	min. value	max. value	the range
(1)	-4	4	$[-4, 4]$
(2)	-4	4	$[-4, 4]$
(3)	-2	2	$[-2, 2]$
(4)	-3	3	$[-3, 3]$

3

(1) $\because 0° \le \theta \le 120°$ $\quad\therefore 0° \le 3\theta \le 360°$

By giving to 3θ some values to some special angles :

$0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \frac{4\pi}{6}, ..., 2\pi$

$\therefore \theta = 0, \frac{\pi}{18}, \frac{2\pi}{18}, \frac{3\pi}{18}, \frac{4\pi}{18}, ..., \frac{12\pi}{18}$

$\because y = \cos 3\theta$

form the table , then draw the graph by yourself , from the graph we get the max. value = 1 , the min. value – 1 and the range = $[-1, 1]$

(2) $\because 0° \le \theta \le 180°$ $\quad\therefore 0° \le 2\theta \le 360°$

By giving 2θ some values to some special angles : $0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, ..., 2\pi$

$\therefore \theta = 0, \frac{\pi}{12}, \frac{2\pi}{12}, \frac{3\pi}{12}, ..., \pi$

$\because y = 5 \sin 2\theta$

form the table , then draw the graph by yourself , from the graph we get the min. value = – 5 the max. value = 5 and the range = $[-5, 5]$

4

Draw by yourself , from the graph we get the range of $y = 4\cos\theta$ is $[-4, 4]$

, the max. value = 4 , the min. value = – 4

, the range of the function : $y = 3\sin\theta$ is $[-3, 3]$

, the max. value = 3 , the min. value = – 3

Third Higher skills

(1) a	(2) c	(3) c	(4) b	(5) d
(6) d	(7) b	(8) d	(9) c	(10) d
(11) d	(12) b			

Instructions to solve :

(1) $\because -1 \le \sin X \le 1$ $\quad\therefore 1 \ge -\sin X \ge -1$

$\therefore -1 \le -\sin X \le 1$ $\quad\therefore 1 \le 2 - \sin X \le 3$

$\therefore \frac{1}{3} \le \frac{2-\sin X}{3} \le 1$ $\quad\therefore \frac{1}{3} \le m \le 1$

(2) The maximum value of the function y is 1

$\therefore \sin\left(\frac{\pi}{4} + X\right) = 1$ $\quad\therefore \frac{\pi}{4} + X = \frac{\pi}{2}$

$\therefore X = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

(3) The period of the function $f(X) = \sin(bX)$ is $\frac{2\pi}{b}$

$\therefore \frac{2\pi}{b} = \frac{2\pi}{3}$ $\quad\therefore b = 3$

(4) The greatest value of the expression $(\cos X_1 - \cos X_2)$

When $\cos X_1 = 1$ and $\cos X_2 = -1$

$\therefore \cos X_1 - \cos X_2 = 1 - (-1) = 2$

(5) $\because f(X) = a\cos bX$ and its period $\frac{2\pi}{b} = \frac{\pi}{2}$

$\therefore b = 4$

, its range $[-1, 1]$ $\quad\therefore a = 1$

$\therefore \frac{a}{b} = \frac{1}{4}$

(6) $\because f(X) = a\cos bX$

$\therefore$ Its period $\frac{2\pi}{b} = \pi$ $\quad\therefore b = 2$

, its range $[-3, 3]$ $\quad\therefore a = 3$

$\therefore a + b = 5$

(7) $a = \sin\frac{\pi}{2} = 1$, $b = \sin\left(\frac{3\pi}{2}\right) = -1$

$\therefore |a| + |b| = |1| + |-1| = 2$

(8) $\because$ The period of the function is : $\frac{2\pi}{\left(\frac{1}{2}\right)} = 4\pi$

, $\because$ from O to B is a whole period

$\therefore$ The X coordinate of B is 4π

(9) $\because B = 2\pi$, $A = -\pi$

$\therefore B - A = 2\pi - (-\pi) = 3\pi$

(10) The number of points which the curve $y = \sin 3X$ intersects X-axis = 2 × number of periods + 1

, $\because$ the function $y = \sin 3$ has period every $\frac{2\pi}{3}$

$\therefore$ Number of periods in the interval $[0, 2\pi] = 2\pi \div \frac{2\pi}{3} = 3$

$\therefore$ Number of intersecting points = $2 \times 3 + 1 = 7$

(11) $\because$ The curve $y = \sin(aX)$

$\therefore$ It makes a complete period for each $\frac{2\pi}{a}$

$\therefore$ The number of the complete period in the interval $[0, 2\pi]$ is a

$\therefore 9 = 2 \times a + 1$ $\quad\therefore a = 4$

(12) $\because$ The curve $f(X) = \sin 2X + 1$ makes a complete period for each $\frac{2\pi}{2} = \pi$

$\therefore$ The number of complete periods in the interval $[0, 2\pi]$ is 2

$\therefore$ The number of required times = 2

Answers of Exercise 12

First Multiple choice questions

(1) a	(2) c	(3) c	(4) c
(5) c	(6) a	(7) b	(8) a
(9) d	(10) a	(11) a	(12) c
(13) a	(14) a	(15) c	(16) b
(17) b	(18) d	(19) c	

Second Essay questions

1

(1) $36^\circ\ 52'\ 12''$ (2) $38^\circ\ 8'\ 25''$

(3) $67^\circ\ 51'\ 34''$

(4) $\because \theta = \tan^{-1}(-0.8227)$, -0.8227 negative.

$\therefore \theta$ lies in 2^{nd} or 4^{th} quad.

$\therefore \theta = 180^\circ - (39^\circ\ 26'\ 39'') = 140^\circ\ 33'\ 21''$

(5) $\because \theta = \sin^{-1}(-0.4652)$, -0.4652 (negative)

$\therefore \theta$ lies in 3^{rd} or 4^{th} quad.

$\therefore \theta = 180^\circ + (27^\circ\ 43'\ 23'') = 207^\circ\ 43'\ 23''$

(6) $\because \theta = \cos^{-1}(-0.5206)$, -0.5206 (negative)

$\therefore \theta$ lies in 2^{nd} or 3^{rd} quad.

$\therefore \theta = 180^\circ - (58^\circ\ 37'\ 39'') = 121^\circ\ 22'\ 21''$

(7) $15^\circ\ 26'\ 7''$

(8) $\because \theta = \cot^{-1}(-1.4612)$, -1.4612 (negative)

$\therefore \theta$ lies in 2^{nd} or 4^{th} quad.

$\therefore \theta = 180^\circ - (34^\circ\ 23'\ 12'') = 145^\circ\ 36'\ 48''$

(9) $17^\circ\ 22'\ 23''$

(10) $\because \theta = \csc^{-1}(-2.5466)$, -2.5466 (negative)

$\therefore \theta$ lies in 3^{rd} or 4^{th} quad.

$\therefore \theta = 180^\circ + (23^\circ\ 7'\ 17'') = 203^\circ\ 7'\ 17''$

(11) $\because \theta = \sec^{-1}(-3.57)$, -3.57 (negative)

$\therefore \theta$ lies in 2^{nd} or 3^{rd} quad.

$\therefore \theta = 180^\circ - (73^\circ\ 43'\ 59'') = 106^\circ\ 16'\ 1''$

(12) $19^\circ\ 35'\ 59''$

2

(1) $\because \theta = \sin^{-1} 0.86603$, 0.86603 (positive)

$\therefore \theta$ lies in 1^{st} or 2^{nd} quad.

$\therefore \theta = 60^\circ\ 0'\ 2''$ or $\theta = 180^\circ - 60^\circ\ 0'\ 2'' = 119^\circ\ 59'\ 58''$

(2) $\because \theta = \cos^{-1}(-0.4752)$, -0.4752 (negative)

$\therefore \theta$ lies in 2^{nd} or 3^{rd} quad.

$\therefore \theta = 180^\circ - (61^\circ\ 37'\ 39'') = 118^\circ\ 22'\ 21''$

or $\theta = 180^\circ + (61^\circ\ 37'\ 39'') = 241^\circ\ 37'\ 39''$

(3) $\because \theta = \csc^{-1}(-1.2576)$, -1.2576 (negative)

$\therefore \theta$ lies in 3^{rd} or 4^{th} quad.

$\therefore \theta = 180^\circ + (52^\circ\ 40'\ 15'') = 232^\circ\ 40'\ 15''$

or $\theta = 360^\circ - (52^\circ\ 40'\ 15'') = 307^\circ\ 19'\ 45''$

(4) $\because \theta = \tan^{-1} 1.5417$, 1.5417 (positive)

$\therefore \theta$ lies in 1^{st} or 3^{rd} quad. $\therefore \theta = 57^\circ\ 1'\ 52''$

or $\theta = 180^\circ + (57^\circ\ 1'\ 52'') = 237^\circ\ 1'\ 52''$

(5) $\because \theta = \cos^{-1}(-0.642)$, -0.642 (negative)

$\therefore \theta$ lies in 2^{nd} or 3^{rd} quad.

$\therefore \theta = 180^\circ - (50^\circ\ 3'\ 32'') = 129^\circ\ 56'\ 28''$

or $\theta = 180^\circ + (50^\circ\ 3'\ 32'') = 230^\circ\ 3'\ 32''$

(6) $\because \theta = \sec^{-1}(2.0515)$, 2.0515 (positive)

$\therefore \theta$ lies in 1^{st} or 4^{th} quad.

$\therefore \theta = 60^\circ\ 49'\ 37''$

or $\theta = 360^\circ - (60^\circ\ 49'\ 37'') = 299^\circ\ 10'\ 23''$

(7) $\because \theta = \csc^{-1}(-1.8715)$, -1.8715 (negative)

$\therefore \theta$ lies in 3^{rd} or 4^{th} quad.

$\therefore \theta = 180^\circ + (32^\circ\ 17'\ 55'') = 212^\circ\ 17'\ 55''$

or $\theta = 360^\circ - (32^\circ\ 17'\ 55'') = 327^\circ\ 42'\ 5''$

(8) $\because \theta = \cot^{-1}(-2.7012)$, -2.7012 (negative)

$\therefore \theta$ lies in 2^{nd} or 4^{th} quad.

$\therefore \theta = 180^\circ - (20^\circ\ 18'\ 53'') = 159^\circ\ 41'\ 7''$

or $\theta = 360^\circ - (20^\circ\ 18'\ 53'') = 339^\circ\ 41'\ 7''$

(9) $\because \theta = \tan^{-1}(-2.1456)$, -2.1456 (negative)

$\therefore \theta$ lies in 2^{nd} or 4^{th} quad.

$\therefore \theta = 180^\circ - (65^\circ\ 0'\ 40'') = 114^\circ\ 59'\ 20''$

or $\theta = 360^\circ - (65^\circ\ 0'\ 40'') = 294^\circ\ 59'\ 20''$

3

(1) $\because B\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ lies in 1^{st} quad.

$\therefore \theta$ lies in the 1^{st} quad.

$\because \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$ $\therefore \theta = 30^\circ$

(2) $\because B\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ lies in 2^{nd} quad.

$\therefore \theta$ lies in 2^{nd} quad.

$\because \sin^{-1}\frac{1}{\sqrt{2}} = 45^\circ$ $\therefore \theta = 180^\circ - 45^\circ = 135^\circ$

(3) $\because B\left(\frac{6}{10}, \frac{-8}{10}\right)$ lies in the 4th quad.

$\therefore \theta$ lies in the 4th quad.

$\because \cos^{-1}\frac{6}{10} \simeq 53^\circ\, 7'\, 48''$

$\therefore \theta = 360^\circ - (53^\circ\, 7'\, 48'') = 306^\circ\, 52'\, 12''$

4

(1) $\because \tan\theta = \frac{8}{5}$ $\quad\therefore \theta = \tan^{-1}\frac{8}{5}$

$\therefore \theta \simeq 57^\circ\, 59'\, 41''$

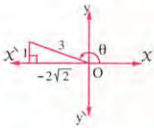
(2) $\because \cos\theta = \frac{7}{9}$ $\quad\therefore \theta = \cos^{-1}\frac{7}{9}$

$\therefore \theta \simeq 38^\circ\, 56'\, 33''$

(3) $\because \sin\theta = \frac{4}{9}$ $\quad\therefore \theta = \sin^{-1}\frac{4}{9}$

$\therefore \theta \simeq 26^\circ\, 23'\, 16''$

5



(1) $\because 90^\circ \le \theta \le 180^\circ$

$\therefore \theta$ lies in 2nd quad.

$\because \sin\theta = \frac{1}{3}$

$\therefore \theta = \sin^{-1}\left(\frac{1}{3}\right)$

$\therefore \theta = 180^\circ - (19^\circ\, 28'\, 16'') = 160^\circ\, 31'\, 44''$

(2) $\cos\theta = \frac{-2\sqrt{2}}{3}$, $\tan\theta = \frac{-1}{2\sqrt{2}}$

, $\sec\theta = \frac{-3}{2\sqrt{2}}$

6

$\because \cos A = -0.5807$ $\quad\therefore m(\angle A) = 125^\circ\, 30'$

$\because \tan B = 0.4578$ $\quad\therefore m(\angle B) = 24^\circ\, 36'$

$\therefore m(\angle A) + m(\angle B) = 150^\circ\, 6'$

$\therefore m(\angle C) = 180^\circ - 150^\circ\, 6' = 29^\circ\, 54'$

7

$\because \tan\theta = 0.499$ (positive)

$\therefore \theta$ lies in 1st or 3rd quad.

$\because \theta = \tan^{-1} 0.499$ $\quad\therefore \theta = 26^\circ\, 31'$

or $\theta = 180^\circ + 26^\circ\, 31' = 206^\circ\, 31'$

8

$\because \cos\theta = -0.3564$ (negative)

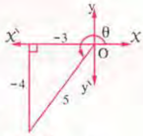
$\therefore \theta$ lies in 2nd or 3rd quad

Let $\cos\theta = 0.3564$ $\quad\therefore \theta = 69^\circ\, 7'$

$\therefore \theta = 180^\circ - 69^\circ\, 7' = 110^\circ\, 53'$

or $\theta = 180^\circ + 69^\circ\, 7' = 249^\circ\, 7'$

9



$\because \tan\theta = \frac{4}{3}$ positive

$\therefore \theta$ lies in 1st quad or 3rd quad.

$\because \theta$ is the greatest positive angle

, $\theta \in]0^\circ, 360^\circ[$ $\quad\therefore \theta$ lies in 3rd quad.

$\therefore \sin\alpha = \sin(180^\circ - 30^\circ)(-\sin\theta) + \frac{1}{5}(-\csc\theta)$

$\times \tan(180^\circ + 45^\circ) = \sin 30^\circ\left(\frac{4}{5}\right) + \frac{1}{5}\left(\frac{5}{4}\right)\tan 45^\circ$

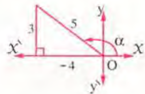
$= \frac{1}{2} \times \frac{4}{5} + \frac{1}{4} = \frac{8+5}{20} = \frac{13}{20}$

$\therefore \sin\alpha = \frac{13}{20}$ (positive) $\quad\therefore \alpha$ lies in 1st or 2nd quad.

$\therefore \alpha = \sin^{-1}\frac{13}{20}$ $\quad\therefore \alpha = 40^\circ\, 32'$

or $\alpha = 180^\circ - 40^\circ\, 32' = 139^\circ\, 28'$

10



$\sin\alpha = \frac{3}{5}$

$\because 90^\circ < \alpha < 180^\circ$

$\therefore \alpha$ lies in the 2nd quad.

$\because \frac{-5}{4}\cos(360^\circ - \alpha) + \cot(270^\circ - \theta) = 2$

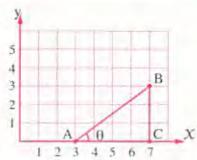
$\therefore \frac{-5}{4}\cos\alpha + \tan\theta = 2$

$-\frac{5}{4} \times \frac{-4}{5} + \tan\theta = 2$ $\quad\therefore 1 + \tan\theta = 2$

$\therefore \tan\theta = 1$ (positive) $\quad\therefore \theta$ lies in 1st or 3rd quad.

$\because \tan 45^\circ = 1$ $\quad\therefore \theta = 45^\circ$ or $\theta = 180^\circ + 45^\circ = 225^\circ$

11



From the graph :

AC = 4 unit length.

BC = 3 unit length.

$\therefore \tan\theta = \frac{3}{4}$

$\therefore \theta \simeq 36^\circ\, 52'\, 12''$

12

Karim's answer is the right because

$\csc\theta = \frac{13}{7}$ or $\sec\theta \ne \frac{13}{7}$

Third **Higher skills**

(1) c	(2) b	(3) a	(4) b
(5) b	(6) a	(7) c	

Instructions to solve :

(1) $\tan(\angle ABC) = \frac{3}{4}$

$\therefore m(\angle ABC) = \tan^{-1}\left(\frac{3}{4}\right)$

(2) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$ $\quad \therefore \sin(30°) = \frac{1}{2}$

(3) $\cos^{-1}(\text{zero}) = \frac{\pi}{2}$

$\therefore \csc\left(\frac{\pi}{2}\right) = \frac{1}{\sin\left(\frac{\pi}{2}\right)} = 1$

(4) In ΔABC :

$m(\angle B) = 90°$

$\therefore AC = \sqrt{(5)^2 + (12)^2} = 13$ cm.

$\tan^{-1}\left(\frac{5}{12}\right) = m(\angle ACB)$

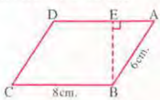
$\therefore \sin(\angle ACB) = \frac{5}{13}$

(5) $\because$ The area of parallelogram ABCD = 40 cm^2

$\therefore BE = \frac{40}{8} = 5$ cm.

$\therefore \sin A = \frac{5}{6}$

$\therefore m(\angle A) \simeq 56°$



(6) $\because \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$ $\quad \therefore \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

, $\because \cot\left(\frac{\pi}{6}\right) = \sqrt{3}$ $\quad \therefore \cot^{-1}\left(\sqrt{3}\right) = \frac{\pi}{6}$

, $\because \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\sqrt{3}\right) = \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

(7) Let $\cos^{-1} x = \alpha$ $\quad \therefore \cos\alpha = x$

, let $\sin^{-1} x = B$ $\quad \therefore \sin B = x$

$\therefore \cos\alpha = \sin B$

$\therefore \alpha + B = \frac{\pi}{2}$ $\quad \therefore \cos^{-1} x + \sin^{-1} x = \frac{\pi}{2}$

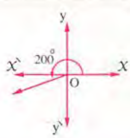
Answers of Life Applications on Unit Two

1

The radian measure

$= 200° \times \frac{\pi}{180°}$

$\simeq 3.49^{rad}$



2

The measure of the angle which the hand made after 10 minutes = 60°

$\therefore$ The covered distance by the point

$= 60° \times \frac{\pi}{180°} \times 6 = 2\pi$ cm.

3

The distance covered during one revolution

$= 2\pi \times 9000 \simeq 56548.67$ km.

$\therefore$ The speed of the satellite $= \frac{56548.67}{6}$

$= 9424.78$ km/hour.

4

The radius length of the circle of the satellite path

= 6400 + 3600 = 10000 km.

$\therefore$ The distance covered during one revolution

$= 2\pi \times 10000 \simeq 62831.85$

$\therefore$ The distance covered during one hour

$= \frac{62831.85}{3} \simeq 20944$ km.

5

(1) The measure of the angle which the shadow rotates after 4 hours

$= 15° \times 4 \times \frac{\pi}{180°} \simeq 1.05^{rad}$

(2) The degree measure of the angle

$= \frac{2\pi}{3} \times \frac{180°}{\pi} = 120°$

$\therefore$ The number of hours = 120° ÷ 15° = 8 hours.

(3) The radian measure of the angle which is made by the shadow after 10 hours

$= 15° \times 10 \times \frac{\pi}{180°} = \frac{5\pi}{6}$

$\therefore$ The length of the arc $= \frac{5\pi}{6} \times 24 = 20\pi$ cm.

6

$\because \sin\theta_1 = k \sin\theta_2$

$\therefore \sin\theta_2 = \frac{\sin 60°}{\sqrt{3}} = \frac{1}{2}$ $\qquad \therefore \theta_2 = 30°$

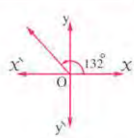
7

(1)The related angle

= 180° − 132°

= 48°

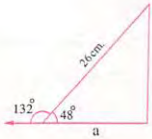
(There are other solutions)



(2) $\cos 48° = \frac{a}{26}$

$\therefore a = 26 \cos 48°$

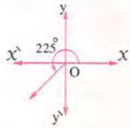
$\simeq 17$ cm.



8

(1) $\frac{5\pi}{4} = \frac{5 \times 180°}{4}$

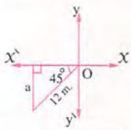
$= 225°$



(2) $\sin 45° = \frac{a}{12}$

$\therefore a = 12 \sin 45°$

$\simeq 8.49$ m



9

At : S = 10 $\quad \therefore 10 = 6 \sin (15\,n)° + 10$

$\therefore 6 \sin (15\,n)° = 0 \quad \therefore \sin (15\,n)° = 0$

$\therefore 15\,n = 0 \quad \therefore n = 0$

or $15\,n = 180 \quad \therefore n = 12$

or $15\,n = 360 \quad \therefore n = 24$

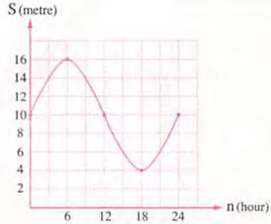
or $15\,n = 540 \quad \therefore n = 36$ (refused)

$\therefore$ The depth of water = 10 m.

at n = 0 , 12 , 24 hours.

$S = 6 \sin (15\,n) + 10$

n in hour	0	6	12	18	24
s in metre	10	16	10	4	10



The ship enters the port at $n \in [0 , 12]$

$\therefore$ Number of hours = 12 hr.

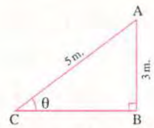
10

$\because \sin \theta = \frac{AB}{AC} = \frac{3}{5}$

$\therefore \theta = \sin^{-1} \frac{3}{5}$

$\therefore \theta \simeq 36° \, 52' \, 12''$

$\therefore \theta^{rad} = 36° \, 52' \, 12'' \times \frac{\pi}{180°} \simeq 0.644^{rad}$

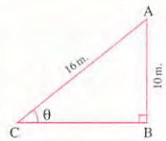


11

$\because \sin \theta = \frac{AB}{AC} = \frac{10}{16} = \frac{5}{8}$

$\therefore \theta = \sin^{-1} \frac{5}{8}$

$\therefore \theta \simeq 38.682°$

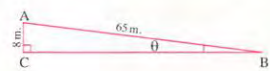


12

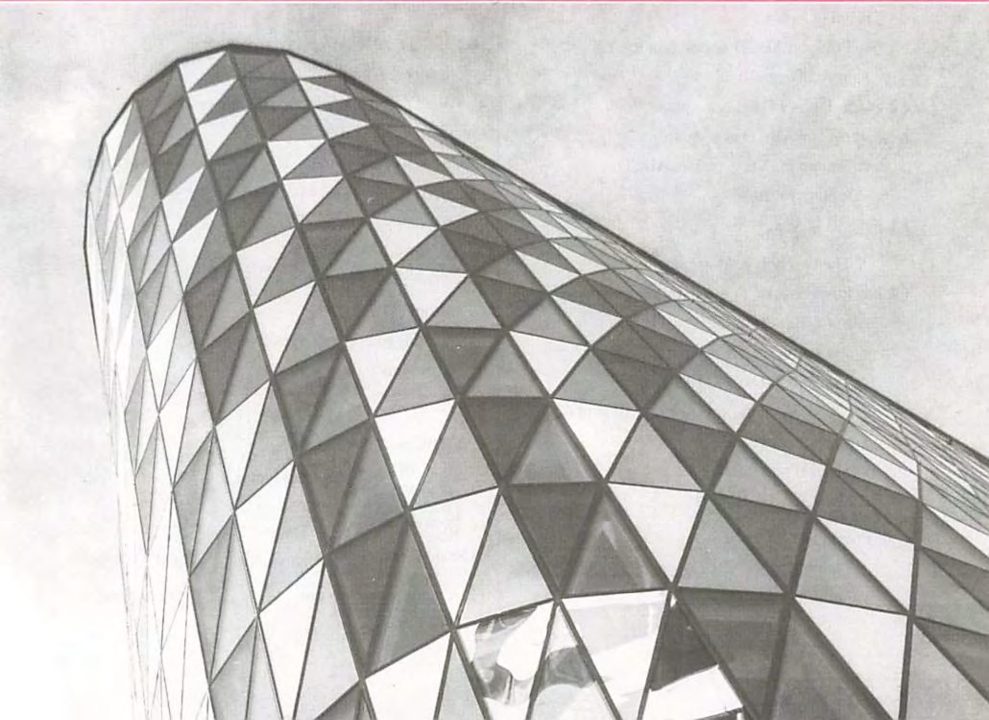
$\because \sin \theta = \frac{AC}{AB}$

$= \frac{8}{65}$

$\therefore \theta = \sin^{-1} \frac{8}{65} \qquad \therefore \theta \simeq 7° \, 4' \, 11''$



Second Geometry



Guide Answers of "Unit Three"

Answers of Exercise 1

First Multiple choice questions

(1) c	(2) b	(3) b	(4) b
(5) d	(6) c	(7) c	(8) c
(9) a	(10) c	(11) c	(12) c
(13) d	(14) a	(15) a	(16) a
(17) b	(18) d	(19) a	(20) c
(21) d	(22) b	(23) c	(24) c
(25) b	(26) a	(27) a	(28) c

Second Essay questions

1

(1) $\because$ m ($\angle$ B) = m ($\angle$ X)

, m ($\angle$ C) = m ($\angle$ Y) , m ($\angle$ D) = m ($\angle$ Z)

$\therefore$ m ($\angle$ A) = m ($\angle$ L) (1)

, $\because \frac{AB}{LX} = \frac{BC}{XY} = \frac{CD}{YZ} = \frac{AD}{LZ} = \frac{5}{4}$ (2)

From (1) , (2) :

$\therefore$ Polygon ABCD ~ polygon LXYZ

, similarity ratio = $\frac{5}{4}$

(2) $\because$ Polygon FGXE is a square

, polygon ABCD is a square.

$\therefore$ Square FGXE ~ square ABCD

, similarity ratio = $\frac{8}{5}$

(3) $\because \frac{AB}{XY} \neq \frac{BC}{YZ}$

$\therefore$ The two polygons are not similar

(4) $\because$ Polygon ABCD is a parallelogram

$\therefore$ m ($\angle$ B) = 180° – 70° = 110°

, $\because$ polygon GFEX is a parallelogram

$\therefore$ m ($\angle$ G) = 180° – 110° = 70°

, $\because$ m ($\angle$ A) = m ($\angle$ G) , m ($\angle$ B) = m ($\angle$ F)

, m ($\angle$ C) = m ($\angle$ E)

, m ($\angle$ D) = m ($\angle$ X) (1)

, $\frac{AB}{GF} = \frac{BC}{FE} = \frac{CD}{EX} = \frac{AD}{GX} = \frac{3}{4}$ (2)

From (1) , (2) :

$\therefore$ Parallelogram ABCD ~ parallelogam GFEX

, similarity ratio = $\frac{3}{4}$

(5) $\because$ Polygon XYZL is a rectangle

$\therefore$ XY = LZ = 30 cm.

, $\because \frac{AB}{XY} \neq \frac{BC}{YZ}$

$\therefore$ The two polygons are not similar

(6) $\because$ Polygon ABCD is a rhombus.

$\therefore$ m ($\angle$ A) = m ($\angle$ C) = $\frac{360° - 140°}{2}$ = 110°

, $\because$ polygon YXLZ is a rhombus

$\therefore$ m ($\angle$ X) = m ($\angle$ Z) = $\frac{360° - 220°}{2}$ = 70°

, $\because$ m ($\angle$ A) = m ($\angle$ Y) , m ($\angle$ B) = m ($\angle$ X)

, m ($\angle$ C) = m ($\angle$ L) , m ($\angle$ D) = m ($\angle$ Z) (1)

, $\frac{AB}{YX} = \frac{BC}{XL} = \frac{CD}{LZ} = \frac{AD}{YZ} = \frac{10}{7}$ (2)

From (1) , (2) :

$\therefore$ Rhombus ABCD ~ rhombus YXLZ

, similarity ratio = $\frac{10}{7}$

(7) $\because$ m ($\angle$ B) = m ($\angle$ D) , m ($\angle$ C) = m ($\angle$ F)

$\therefore$ m ($\angle$ A) = m ($\angle$ E) (1)

, $\because \frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF} = \frac{7}{12}$ (2)

From (1) , (2) : $\therefore \Delta$ ABC ~ Δ EDF

, similarity ratio = $\frac{7}{12}$

(8) $\because \overline{AD} \parallel \overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ A) = 180° – m ($\angle$ B)

, $\because \overline{YZ} \parallel \overline{XL}$, $\overleftrightarrow{LZ}$ is a transversal

$\therefore$ m ($\angle$ Z) = 180° – m ($\angle$ L)

, $\because$ m ($\angle$ B) = m ($\angle$ L) $\therefore$ m ($\angle$ A) = m ($\angle$ Z)

, $\because$ m ($\angle$ A) = m ($\angle$ Z) , m ($\angle$ B) = m ($\angle$ L)

, m ($\angle$ C) = m ($\angle$ X)

$\therefore$ m ($\angle$ D) = m ($\angle$ Y) (1)

, $\because \frac{AB}{ZL} = \frac{BC}{LX} = \frac{CD}{XY} = \frac{AD}{ZY} = \frac{5}{4}$ (2)

From (1) , (2) :

$\therefore$ Polygon ABCD ~ polygon ZLXY

, similarity ratio = $\frac{5}{4}$

2

$\because \Delta$ ABC ~ Δ NML

$\therefore \frac{AB}{NM} = \frac{BC}{ML} = \frac{AC}{NL}$ = scale factor

$\therefore \frac{15}{10} = \frac{12}{x} = \frac{14}{y}$

$\therefore$ Scale factor = $\frac{15}{10} = \frac{3}{2}$ (First req.)

, x = 8 cm. , y = $9\frac{1}{3}$ cm. (Second req.)

3

$\because$ Polygon ABCD $\sim$ polygon EFGH

$\therefore \frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{EH}$ = scale factor

$\therefore \frac{(y+2)}{6} = \frac{BC}{FG} = \frac{15}{x} = \frac{12}{8}$

$\therefore$ Scale factor $= \frac{12}{8} = \frac{3}{2}$ (First req.)

, $x = 10$ cm. , $y + 2 = 9$

$\therefore y = 7$ cm. (Second req.)

4

$\because \Delta ADE \sim \Delta ABC$

$\therefore m(\angle ADE) = m(\angle B)$ and they are corresponding angles.

$\therefore \overline{DE} \parallel \overline{BC}$ (First req.)

$\because \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ $\therefore \frac{6}{AB} = \frac{4}{12} = \frac{5}{AC}$

$\therefore AB = 18$ cm. $\therefore BD = 12$ cm.

, $AC = 15$ cm. $\therefore CE = 10$ cm. (Second req.)

5

$\because \Delta ABC \sim \Delta DEF$

$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta DEF}$

$\therefore \frac{AB}{8} = \frac{BC}{9} = \frac{AC}{10} = \frac{81}{27}$

$\therefore AB = 24$ cm. , $BC = 27$ cm.

, $AC = 30$ cm. (The req.)

6

Let the two dimensions of the second rectangle be x cm. and y cm.

$\because$ The two rectangles are similar.

$\therefore \frac{8}{x} = \frac{12}{y} = \frac{40}{200}$

$\therefore x = 40$ cm. , $y = 60$ cm.

$\therefore$ Area of second rectangle $= 40 \times 60 = 2400$ cm^2.

(The req.)

7

$\because$ Polygon ABCD $\sim$ polygon XYZL

$\therefore m(\angle A) = m(\angle X) = 115°$

$\therefore m(\angle XLZ) = 360° - (115° + 85° + 70°) = 90°$

, $\because$ polygon ABCD $\sim$ polygon XYZL

$\therefore \frac{AD}{XL} = \frac{BC}{YZ} = \frac{\text{perimeter of polygon ABCD}}{\text{perimeter of polygon XYZL}}$

$\therefore \frac{AD}{4.8} = \frac{6}{8} = \frac{19.5}{\text{perimeter of polygon XYZL}}$

$\therefore AD = 3.6$ cm. (First req.)

, perimeter of polygon XYZL = 26 cm. (Second req.)

8

(1) XY (2) CD

(3) AD (4) XYZL , ABCD

9

$\because \Delta MAB \sim \Delta MDC$

$\therefore m(\angle A) = m(\angle D)$ and they are alternate angles.

$\therefore \overline{AB} \parallel \overline{DC}$ (First req.)

$\because \Delta MAB \sim \Delta MDC$ $\therefore \frac{MA}{MD} = \frac{MB}{MC} = \frac{5}{3}$

$\therefore MA = 5k$, $MD = 3k$

, $\because AD = MA + MD$, $AD = 6$ cm.

$\therefore 5k + 3k = 6$ $\therefore 8k = 6$

$\therefore k = \frac{3}{4}$

$\therefore AM = \frac{5 \times 3}{4} = 3\frac{3}{4}$ cm. (Second req.)

10

$\because \Delta MAB \sim \Delta MCD$

$\therefore m(\angle A) = m(\angle C)$ (They are drawn on $\overline{BD}$ and on the same side of it)

$\therefore$ The figure ABDC is a cyclic quadrilateral

(First req.)

, $\because \Delta MAB \sim \Delta MCD$

$\therefore \frac{MA}{MC} = \frac{AB}{CD} = \frac{MB}{MD}$ $\therefore \frac{4.8}{MC} = \frac{8}{4} = \frac{MB}{2.5}$

$\therefore MC = 2.4$ cm. , $MB = 5$ cm.

$\therefore BC = 2.4 + 5 = 7.4$ cm. (Second req.)

11

(1) Notice that the required triangle is an enlargement of ΔABC and let $\Delta \grave{A}\grave{B}\grave{C} \sim \Delta ABC$

$\therefore \frac{\grave{A}\grave{B}}{AB} = \frac{\grave{B}\grave{C}}{BC} = \frac{\grave{A}\grave{C}}{AC}$ = scale factor

$\therefore \frac{\grave{A}\grave{B}}{5} = \frac{\grave{B}\grave{C}}{6} = \frac{\grave{A}\grave{C}}{9} = 2.5$

$\therefore \grave{A}\grave{B} = 12.5$ cm. , $\grave{B}\grave{C} = 15$ cm.

, $\grave{A}\grave{C} = 22.5$ cm. (The req.)

(2) Notice that the required triangle is a shrinking of ΔABC and let $\Delta A'B'C' \sim \Delta ABC$

$\therefore \frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$ = scale factor.

$\therefore \frac{A'B'}{5} = \frac{B'C'}{6} = \frac{A'C'}{9} = 0.6$

$\therefore A'B' = 3$ cm. , $B'C' = 3.6$ cm.

, $A'C' = 5.4$ cm. (The req.)

12

(1) Notice that the required rectangle is an enlargement of the given rectangle and let rectangle $A'B'C'D' \sim$ rectangle ABCD

$\therefore \frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{\text{perimeter of rectangle } A'B'C'D'}{\text{perimeter of rectangle } ABCD}$ = scale factor.

$\therefore \frac{A'B'}{10} = \frac{B'C'}{6} = \frac{\text{perimeter of rectangle } A'B'C'D'}{32} = 3$

$\therefore A'B' = 30$ cm. , $B'C' = 18$ cm.

, perimeter of rectangle $A'B'C'D' = 96$ cm.

, area of rectangle $A'B'C'D' = 30 \times 18 = 540$ cm.^{2}

(The req.)

(2) Notice that the required rectangle is a shrinking of the given rectangle and let rectangle $A'B'C'D'$ $\sim$ rectangle ABCD

$\therefore \frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{\text{perimeter of rectangle } A'B'C'D'}{\text{perimeter of rectangle } ABCD}$ = scale factor.

$\therefore \frac{A'B'}{10} = \frac{B'C'}{6} = \frac{\text{perimeter of rectangle } A'B'C'D'}{32} = 0.4$

$\therefore A'B' = 4$ cm. , $B'C' = 2.4$ cm.

, perimeter of rectangle $A'B'C'D' = 12.8$ cm.

, area of rectangle $A'B'C'D' = 4 \times 2.4 = 9.6$ cm.^{2}

(The req.)

13

$\because \Delta ABC \sim \Delta DBA$

$\therefore m(\angle C) = m(\angle DAB)$

$\therefore \overline{AB}$ is a tangent to the circle passing through the vertices of ΔADC (First req.)

$\because \Delta ABC \sim \Delta DBA$ $\therefore \frac{AB}{DB} = \frac{BC}{BA}$

$\therefore (AB)^2 = DB \times BC$

$\therefore$ AB is a mean proportional between BD and BC

(Second req.)

$\because \Delta ABC \sim \Delta DBA$ $\therefore \frac{AB}{DB} = \frac{BC}{BA} = \frac{AC}{DA}$

$\therefore \frac{6}{DB} = \frac{9}{6} = \frac{7.5}{DA}$

$\therefore DA = 5$ cm. , $DB = 4$ cm.

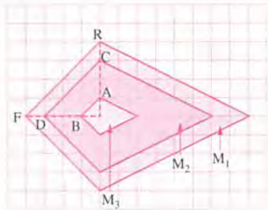
$\therefore CD = 9 - 4 = 5$ cm. (Third req.)

14

Let side length of square of net = unit length.

$\therefore$ length of diagonal of square $= \sqrt{2}$ unit length.

(1)



From Pythagoras :

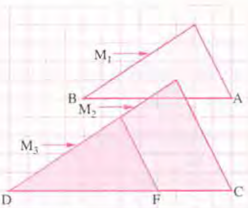
$\therefore AB = \sqrt{2}$ unit length , $CD = 3\sqrt{2}$ unit length

, $FR = 4\sqrt{2}$ unit length.

$\therefore$ The scale factor of similarity of polygon M_1 to polygon $M_3 = \frac{FR}{AB} = \frac{4\sqrt{2}}{\sqrt{2}} = 4$

, the scale factor of similarity of polygon M_2 to polygon $M_3 = \frac{CD}{AB} = \frac{3\sqrt{2}}{\sqrt{2}} = 3$

(2)



$\because AB = 8$ unit length , $CD = 12$ unit length

, $FD = 8$ unit length.

$\therefore$ The scale factor of similarity of polygon M_1 to polygon $M_3 = \frac{AB}{FD} = \frac{8}{8} = 1$

, the scale factor of similarity of polygon M_2 to polygon $M_3 = \frac{CD}{FD} = \frac{12}{8} = \frac{3}{2}$

Third **Higher skills**

$\because$ Rectangle ABCD ~ rectangle AEON

$\therefore \frac{\text{perimeter of rectangle ABCD}}{\text{perimeter of rectangle AEON}}$

$= \frac{AB}{AE} = \frac{AD}{AN} = \frac{AB - AD}{AE - AN}$ (Q.E.D.)

Answers of Exercise 2

First **Multiple choice questions**

(1) b	(2) c	(3) a	(4) d
(5) c	(6) b	(7) c	(8) a
(9) d	(10) d	(11) b	(12) c
(13) c	(14) a	(15) b	(16) b
(17) b	(18) d	(19) c	(20) c
(21) d	(22) c	(23) b	(24) a
(25) b	(26) c	(27) b	(28) d
(29) a	(30) b	(31) c	(32) b
(33) c	(34) c	(35) b	(36) a
(37) c	(38) b	(39) b	(40) d
(41) d	(42) d	(43) d	(44) b
(45) c	(46) d	(47) b	(48) b
(49) c	(50) c	(51) c	(52) c
(53) a	(54) b	(55) b	(56) d
(57) a	(58) a		

Second **Essay questions**

1

(1) In Δ ABC :

m ($\angle$ A) = 180° – (80° + 55°) = 45°

$\therefore$ m ($\angle$ A) = m ($\angle$ D) = 45°

, m ($\angle$ C) = m ($\angle$ F) = 55°

$\therefore \Delta$ ABC ~ Δ DEF

(2) $\because \Delta\Delta$ XYZ , NLM are right-angled triangles

, m ($\angle$ Z) = m ($\angle$ M) = 25°

$\therefore \Delta$ XYZ ~ Δ NLM

(3) In Δ ABC :

m ($\angle$ B) = 180° – (65° + 30°) = 85°

, in Δ XYZ :

m ($\angle$ X) = 180° – (75° + 65°) = 40°

$\therefore$ In $\Delta\Delta$ ABC , XYZ :

only m ($\angle$ A) = m ($\angle$ Y)

$\therefore$ The two triangles are not similar.

(4) $\because \overline{AC} \parallel \overline{DB}$ $\therefore \Delta$ AEC ~ Δ BED

(5) $\because \Delta\Delta$ ABC , DEF are two equilateral triangles

$\therefore \Delta$ ABC ~ Δ DEF

(6) $\because \Delta\Delta$ ABC , XZY are isosceles triangles

, m ($\angle$ B) = m ($\angle$ Z) = 70°

$\therefore \Delta$ ABC ~ Δ XZY

(7) $\because \frac{AD}{AB} \neq \frac{AE}{AC}$ $\therefore \Delta\Delta$ ADE , ABC aren't similar

(8) Δ XYZ ~ Δ NLM because : $\frac{XY}{NL} = \frac{YZ}{LM} = \frac{XZ}{NM} = \frac{3}{2}$

(9) Δ AEC ~ Δ BED because : $\frac{AE}{BE} = \frac{CE}{DE} = \frac{1}{2}$

, m ($\angle$ AEC) = m ($\angle$ BED) (V.O.A.)

(10) Δ XYZ ~ Δ LYM because : $\frac{XY}{LY} = \frac{XZ}{LM} = \frac{8}{3}$

, m ($\angle$ X) = m ($\angle$ YLM)

2

$\because \overline{AD} \parallel \overline{BC}$

$\therefore \Delta$ AHD ~ Δ BHC (Q.E.D. 1)

$\therefore \frac{AH}{BH} = \frac{HD}{HC}$

$\therefore$ AH × HC = DH × HB (Q.E.D. 2)

3

$\because \frac{3}{4} = \frac{4.5}{6} = \frac{6}{8}$ $\therefore \frac{AB}{EF} = \frac{BC}{DE} = \frac{CA}{FD}$

$\therefore \Delta$ CAB ~ Δ DFE (Q.E.D.)

4

$\because \frac{XB}{AB} = \frac{9}{12} = \frac{3}{4}$, $\frac{BY}{BC} = \frac{18}{24} = \frac{3}{4}$, $\frac{XY}{AC} = \frac{13.5}{18} = \frac{3}{4}$

$\therefore \frac{XB}{AB} = \frac{BY}{BC} = \frac{XY}{AC}$

$\therefore \Delta$ XBY ~ Δ ABC (Q.E.D. 1)

We deduce that :

m ($\angle$ XBY) = m ($\angle$ ABC)

$\therefore \overrightarrow{BC}$ bisects $\angle$ ABX (Q.E.D. 2)

5

$\because \frac{AB}{DB} = \frac{6}{4} = \frac{3}{2}$, $\frac{BC}{BA} = \frac{9}{6} = \frac{3}{2}$, $\frac{AC}{DA} = \frac{7.5}{5} = \frac{3}{2}$

$\therefore \frac{AB}{DB} = \frac{BC}{BA} = \frac{AC}{DA}$

$\therefore \Delta ABC \sim \Delta DBA$ (Q.E.D. 1)

We deduce that : m ($\angle$ ABD) = m ($\angle$ ABC)

$\therefore \overrightarrow{BA}$ bisects $\angle$ DBC (Q.E.D. 2)

6

AE = 6 – 2 = 4 cm. $\therefore \frac{AE}{AB} = \frac{4}{8} = \frac{1}{2}$, $\frac{AD}{AC} = \frac{3}{6} = \frac{1}{2}$

$\therefore$ In $\Delta\Delta$ AED , ABC :

$\because \angle$ A is common , $\frac{AE}{AB} = \frac{AD}{AC} = \frac{1}{2}$

$\therefore \Delta AED \sim \Delta ABC$ (Q.E.D.)

7

$\because \frac{AE}{DE} = \frac{7.5}{10} = \frac{3}{4}$, $\frac{BE}{EC} = \frac{9}{12} = \frac{3}{4}$

$\therefore$ In $\Delta\Delta$ ABE , DCE : $\frac{AE}{DE} = \frac{BE}{EC} = \frac{3}{4}$

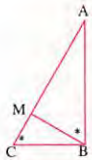
, m ($\angle$ AEB) = m ($\angle$ CED) (V.O.A.)

$\therefore \Delta ABE \sim \Delta DCE$ (First req.)

$\therefore \frac{AB}{DC} = \frac{BE}{EC}$ $\quad \therefore \frac{6}{DC} = \frac{3}{4}$

$\therefore$ DC = 8 cm. (Second req.)

8



In $\Delta\Delta$ ABM , ACB :

$\because \angle$ A is a common angle

, m ($\angle$ ABM) = m ($\angle$ C)

$\therefore \Delta ABM \sim \Delta ACB$

$\therefore \frac{AB}{AC} = \frac{AM}{AB}$

$\therefore (AB)^2 = AM \times AC$ (Q.E.D.)

9

(1) $\Delta ADE \sim \Delta ABC$

, $\Delta ADX \sim \Delta ABY$, $\Delta AXE \sim \Delta AYC$

(2) $\because \Delta ADE \sim \Delta ABC$

$\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$ (1)

$\because \Delta ADX \sim \Delta ABY$

$\therefore \frac{AD}{AB} = \frac{DX}{BY} = \frac{AX}{AY}$ (2)

$\because \Delta AXE \sim \Delta AYC$

$\therefore \frac{AX}{AY} = \frac{XE}{YC} = \frac{AE}{AC}$ (3)

from (1) , (2) , (3) :

$\therefore \frac{DX}{BY} = \frac{XE}{YC} = \frac{DE}{BC}$ (Q.E.D.)

10

$\because \frac{AB}{DB} = \frac{6}{4.5} = \frac{4}{3}$, $\frac{BC}{BF} = \frac{12}{9} = \frac{4}{3}$, $\frac{AC}{DF} = \frac{8}{6} = \frac{4}{3}$

$\therefore \frac{AB}{DB} = \frac{BC}{BF} = \frac{AC}{DF}$

$\therefore \Delta ABC \sim \Delta DBF$ (Q.E.D. 1)

$\therefore$ m ($\angle$ C) = m ($\angle$ BFD)

$\because$ m ($\angle$ BFD) = m ($\angle$ EFC) (V.O.A.)

$\therefore$ m ($\angle$ C) = m ($\angle$ EFC)

$\therefore \Delta$ EFC is isosceles (Q.E.D. 2)

11

$\because \frac{AB}{DA} = \frac{CE}{BC}$ $\quad \therefore \frac{AB}{CE} = \frac{DA}{BC}$

$\because \frac{BD}{DA} = \frac{EB}{BC}$ $\quad \therefore \frac{BD}{EB} = \frac{DA}{BC}$

$\therefore \frac{AB}{CE} = \frac{DA}{BC} = \frac{BD}{EB}$

$\therefore \Delta DBA \sim \Delta BEC$ We deduce that

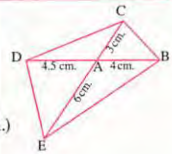
m ($\angle$ ADB) = m ($\angle$ CBE) and they are alternate angles

$\therefore \overline{AD} \parallel \overline{BC}$ (Q.E.D. 1)

m ($\angle$ ABD) = m ($\angle$ CEB) and they are alternate angles

$\therefore \overline{AB} \parallel \overline{CE}$ (Q.E.D. 2)

12



In $\Delta\Delta$ ABC , AED :

$\because \frac{AB}{AE} = \frac{AC}{AD}$ $\left(\text{each} = \frac{2}{3}\right)$

, m ($\angle$BAC) = m ($\angle$EAD) (V.O.A.)

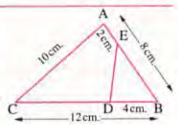
$\therefore \Delta ABC \sim \Delta AED$

We deduce that m ($\angle$ ACB) = m ($\angle$ ADE)

and they are drawn on $\overline{BE}$ and on the same side from it

$\therefore$ BCDE is a cyclic quadrilateral. (Q.E.D.)

13



In $\Delta\Delta$ BDE , BAC :

$\because \frac{BD}{BA} = \frac{4}{8} = \frac{1}{2}$, $\frac{BE}{BC} = \frac{6}{12} = \frac{1}{2}$

$\therefore \frac{BD}{BA} = \frac{BE}{BC}$

, $\angle$ B is common $\quad \therefore \Delta BDE \sim \Delta BAC$

, $\frac{DE}{AC} = \frac{1}{2}$ $\quad \therefore \frac{DE}{10} = \frac{1}{2}$

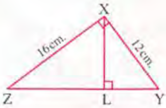
$\therefore$ DE = 5 cm. (Q.E.D. 1)

We deduce that from similarity

m ($\angle$ BDE) = m ($\angle$ BAC)

$\therefore$ ACDE is a cyclic quadrilateral (Q.E.D. 2)

14



$\because (XY)^2 = YL \times YZ$

$, (XZ)^2 = ZL \times ZY$

$\therefore \frac{(XY)^2}{(XZ)^2} = \frac{YL}{ZL}$ (First req.)

$, \because (YZ)^2 = (XY)^2 + (XZ)^2 = 144 + 256 = 400$

$\therefore YZ = 20$ cm.

$, \because (XY)^2 = YL \times YZ \quad \therefore 144 = YL \times 20$

$\therefore YL = 7.2$ cm. (Second req.)

$, XL = \frac{XY \times XZ}{YZ} = \frac{12 \times 16}{20} = 9.6$ cm. (Third req.)

15

$\because$ ABCD is a parallelogram.

$\therefore \overleftrightarrow{AD} \,/\!/\, \overleftrightarrow{BC} \quad \therefore \overline{AE} \,/\!/\, \overline{BC}$

$\therefore \Delta AHE \sim \Delta CHB$ (Q.E.D. 1)

$\therefore \frac{AH}{CH} = \frac{HE}{HB}$ (1)

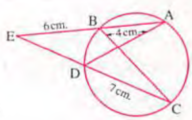
$, \because \overline{AB} \,/\!/\, \overline{CO} \quad \therefore \Delta ABH \sim \Delta COH$

$\therefore \frac{BH}{OH} = \frac{AH}{CH}$ (2)

From (1) , (2) : $\therefore \frac{HE}{HB} = \frac{BH}{OH}$

$\therefore (HB)^2 = HE \times OH$ (Q.E.D. 2)

16



$\because \angle A , \angle C$ subtended $\overset{\frown}{BD}$

$\therefore m(\angle A) = m(\angle C)$

$, \because \angle E$ is a common angle

$\therefore \Delta ADE \sim \Delta CBE$ (First req.)

$\therefore \frac{DE}{BE} = \frac{AE}{CE} \quad \therefore \frac{DE}{6} = \frac{10}{7 + DE}$

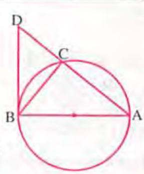
$\therefore 7\,DE + (DE)^2 = 60$

$\therefore (DE)^2 + 7\,DE - 60 = 0$

$\therefore (DE + 12)(DE - 5) = 0 \quad \therefore DE = 5$ cm.

$\therefore CE = 12$ cm. (Second req.)

17



$\overleftrightarrow{BD}$ is a tangent to the circle at B

$\therefore \overline{AB} \perp \overleftrightarrow{BD}$

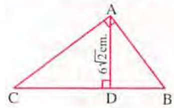
$, \because \overline{AB}$ is a diameter in the circle

$\therefore m(\angle ACB) = 90°$

$\therefore \Delta ABD$ is right-angled at B , $\overline{BC} \perp \overline{AD}$

$\therefore (BC)^2 = CA \times CD$ (Q.E.D.)

18



$\because DC = 2\,BD$

$, AD = 6\sqrt{2}$ cm.

$, \because (AD)^2 = DB \times DC$

$\therefore \left(6\sqrt{2}\right)^2 = DB \times 2\,DB \quad \therefore 72 = 2\,(BD)^2$

$\therefore (BD)^2 = 36 \quad \therefore BD = 6$ cm.

$\therefore CD = 12$ cm. $\quad \therefore (AB)^2 = 6 \times 18 = 108$

$\therefore AB = 6\sqrt{3}$ cm. $, (AC)^2 = 12 \times 18 = 216$

$\therefore AC = 6\sqrt{6}$ cm. (The req.)

19

$\because \overline{BC} \,/\!/\, \overline{AD} , \overleftrightarrow{AB}$ is transversal.

$\therefore m(\angle A) = m(\angle B) = 90°$ (alternate angles)

$\therefore$ In $\Delta\Delta$ ABC , EAD :

$\frac{AB}{EA} = \frac{2}{1} = 2$ (Because E is the midpoint of $\overline{AB}$)

$, \frac{BC}{AD} = \frac{12}{6} = 2$

$, m(\angle A) = m(\angle B) = 90°$

$\therefore \Delta ABC \sim \Delta EAD$ (Q.E.D. 1)

We deduce that : $m(\angle BAC) = m(\angle AED)$ and they are alternate angles.

$\therefore \overline{AC} \,/\!/\, \overline{DE}$ (Q.E.D. 2)

20

$\because \frac{AB}{BD} = \frac{6}{4} = \frac{3}{2} , \frac{BC}{BA} = \frac{9}{6} = \frac{3}{2}$

$\therefore$ In $\Delta\Delta$ ABC , DBA :

$\frac{AB}{BD} = \frac{BC}{BA} = \frac{3}{2} , \angle B$ is common.

$\therefore \Delta ABC \sim \Delta DBA$ (First req.)

$\therefore \frac{AB}{DB} = \frac{AC}{AD} \quad \therefore \frac{6}{4} = \frac{8}{AD}$

$\therefore AD = 5\frac{1}{3}$ cm. (Second req.)

From similarity we deduce $m(\angle BAD) = m(\angle C)$

$\therefore \overline{AB}$ is a tangent segment for the circle passing through the vertices of Δ ADC

(Third req.)

21

$\because \frac{KO}{LE} = \frac{4.5}{9} = \frac{1}{2} , \frac{OE}{LM} = \frac{6}{12} = \frac{1}{2} \quad \therefore \frac{KE}{ME} = \frac{4}{8} = \frac{1}{2}$

$\therefore \Delta KOE \sim \Delta ELM$

$\therefore m(\angle OKE) = m(\angle LEM)$

and they are corresponding angles

$\therefore \overline{OK} \parallel \overline{LE}$ (First req.)

$m(\angle OEK) = m(\angle LME)$ and they are corresponding angles.

$\therefore \overline{EO} \parallel \overline{LM}$ (Second req.)

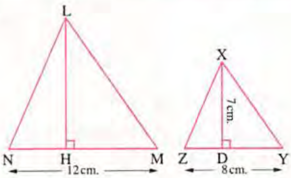
$, \because \overline{LE} \parallel \overline{OK}$ $\quad \therefore \Delta NKO \sim \Delta NEL$

$\therefore \frac{NK}{NE} = \frac{KO}{EL} = \frac{4.5}{9}$ $\quad \therefore \frac{NK}{NK+4} = \frac{1}{2}$

$\therefore 2NK = NK + 4$

$\therefore NK = 4$ cm. (Third req.)

22



$\because \Delta\Delta$ XYZ , LMN have equal measures of corresponding angles.

$\therefore \Delta XYZ \sim \Delta LMN$ $\quad \therefore \frac{XY}{LM} = \frac{YZ}{MN} = \frac{8}{12} = \frac{2}{3}$

In $\Delta\Delta$ XYD , LMH :

$\because m(\angle Y) = m(\angle M)$ (Given)

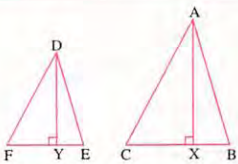
$, m(\angle XDY) = m(\angle LHM) = 90°$

$\therefore \Delta XYD \sim \Delta LMH$

$\therefore \frac{XD}{LH} = \frac{XY}{LM}$ $\quad \therefore \frac{7}{LH} = \frac{2}{3}$

$\therefore LH = 10.5$ cm. (The req.)

23



$\because \Delta ABC \sim \Delta DEF$

$\therefore m(\angle B) = m(\angle E) , m(\angle C) = m(\angle F)$

$\therefore$ In $\Delta\Delta$ ABX , DEY :

$\because m(\angle B) = m(\angle E)$

$, m(\angle BXA) = m(\angle EYD) = 90°$

$\therefore \Delta ABX \sim \Delta DEY$

$\therefore \frac{BX}{EY} = \frac{AX}{DY}$ (1)

In $\Delta\Delta$ AXC , DYF :

$\because m(\angle C) = m(\angle F)$

$, m(\angle AXC) = m(\angle DYF) = 90°$

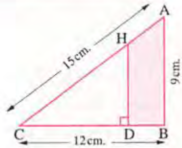
$\therefore \Delta AXC \sim \Delta DYF$

$\therefore \frac{AX}{DY} = \frac{XC}{YF}$ (2)

From (1) , (2) : $\therefore \frac{BX}{EY} = \frac{XC}{YF}$

$\therefore BX \times YF = XC \times YE$ (Q.E.D.)

24



$\because (AC)^2 = 225$

$, (AB)^2 + (BC)^2 = 225$

$\therefore \angle B$ is a right angle.

$\therefore \overline{DH} \parallel \overline{AB}$

$\therefore \Delta CHD \sim \Delta CAB$

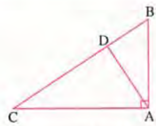
$\therefore \frac{CD}{CB} = \frac{HD}{AB}$ $\quad \therefore \frac{3}{4} = \frac{HD}{9}$

$\therefore HD = 6\frac{3}{4}$ cm. , $BD = 12 \times \frac{1}{4} = 3$ cm.

$\therefore$ Figure ABDH is a trapezium of area.

$\frac{AB + DH}{2} \times BD = \frac{9 + 6\frac{3}{4}}{2} \times 3 = 23\frac{5}{8}$ cm.^{2} (The req.)

25



In $\Delta\Delta$ DBA , ABC :

$\because \frac{DB}{AB} = \frac{BA}{BC}$,

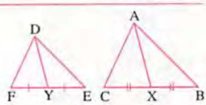
$\angle B$ is common

$\therefore \Delta DBA \sim \Delta ABC$ (Q.E.D. 1)

We deduce that : $m(\angle ADB) = m(\angle CAB) = 90°$

$\therefore \overline{AD} \perp \overline{BC}$ (Q.E.D. 2)

26



$\because \Delta ABC \sim \Delta DEF$

$\therefore m(\angle B) = m(\angle E)$ (1)

$, \frac{AB}{DE} = \frac{BC}{EF} = \frac{2\,BX}{2\,EY} = \frac{BX}{EY}$ (2)

From (1) , (2) we deduce that :

$\Delta ABX \sim \Delta DEY$ (Q.E.D.)

27

In $\Delta\Delta$ ABE , DBC :

$\because \frac{AB}{BD} = \frac{AE}{DC}$

$, m(\angle BAE) = m(\angle BDC)$

two inscribed angles subtended by $\overset{\frown}{BC}$

$\therefore \Delta ABE \sim \Delta DBC$ (Q.E.D. 1)

$\therefore m(\angle ABE) = m(\angle DBC)$

$\therefore \overrightarrow{BD}$ bisects $\angle ABC$ (Q.E.D. 2)

28

$\because \angle C$ complements $\angle DAC$

, $\angle EAD$ complements $\angle DAC$

$\therefore m(\angle C) = m(\angle EAD)$

, $m(\angle DEA) = m(\angle DFC) = 90°$

$\therefore \Delta ADE \sim \Delta CDF$ (Q.E.D. 1)

$\because (DE)^2 = AE \times EB$ $\therefore DE = \sqrt{AE \times EB}$

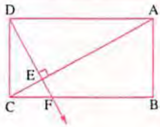
, $(DF)^2 = AF \times FC$ $\therefore DF = \sqrt{AF \times FC}$

$\therefore$ Area of rectangle AEDF $= DE \times DF$

$= \sqrt{AE \times EB \times AF \times FC}$

(Q.E.D. 2)

29



$\because$ ABCD is a rectangle.

$\therefore m(\angle ADC) = m(\angle BCD)$

$= 90°$

$\therefore$ In ΔADC :

$m(\angle ADC) = 90°$, $\overline{DE} \perp \overline{AC}$

$\therefore \Delta ADC \sim \Delta AED$

$\therefore \frac{AD}{AE} = \frac{AC}{AD}$ $\therefore (AD)^2 = AE \times AC$

$\therefore AD = \sqrt{AE \times AC}$

, in ΔDCF : $m(\angle DCF) = 90°$, $\overline{CE} \perp \overline{DF}$

$\therefore \Delta DCF \sim \Delta DEC$ $\therefore \frac{DC}{DE} = \frac{DF}{DC}$

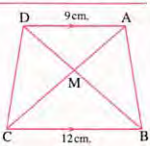
$\therefore (DC)^2 = DE \times DF$ $\therefore DC = \sqrt{DE \times DF}$

$\therefore$ The area of the rectangle ABCD $= AD \times DC$

$= \sqrt{AE \times AC} \times \sqrt{DE \times DF}$

$= \sqrt{AE \times AC \times DE \times DF}$ (Q.E.D.)

30



$\because \overline{AD} \parallel \overline{BC}$

$\therefore \Delta MAD \sim \Delta MCB$

$\therefore \frac{MA}{MC} = \frac{MD}{MB}$

$\therefore MA \times MB = MC \times MD$ (First req.)

From similarity , we get : $\frac{MA}{MC} = \frac{AD}{CB} = \frac{9}{12} = \frac{3}{4}$

Let $MA = 3k$, $MC = 4k$

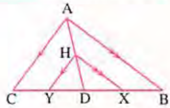
, $\because AC = MA + MC$, $AC = 14$ cm.

$\therefore 3k + 4k = 14$ $\therefore 7k = 14$

$\therefore k = 2$

$\therefore MA = 2 \times 3 = 6$ cm. (Second req.)

31



In $\Delta\Delta$ ABC , HXY :

$\because m(\angle B) = m(\angle HXY)$

(corresponding angles)

, $m(\angle C) = m(\angle HYX)$ (corresponding angles)

$\therefore \Delta ABC \sim \Delta HXY$ (Q.E.D. 1)

$\therefore \frac{AB}{HX} = \frac{BC}{XY}$ (1)

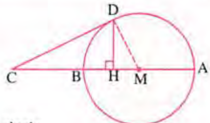
, $\because \overline{HX} \parallel \overline{AB}$ $\therefore \Delta DAB \sim \Delta DHX$

$\therefore \frac{DA}{DH} = \frac{AB}{HX}$ (2)

From (1) , (2) : $\therefore \frac{AD}{DH} = \frac{BC}{XY}$

$\therefore XY \times AD = BC \times DH$ (Q.E.D. 2)

32



Construction :

Draw $\overline{MD}$

Proof :

$\because \overline{CD}$ is a tangent to the circle.

$\therefore \angle CDM$ is a right angle.

$\therefore (CD)^2 = CH \times CM$ (1)

but $(CD)^2 = (CM)^2 - (MD)^2 = (CM)^2 - (MB)^2$

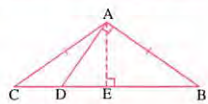
$= (CM - MB)(CM + MB) = CB(CM + MA)$

$= CB \times CA$ (2)

From (1) , (2) :

$\therefore (CD)^2 = CH \times CM = CB \times CA$ (Q.E.D.)

33



Construction :

Draw $\overline{AE} \perp \overline{BC}$

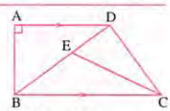
Proof :

$\because AB = AC$, $\overline{AE} \perp \overline{BC}$ $\therefore BE = \frac{1}{2} BC$

, $\because (AB)^2 = BE \times BD$ $\therefore (AB)^2 = \frac{1}{2} BC \times BD$

$\therefore 2(AB)^2 = BD \times BC$ (Q.E.D.)

34



$\because AB \times EC = DE \times BD$

$\therefore \frac{BD}{EC} = \frac{AB}{DE}$

, $\because CD \times BD = DA \times EC$ $\therefore \frac{BD}{EC} = \frac{DA}{CD}$

$\therefore \frac{BD}{EC} = \frac{AB}{DE} = \frac{DA}{CD}$ $\therefore \Delta ADB \sim \Delta DCE$

We deduce that :

m ($\angle$ CDE) = m ($\angle$ A) = 90°

$\therefore$ In Δ BCD : $(BC)^2 = (DB)^2 + (CD)^2$

while $(BD)^2 = (AB)^2 + (AD)^2$

$\therefore (BC)^2 = (AB)^2 + (AD)^2 + (CD)^2$ (Q.E.D.)

35

In $\Delta\Delta$ BXA , CDA : $\because \frac{BX}{CD} = \frac{BA}{CA}$

, m ($\angle$ B) = m ($\angle$ C)

two inscribed angles subtended by $\overset{\frown}{AD}$

$\therefore \Delta BXA \sim \Delta CDA$ (Q.E.D. 1)

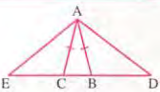
We deduce that :

m ($\angle$ AXB) = m ($\angle$ ADC)

$\therefore$ m ($\angle$ ADC) = 90°

$\therefore \overline{AC}$ is a diameter in the circle (Q.E.D. 2)

36



$\because$ AB = AC

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ACE) (1)

$\because (AB)^2 = DB \times CE$

$\therefore \frac{DB}{AB} = \frac{AB}{CE}$ $\therefore \frac{DB}{AC} = \frac{AB}{CE}$ (2)

From (1) , (2) : $\therefore \Delta ABD \sim \Delta ECA$ (Q.E.D.)

Third **Higher skills**

(1) d	(2) c	(3) b	(4) d
(5) c	(6) d	(7) b	(8) b
(9) c	(10) c	(11) b	(12) b
(13) b	(14) b	(15) b	(16) c
(17) d	(18) c		

Instructions to solve :

(1) $\because \frac{x - y}{x + y} = \frac{2}{7}$ $\therefore 7x - 7y = 2x + 2y$

$\therefore 5x = 9y$

, $\because \overline{DE} \parallel \overline{BC}$ $\therefore \Delta AED \sim \Delta ACB$

$\therefore \frac{AE}{AC} = \frac{DE}{BC}$ $\therefore \frac{AE}{AE + 8} = \frac{y}{x} = \frac{5}{9}$

$\therefore 9 AE = 5 AE + 40$ $\therefore 4 AE = 40$

$\therefore$ AE = 10 cm.

(2) $\because$ M is the point of concurrent of medians of Δ ABC

$\therefore \frac{AM}{AD} = \frac{2}{3}$, $\overline{AD}$ is median in Δ ABC

$\therefore$ D is the midpoint of $\overline{BC}$

, $\because \overline{ED} \parallel \overline{AC}$ $\therefore ED = \frac{1}{2} AC = 9$ cm.

, in Δ AED : $\because \overline{FM} \parallel \overline{ED}$

$\therefore \Delta AFM \sim \Delta AED$ $\therefore \frac{FM}{ED} = \frac{AM}{AD}$

$\therefore \frac{FM}{9} = \frac{2}{3}$ $\therefore$ FM = 6 cm.

(3) In $\Delta\Delta$ ABC , DBA

$\because$ m ($\angle$ BAC) = m ($\angle$ D) , $\angle$ B is common.

$\therefore \frac{AB}{DB} = \frac{BC}{BA} = \frac{AC}{DA}$ $\therefore \frac{6}{5 + BC} = \frac{BC}{6}$

$\therefore 36 = 5\, BC + (BC)^2$

$\therefore (BC)^2 + 5\, BC - 36 = 0$

$\therefore (BC - 4)(BC + 9)$ $\therefore$ BC = 4 cm.

(4) In $\Delta\Delta$ ACD , ABC :

m ($\angle$ ACD) = m ($\angle$ B) , $\angle$ A is common angle.

$\therefore \Delta ACD \sim \Delta ABC$ $\therefore \frac{AC}{AB} = \frac{CD}{BC} = \frac{AD}{AC}$

$\therefore \frac{x}{y + z} = \frac{y}{x}$ $\therefore x^2 = y^2 + yz$

$\therefore x^2 - y^2 = yz = 16$

(5) In Δ ABC : $\because \overline{XD} \parallel \overline{BC}$

$\therefore \frac{AE}{AC} = \frac{DE}{BC}$ $\therefore \frac{2}{10} = \frac{DE}{15}$

$\therefore$ DE = 3 cm.

In Δ EXC :

$\because$ m ($\angle$ EXC) = m ($\angle$ XCB) (alternate angles)

, $\because \overrightarrow{CX}$ bisects $\angle$ ACB

$\therefore$ m ($\angle$ EXC) = m ($\angle$ ECX)

$\therefore$ XE = EC = 8 cm. $\therefore$ XD = 8 – 3 = 5 cm.

(6) In Δ ADE : $\because$ AD = AE

$\therefore$ m ($\angle$ ADE) = m ($\angle$ AED)

$\therefore$ m ($\angle$ ADB) = m ($\angle$ AEC)

In $\Delta\Delta$ BDA , AEC : $\because$ m ($\angle$ B) = m ($\angle$ EAC)

, m ($\angle$ ADB) = m ($\angle$ AEC)

$\therefore \Delta BDA \sim \Delta AEC$ $\therefore \frac{BD}{AE} = \frac{DA}{EC}$

, $\because$ AD = AE $\therefore \frac{9}{AD} = \frac{AD}{4}$

$\therefore (AD)^2 = 36$ $\therefore$ AD = 6 cm.

(7) $\because$ Δ BDE is an equilateral triangle.

$\therefore$ m ($\angle$ BDE) = m ($\angle$ BED) = m ($\angle$ DBE) = 60°

$\therefore$ m ($\angle$ BDA) = m ($\angle$ BEC) = 120°

, $\because$ m ($\angle$ DBE) = 60°

$\therefore$ m ($\angle$ ABD) + m ($\angle$ CBE) = 60°

, $\because$ m ($\angle$ BAD) + m ($\angle$ ABD) = 180° – 120° = 60°

$\therefore$ m ($\angle$ BAD) = m ($\angle$ CBE)

In $\Delta\Delta$ DAB , EBC :

m ($\angle$ ADB) = m ($\angle$ BEC) = 120°

, m ($\angle$ BAD) = m ($\angle$ CBE)

$\therefore$ Δ DAB ~ Δ EBC $\quad \therefore \frac{DA}{EB} = \frac{DB}{EC}$

$\therefore \frac{9}{x} = \frac{x}{4} \quad \therefore x^2 = 36$

$\therefore x = 6$ cm.

(8) $\because$ $\angle$ EDF is an exterior angle of the triangle ADC

$\therefore$ m ($\angle$ EDF) = m ($\angle$ 3) + m ($\angle$ CAD)

, $\because$ m ($\angle$ 3) = m ($\angle$ 1)

$\therefore$ m ($\angle$ EDF) = m ($\angle$ 1) + m ($\angle$ CAD) = m ($\angle$ CAB)

Similarly ; m ($\angle$ DFE) = m ($\angle$ ABC)

$\therefore$ Δ DEF ~ Δ ACB

$\therefore$ DE : EF : FD = AC : CB : BA = 12 : 11 : 7

(9) In Δ ADC : $\because$ AD = AC

$\therefore$ m ($\angle$ ADC) = m ($\angle$ ACD)

, $\because$ m ($\angle$ BDE) = m ($\angle$ ADC) (V.O.A)

$\therefore$ m ($\angle$ BDE) = m ($\angle$ ACD)

In $\Delta\Delta$ BDE , BCA :

m ($\angle$ ABC) = m ($\angle$ EBD)

, m ($\angle$ BDE) = m ($\angle$ ACB)

$\therefore$ Δ BDE ~ Δ BCA $\quad \therefore \frac{BD}{BC} = \frac{DE}{CA} = \frac{BE}{BA}$

$\therefore \frac{9}{15} = \frac{3}{CA} \quad \therefore$ CA = 5 cm.

$\therefore$ The perimeter of Δ ADC = 5 + 5 + 6 = 16 cm.

(10) In Δ DAE : $\because \overline{XY} \parallel \overline{AE}$

$\therefore$ Δ DXY ~ Δ DAE $\quad \therefore \frac{DX}{DA} = \frac{XY}{AE}$

$\therefore \frac{DX}{DX + 4} = \frac{6}{9} = \frac{2}{3}$

$\therefore$ 3 DX = 2 DX + 8 $\quad \therefore$ DX = 8 cm.

In Δ ABC : $\because \overline{DE} \parallel \overline{BC}$

$\therefore$ Δ ADE ~ Δ ABC $\quad \therefore \frac{AE}{AC} = \frac{AD}{AB}$

$\therefore \frac{9}{12} = \frac{12}{AB} \quad \therefore$ AB = 16 cm.

$\therefore$ DB = 16 – 12 = 4 cm.

(11) In $\Delta\Delta$ ABC , CED :

$\because$ m ($\angle$ ACB) + m ($\angle$ ECD) = 90°

In Δ ABC : m ($\angle$ B) = 90°

$\therefore$ m ($\angle$ ACB) + m ($\angle$ CAB) = 90°

$\therefore$ m ($\angle$ CAB) = m ($\angle$ ECD)

$\therefore$ Δ ABC ~ Δ CDE $\quad \therefore \frac{AB}{CD} = \frac{BC}{DE} = \frac{AC}{CE}$

$\therefore \frac{3}{6} = \frac{x}{y} \quad \therefore y = 2x$

$\because x^2 + y^2 = \left(5\sqrt{5}\right)^2 \quad \therefore x^2 + (2x)^2 = 125$

$\therefore 5x^2 = 125 \quad \therefore x^2 = 25$

$\therefore x = 5 \quad \therefore y = 10$

$\therefore x + y = 5 + 10 = 15$ cm.

(12) In the quadrilateral AXFZ :

$\because$ m ($\angle$ AXF) + m ($\angle$ AZF) = 180°

$\therefore$ AXFZ is a cyclic quadrilateral

$\therefore$ m ($\angle$ DFE) = m ($\angle$ A)

Similarly : m ($\angle$ FDE) = m ($\angle$ B)

$\therefore$ Δ ABC ~ Δ FDE $\quad \therefore \frac{AB}{FD} = \frac{BC}{DE} = \frac{AC}{FE}$

$\therefore \frac{12}{4} = \frac{9}{FE} \quad \therefore$ FE = 3 cm.

(13) In $\Delta\Delta$ ADE , CBD :

$\because$ m ($\angle$ ADE) = m ($\angle$ DBC) (alternate angles)

$\therefore$ Δ ADE ~ Δ CBD $\quad \therefore \frac{AD}{CB} = \frac{DE}{BD} = \frac{AE}{CD}$

, $\because$ BE = 2 ED $\quad \therefore \frac{DE}{BD} = \frac{1}{3}$

$\therefore \frac{1}{3} = \frac{AE}{6} \quad \therefore$ AE = 2 cm.

(14) In Δ ABC : $\because$ m ($\angle$ A) = 90°

$\therefore$ $\angle$ B complements $\angle$ C

In Δ YFC : $\because$ m ($\angle$ F) = 90°

$\therefore$ $\angle$ C complements $\angle$ FYC

$\therefore$ m ($\angle$ B) = m ($\angle$ FYC)

In $\Delta\Delta$ BED , YFC :

$\because$ m ($\angle$ DEB) = m ($\angle$ YFC) = 90°

, m ($\angle$ B) = m ($\angle$ FYC)

$\therefore$ Δ BED ~ Δ YFC $\quad \therefore \frac{BE}{YF} = \frac{ED}{FC} = \frac{BD}{YC}$

$\therefore \frac{8}{YF} = \frac{ED}{2} \quad \therefore$ YF × ED = 16

$\therefore$ The area of the square DEFY = 16 cm².

(15) In Δ ABC :

$\because \overline{EF} \parallel \overline{AB} \quad \therefore \frac{CF}{CB} = \frac{EF}{3}$ (1)

, in Δ DBC :

$\because \overline{EF} \parallel \overline{DC} \quad \therefore \frac{BF}{BC} = \frac{FE}{6}$ (2)

Adding (1) , (2) : $\therefore \frac{CF}{BC} + \frac{BF}{BC} = \frac{EF}{3} + \frac{EF}{6}$

$\therefore \frac{BC}{BC} = \frac{2\,EF + EF}{6}$ $\quad \therefore \frac{3\,EF}{6} = 1$

$\therefore$ EF = 2 cm.

(16) In Δ ABC :

$\because \overline{DE} \parallel \overline{AC}$ $\quad \therefore \frac{BE}{BA} = \frac{BD}{BC}$ (1)

, in Δ ADB :

$\because \overline{FE} \parallel \overline{DB}$ $\quad \therefore \frac{AE}{AB} = \frac{FE}{DB}$ (2)

By adding (1) , (2) : $\therefore \frac{BE}{AB} + \frac{AE}{AB} = \frac{BD}{BC} + \frac{FE}{DB}$

$\therefore \frac{AB}{AB} = \frac{6}{14} + \frac{FE}{6}$ $\quad \therefore \frac{FE}{6} = 1 - \frac{3}{7} = \frac{4}{7}$

$\therefore FE = \frac{24}{7}$ cm.

(17) $\because$ AEC supplements $\angle$ BEC

, $\angle$ ACB supplements $\angle$ ACD

, $\because$ m ($\angle$ BEC) = m ($\angle$ ACD)

$\therefore$ m ($\angle$ AEC) = m ($\angle$ ACB)

In $\Delta\Delta$ AEC , ACB

$\because$ m ($\angle$ AEC) = m ($\angle$ ACB)

, $\angle$ A is a common angle.

$\therefore \Delta AEC \sim \Delta ACB$ $\quad \therefore \frac{AE}{AC} = \frac{EC}{CB} = \frac{AC}{AB}$

$\therefore \frac{4}{8} = \frac{6}{CB} = \frac{8}{AB}$

$\therefore$ CB = 12 cm. , AB = 16 cm.

$\therefore$ BE = 16 − 4 = 12 cm.

$\therefore$ BE + BC = 12 + 12 cm. = 24 cm.

(18) $\because \overline{AB} \parallel \overline{DC}$ $\quad \therefore \Delta AEB \sim \Delta CED$

$\therefore \frac{AE}{CE} = \frac{EB}{ED} = \frac{AB}{CD} = \frac{4}{9}$

$\therefore AE = 4\,X$, $EC = 9\,X$

, $\because \Delta$ ABC is right angled triangle at B

, $\overline{BE} \perp \overline{AC}$ $\quad \therefore (AB)^2 = AE \times AC$

$\therefore 16 = 4\,X \times 13\,X$ $\quad \therefore X^2 = \frac{4}{13}$

, $(BC)^2 = CE \times CA = 9\,X \times 13\,X = 9 \times 13\,X^2$

$= 9 \times 13 \times \frac{4}{13}$

$= 36$

$\therefore$ BC = 6 cm.

$\therefore$ The area of the trapezium ABCD $= \frac{4+9}{2} \times 6$

$= 39$ cm^2.

Answers of Exercise 3

First | Multiple choice questions

(1) d	(2) d	(3) a	(4) c
(5) c	(6) c	(7) a	(8) d
(9) a	(10) d	(11) c	(12) d
(13) b	(14) c	(15) a	(16) a
(17) d	(18) d	(19) d	(20) a
(21) a	(22) c	(23) b	(24) a
(25) b	(26) d	(27) c	(28) c
(29) d	(30) b	(31) a	(32) c
(33) b	(34) c	(35) c	

Second | Essay questions

1

$\because \frac{\text{Area of the 1}^{st}\text{ triangle}}{\text{Area of the 2}^{nd}\text{ triangle}} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

Let area of the 1st triangle = $9\,X$

$\therefore$ Area of the 2nd triangle = $4\,X$

$\therefore 9\,X + 4\,X = 130$ $\quad \therefore 13\,X = 130$

$\therefore X = 10$

$\therefore$ Area of the 1st triangle = 90 cm^2.

, area of the 2nd triangle = 40 cm^2. (The req.)

2

$\because$ Ratio between lengths of two corresponding sides = 1 : 3

$\therefore$ Ratio between areas of the two polygons = 1 : 9

Let the area of the 1st polygon = X

$\therefore$ Area of the 2nd polygon = $9\,X$

$\therefore 9\,X - X = 32$ $\quad \therefore 8\,X = 32$ $\quad \therefore X = 4$

$\therefore$ Area of the 1st polygon = 4 cm^2.

, area of the 2nd polygon = 36 cm^2. (The req.)

3

$\because \overline{AB} \parallel \overline{DC}$

$\therefore$ m ($\angle$ B) = m ($\angle$ DCE) (corresponding angles) (1)

, $\because \overline{AC} \parallel \overline{DE}$

$\therefore$ m ($\angle$ BCA) = m ($\angle$ E) (corresponding angles) (2)

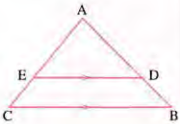
From (1) , (2) : $\therefore \Delta ABC \sim \Delta DCE$

$\therefore \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DCE} = \left(\dfrac{AB}{DC}\right)^2 = \left(\dfrac{3}{2}\right)^2$

$\therefore \dfrac{\text{Area of } \Delta ABC}{16} = \dfrac{9}{4}$

$\therefore$ Area of $\Delta ABC = 36 \text{ cm}^2$ (The req.)

4



$\because \overline{DE} \,//\, \overline{BC}$

$\therefore \Delta ADE \sim \Delta ABC$

$\therefore \dfrac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ABC} = \left(\dfrac{AD}{AB}\right)^2 = \left(\dfrac{2}{3}\right)^2$

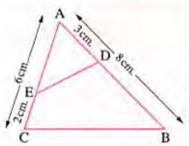
$\therefore \dfrac{60}{\text{Area of } \Delta ABC} = \dfrac{4}{9}$

$\therefore$ Area of $\Delta ABC = 135 \text{ cm}^2$

$\therefore$ Area of trapezium DBCE $= 135 - 60 = 75 \text{ cm}^2$

(The req.)

5



$\because \Delta\Delta$ ADE , ACB have :

$\dfrac{AD}{AC} = \dfrac{3}{6} = \dfrac{1}{2}$

, $\dfrac{AE}{AB} = \dfrac{4}{8} = \dfrac{1}{2}$

, $\angle A$ is common. $\therefore \Delta ADE \sim \Delta ACB$

$\therefore \dfrac{\text{Area of } \Delta ADE}{\text{Area of } \Delta ACB} = \left(\dfrac{AD}{AC}\right)^2 = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

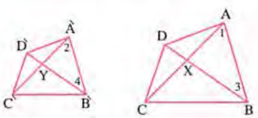
Let area of $\Delta ADE = x$

$\therefore$ Area of $\Delta ACB = 4x$

$\therefore$ Area of figure DBCE $= 4x - x = 3x$

$\therefore \dfrac{\text{Area of } \Delta ADE}{\text{Area of figure DBCE}} = \dfrac{x}{3x} = \dfrac{1}{3}$ (The req.)

6



$\because$ The two polygons are similar.

$\therefore \Delta ABC \sim \Delta A'B'C'$

$\therefore m(\angle 1) = m(\angle 2)$

, $\Delta ABD \sim \Delta A'B'D'$ $\therefore m(\angle 3) = m(\angle 4)$

$\therefore \Delta ABX \sim \Delta A'B'Y$ $\therefore \dfrac{BX}{B'Y} = \dfrac{AB}{A'B'}$

$\therefore \dfrac{\text{a (polygon ABCD)}}{\text{a (polygon } A'B'C'D')} = \dfrac{(AB)^2}{(A'B')^2} = \dfrac{(BX)^2}{(B'Y)^2}$ (Q.E.D.)

7

$\because \Delta\Delta$ ABC , DBA have :

$\angle B$ is common , $m(\angle C) = m(\angle DAB)$

$\therefore \Delta ABC \sim \Delta DBA$ (First req.)

$\therefore \dfrac{AB}{DB} = \dfrac{BC}{BA}$

$\therefore (AB)^2 = DB \times BC = 6 \times 9$

$\therefore AB = 3\sqrt{6}$ cm. (Second req.)

$\therefore \dfrac{\text{Area of } (\Delta ABC)}{\text{Area of } (\Delta DBA)} = \left(\dfrac{BC}{BA}\right)^2 = \left(\dfrac{9}{3\sqrt{6}}\right)^2 = \dfrac{3}{2}$

(Third req.)

8

$\because \Delta BEO \sim \Delta ADO$

$\therefore \dfrac{\text{Area of } (\Delta BEO)}{\text{Area of } (\Delta ADO)} = \left(\dfrac{BO}{AO}\right)^2 = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

, $\because$ area of $(\Delta BEO) = 9 \text{ cm}^2$

$\therefore$ Area of $(\Delta ADO) = 9 \times 4 = 36 \text{ cm}^2$ (1)

, $\because \Delta BEO \sim \Delta CED$

$\therefore \dfrac{\text{Area of } (\Delta BEO)}{\text{Area of } (\Delta CED)} = \left(\dfrac{BO}{CD}\right)^2 = \left(\dfrac{BO}{AB}\right)^2 = \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$

, $\because$ area of $(\Delta BEO) = 9 \text{ cm}^2$

$\therefore$ Area of $(\Delta CED) = 9 \times 9 = 81 \text{ cm}^2$

$\therefore$ Area of polygon BODC $= 81 - 9 = 72 \text{ cm}^2$ (2)

Adding (1) , (2) :

$\therefore$ Area of parallelogram ABCD $= 108 \text{ cm}^2$

(The req.)

9

$\because \overline{FC} \,//\, \overline{AD}$, $\overleftrightarrow{DF}$ is a transversal

$\therefore m(\angle F) = m(\angle ADE)$ (Alternate angles)

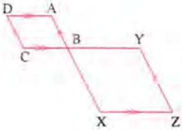
, $\because m(\angle C) = m(\angle A)$ (properties of a parallelogram)

$\therefore \Delta DCF \sim \Delta EAD$

$\therefore \dfrac{\text{Area of } (\Delta DCF)}{\text{Area of } (\Delta EAD)} = \left(\dfrac{DC}{EA}\right)^2 = \left(\dfrac{AB}{EA}\right)^2 = \dfrac{25}{9}$

(The req.)

10



$\because m(\angle ABC)$

$= m(\angle XBY)$ (V.O.A.)

$\therefore m(\angle A) = m(\angle X)$

$, m(\angle C) = m(\angle Y)$

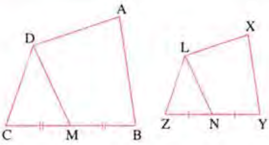
$, m(\angle D) = m(\angle Z)$

$, \because \frac{AB}{XB} = \frac{1}{2}, \frac{BC}{BY} = \frac{1}{2} \qquad \therefore \frac{CD}{YZ} = \frac{1}{2}, \frac{AD}{XZ} = \frac{1}{2}$

$\therefore$ Parallelogram ABCD ~ parallelogram XBYZ

$\therefore \frac{a\,(\text{parallelogram ABCD})}{a\,(\text{parallelogram XBYZ})} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ (Q.E.D.)

11



$\because$ The two polygons are similar.

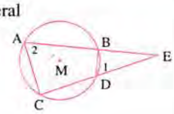
$\therefore \Delta MDC \sim \Delta NLZ \qquad \therefore \frac{MD}{NL} = \frac{DC}{LZ}$

$\therefore \frac{a\,(\text{polygon ABCD})}{a\,(\text{polygon XYZL})} = \left(\frac{DC}{LZ}\right)^2 = \left(\frac{MD}{NL}\right)^2$

$\therefore$ a (polygon ABCD) : a (polygon XYZL)

$= (MD)^2 : (NL)^2$ (Q.E.D.)

12



$\because$ ABDC is a cyclic quadrilateral

$, D \in \overline{CE}$

$\therefore m(\angle 1) = m(\angle 2)$

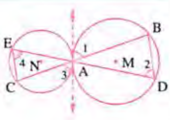
$\therefore$ In $\Delta\Delta$ EBD , ECA :

$m(\angle 1) = m(\angle 2)$, $\angle E$ is common

$\therefore \Delta EBD \sim \Delta ECA$

$\therefore \frac{a\,(\Delta EBD)}{a\,(\Delta ECA)} = \left(\frac{BD}{AC}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$ (The req.)

13



Const. : Draw the common tangent of the two circles at A

Proof :

$\because m(\angle 1) = m(\angle 2)$, $m(\angle 3) = m(\angle 4)$

$, m(\angle 1) = m(\angle 3)$

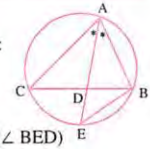
$\therefore m(\angle 2) = m(\angle 4)$

$, \because m(\angle BAD) = m(\angle CAE)$ (V.O.A.)

$\therefore \Delta ABD \sim \Delta ACE$

$\therefore \frac{a\,(\Delta ABD)}{a\,(\Delta ACE)} = \frac{(BD)^2}{(CE)^2}$ (Q.E.D.)

14



$\because \Delta\Delta\Delta$ ABE , ADC , BDE have :

$m(\angle BAE) = m(\angle DAC)$

$= m(\angle DBE)$

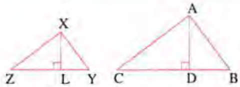
$, m(\angle AEB) = m(\angle ACD) = m(\angle BED)$

$\therefore \Delta ABE \sim \Delta ADC \sim \Delta BDE$

$\therefore a(\Delta ABE) : a(\Delta ADC) : a(\Delta BDE)$

$= (BE)^2 : (DC)^2 : (DE)^2$ (Q.E.D.)

15



$\because \Delta ABC \sim \Delta XYZ$

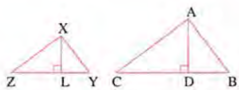
$\therefore \frac{a\,(\Delta ABC)}{a\,(\Delta XYZ)} = \left(\frac{BC}{YZ}\right)^2$, but $\frac{a\,(\Delta ABC)}{a\,(\Delta XYZ)} = \frac{\frac{1}{2} BC \times AD}{\frac{1}{2} YZ \times XL}$

$\therefore \left(\frac{BC}{YZ}\right)^2 = \frac{BC \times AD}{YZ \times XL} \qquad \therefore \frac{BC}{YZ} = \frac{AD}{XL}$

$\therefore BC \times XL = AD \times YZ$ (Q.E.D.)

16

(1)



Let $\Delta ABC \sim \Delta XYZ$

and height $\overline{AD}$ is corresponding to height $\overline{XL}$

$\therefore \frac{a\,(\Delta ABC)}{a\,(\Delta XYZ)} = \frac{(BC)^2}{(YZ)^2}$

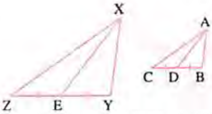
$\therefore \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times YZ \times LX} = \frac{(BC)^2}{(YZ)^2} \qquad \therefore \frac{AD}{LX} = \frac{BC}{YZ}$

$\therefore$ The ratio between the two corresponding heights equals the ratio between the two corresponding sides.

$\therefore$ The ratio between the areas of the two similar triangles equals the square of the ratio of any two corresponding heights.

(Q.E.D.)

(2)



Let $\Delta ABC \sim \Delta XYZ$, $\overline{AD}$, $\overline{XE}$ are two corresponding medians in them.

$\therefore \frac{AB}{XY} = \frac{BC}{YZ}$, $m(\angle B) = m(\angle Y)$

$\therefore \frac{AB}{XY} = \frac{\frac{1}{2}BC}{\frac{1}{2}YZ}$ $\qquad \therefore \frac{AB}{XY} = \frac{BD}{YE}$

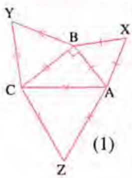
$\therefore \Delta ABD \sim \Delta XYE$ $\qquad \therefore \frac{AB}{XY} = \frac{AD}{XE}$

$\therefore$ The ratio between the lengths of two corresponding sides in the two triangles ABC , XYZ equals the ratio of the lengths of the two corresponding medians : $\overline{AD}$, $\overline{XE}$

$\therefore \frac{\text{Area of }(\Delta ABC)}{\text{Area of }(\Delta XYZ)} = \left(\frac{AB}{XY}\right)^2 = \left(\frac{AD}{XE}\right)^2$

i.e. The square of the ratio of the lengths of the two corresponding medians. (Q.E.D.)

17



$\because \Delta\Delta\Delta$ ABX , BCY , ACZ are equilateral triangles

$\therefore \Delta ABX \sim \Delta BCY \sim \Delta ACZ$

$\therefore \frac{a(\Delta ABX)}{a(\Delta ACZ)} = \left(\frac{AB}{AC}\right)^2 = \frac{(AB)^2}{(AC)^2}$ (1)

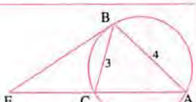
, $\frac{a(\Delta BCY)}{a(\Delta ACZ)} = \left(\frac{BC}{AC}\right)^2 = \frac{(BC)^2}{(AC)^2}$ (2)

Adding (1) , (2) :

$\therefore \frac{a(\Delta ABX) + a(\Delta BCY)}{a(\Delta ACZ)} = \frac{(AB)^2 + (BC)^2}{(AC)^2} = \frac{(AC)^2}{(AC)^2}$

$\therefore a(\Delta ABX) + a(\Delta BCY) = a(\Delta ACZ)$ (Q.E.D.)

18



In $\Delta\Delta$ BCE , ABE :

$\because m(\angle CBE)$ (tangency) $= m(\angle A)$ (inscribed)

, $\angle E$ is common

$\therefore \Delta BCE \sim \Delta ABE$

$\therefore \frac{a(\Delta BCE)}{a(\Delta ABE)} = \left(\frac{BC}{AB}\right)^2 = \frac{9}{16}$

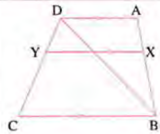
$\therefore a(\Delta BCE) = 9x$ $\qquad \therefore a(\Delta ABE) = 16x$

$\therefore a(\Delta ABC) = a(\Delta ABE) - a(\Delta BCE)$

$= 16x - 9x = 7x$

$\therefore \frac{a(\Delta ABC)}{a(\Delta ABE)} = \frac{7x}{16x} = \frac{7}{16}$ (Q.E.D.)

19



$\because$ Polygon AXYD $\sim$ polygon XBCY

$\therefore \frac{a(\text{polygon AXYD})}{a(\text{polygon XBCY})} = \frac{(AD)^2}{(XY)^2}$

but from similarity $\frac{AD}{XY} = \frac{XY}{BC}$

$\therefore (XY)^2 = AD \times BC$

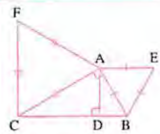
$\therefore \frac{a(\text{polygon AXYD})}{a(\text{polygon XBCY})} = \frac{(AD)^2}{AD \times BC} = \frac{AD}{BC}$ (1)

$\because \frac{a(\Delta ABD)}{a(\Delta DBC)} = \frac{AD}{BC}$ (have equal heights) (2)

From (1) , (2) :

$\therefore \frac{a(\text{polygon AXYD})}{a(\text{polygon XBCY})} = \frac{a(\Delta ABD)}{a(\Delta BDC)}$ (Q.E.D.)

20



$\because \Delta ADB \sim \Delta CDA$

$\therefore \frac{AD}{CD} = \frac{DB}{DA} = \frac{AB}{CA}$ (1)

, $\because \Delta ABE \sim \Delta CAF$

$\therefore \frac{AB}{CA} = \frac{BE}{AF} = \frac{AE}{CF}$ (2)

From (1) , (2) :

$\therefore$ Lengths of corresponding sides in the two polygons ADBE , CDAF are proportional.

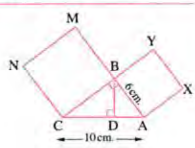
, $\because$ measures of corresponding angles in the two polygons ADBE , CDAF are equal (Why) ?

$\therefore$ Polygon ADBE $\sim$ Polygon CDAF (Q.E.D.1)

$\therefore \frac{a(\text{polygon ADBE})}{a(\text{polygon CDAF})} = \left(\frac{AD}{CD}\right)^2 = \frac{(AD)^2}{(CD)^2}$

$= \frac{BD \cdot DC}{(CD)^2} = \frac{BD}{CD}$ (Q.E.D.2)

21



$\because \Delta ABD \sim \Delta BCD$

$\therefore \frac{AB}{BC} = \frac{BD}{CD} = \frac{AD}{BD}$

$\therefore \frac{AB}{BC} = \frac{AX}{BM} = \frac{XY}{MN} = \frac{BY}{CN}$

$\therefore \frac{BD}{CD} = \frac{AD}{BD} = \frac{AB}{BC} = \frac{AX}{BM} = \frac{XY}{MN} = \frac{BY}{CN}$

$\therefore$ Lengths of corresponding sides in the two polygons DAXYB , DBMNC are proportional.

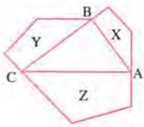
, $\because$ measures of corresponding angles in the two polygons DAXYB , DBMNC are equal (Why) ?

$\therefore$ Polygon DAXYB ~ Polygon DBMNC (First req.)

$\because BC = \sqrt{100 - 36} = 8$ cm.

$\therefore \frac{a\,(\text{polygon DAXYB})}{a\,(\text{polygon DBMNC})} = \left(\frac{6}{8}\right)^2 = \frac{36}{64} = \frac{9}{16}$ (Second req.)

22



$\because$ Polygon X ~ Polygon Z

$\therefore \frac{a\,(\text{polygon X})}{a\,(\text{polygon Z})} = \left(\frac{AB}{AC}\right)^2 = \frac{(AB)^2}{(AC)^2}$ (1)

$\because$ Polygon Y ~ Polygon Z

$\therefore \frac{a\,(\text{polygon Y})}{a\,(\text{polygon Z})} = \left(\frac{BC}{AC}\right)^2 = \frac{(BC)^2}{(AC)^2}$ (2)

Adding (1) , (2) :

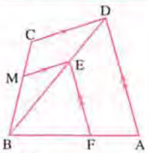
$\therefore \frac{a\,(\text{polygon X}) + a\,(\text{polygon Y})}{a\,(\text{polygon Z})} = \frac{(AB)^2 + (BC)^2}{(AC)^2}$

$\therefore \frac{40 + 85}{125} = \frac{(AB)^2 + (BC)^2}{(AC)^2}$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta$ ABC is a right-angled triangle at B (Q.E.D.)

23



The two polygons BCDA , BMEF have :

(1) The measures of their corresponding angles are equal because $\angle$ B is common

, m ($\angle$ A) = m ($\angle$ BFE) (corresponding angles)

, m ($\angle$ C) = m ($\angle$ BME) (corresponding angles)

, m ($\angle$ ADC) = m ($\angle$ FEM)

(2) Lengths of their corresponding sides are proportional : Δ BAD ~ Δ BFE

$\therefore \frac{BA}{BF} = \frac{AD}{FE} = \frac{BD}{BE}$, Δ BDC ~ Δ BEM

$\therefore \frac{BD}{BE} = \frac{DC}{EM} = \frac{BC}{BM}$

$\therefore \frac{BA}{BF} = \frac{AD}{FE} = \frac{DC}{EM} = \frac{BC}{BM}$ (1)

$\therefore$ Polygon BCDA ~ polygon BMEF

$\therefore$ From (1) : $\frac{BF}{BA} = \frac{BM}{BC}$

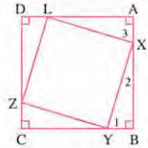
$\therefore \frac{BF}{BA} \times \frac{BF}{BA} = \frac{BM}{BC} \times \frac{BF}{BA}$

$\therefore \left(\frac{BF}{BA}\right)^2 = \frac{BM \times BF}{BC \times BA}$

, $\because \frac{a\,(\text{polygon BMEF})}{a\,(\text{polygon BCDA})} = \left(\frac{BF}{BA}\right)^2$

$\therefore \frac{a\,(\text{polygon BMEF})}{a\,(\text{polygon BCDA})} = \frac{BM \times BF}{BC \times BA}$ (Q.E.D.)

24



Let square ABCD have side of length k unit length.

$\therefore AX = \frac{1}{4}$ k unit length

, $BX = \frac{3}{4}$ k unit length , $BY = \frac{1}{4}$ k unit length.

, $AL = \frac{3}{4}$k unit length.

$\therefore \Delta\Delta$ AXL , BYX right-angled triangles have :

XB = AL , BY = AX $\therefore \Delta AXL \equiv \Delta BYX$

$\therefore$ XL = XY , similar we can prove that :

LZ = ZY , m ($\angle$ 1) = m ($\angle$ 3)

, $\because$ m ($\angle$ 1) + m ($\angle$ 2) = 90°

$\therefore$ m ($\angle$ 2) + m ($\angle$ 3) = 90°

$\therefore$ m ($\angle$ LXY) = 90°

$\therefore$ XYZL is a square. (Q.E.D.1)

its side length $= \sqrt{\left(\frac{1}{4}k\right)^2 + \left(\frac{3}{4}k\right)^2} = \frac{\sqrt{10}}{4}$ k unit length.

, $\because$ all squares are similar.

$\therefore \frac{a\,(\text{the square XYZL})}{a\,(\text{the square ABCD})} = \left(\frac{\frac{\sqrt{10}\,k}{4}}{k}\right)^2 = \frac{5}{8}$ (Q.E.D.2)

25

$\because$ m ($\angle$ CBY) = m ($\angle$ CDY)

(two inscribed angles on same arc $\overset{\frown}{CY}$)

, m ($\angle$ CDY) = m ($\angle$ X) (corresponding angles)

$\therefore$ m ($\angle$ CBY) = m ($\angle$ X)

, $\because$ m ($\angle$ BDX) = m ($\angle$ BCY)

Exterior of the cyclic quadrilateral BCYD

$\therefore \Delta$ DBX $\sim \Delta$ CYB

$\therefore \frac{a(\Delta DBX)}{a(\Delta CYB)} = \frac{(BX)^2}{(YB)^2}$ (Q.E.D.)

Third **Higher skills**

1

(1) b (2) c (3) c (4) c (5) b (6) c

(7) c (8) c (9) a (10) d (11) b (12) d

Instructions to solve 1 :

(1) $\because \overline{FY} // \overline{CD}$ $\therefore \Delta AFY \sim \Delta ACD$

$\therefore \frac{a(\Delta AFY)}{a(\Delta ACD)} = \left(\frac{AF}{AC}\right)^2$ $\therefore \left(\frac{AF}{AC}\right)^2 = \frac{5}{5+40} = \frac{1}{9}$

, $\because \overline{BC} // \overline{EF}$ $\therefore \Delta AEF \sim \Delta ABC$

$\therefore \frac{a(\Delta AEF)}{a(\Delta ABC)} = \left(\frac{AF}{AC}\right)^2$ $\therefore \frac{a(\Delta AEF)}{a(\Delta ABC)} = \frac{1}{9}$

$\therefore \frac{a(\Delta AEF)}{a(\Delta ABC) - a(\Delta AEF)} = \frac{1}{9-1} = \frac{1}{8}$

$\therefore \frac{a(\Delta AEF)}{32} = \frac{1}{8}$ $\therefore a(\Delta AEF) = 4 \text{ cm}^2$

(2) $\because \overline{XY} // \overline{BC}$ $\therefore \Delta AXY \sim \Delta ABC$

$\therefore \frac{a(\Delta AXY)}{a(\Delta ABC)} = \left(\frac{AY}{AC}\right)^2$

$\therefore \left(\frac{AY}{AC}\right)^2 = \frac{40}{40+50} = \frac{4}{9}$

, $\because \overline{YZ} // \overline{CD}$ $\therefore \frac{AY}{AC} = \frac{AZ}{AD}$

$\therefore \left(\frac{AZ}{AD}\right)^2 = \frac{4}{9}$ $\therefore \frac{AZ}{AD} = \frac{2}{3}$

$\therefore \frac{DZ}{DA} = \frac{1}{3}$

, $\because \overline{ZM} // \overline{AE}$ $\therefore \Delta DZM \sim \Delta DAE$

$\therefore \frac{a(\Delta DZM)}{a(\Delta DAE)} = \left(\frac{DZ}{DA}\right)^2$

$\therefore \frac{13}{13 + a(\text{The quadrilateral AEMZ})} = \frac{1}{9}$

$\therefore$ 13 + a (the quadrilateral AEMZ) = 117

$\therefore$ a (the quadrilateral AEMZ) = 117 – 13 = 104 cm.^{2}

(3) In Δ AED , Δ BCA :

$\because$ m ($\angle$ AED) = m ($\angle$ ACB) = 90°

, m ($\angle$ DAE) = m ($\angle$ CBA)

$\therefore \Delta AED \sim \Delta BCA$ $\therefore \frac{a(\Delta AED)}{a(\Delta BCA)} = \left(\frac{AD}{AB}\right)^2$

$\therefore \frac{a(\Delta AED)}{a(\Delta BCA)} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$\therefore \frac{6}{a(\Delta BCA)} = \frac{1}{9}$ $\therefore a(\Delta ACB) = 54 \text{ cm}^2$

$\therefore$ The area of the shaded region = 54 – 6

= 48 cm.^{2}

(4) In Δ AXY :

$\because \overline{XY} // \overline{DE}$ $\therefore \Delta AED \sim \Delta AXY$

$\therefore \frac{a(\Delta ADE)}{a(\Delta AXY)} = \left(\frac{AE}{AY}\right)^2 = \left(\frac{2}{8}\right)^2 = \frac{1}{16}$

$\therefore \frac{a(\Delta ADE)}{a(\Delta ADE) + a(\text{figure DXYE})} = \frac{1}{16}$

$\therefore \frac{a(\Delta ADE)}{a(\Delta ADE) + 30} = \frac{1}{16}$

$\therefore$ 16 a (Δ ADE) = a (Δ ADE) + 30

$\therefore$ 15 a (Δ ADE) = 30 $\therefore$ a (Δ ADE) = 2 cm.^{2}

In Δ ABC :

$\because \overline{XY} // \overline{BC}$ $\therefore \Delta AXY \sim \Delta ABC$

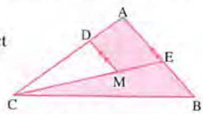
$\therefore \frac{a(\Delta AXY)}{a(\Delta ABC)} = \left(\frac{AY}{AC}\right)^2$ $\therefore \frac{2+30}{a(\Delta ABC)} = \left(\frac{8}{10}\right)^2$

$\therefore a(\Delta ABC) = \frac{25 \times 32}{16} = 50 \text{ cm}^2$

$\therefore$ The area of figure XBCY = 50 – 32 = 18 cm.^{2}

(5) **Construction :**

Draw $\overrightarrow{CM}$ to intersect $\overline{AB}$ at E



Proof :

$\because$ M is the point of intersection of medians of Δ ABC

$\therefore \overline{CE}$ is a median.

$\therefore a(\Delta AEC) = \frac{1}{2} a(\Delta ABC) = \frac{1}{2} \times 36 = 18 \text{ cm.}^2$

, $\because \frac{CM}{CE} = \frac{2}{3}$

, $\because \overline{MD} // \overline{AE}$ $\therefore \Delta CMD \sim \Delta CEA$

$\therefore \frac{a(\Delta CMD)}{a(\Delta CEA)} = \left(\frac{CM}{CE}\right)^2$

$\therefore \frac{a(\Delta CMD)}{18} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$\therefore$ a (Δ CMD) = 8 cm.^{2}

$\therefore$ The area of the shaded region = 36 – 8

= 28 cm.^{2}

(6) In $\Delta\Delta$ FDE , ABC :

$\because$ m ($\angle$ FDE) = m ($\angle$ ABC) (corresponding angles)

, m ($\angle$ FED) = m ($\angle$ ACB) (corresponding angles)

$\therefore \Delta FDE \sim \Delta ABC$ $\therefore \frac{a(\Delta FDE)}{a(\Delta ABC)} = \left(\frac{DE}{BC}\right)^2$

$\therefore \frac{6}{a(\Delta ABC)} = \left(\frac{3}{9}\right)^2 = \frac{1}{9}$

$\therefore a(\Delta ABC) = 54 \text{ cm}^2$

$\therefore$ The area of the shaded region $= 54 - 6$

$= 48 \text{ cm}^2$

(7) $\because \Delta ABC \sim \Delta DEF \qquad \therefore \frac{a(\Delta ABC)}{a(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2$

$\therefore \frac{x+2}{x+7} = \left(\frac{x}{x+1}\right)^2$

$\therefore \frac{x+2}{x+7} = \frac{x^2}{x^2+2x+1}$

$\therefore \frac{x+2}{(x+7)-(x+2)} = \frac{x^2}{(x^2+2x+1)-x^2}$

$\therefore \frac{x+2}{5} = \frac{x^2}{2x+1}$

$\therefore (x+2)(2x+1) = 5x^2$

$\therefore 2x^2+5x+2 = 5x^2$

$\therefore 3x^2-5x-2 = 0$

$\therefore (3x+1)(x-2) = 0$

$\therefore x = -\frac{1}{3}$ (refused) or $x = 2$

(8) In ΔABC :

$\because \overline{DE} \parallel \overline{BC} \qquad \therefore \Delta ADE \sim \Delta ABC$

$\therefore \frac{a(\Delta ADE)}{a(\Delta ABC)} = \left(\frac{AD}{AB}\right)^2$

$\therefore \frac{a(\Delta ADE)}{a(\Delta ABC)} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$

$\therefore a(\Delta ADE) = \frac{4}{25} \times a(\Delta ABC)$

$, \because \overline{EF} \parallel \overline{AB} \qquad \therefore \Delta CFE \sim \Delta CBA$

$\therefore \frac{a(\Delta CFE)}{a(\Delta CBA)} = \left(\frac{FE}{BA}\right)^2$

$\therefore \frac{a(\Delta CFE)}{a(\Delta CBA)} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$

$\therefore a(\Delta CFE) = \frac{9}{25} \times a(\Delta CBA)$

$\therefore a(\square DBFE)$

$= a(\Delta ABC) - \big(a(\Delta ADE) + a(\Delta CFE)\big)$

$= a(\Delta ABC) - \left(\frac{4}{25} \times a(\Delta ABC) + \frac{9}{25} \times (\Delta ABC)\right)$

$= \frac{12}{25} \times a(\Delta ABC)$

$\therefore \frac{a(\square DBFE)}{a(\Delta ABC)} = \frac{12}{25}$

(9) The area of the square ABCD $= 6 \times 6 = 36 \text{ cm}^2$

$\therefore$ The area of $\Delta DBC = \frac{1}{2} \times 36 = 18 \text{ cm}^2$

$, \because \overline{FY} \parallel \overline{BC} \qquad \therefore \Delta DFY \sim \Delta DCB$

$\therefore \frac{a(\Delta DFY)}{a(\Delta DCB)} = \left(\frac{DE}{DC}\right)^2 \quad \therefore \frac{a(\Delta DFY)}{18} = \left(\frac{2}{3}\right)^2$

$\therefore a(\Delta DFY) = 8 \text{ cm}^2$

$, \because \overline{XE} \parallel \overline{YF} \qquad \therefore \Delta DEX \sim \Delta DFY$

$\therefore \frac{a(\Delta DEX)}{a(\Delta DFY)} = \left(\frac{DE}{DF}\right)^2 \quad \therefore \frac{a(\Delta DEX)}{8} = \left(\frac{1}{2}\right)^2$

$\therefore a(\Delta DEX) = 2 \text{ cm}^2$

$\therefore$ The area (figure XYFE) $= 8 - 2 = 6 \text{ cm}^2$

(10) $x + y = \sqrt{(12)^2 + 9^2} = 15$ cm.

$, z = \frac{12 \times 9}{15} = 7.2$ cm.

$\therefore x + y + z = 22.2$ cm.

(11) In $\Delta\Delta$ ABE , EBF :

$\overline{AE}$, $\overline{EF}$ on the same straight line

, B is a common vertex.

$\therefore \frac{a(\Delta ABE)}{a(\Delta EBF)} = \frac{AE}{EF} \qquad \therefore \frac{AE}{EF} = \frac{2}{3}$

$, \because \overline{BA} \parallel \overline{FD} \qquad \therefore \Delta AEB \sim \Delta FED$

$\therefore \frac{a(\Delta AEB)}{a(\Delta FED)} = \left(\frac{AE}{FE}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

$\therefore \frac{2}{a(\Delta FED)} = \frac{4}{9} \qquad \therefore a(\Delta FED) = 4.5 \text{ cm}^2$

, the area of ΔCBD = the area of ΔBFD

$= 3 + 4.5 = 7.5 \text{ cm}^2$

$\therefore$ The area of the shaded region $= 7.5 - 2$

$= 5\frac{1}{2} \text{ cm}^2$

(12) $\because$ The scale factor of similarity of polygon P_1 to the polygon P_2 is $\frac{2}{3}$

$\because \frac{\text{Area}(P_1)}{\text{Area}(P_2)} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

, the scale factor of similarity of polygon P_3 to the polygon P_2 is $\frac{1}{3}$

$\therefore \frac{\text{Area}(P_3)}{\text{Area}(P_2)} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

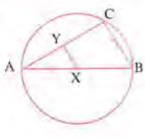
$\therefore$ Area (P_1) : Area (P_2) : Area (P_3)

$4 \quad : \quad 9 \quad : \quad 1$

$\therefore \sqrt{\text{Area}(P_1)} + \sqrt{\text{Area}(P_3)} = \sqrt{4k} + \sqrt{k} = 3\sqrt{k}$

$, \sqrt{\text{Area}(P_2)} = \sqrt{9k} = 3\sqrt{k}$

2



$\because \overline{XY} \parallel \overline{BC}$

$\therefore \Delta ABC \sim \Delta AXY$

$\therefore \frac{a(\Delta ABC)}{a(\Delta AXY)} = \frac{(AB)^2}{(AX)^2} = \frac{(AB)^2}{(CB)^2}$

$\therefore$ a (the polygon XBCY) = a (Δ ABC) − a (Δ AXY)

$\therefore \frac{\text{a (the polygon XBCY)}}{\text{a }(\Delta \text{ ABC})} = \frac{\text{a }(\Delta \text{ ABC}) - \text{a }(\Delta \text{ AXY})}{\text{a }(\Delta \text{ ABC})}$

$= 1 - \frac{\text{a }(\Delta \text{ AXY})}{\text{a }(\Delta \text{ ABC})} = 1 - \frac{(CB)^2}{(AB)^2} = \frac{(AB)^2 - (BC)^2}{(AB)^2}$

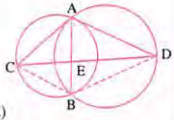
, $\because \angle$ ACB is a right angle (inscribed on a semicircle)

$\therefore (AB)^2 - (BC)^2 = (AC)^2$

$\therefore \frac{\text{a (the polygon XBCY)}}{\text{a }(\Delta \text{ ABC})} = \frac{(AC)^2}{(AB)^2}$

$\therefore \frac{\text{a }(\Delta \text{ ABC})}{\text{a (the polygon XBCY)}} = \frac{(AB)^2}{(AC)^2}$ (Q.E.D.)

3



Construction :

Draw $\overline{BC}$, $\overline{DB}$

Proof :

$\because \frac{\text{a }(\Delta \text{ ADE})}{\text{a }(\Delta \text{ ACE})} = \frac{DE}{EC} = \frac{\text{a }(\Delta \text{ BDE})}{\text{a }(\Delta \text{ BCE})}$

$\therefore \frac{\text{a }(\Delta \text{ ADE}) + \text{a }(\Delta \text{ BDE})}{\text{a }(\Delta \text{ ACE}) + \text{a }(\Delta \text{ BCE})} = \frac{DE}{EC}$

$\therefore \frac{\text{a }(\Delta \text{ ABD})}{\text{a }(\Delta \text{ ABC})} = \frac{DE}{EC}$ (1)

, $\because$ m ($\angle$ DAB) = m ($\angle$ ACB)

(Tangency and inscribed angles subtended same arc $\overset{\frown}{AB}$)

, similarly m ($\angle$ ADB) = m ($\angle$ BAC)

$\therefore \Delta$ ADB $\sim \Delta$ CAB $\therefore \frac{\text{a }(\Delta \text{ ADB})}{\text{a }(\Delta \text{ CAB})} = \frac{(AD)^2}{(CA)^2}$ (2)

From (1) , (2) : $\therefore \frac{DE}{EC} = \frac{(AD)^2}{(CA)^2}$

$\therefore \frac{CE}{ED} = \frac{(AC)^2}{(AD)^2}$ (Q.E.D.)

4

$\because$ Any two regular polygons having the same number of sides are similar.

$\therefore \frac{\text{a (square ABCD)}}{\text{a (square } A'B'C'D')} = \frac{(AB)^2}{(A'B')^2}$

Let the length of the radius of the circle = r

$\therefore AB = r\sqrt{2}$ (because the diagonal of square ABCD is a diameter in a circle).

, $A'B' = 2$ r (because the length of the side of square $A'B'C'D'$ equals the diameter of the circle).

$\therefore \frac{\text{a (square ABCD)}}{\text{a (square } A'B'C'D')} = \frac{\left(r\sqrt{2}\right)^2}{(2\,r)^2} = \frac{1}{2}$ (The req.)

Answers of Exercise 4

First Multiple choice questions

(1) b	(2) d	(3) a	(4) c
(5) a	(6) d	(7) a	(8) d
(9) b	(10) a	(11) c	(12) c
(13) a	(14) c	(15) b	(16) a
(17) a	(18) c	(19) b	(20) a
(21) c	(22) c	(23) a	(24) a
(25) d	(26) d	(27) d	(28) b
(29) c	(30) b	(31) c	(32) b
(33) d	(34) b	(35) d	(36) a
(37) a	(38) b	(39) b	(40) a
(41) c			

Second Essay questions

1

(1) $\because$ AE × EB = 6 × 7 = 42

, CE × ED = 5 × 8.4 = 42

$\therefore$ AE × EB = CE × ED

$\therefore$ The points A , B , C , D lie on one circle.

(2) , (3) The points A , B , C , D are not lie on one circle because points A , B , D lie on one straight line.

(4) $\because$ AE × EB = 5 × 20 = 100

, CE × ED = 10 × 10 = 100

$\therefore$ AE × EB = CE × ED

$\therefore$ The points A , B , C , D lie on same circle.

(5) $\because$ AE × BE = 12 × 3 = 36

, CE × DE = 9 × 4 = 36

$\therefore$ The points A , B , C , D lie on one circle.

(6) $\because$ AE × BE = 6 × 3.6 = 21.6

, CE × DE = 7.2 × 2.8 = 20.16

$\therefore$ AE × BE $\neq$ CE × DE

$\therefore$ The points A , B , C , D are not lie on one circle.

2 (1) , (4) , (6)

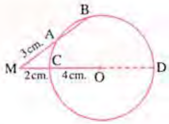
3

$\because (XA)^2 = XB \times XC$ $\therefore (15)^2 = 9\,(9 + BC)$

$\therefore 225 = 9\,(9 + 2\,r)$ $\therefore 25 = 9 + 2\,r$

$\therefore$ r = 8 cm. (The req.)

4



Draw $\overrightarrow{MO}$ to intersect the circle at C , D

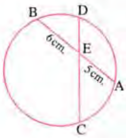
$\therefore$ MD = 6 + 4 = 10 cm.

, $\because$ MA × MB = MC × MD

$\therefore 3 \times MB = 2 \times 10$ $\qquad \therefore MB = 6\frac{2}{3}$ cm.

$\therefore AB = 6\frac{2}{3} - 3 = 3\frac{2}{3}$ cm. (The req.)

5



Let CE = x cm.

$\therefore$ DE = $(11.5 - x)$ cm.

, $\because$ AE × EB = CE × ED

$\therefore 5 \times 6 = x\left(\frac{23}{2} - x\right)$

$\therefore 2x^2 - 23x + 60 = 0$

$\therefore (2x - 15)(x - 4) = 0$

$\therefore$ The lengths of $\overline{CE}$, $\overline{ED}$ are 7.5 cm. , 4 cm.

(The req.)

6

$\because (AB)^2 = AC \times AD \quad \therefore \left(5\sqrt{2}\right)^2 = \frac{1}{2} \times AD \times AD$

$\therefore 50 = \frac{1}{2}(AD)^2 \qquad \therefore (AD)^2 = 100$

$\therefore$ AD = 10 cm. (The req.)

7

$\because \overline{AD}$ is a tangent , $\overline{AB}$ is a diameter

$\therefore m(\angle DAB) = 90°$

, $\because (AD)^2 = DC \times DB = 4 \times 16 = 64$

$\therefore (AB)^2 = (DB)^2 - (AD)^2$

$\therefore (AB)^2 = (16)^2 - 64 = 192 = (2r)^2$

$\therefore r^2$ of the circle $= \frac{1}{4} \times 192 = 48$

$\therefore$ Area of the circle M $= r^2\pi = 48\pi$ cm^2. (The req.)

8

From the major circle : $(XY)^2 = XC \times XD$ (1)

From the minor circle : $(XY)^2 = XA \times XB$ (2)

From (1) , (2) : $\therefore XC \times XD = XA \times XB$

$\therefore \frac{XC}{XB} = \frac{XA}{XD}$ (Q.E.D.)

9

$\because DN \times NB = EN \times NF \qquad \therefore DN \times 6 = 2 \times 9$

$\therefore$ DN = 3 cm.

, $\because$ AB = DN $\qquad \therefore$ AB = 3 cm.

$\therefore (AC)^2 = AB \times AD = 3 \times 12$

$\therefore$ AC = 6 cm. (First req.)

In $\Delta\Delta$ ACB , ADC : $\angle$ A common angle

, m ($\angle$ ACB) tangency = m ($\angle$ D) inscribed.

$\therefore \Delta ACB \sim \Delta ADC$

$\therefore \frac{\text{Area of } \Delta ACB}{\text{Area of } \Delta ADC} = \left(\frac{AC}{AD}\right)^2 = \left(\frac{6}{12}\right)^2 = \frac{1}{4}$

(Second req.)

10

$\because MB \times MA = MY \times MX$ (1)

, $\because MC \times MD = MY \times MX$ (2)

From (1) , (2) : $\therefore MB \times MA = MC \times MD$

$\therefore$ A , B , C , D lie on one circle. (Q.E.D.)

11

$\because \Delta\Delta$ XLM , XZY have :

$\frac{XL}{XZ} = \frac{4}{8} = \frac{1}{2}$, $\frac{XM}{XY} = \frac{6}{12} = \frac{1}{2}$

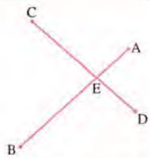
, $\angle$ X is a common angle.

$\therefore \Delta XLM \sim \Delta XZY$ (Q.E.D. 1)

, $\because \frac{XL}{XZ} = \frac{XM}{XY} \qquad \therefore XL \times XY = XM \times XZ$

$\therefore$ Figure LYZM is a cyclic quadrilateral (Q.E.D. 2)

12



$\because AE = \frac{5}{12}$ BE , BE = 6 cm.

$\therefore$ AE = 2.5 cm.

, $\because DE = \frac{3}{5}$ CE , EC = 5 cm.

$\therefore$ DE = 3 cm.

$\because AE \times BE = 2.5 \times 6 = 15$, $DE \times EC = 3 \times 5 = 15$

$\therefore AE \times BE = DE \times EC$

$\therefore$ The points A , B , C , D lie on the same circle.

(Q.E.D.)

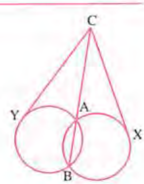
13

$\because (NX)^2 = NB \times NA$, $(NX)^2 = NC \times ND$

$\therefore NB \times NA = NC \times ND$

$\therefore \frac{NB}{NC} = \frac{ND}{NA}$ (Q.E.D.)

14



$\because (CX)^2 = CA \times CB$

, $(CY)^2 = CA \times CB$

$\therefore$ CX = CY (Q.E.D.)

15

In circle M :

$\because (AB)^2 = AF \times AE \quad \therefore (AB)^2 = 4 \times 9 = 36$

$\therefore AB = 6$ cm. (1)

In circle N $= (AC)^2 = AE \times AD = 9 \times 16 = 144$

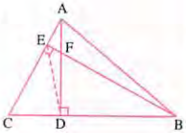
$\therefore AC = 12$ cm. (2)

From (1) , (2) : $\therefore AB = \frac{1}{2} AC$

$\therefore$ B is the midpoint of $\overline{AC}$ (Q.E.D.)

16

Const. : Draw $\overline{ED}$



Proof : $\because$ The figure FDCE is a cyclic quadrilateral

$\therefore AE \times AC = AF \times AD$

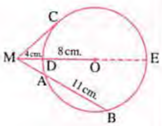
$\because$ The figure ABDE is a cyclic quadrilateral.

$\therefore BF \times FE = AF \times FD$

Dividing : $\frac{AE \times AC}{BF \times FE} = \frac{AD}{FD}$ (Q.E.D.)

17

Draw $\overrightarrow{MO}$ to intersect the circle at D , E



$\therefore ME = 12 + 8 = 20$ cm.

, $\because MA \times MB = MD \times ME$

$\therefore MA\,(MA + 11) = 4 \times 20$

$\therefore (MA)^2 + 11\,MA - 80 = 0$

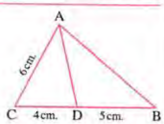
$\therefore (MA + 16)\,(MA - 5) = 0$

$\therefore MA = 5$ cm. (First req.)

$\because (MC)^2 = MD \times ME = 4 \times 20 = 80$

$\therefore MC = \sqrt{80} = 4\sqrt{5}$ cm. (Second req.)

18



$\because (AC)^2 = CD \times BC$

$\therefore \overline{AC}$ is a tangent to the circle passing through the points A , B , D (Q.E.D. 1)

, $\because \Delta\Delta$ ACD , BCA have :

m ($\angle$ DAC) = m ($\angle$ B)

(tangency and inscribed angles subtended by $\overset{\frown}{AD}$)

, $\angle$ C is a common angle.

$\therefore \Delta ACD \sim \Delta BCA$ (Q.E.D. 2)

$\because \frac{a(\Delta ACD)}{a(\Delta BCA)} = \left(\frac{CD}{CA}\right)^2 = \left(\frac{4}{6}\right)^2 = \frac{4}{9}$

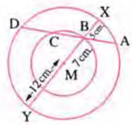
$\therefore a(\Delta ACD) = 4k$, $a(\Delta BCA) = 9k$

$\therefore a(\Delta ABD) = 5k$

$\therefore \frac{a(\Delta ABD)}{a(\Delta ABC)} = \frac{5k}{9k} = \frac{5}{9}$ (Q.E.D. 3)

19 Construction :

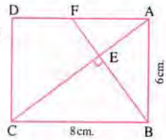
Draw the diameter $\overline{XY}$ in the major circle , intersecting the minor circle at B



Proof : $\because \overline{AD} \cap \overline{XY} = \{B\}$

$\therefore AB \times BD = XB \times BY = 5 \times 19 = 95$ (Q.E.D.)

20



$\because \Delta ABC$ is right-angled at B , $\overline{BE} \perp \overline{AC}$

$\therefore (AB)^2 = AE \times AC$ (1)

$\because$ Figure FECD is a cyclic quadrilateral.

(because m ($\angle$ D) + m ($\angle$ FEC) = 180°)

$\therefore AF \times AD = AE \times AC$ (2)

From (1) , (2) :

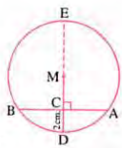
$\therefore (AB)^2 = AF \times AD$ (First req.)

$\therefore (6)^2 = AF \times 8$

$\therefore AF = 4.5$ cm. (Second req.)

21

$\because \overline{MC} \perp \overline{AB}$



$\therefore$ C is the midpoint of $\overline{AB}$

$\therefore AC = CB = 4$ cm.

Draw $\overline{DE}$ a diameter in the circle.

Let the radius length of the circle = r

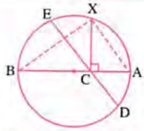
$\therefore DE = 2r$

$\therefore EC = DE - DC = (2r - 2)$ cm.

, $\because AC \times CB = DC \times CE$

$\therefore 4 \times 4 = 2\,(2r - 2) \quad \therefore r = 5$ cm. (The req.)

22



Construction :

Draw $\overline{XA}$, $\overline{XB}$

Proof :

$\because \angle$ AXB is a right angle (inscribed in a semicircle)

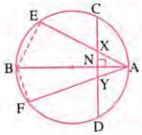
, $\overline{XC} \perp \overline{AB}$, $\overline{XC} \perp \overline{AB}$

$\because (XC)^2 = AC \times CB$

but $AC \times CB = DC \times CE$

$\therefore (XC)^2 = DC \times CE$ (Q.E.D.)

23



Construction :

Draw $\overline{BE}$, $\overline{BF}$

Proof : $\because$ $\overline{AB}$ is a diameter

$\therefore \angle E$ is a right angle ,

$\angle F$ is a right angle

, $\because \angle XNB$ is a right angle ,

$\angle YNB$ is also a right angle

$\therefore$ Both figures XNBE , YNBF are cyclic quadrilaterals

, $\because$ XNBE is a cyclic quadrilateral.

$\therefore AX \times AE = AN \times AB$ (1)

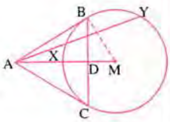
$\because$ YNBF is a cyclic quadrilateral

$\therefore AY \times AF = AN \times AB$ (2)

From (1) , (2) :

$\therefore AX \times AE = AY \times AF$ (Q.E.D.)

24



Construction :

Draw $\overline{MB}$

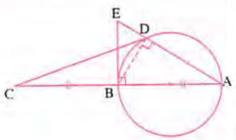
Proof : $\because m(\angle MBA) = 90°$

, $\overline{BC} \perp \overline{MA}$

$\therefore (AB)^2 = AD \times AM$ but $(AB)^2 = AX \times AY$

$\therefore AX \times AY = AD \times AM$ (Q.E.D.)

25



Construction :

Draw $\overline{BD}$

Proof :

In ΔABE :

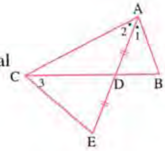
$\because m(\angle ABE) = 90°$

, $\overline{BD} \perp \overline{AE}$ $\therefore (AB)^2 = AD \times AE$

, $\because (CD)^2 = CB \times CA = AB \times 2\,AB = 2\,(AB)^2$

$\therefore (CD)^2 = 2\,AD \times AE$ (Q.E.D.)

26



$\because (AD)^2 = DB \times DC$

$\therefore AD \times DE = DB \times DC$

$\therefore$ ABEC is a cyclic quadrilateral

$\therefore m(\angle 1) = m(\angle 3)$

but $m(\angle 1) = m(\angle 2)$

$\therefore m(\angle 2) = m(\angle 3)$

$\therefore \Delta\Delta$ ECD , EAC have :

$m(\angle 2) = m(\angle 3)$, $\angle E$ is a common angle

$\therefore \Delta ECD \sim \Delta EAC$ (Q.E.D. 1)

From this similarity we get : $\frac{EC}{EA} = \frac{ED}{EC}$

$\therefore (EC)^2 = ED \times EA = ED \times 2\,ED$

$= 2\,(ED)^2$ (Q.E.D. 2)

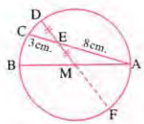
Third Higher skills

1

(1) c (2) d (3) c (4) b (5) a (6) c

(7) a (8) c (9) c (10) b (11) b (12) b

Instructions to solve 1 :



(1) Let $ED = EM = x$

$\therefore MD = 2x = r$

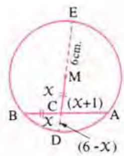
$\therefore FM = 2x = r$

, $\because EA \times EC = ED \times EF$

$\therefore 8 \times 3 = x \times 3x$ $\therefore 3x^2 = 24$

$\therefore x^2 = 8$ $\therefore x = 2\sqrt{2}$

$\therefore ME = 2\sqrt{2}$ cm.



(2) Let $MC = CB = x$

$\therefore CE \times CD = CA \times CB$

$\therefore (6 + x)(6 - x) = (x + 1)(x)$

$\therefore 36 - x^2 = x^2 + x$

$\therefore 2x^2 + x - 36 = 0$

$\therefore (2x + 9)(x - 4) = 0$

$\therefore x = \frac{-9}{2}$ (Refused) or $x = 4$

$\therefore AC = 4 + 1 = 5$, $CB = 4$

$\therefore AB = 5 + 4 = 9$ cm.

(3) $\because \overline{CX}$ is a tangent segment

$\therefore (CX)^2 = CB \times CA$

$\therefore (8)^2 = CB \times (CB + 30)$

$\therefore (CB)^2 + 30\,(CB) - 64 = 0$

$\therefore \big((CB) - 2\big)\big((CB) + 32\big) = 0$

$\therefore$ CB = 2 cm or CB = – 32 (refused)

, $\because \overline{DY}$ is a tangent segment

$\therefore (DY)^2 = DB \times DA$

$\therefore (20)^2 = DB\,(DB + 30)$

$\therefore (DB)^2 + 30\,(DB) - 400 = 0$

$\therefore ((DB) - 10)\,((DB) + 40) = 0$

$\therefore$ DB = 10 cm. or DB = – 40 (Refused)

$\therefore$ DC = 10 – 2 = 8 cm.

(4) $\because \overrightarrow{FE}$ is a tangent to the bigger circle at E

$\therefore (FE)^2 = FC \times FD \qquad \therefore (FE)^2 = 4 \times 9 = 36$

$\therefore$ FE = 6 cm.

, $\because FE \times FB = FC \times FA \;\therefore 6 \times FB = 4 \times 3$

$\therefore$ FB = 2 cm. $\qquad \therefore$ BE = 2 + 6 = 8 cm.

(5) $\because \overrightarrow{AB}$, $\overrightarrow{AD}$ are two tangents to the smaller circle at B and D

$\therefore AB = AD = x$

$\therefore AC = x - 1$, $AE = x + 2$

, $\because (AB)^2 = AC \times AE \quad \therefore x^2 = (x-1)(x+2)$

$\therefore x^2 = x^2 + x - 2 \qquad \therefore x - 2 = 0$

$\therefore x = 2$

(6) $\because \overline{CB}$ is a tangent to the circle

$\therefore (CB)^2 = CE \times CD$

$\therefore (CB)^2 = 3 \times (3 + 18) = 63$

$\therefore CB = \sqrt{63} = 3\sqrt{7}$ cm.

, $\because$ AB = AD (are two tangents)

$\therefore AC - AD = AC - AB = CB = 3\sqrt{7}$ cm.

(7) Draw $\overline{AD}$

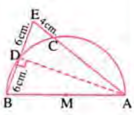
, $\because \overline{AB}$ is a diameter in the semicircle (M)

$\therefore \overline{AD} \perp \overline{BE}$

In Δ ABE :

$\because$ BD = DE = 6 cm.

, $\overline{AD} \perp \overline{BE}$



$\therefore \Delta$ ABE is an isosceles triangle

$\therefore$ AE = AB

, $\because EC \times EA = ED \times EB$

$\therefore 4 \times EA = 6 \times 12 \qquad \therefore$ EA = 18 cm.

$\therefore$ AB = 18 cm. $\qquad \therefore r = 18 \div 2 = 9$ cm.

(8) In Δ ABC : m ($\angle$ B) = 90°

$\therefore (AC)^2 = (12)^2 + (9)^2 = 225$

$\therefore AC = \sqrt{225} = 15$ cm.

, $\because AE \times AB = AD \times AC$

$\therefore 5 \times 12 = AD \times 15 \qquad \therefore$ AD = 4 cm.

$\therefore$ DC = 15 – 4 = 11 cm.

(9) $\because \frac{XE}{EY} = \frac{2}{3}$

$\therefore$ XE = 2 k , EY = 3 k

$\because EA \times EB = EX \times ED$

$\therefore EA \times EB = 2\,k \times (3\,k + 6)$ (1)

, $\because EA \times EB = EY \times EC$

$\therefore EA \times EB = 3\,k\,(2\,k + (CX))$ (2)

From (1) , (2) :

$\therefore 2\,k\,(3\,k + 6) = 3\,k\,(2\,k + (CX))$

$\therefore 6\,k^2 + 12\,k = 6\,k^2 + 3\,k\,(CX)$

$\therefore 12\,k = 3\,k\,(CX) \qquad \therefore$ CX = 4 cm.

(10) $\because$ The perimeter of Δ EMC = 20 cm.

$\therefore 4 + 6 + 2\,r + DC = 20$

$\therefore DC = 10 - 2\,r$ (1)

, $\because EA \times EB = ED \times EC$

$\therefore 4 \times (4 + 2\,r) = 6 \times (6 + DC)$ (2)

By substituting from (1) in (2) :

$\therefore 16 + 8\,r = 36 + 6\,(10 - 2\,r)$

$\therefore 16 + 8\,r = 36 + 60 - 12\,r$

$\therefore 20\,r = 80$

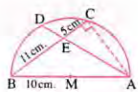
$\therefore r = 4$ cm.

$\therefore$ The perimeter of Δ EMC is sufficient to find the length of the radius.

(11) Draw $\overline{AC}$

$\because \overline{AB}$ is a diameter in the semicircle

$\therefore$ m ($\angle$ ACB) = 90°



In Δ ACB : $AC = \sqrt{20^2 - 16^2}$

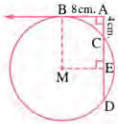
$\therefore$ AC = 12 cm.

In Δ ACE : AE $= \sqrt{12^2 + 5^2} = 13$ cm.

, $\because$ EC $\times$ EB $=$ EA $\times$ ED

$\therefore 5 \times 11 = 13 \times$ ED

$\therefore$ ED $= \frac{55}{13}$ cm.



(12) $\because \overrightarrow{AB}$ is a tangent

$\therefore (AB)^2 = AC \times AD$

$\therefore 8^2 = 4 \times AD$

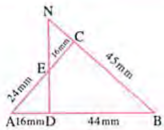
$\therefore$ AD = 16 cm.

$\therefore$ CD = 16 − 4 = 12 cm.

$\therefore \overline{ME} \perp \overline{CD}$ $\quad \therefore$ E is the midpoint of $\overline{CD}$

$\therefore$ EC = 6 cm. $\quad \therefore$ r = BM = 4 + 6 = 10 cm.

2



$\Delta\Delta$ ADE , ACB have :

$\frac{AD}{AC} = \frac{16}{40} = \frac{2}{5}$

, $\frac{AE}{AB} = \frac{24}{60} = \frac{2}{5}$

, $\angle$ A is a common angle.

$\therefore \Delta ADE \sim \Delta ACB$

$\therefore \frac{AD}{AC} = \frac{DE}{CB} = \frac{AE}{AB}$ $\quad \therefore \frac{16}{40} = \frac{DE}{45}$

$\therefore$ DE = 18 mm. (First req.)

, AD $\times$ AB = AE $\times$ AC

$\therefore$ BDEC is a cyclic quadrilateral.

$\therefore$ m ($\angle$ B) = m ($\angle$ CEN)

$\therefore \Delta\Delta$ DNB , CNE have :

m ($\angle$ CEN) = m ($\angle$ B) , $\angle$ N is a common angle

$\therefore \Delta DNB \sim \Delta CNE$ $\quad \therefore \frac{DN}{CN} = \frac{NB}{NE} = \frac{DB}{CE}$

$\therefore \frac{NB}{NE} = \frac{44}{16} = \frac{11}{4}$ $\quad \therefore \frac{NC + 45}{NE} = \frac{11}{4}$

$\therefore$ 11 NE − 4 NC = 180 (1)

, $\frac{DN}{CN} = \frac{11}{4}$ $\quad \therefore \frac{18 + EN}{CN} = \frac{11}{4}$

$\therefore$ 11 CN − 4 EN = 72 (2)

Solving the two equations (1) , (2) together :

$\therefore$ CN = 14.4 mm. , NE = 21.6 mm. (Second req.)

Answers of Life Applications on Unit Three

1

$\because$ The scale factor = drawing scale of the house

$\therefore$ The scale factor $= \frac{1}{150}$

$\therefore$ The dimensions of reciption are :

$5.6 \times 150 = 840$ cm. = 8.4 m.

, $3.4 \times 150 = 510$ cm. = 5.1 m. (First req.)

, the dimensions of the bedroom are :

$2.6 \times 150 = 390$ cm. = 3.9 m.

, $3.4 \times 150 = 510$ cm. = 5.1 m. (Second req.)

, the dimensions of the living room are :

$2.4 \times 150 = 360$ cm. = 3.6 m.

, $3.6 \times 150 = 540$ cm. = 5.4 m

$\therefore$ The area of the living room $= 3.6 \times 5.4 = 19.44$ m.^{2}

(Third req.)

The length of the bath room , the kitchen and the living room $= (2.6 + 2.6 + 3.6) \times 150 = 1320$ cm.

= 13.2 m.

and the width of this part $= 2.4 \times 150 = 360$ cm. = 3.6 m.

$\therefore$ The area of this part $= 3.6 \times 13.2 = 47.52$ m^2.

The length of bedroom and the reciption

$= (2.6 + 5.6) \times 150 = 1230$ cm. = 12.3 m.

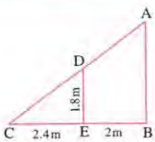
, the width of this part $= 3.4 \times 150 = 510$ cm. = 5.1 m.

$\therefore$ The area of this part $= 12.3 \times 5.1 = 62.73$ m.^{2}

$\therefore$ The area of the house $= 47.52 + 62.73 = 110.25$ m.^{2}

(Fourth req.)

2



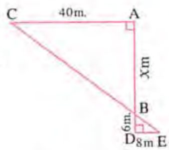
$\because \overline{DE} \parallel \overline{AB}$

$\therefore \Delta ABC \sim \Delta DEC$

$\therefore \frac{AB}{DE} = \frac{BC}{EC}$ $\quad \therefore \frac{AB}{1.8} = \frac{4.4}{2.4}$

$\therefore$ AB = 3.3 m. (The req.)

3



(1) In $\Delta\Delta$ ABC , DBE :

m ($\angle$ A) = m ($\angle$ D) = 90°

, m ($\angle$ ABC)

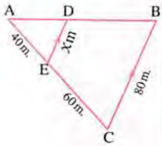
= m ($\angle$ DBE) (V.O.A)

$\therefore \Delta ABC \sim \Delta DBE$

$\therefore \frac{AB}{DB} = \frac{AC}{DE}$ $\quad \therefore \frac{x}{6} = \frac{40}{8}$

$\therefore x = 30$ m. (The req.)

(2) $\because \overline{DE} \parallel \overline{BC}$



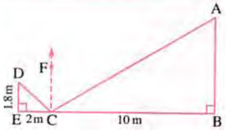
$\therefore \Delta ABC \sim \Delta ADE$

$\therefore \frac{BC}{DE} = \frac{AC}{AE}$

$\therefore \frac{80}{x} = \frac{100}{40}$

$\therefore x = 32$ m. (The req.)

4



In $\Delta\Delta$ ABC , DEC :

$\because$ m ($\angle$ ACF)

= m ($\angle$ DCF)

(measure of incidence angle = measure of reflection angle).

$\therefore$ m ($\angle$ ACB) = m ($\angle$ DCE)

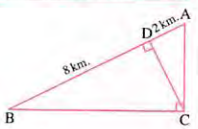
, m ($\angle$ B) = m ($\angle$ E) = 90°

$\therefore \Delta ABC \sim \Delta DEC$

$\therefore \frac{AB}{DE} = \frac{BC}{EC}$ $\therefore \frac{AB}{1.8} = \frac{10}{2}$

$\therefore$ AB = 9 m. (The req.)

5



$\because \Delta ABC$ is a right angled triangle at C

, $\overline{CD} \perp \overline{AB}$

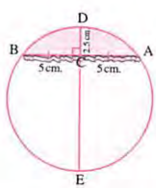
$\therefore (CD)^2 = AD \times DB = 2 \times 8 = 16$

$\therefore$ CD = 4 km. (First req.)

, $(BC)^2 = BD \times BA = 8 \times 10 = 80$

$\therefore BC = 4\sqrt{5}$ km. (Second req.)

6



$\because$ C is the midpoint of $\overline{AB}$,

$\overline{CD} \perp \overline{AB}$

$\therefore \overleftrightarrow{DC}$ passes through the centre of the circle.

$\therefore AC \times CB = DC \times CE$

$\therefore 5 \times 5 = 2.5 \times CE$

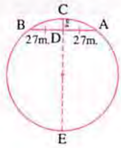
$\therefore$ CE = 10 cm.

$\therefore$ DE = 10 + 2.5 = 12.5 cm.

$\therefore$ The length of the radius of the disc

$= \frac{1}{180} = 6.25$ cm. (The req.)

7



$\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{CD} \perp \overline{AB}$

$\therefore \overleftrightarrow{CD}$ passes through the center of the circle and intersects it at E

$\therefore AD \times DB = CD \times DE$

$\therefore 27 \times 27 = 9 \times DE$ $\therefore$ DE = 81 m.

$\therefore$ CE = 81 + 9 = 90 m.

$\therefore$ length of the radius of the arc is circle

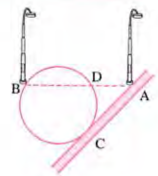
$= \frac{90}{2} = 45$ m. (The req.)

8

$\because 15 \times x = 10 \times 12$ $\therefore x = 8$

$\therefore$ The fountain is at a distance 8 metres from the entrance. (The req.)

9



To find the length of the wire $\overline{AB}$ as in the opposite figure.

Measure the length of the road $\overline{AC}$, $\overline{AD}$

and substitute in the law :

$(AC)^2 = AD \times AB$

$\therefore AB = \frac{(AC)^2}{AD}$ (Q.E.D.)

Guide Answers of "Unit Four"

Answers of Exercise 5

First Multiple choice questions

(1) First : b	Second : d	Third : b	(2) d
(3) c	(4) b	(5) b	(6) b
(7) d	(8) c	(9) c	(10) a
(11) d	(12) a	(13) c	(14) d
(15) c	(16) c	(17) d	(18) b
(19) b	(20) d	(21) b	(22) d
(23) b	(24) a	(25) b	(26) c

Second Essay questions

1

(1) $\because \frac{AD}{DB} = \frac{15}{9} = \frac{5}{3}$, $\frac{AE}{EC} = \frac{18}{12} = \frac{3}{2}$

$\therefore \frac{AD}{DB} \neq \frac{AE}{EC}$ $\therefore \overline{DE}$ is not parallel to $\overline{BC}$

(2) $\because \frac{CA}{AE} = \frac{45}{63} = \frac{5}{7}$, $\frac{BA}{AD} = \frac{55}{77} = \frac{5}{7}$

$\therefore \frac{CA}{AE} = \frac{BA}{AD}$ $\therefore \overline{DE} \parallel \overline{BC}$

(3) $\because \frac{DA}{AB} = \frac{3}{4}$, $\frac{EA}{AC} = \frac{6}{8} = \frac{3}{4}$

$\therefore \frac{DA}{AB} = \frac{EA}{AC}$ $\therefore \overline{DE} \parallel \overline{BC}$

(4) $\because \frac{AD}{DB} = \frac{6}{10} = \frac{3}{5}$, $\frac{AE}{EC} = \frac{9}{15} = \frac{3}{5}$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ $\therefore \overline{DE} \parallel \overline{BC}$

(5) $\because \frac{AD}{DB} = \frac{28}{20} = \frac{7}{5}$, $\frac{AE}{EC} = \frac{42}{24} = \frac{7}{4}$

$\therefore \frac{AD}{DB} \neq \frac{AE}{EC}$ $\therefore \overline{DE}$ is not parallel to $\overline{BC}$

(6) In the right-angled triangle AED at E :

$(AD)^2 = (AE)^2 + (ED)^2 = 225 + 400 = 625$

$\therefore AD = 25$ cm.

$, \because \frac{AE}{EC} = \frac{15}{9} = \frac{5}{3}$, $\frac{AD}{DB} = \frac{25}{15} = \frac{5}{3}$

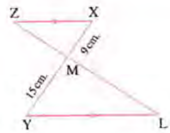
$\therefore \frac{AE}{EC} = \frac{AD}{DB}$ $\therefore \overline{DE} \parallel \overline{BC}$

2

$\because \frac{AE}{ED} = \frac{5}{15} = \frac{1}{3}$, $\frac{BE}{EC} = \frac{4}{12} = \frac{1}{3}$

$\therefore \frac{AE}{ED} = \frac{BE}{EC}$ $\therefore \overline{AB} \parallel \overline{CD}$ (Q.E.D.)

3



$\because \overline{XZ} \parallel \overline{LY}$

$\therefore \frac{ZM}{ZL} = \frac{XM}{XY}$

$\therefore \frac{ZM}{36} = \frac{9}{24}$

$\therefore ZM = 13.5$ cm. (The req.)

4

$\because \overline{BC} \parallel \overline{ED}$ $\therefore \frac{BA}{AD} = \frac{CA}{AE}$

$\therefore \frac{6}{12} = \frac{5}{AE}$ $\therefore AE = 10$ cm. (First req.)

$\because \overline{XY} \parallel \overline{DE}$ $\therefore \frac{DX}{AD} = \frac{YE}{AE}$

$\therefore \frac{DX}{12} = \frac{4}{10}$ $\therefore DX = 4.8$ cm. (Second req.)

5

(1) $\because \overline{DE} \parallel \overline{BC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$

$\therefore \frac{4}{8} = \frac{x}{6}$ $\therefore x = 3$

(2) $\because \overline{DE} \parallel \overline{BC}$ $\therefore \frac{AE}{EC} = \frac{AD}{DB}$

$\therefore \frac{x}{5} = \frac{x-2}{3}$ $\therefore 5x - 10 = 3x$

$\therefore 2x = 10$ $\therefore x = 5$

(3) $\because \overline{DF} \parallel \overline{AC}$ $\therefore \frac{AD}{AB} = \frac{CF}{CB}$

$\therefore \frac{x}{21} = \frac{6}{14}$ $\therefore x = 9$

(4) $\because 2\,DB = 12$ $\therefore DB = 6$ cm.

$, \because 3\,FC = 12$ $\therefore FC = 4$ cm.

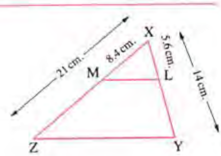
$, \because \overline{DF} \parallel \overline{AC}$ $\therefore \frac{AD}{DB} = \frac{CF}{FB}$

$\therefore \frac{x}{6} = \frac{4}{x+5}$ $\therefore x(x+5) = 24$

$\therefore x^2 + 5x - 24 = 0$ $\therefore (x+8)(x-3) = 0$

$\therefore x = -8$ (refused) or $x = 3$

6



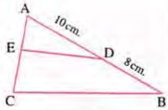
$\because \frac{XL}{XY} = \frac{5.6}{14} = \frac{2}{5}$

$, \frac{XM}{XZ} = \frac{8.4}{21} = \frac{2}{5}$

$\therefore \frac{XL}{XY} = \frac{XM}{XZ}$

$\therefore \overline{LM} \parallel \overline{YZ}$ (Q.E.D.)

7



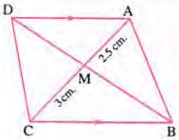
$\because 5\ AE = 4\ EC$

$\therefore \frac{AE}{EC} = \frac{4}{5}$

$, \because \frac{AD}{DB} = \frac{10}{8} = \frac{5}{4}$ $\qquad \therefore \frac{AE}{EC} \neq \frac{AD}{DB}$

$\therefore \overline{DE}$ is not parallel to $\overline{BC}$ (Q.E.D.)

8



$\because \overline{AD} \parallel \overline{BC}$

$\therefore \frac{MA}{AC} = \frac{MD}{DB}$

$\therefore \frac{2.5}{2.5+3} = \frac{MD}{7\frac{1}{3}}$

$\therefore MD = 3\frac{1}{3}$ cm. (First req.)

$\therefore MB = 7\frac{1}{3} - 3\frac{1}{3} = 4$ cm. (Second req.)

9

In Δ ABC : $\because \overline{DF} \parallel \overline{BC}$

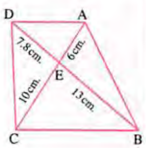
$\therefore \frac{AD}{DB} = \frac{AF}{FC}$ $\qquad \therefore \frac{6}{5} = \frac{AF}{5.5}$

$\therefore AF = 6.6$ cm. $\qquad \therefore EF = 6.6 - 3.6 = 3$ cm.

, in Δ ABF : $\frac{AD}{DB} = \frac{6}{5}$, $\frac{AE}{EF} = \frac{3.6}{3} = \frac{6}{5}$

$\therefore \frac{AD}{DB} = \frac{AE}{EF}$ $\qquad \therefore \overline{DE} \parallel \overline{BF}$ (Q.E.D.)

10



$\because \frac{AE}{EC} = \frac{6}{10} = \frac{3}{5}$

$, \because \frac{DE}{EB} = \frac{7.8}{13} = \frac{3}{5}$

$\therefore \frac{AE}{EC} = \frac{DE}{EB}$

$\therefore \overline{AD} \parallel \overline{BC}$

$\therefore$ ABCD is a trapezium. (Q.E.D.)

11

In Δ DAE , which is right at A :

$\because (AD)^2 = (DE)^2 - (AE)^2 = 25 - 16 = 9$

$\therefore AD = 3$ cm.

In Δ ABC : $\because \frac{AD}{DB} = \frac{3}{6} = \frac{1}{2}$, $\frac{AE}{EC} = \frac{4}{8} = \frac{1}{2}$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ $\qquad \therefore \overline{DE} \parallel \overline{BC}$ (First req.)

In Δ ABC which is right at A :

$\because (BC)^2 = (AB)^2 + (AC)^2 = 81 + 144 = 225$

$\therefore BC = 15$ cm. (Second req.)

12

$\because \overline{XY} \parallel \overline{BC}$ $\qquad \therefore \frac{AX}{AB} = \frac{AY}{AC}$

, from the properties of the proportion :

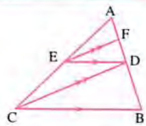
$\therefore \frac{AX}{AB} = \frac{AY}{AC} = \frac{AX+AY}{AB+AC} = \frac{3}{5}$

$\therefore \frac{AX}{AX+3} = \frac{6}{6+CY} = \frac{3}{5}$ $\qquad \therefore 5\ AX = 3\ AX + 9$

$\therefore AX = 4.5$ cm. (First req.)

$, 18 + 3\ CY = 30$ $\quad \therefore CY = 4$ cm. (Second req.)

13



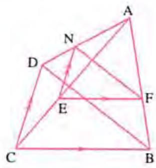
$\because \overline{DE} \parallel \overline{BC}$

$\therefore \frac{AD}{AB} = \frac{AE}{AC}$

$, \because \overline{EF} \parallel \overline{CD}$ $\qquad \therefore \frac{AF}{AD} = \frac{AE}{AC}$

$\therefore \frac{AF}{AD} = \frac{AD}{AB}$ $\qquad \therefore (AD)^2 = AF \times AB$ (Q.E.D.)

14



$\because \overline{EF} \parallel \overline{CB}$

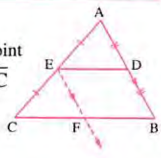
$\therefore \frac{AE}{AC} = \frac{AF}{AB}$

$, \because \overline{EN} \parallel \overline{CD}$

$\therefore \frac{AE}{AC} = \frac{AN}{AD}$ $\qquad \therefore \frac{AF}{AB} = \frac{AN}{AD}$

$\therefore$ In Δ ABD : $\overline{FN} \parallel \overline{BD}$ (Q.E.D.)

15



Given : Δ ABC , D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

R.T.P. : (1) $\overline{DE} \parallel \overline{BC}$

(2) $DE = \frac{1}{2}$ BC

Construction : Draw $\overrightarrow{EF} \parallel \overline{AB}$ to intersect $\overline{BC}$ at F

Proof : $\because \frac{AD}{DB} = 1$, $\frac{AE}{EC} = 1$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ $\qquad \therefore \overline{DE} \parallel \overline{BC}$ (Q.E.D. 1)

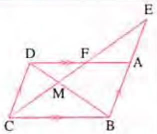
$\because \overline{EF} \parallel \overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore$ F is the midpoint of $\overline{BC}$ $\quad \therefore BF = \frac{1}{2} BC$

, $\because$ the figure BDEF is a parallelogram.

$\therefore DE = BF = \frac{1}{2} BC$ (Q.E.D. 2)

16



$\because \overline{DF} \parallel \overline{BC}$

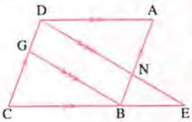
$\therefore \frac{CM}{MF} = \frac{BM}{MD}$

, $\because \overline{CD} \parallel \overline{BE}$

$\therefore \frac{ME}{MC} = \frac{MB}{MD}$ $\qquad \therefore \frac{CM}{MF} = \frac{ME}{MC}$

$\therefore (CM)^2 = MF \times ME$ (Q.E.D.)

17



$\because \overline{AD} \parallel \overline{BE}$

$\therefore \frac{AN}{NB} = \frac{DN}{NE}$

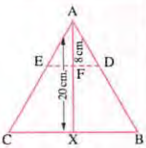
, $\because \overline{NB} \parallel \overline{CD}$

$\therefore \frac{DN}{NE} = \frac{BC}{BE}$ $\qquad \therefore \frac{AN}{NB} = \frac{BC}{BE}$

, $\because \overline{BG} \parallel \overline{DE}$ $\qquad \therefore \frac{BC}{BE} = \frac{CG}{GD}$

$\therefore \frac{AN}{NB} = \frac{CG}{GD}$ (Q.E.D.)

18



In Δ ABC : $\because$ 3 AD = 2 DB

$\therefore \frac{AD}{DB} = \frac{2}{3}$

, $\because \frac{AF}{FX} = \frac{8}{12} = \frac{2}{3}$

$\therefore \frac{AD}{DB} = \frac{AF}{FX}$ $\qquad \therefore \overline{DF} \parallel \overline{BX}$ (1)

In Δ AXC : $\because$ 5 CE = 3 AC $\qquad \therefore \frac{AC}{CE} = \frac{5}{3}$

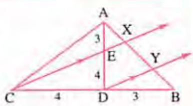
$\therefore \frac{AX}{XF} = \frac{20}{12} = \frac{5}{3}$ $\qquad \therefore \frac{AC}{CE} = \frac{AX}{XF}$

$\therefore \overline{FE} \parallel \overline{XC}$ (2)

From (1) , (2) : $\therefore \overline{DE} \parallel \overline{BC}$

$\therefore$ The points D , F and E are collinear (Q.E.D.)

19



In Δ ADY : $\because \overline{EX} \parallel \overline{DY}$

$\therefore \frac{AE}{ED} = \frac{AX}{XY} = \frac{3}{4}$ (1)

In Δ BCX : $\because \overline{DY} \parallel \overline{CX}$

$\therefore \frac{BD}{DC} = \frac{BY}{XY} = \frac{3}{4}$ (2)

From (1) , (2) : $\therefore \frac{AX}{XY} = \frac{BY}{XY}$

$\therefore AX = BY$ (Q.E.D.)

20

In Δ ABC : $\because \overline{DX} \parallel \overline{AC}$

$\therefore \frac{BX}{BC} = \frac{BD}{BA}$ $\quad \therefore \frac{BX}{13.5} = \frac{2}{5}$ $\quad \therefore BX = 5.4$ cm.

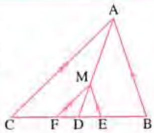
, $\because \overline{EY} \parallel \overline{AB}$ $\therefore \frac{CY}{CB} = \frac{CE}{CA}$ $\quad \therefore \frac{CY}{13.5} = \frac{4}{9}$

$\therefore CY = 6$ cm.

$\therefore XY = BC - (BX + CY) = 13.5 - (5.4 + 6) = 2.1$ cm.

(The req.)

21



$\because \overline{ME} \parallel \overline{AB}$ $\qquad \therefore \frac{DM}{DA} = \frac{DE}{DB}$

, $\because \overline{MF} \parallel \overline{AC}$

$\therefore \frac{DM}{DA} = \frac{DF}{DC}$ $\qquad \therefore \frac{DE}{DB} = \frac{DF}{DC}$

, $\because$ DB = DC $\qquad \therefore$ DE = DF

$\therefore$ D is the midpoint of $\overline{EF}$ (Q.E.D. 1)

, $\because$ D is the midpoint of $\overline{BC}$

$\therefore \overline{AD}$ is a median of Δ ABC $\quad \therefore DM = \frac{1}{3} AD$

In Δ ABD : $\therefore DE = \frac{1}{3} BD$

, in Δ ACD : $\therefore DF = \frac{1}{3} DC$

By adding : $\therefore DE + DF = \frac{1}{3} (BD + DC)$

$\therefore EF = \frac{1}{3} BC$ (Q.E.D. 2)

22

$\because \frac{\text{The area of } \Delta \text{ ADE}}{\text{The area of } \Delta \text{ ABE}} = \frac{AD}{AB}$

(because they have the same height)

, $\frac{\text{the area of } \Delta \text{ ABE}}{\text{the area of } \Delta \text{ ABC}} = \frac{AE}{AC}$

(because they have the same height)

, $\because \frac{AD}{AB} = \frac{AE}{AC}$ (because $\overline{DE} \parallel \overline{BC}$)

$\therefore \frac{\text{The area of } \Delta \text{ ADE}}{\text{The area of } \Delta \text{ ABE}} = \frac{\text{The area of } \Delta \text{ ABE}}{\text{The area of } \Delta \text{ ABC}}$ (Q.E.D.)

Third Higher skills

1

(1) c	(2) b	(3) c
(4) a	(5) b	(6) b

Instructions to solve 1 :

(1) $\because m(\angle YDF) = m(\angle YCB)$ (corresponding angles)

$, \because m(\angle ADY) = m(\angle CDB)$ (V.O.A)

$, \because m(\angle ADY) = m(\angle FDY)$

$\therefore m(\angle CDB) = m(\angle DCB)$

$\therefore BC = BD = 15$ cm.

In $\Delta ABC : \because \overline{DE} \,//\, \overline{BC}$ $\quad \therefore \frac{AD}{AB} = \frac{DE}{BC}$

$\therefore \frac{AD}{AD + 15} = \frac{10}{15} = \frac{2}{3}$

$\therefore 3\,AD = 2\,AD + 30$ $\quad \therefore AD = 30$ cm.

(2) To prove that $\overline{DE} \,//\, \overline{BC}$, it is sufficient to be

$\frac{AD}{AB} = \frac{AE}{AC}$

$, \because \overline{DF} \,//\, \overline{BE}$ $\quad \therefore \frac{AD}{AB} = \frac{AF}{AE}$

i.e. $\frac{AE}{AC} = \frac{AF}{AE}$ $\quad \therefore AF \times AC = (AE)^2$

(3) $\because 2x^2 - 3xy - 5y^2 = 0$

$\therefore (2x - 5y)(x + y) = 0$

$\therefore 2x = 5y$ $\quad$ *i.e.* $\frac{y}{x} = \frac{2}{5}$

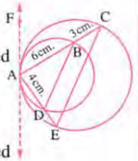
or $x = -y$ (Refused)

In $\Delta ABC : \because \overline{ED} \,//\, \overline{BC}$

$\therefore \frac{AE}{AB} = \frac{ED}{BC}$ $\quad \therefore \frac{AE}{10} = \frac{y}{x} = \frac{2}{5}$

$\therefore AE = 4$ cm. $\quad \therefore EB = 10 - 4 = 6$ cm.

(4) Draw the common tangent $\overleftrightarrow{AF}$



$\because m(\angle FAB) = m(\angle ADB)$

(angle of tangency and inscribed angle subtended the arc $\overset{\frown}{AB}$)

$, \because m(\angle FAC) = m(\angle AEC)$

(angle of tangency and inscribed angle subtended the arc $\overset{\frown}{AC}$)

$\therefore m(\angle ADB) = m(\angle AEC)$

(in corresponding position)

$\therefore \overline{DB} \,//\, \overline{EC}$ $\quad \therefore \frac{AB}{BC} = \frac{AD}{DE}$

$\therefore \frac{6}{3} = \frac{4}{DE}$ $\quad \therefore DE = 2$ cm.

(5) In $\Delta\Delta$ ACE , ECF

$\because \overline{AE} , \overline{EF}$ are on the same straight line and have common vertex C

$\therefore \frac{a.(\Delta ACE)}{a.(\Delta CEF)} = \frac{AE}{EF}$

$\therefore \frac{AE}{EF} = \frac{15}{9} = \frac{5}{3}$ $\quad \therefore \frac{AE}{AF} = \frac{5}{8}$

In $\Delta ABF : \because \overline{DE} \,//\, \overline{BF}$ $\quad \therefore \frac{AD}{AB} = \frac{AE}{AF}$

$\therefore \frac{AD}{16} = \frac{5}{8}$ $\quad \therefore AD = 10$ cm.

(6) In $\Delta\Delta$ CBE , EBA :

$\because \overline{CE} , \overline{EA}$ are on the same straight line and have common vertex B

$\therefore \frac{a.(\Delta ABE)}{a.(\Delta CBE)} = \frac{AE}{CE}$ $\quad \therefore \frac{a.(\Delta ABE)}{9} = \frac{4}{2} = \frac{2}{1}$

$\therefore a(\Delta ABE) = 18\ cm^2.$

In $\Delta ABC : \because \overline{DE} \,//\, \overline{BC}$

$\therefore \frac{AD}{DB} = \frac{AE}{EC} = \frac{4}{2} = \frac{2}{1}$ $\quad \therefore \frac{AD}{AB} = \frac{2}{3}$

In $\Delta\Delta$ ADE , ABE : $\because \overline{AD} , \overline{AB}$ are on the same straight line and have common vertex E.

$\therefore \frac{a.(\Delta ADE)}{a.(\Delta ABE)} = \frac{AD}{AB}$ $\quad \therefore \frac{a.(\Delta ADE)}{18} = \frac{2}{3}$

$\therefore a.(\Delta ADE) = 12\ cm^2.$

2

$\because \frac{AD}{DB} = \frac{CE}{EA}$ $\quad \therefore \frac{AD}{AD + DB} = \frac{CE}{CE + EA}$

$\therefore \frac{AD}{AB} = \frac{CE}{CA}$ (1)

$, \because \overline{EG} \,//\, \overline{BC}$ $\quad \therefore \frac{CE}{CA} = \frac{BG}{BA}$ (2)

From (1) , (2) : $\therefore \frac{AD}{AB} = \frac{BG}{BA}$ $\quad \therefore AD = BG$

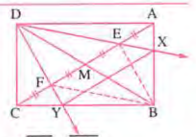
$, \because AX = BX$ (given)

$\therefore AD - AX = BG - BX$ $\quad \therefore DX = XG$

In $\Delta DEG : \because X$ is the midpoint of $\overline{DG}$, $\overline{XF} \,//\, \overline{GE}$

$\therefore F$ is the midpoint of $\overline{DE}$ (Q.E.D.)

3



Construction :

Draw $\overline{BE} , \overline{BF}$

Proof : In the figure BEDF

$\because M$ is the midpoint of each of $\overline{EF} , \overline{BD}$

$\therefore$ The figure BEDF is a parallelogram.

In $\Delta ABF : \because \overline{XE} \,//\, \overline{BF}$

$\therefore \frac{AX}{XB} = \frac{AE}{EF} = \frac{1}{2}$ (1)

, in Δ BCE : $\because \overline{FY} // \overline{EB}$

$\therefore \frac{CY}{YB} = \frac{CF}{FE} = \frac{1}{2}$ (2)

From (1) , (2) : $\therefore \frac{AX}{XB} = \frac{CY}{YB}$ this in the triangle ABC

$\therefore \overline{XY} // \overline{AC}$ (Q.E.D.)

Answers of Exercise 6

First Multiple choice questions

(1) b	(2) d	(3) c	(4) b
(5) b	(6) b	(7) c	(8) b
(9) b	(10) b	(11) b	(12) d
(13) c	(14) c	(15) c	(16) d
(17) a	(18) c	(19) c	(20) d
(21) c			

Second Essay questions

1

(1) EF	(2) DF	(3) DE	(4) DF
(5) ME	(6) DF	(7) ME	(8) MC

2

(1) $\because \overline{AB} // \overline{DE}$, BE = EC $\therefore$ AD = DC

$\therefore 3x - 1 = 2x + 3$ $\therefore x = 4$

, $\because$ BE = EC $\therefore 2y + 7 = 13$

$\therefore 2y = 6$ $\therefore y = 3$

(2) $\because \overline{AD} // \overline{BE} // \overline{CF}$, DE = EF = FM

$\therefore$ AB = BC = CM $\therefore x^2 - 3 = 3x + 1$

$\therefore x^2 - 3x - 4 = 0$ $\therefore (x-4)(x+1) = 0$

$\therefore x = 4$ or $x = -1$ (refused)

, $\because$ BC = CM $\therefore 2y - 1 = 13$

$\therefore 2y = 14$ $\therefore y = 7$

(3) $\because \overline{AB} // \overline{DC} // \overline{EF}$ $\therefore \frac{AM}{BM} = \frac{MD}{MC} = \frac{DF}{CE}$

$\therefore \frac{y-4}{4x-1} = \frac{2}{3} = \frac{y-4}{2x+7}$

$\therefore 4x - 1 = 2x + 7$ $\therefore 2x = 8$

$\therefore x = 4$ $\therefore \frac{y-4}{15} = \frac{2}{3}$

$\therefore y - 4 = 10$ $\therefore y = 14$

(4) $\because \overline{AB} // \overline{CD} // \overline{EF}$, BD = DF

$\therefore$ AC = CE $\therefore 2x - 3 = x + 2$

$\therefore x = 5$

$\because$ BD = DF $\therefore y + 3 = 6$ $\therefore y = 3$

(5) $\because x + 3 = 2y + 5$

$\therefore x - 2y = 2$ (1)

, $x - 3 = y + 2$ $\therefore x - y = 5$ (2)

by subtracting (1) from (2) :

$\therefore y = 3$, by substituting in (2) :

$\therefore x = 8$

(6) $\because \overline{DE} // \overline{BC}$, AD = DB $\therefore$ AE = EC

$\therefore x + 6 = 3x - 2$ $\therefore 2x = 8$ $\therefore x = 4$

In Δ ABC :

$\because$ D , E are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively

$\therefore DE = \frac{1}{2} BC$ $\therefore 3y - 2 = \frac{1}{2}(5y - 1)$

$\therefore 6y - 4 = 5y - 1$ $\therefore y = 3$

(7) $\because 2x + 1 = y - 3$ $\therefore x^2 - 5 = 3x - 1$

$\therefore x^2 - 3x - 4 = 0$ $\therefore (x-4)(x+1) = 0$

$\therefore x = 4$ or $x = -1$ (refused)

$\therefore y - 3 = 9$ $\therefore y = 12$

(8) $\because \frac{3x+2}{15} = \frac{2x+4}{12}$

$\therefore 12(3x+2) = 15(2x+4)$

$\therefore 36x + 24 = 30x + 60$ $\therefore 6x = 36$

$\therefore x = 6$ $\therefore \frac{y}{15} = \frac{11}{12}$ $\therefore y = 13\frac{3}{4}$

(9) $\because \frac{x+1}{9} = \frac{6}{5} = \frac{10}{y}$

$\therefore x + 1 = \frac{54}{5} = 10\frac{4}{5}$ $\therefore x = 9\frac{4}{5}$

, $y = \frac{50}{6} = 8\frac{1}{3}$

3

$\because L_1 // L_2 // L_3 // L_4$ and M , M̀ are two transversals

$\therefore \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZN}$ $\therefore \frac{1.6}{XY} = \frac{2.4}{3.6} = \frac{CD}{4.8}$

$\therefore XY = \frac{1.6 \times 3.6}{2.4} = 2.4$ cm.

, $CD = \frac{2.4 \times 4.8}{2.4} = 3.2$ cm. (The req.)

4

$\because \overleftrightarrow{AB} \parallel \overleftrightarrow{DE} \parallel \overleftrightarrow{XF}$

, $\overleftrightarrow{CB}$, $\overleftrightarrow{CA}$ are two transversals.

$\therefore \frac{CX}{CF} = \frac{XE}{FD} = \frac{BE}{AD}$ $\quad\therefore \frac{5}{7.5} = \frac{4}{FD} = \frac{BE}{6}$

$\therefore FD = 6$ cm. , $BE = 4$ cm. (The req.)

5

$\because \overline{AC} \parallel \overline{FE} \parallel \overline{DB}$ $\quad\therefore \frac{ME}{MF} = \frac{EB}{FD}$

$\therefore \frac{6}{MF} = \frac{9}{15}$ $\quad\therefore MF = 10$ cm. (First req.)

, $\because \frac{AM}{MB} = \frac{CM}{MD}$ $\quad\therefore \frac{AM}{15} = \frac{18}{25}$

$\therefore AM = 10.8$ cm. (Second req.)

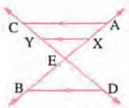
6

$\because \overline{AB} \parallel \overline{CD} \parallel \overline{EF}$ $\quad\therefore \frac{AC}{BD} = \frac{CK}{DK} = \frac{KF}{KE} = \frac{AF}{BE}$

$\therefore \frac{5}{BD} = \frac{10}{DK} = \frac{7.5}{KE} = \frac{22.5}{18}$

$\therefore BD = 4$ cm. , $DK = 8$ cm. , $KE = 6$ cm. (The req.)

7



$\because \overline{XY} \parallel \overline{BD} \parallel \overline{AC}$

$\therefore \frac{AX}{CY} = \frac{EB}{ED}$

$\therefore AX \times ED = CY \times EB$ (Q.E.D.)

8

$\because \overline{AB} \parallel \overline{CD} \parallel \overline{EF} \parallel \overline{XY} \parallel \overline{ZK}$

$\therefore \frac{AC}{BD} = \frac{CE}{DF} = \frac{EX}{FY} = \frac{XZ}{YK}$ $\quad\therefore \frac{2}{2.5} = \frac{CE}{DF} = \frac{EX}{4.5} = \frac{XZ}{3}$

$\therefore EX = 3.6$ cm. , $XZ = 2.4$ cm.

$\therefore CE = 12 - (3.6 + 2.4) = 6$ cm.

$\therefore DF = 7.5$ cm. (The req.)

9

$\because AX : XY : YC = 2 : 3 : 5$

$\therefore BD : DE : EC = 2 : 3 : 5$

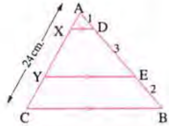
$\therefore \frac{BD}{2} = \frac{DE}{3} = \frac{EC}{5}$ $\quad\therefore \frac{BD}{2} = \frac{7.5}{3} = \frac{EC}{5}$

$\therefore BD = 5$ cm. , $EC = 12.5$ cm.

, $\because \frac{AX}{AC} = \frac{BD}{BC}$ $\quad\therefore \frac{4}{AC} = \frac{5}{5 + 7.5 + 12.5}$

$\therefore AC = 20$ cm. (The req.)

10



$\because \overline{DX} \parallel \overline{EY} \parallel \overline{BC}$

$\therefore \frac{AX}{AD} = \frac{XY}{DE} = \frac{YC}{EB}$

$\therefore \frac{AX}{1} = \frac{XY}{3} = \frac{YC}{2}$

$\therefore \frac{AX + XY + YC}{1 + 3 + 2} = \frac{AC}{6} = \frac{24}{6} = 4$

$\therefore AX = 4$ cm. , $XY = 12$ cm.

$\therefore YC = 8$ cm. (The req.)

11

$\because AB : BC : CD$

$$\begin{array}{ccccc} 1 & : & 2 & & \\ & & 4 & : & 5 \\ \hline 2 & : & 4 & : & 5 \end{array}$$

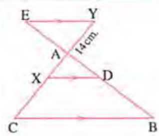
, $\because L_1 \parallel L_2 \parallel L_3 \parallel L_4$ $\quad\therefore \frac{XY}{AB} = \frac{YZ}{BC} = \frac{ZN}{CD}$

$\therefore \frac{XY}{2} = \frac{YZ}{4} = \frac{ZN}{5} = \frac{XY + YZ + ZN}{2 + 4 + 5} = \frac{XN}{11} = \frac{16.5}{11} = \frac{3}{2}$

$\therefore XY = 3$ cm. , $YZ = 6$ cm. , $ZN = 7.5$ cm.

(The req.)

12



$\because BD : DA : AE$

$= 5 : 3 : \frac{BD + DA}{2}$

$= 5 : 3 : 4$

, $\because \overline{BC} \parallel \overline{DX} \parallel \overline{EY}$

$\therefore \frac{CX}{BD} = \frac{XA}{DA} = \frac{AY}{AE}$ $\quad\therefore \frac{CX}{5} = \frac{XA}{3} = \frac{14}{4}$

$\therefore CX = 17.5$ cm. , $XA = 10.5$ cm.

$\therefore AC = 17.5 + 10.5 = 28$ cm. (The req.)

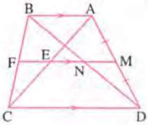
13

$\because \overline{DC} \parallel \overline{FE}$ $\quad\therefore \frac{DG}{GF} = \frac{CG}{GE}$

, $\because \frac{DG}{GF} = \frac{AG}{GC}$ (given) $\quad\therefore \frac{CG}{GE} = \frac{AG}{CG}$

$\therefore (CG)^2 = AG \times GE$ (Q.E.D.)

14



$\because \overline{AB} // \overline{MF} // \overline{DC}$ and $\overleftrightarrow{DA}$

, $\overleftrightarrow{DB}$ are two transversals

$\therefore \frac{DM}{DN} = \frac{MA}{NB}$

, $\because$ MD = MA $\quad \therefore$ DN = NB

$\therefore$ N is the midpoint of $\overline{BD}$

similarly, we prove that E is the midpoint of $\overline{AC}$ and F is the midpoint of $\overline{BC}$ (Q.E.D. 1)

In Δ ADC :

$\because$ M , E are the midpoints of $\overline{AD}$, $\overline{AC}$ respectively.

$\therefore ME = \frac{1}{2} DC$ (1)

, in Δ ABC :

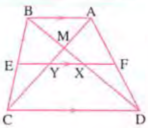
$\because$ E , F are the midpoints of $\overline{AC}$, $\overline{BC}$ respectively

$\therefore EF = \frac{1}{2} AB$ (2)

From (1) , (2) : $\therefore ME + EF = \frac{1}{2} (DC + AB)$

$\therefore MF = \frac{1}{2} (DC + AB)$ (Q.E.D. 2)

15



In Δ ABC :

$\because$ E is the midpoint of $\overline{BC}$

, $\overrightarrow{EY} // \overline{AB}$

$\therefore$ Y is the midpoint of $\overline{AC}$,

$EY = \frac{1}{2} AB$ (Q.E.D. 1)

, $\because \overline{AB} // \overline{FE} // \overline{DC}$ and $\overleftrightarrow{AC}$, $\overleftrightarrow{DB}$ are two transversals

$\therefore \frac{AM}{BM} = \frac{MY}{MX} = \frac{YC}{XD}$ $\qquad \therefore \frac{AM + MY}{BM + MX} = \frac{MY + YC}{MX + XD}$

$\therefore \frac{AY}{BX} = \frac{MC}{MD}$ $\qquad \therefore \frac{AY}{MC} = \frac{BX}{DM}$ (Q.E.D. 2)

16

It is possible to find $\frac{AB}{BC}$ by three methods :

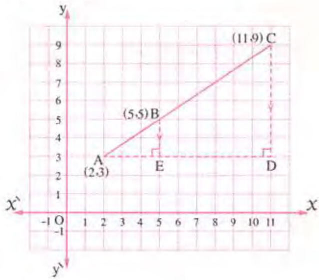
First method : Using the distance between two points in the cartesian plane :

$\therefore AB = \sqrt{(5-2)^2 + (5-3)^2} = \sqrt{13}$ length unit.

, $BC = \sqrt{(11-5)^2 + (9-5)^2} = 2\sqrt{13}$ length unit.

$\therefore \frac{AB}{BC} = \frac{\sqrt{13}}{2\sqrt{13}} = \frac{1}{2}$

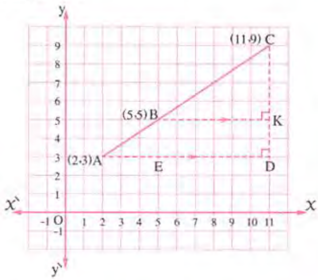
Second method :



Make $\overline{AC}$ as a hypotenuse in a right-angled triangle at D where D (11 , 3) , then draw $\overrightarrow{BE} // \overline{CD}$ to intersect $\overline{AD}$ at E (5 , 3)

In Δ ADC : $\because \overline{BE} // \overline{CD}$ $\qquad \therefore \frac{AB}{BC} = \frac{AE}{ED} = \frac{3}{6} = \frac{1}{2}$

Third method :



as in the previous but draw $\overrightarrow{BK} // \overline{AD}$ to intersect $\overline{CD}$ at K (11 , 5)

In Δ ADC : $\therefore \overline{BK} // \overline{AD}$ $\qquad \therefore \frac{AB}{BC} = \frac{DK}{KC} = \frac{2}{4} = \frac{1}{2}$

Third Higher skills

1 (1) b $\quad$ (2) b $\quad$ (3) c $\quad$ (4) d

Instructions to solve 1 :

(1) $\because \overleftrightarrow{AB} // \overleftrightarrow{CD} // \overleftrightarrow{EF}$

$\therefore \frac{x}{4} = \frac{3}{y}$ $\qquad \therefore x y = 12$

, $(x + y)^2 = x^2 + y^2 + 2 x y = 57 + 2 \times 12 = 81$

$\therefore x + y = 9$ cm.

(2) The distance between the two points

$(0 , 6) , (-2 , 2)$

$= \sqrt{(0+2)^2 + (6-2)^2} = 2\sqrt{5}$ cm.

, the distance between the two points

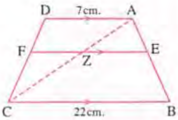
$(-2 , 2) , (-3 , 0)$

$= \sqrt{(-2+3)^2 + (2-0)^2} = \sqrt{5}$ cm.

$\therefore \frac{x}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5}}$ $\qquad \therefore x = 2\sqrt{5}$

(3) Draw $\overline{AC}$ to intersect $\overline{EF}$ at Z

In Δ ABC : $\because \overline{EZ} \parallel \overline{BC}$



$\therefore \frac{AE}{AB} = \frac{EZ}{BC} = \frac{AZ}{AC}$

$\therefore \frac{2}{5} = \frac{EZ}{22} = \frac{AZ}{AC}$

$\therefore$ EZ = 8.8 cm.

In Δ ADC : $\because \overline{ZF} \parallel \overline{AD}$

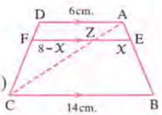
$\therefore \frac{CZ}{CA} = \frac{ZF}{AD}$

$\therefore \frac{3}{5} = \frac{ZF}{7}$ $\qquad \therefore$ ZF = 4.2 cm.

$\therefore$ EF = 8.8 + 4.2 = 13 cm.

(4) Draw $\overline{AC}$ to intersect $\overline{EF}$ at Z let EZ = x

In Δ ABC : $\because \overline{EZ} \parallel \overline{BC}$



$\therefore \frac{AZ}{AC} = \frac{EZ}{BC}$

$\therefore \frac{AZ}{AC} = \frac{x}{14}$ $\qquad$ (1)

In Δ ADC : $\because \overline{ZF} \parallel \overline{AD}$

$\therefore \frac{CZ}{CA} = \frac{ZF}{AD}$

$\therefore \frac{CZ}{CA} = \frac{8-x}{6}$ $\qquad$ (2)

By adding (1) , (2) : $\therefore \frac{AZ}{AC} + \frac{CZ}{CA} = \frac{x}{14} + \frac{8-x}{6}$

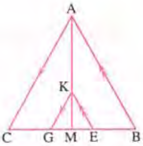
$\therefore \frac{AC}{AC} = \frac{6x}{84} + \frac{112-14x}{84}$

$\therefore \frac{112-8x}{84} = 1$ $\qquad \therefore 112 - 8x = 84$

$\therefore 8x = 28$ $\qquad \therefore x = 3\frac{1}{2}$

$, \because \frac{AE}{AB} = \frac{EZ}{BC} = \frac{3\frac{1}{2}}{14} = \frac{1}{4}$ $\qquad \therefore \frac{AE}{EB} = \frac{1}{3}$

2



$\because \overline{AB} \parallel \overline{KE}$

$\therefore \frac{ME}{MK} = \frac{EB}{KA} = \frac{MB}{MA}$ $\qquad$ (1)

$\because \overline{KG} \parallel \overline{AC}$

$\therefore \frac{MG}{MK} = \frac{GC}{KA} = \frac{MC}{MA}$ $\qquad$ (2)

From (1) , (2) : $\because$ MB = MC

$\therefore \frac{ME}{MK} = \frac{EB}{KA} = \frac{MG}{MK} = \frac{GC}{KA}$ $\qquad \therefore$ ME = MG

$\therefore$ M is the midpoint of $\overline{EG}$ $\qquad$ (Q.E.D. 1)

$\therefore \frac{MK}{KA} = \frac{ME}{EB}$

, $\because$ K is the point of intersection of the medians of Δ ABC

$\therefore \frac{MK}{KA} = \frac{1}{2}$, $\frac{ME}{EB} = \frac{1}{2}$ $\qquad \therefore$ ME = $\frac{1}{2}$ EB

, $\because$ ME = MG

$\therefore$ ME + MG = $\frac{1}{2}$ EB + $\frac{1}{2}$ EB

$\therefore$ EG = EB $\qquad$ (3)

$, \because \frac{MG}{GC} = \frac{MK}{KA} = \frac{1}{2}$ $\qquad \therefore$ MG = $\frac{1}{2}$ GC

, $\because$ MG = ME $\qquad \therefore$ MG + ME = $\frac{1}{2}$ GC + $\frac{1}{2}$ GC

$\therefore$ EG = GC $\qquad$ (4)

From (3) , (4) : $\therefore$ BE = EG = GC = $\frac{1}{3}$ BC (Q.E.D. 2)

3

$\because \overline{BC} \parallel \overline{ED}$ and $\overleftrightarrow{FE}$, $\overleftrightarrow{FD}$ are two transversals

$\therefore \frac{FB}{FE} = \frac{FC}{FD}$ $\qquad$ (1)

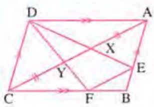
, $\because \overline{BD} \parallel \overline{EX}$ and $\overleftrightarrow{FE}$, $\overleftrightarrow{FX}$ are two transversals

$\therefore \frac{FB}{FE} = \frac{FC}{FX}$ $\qquad$ (2)

From (1) , (2) , by multiplying

$\therefore \left(\frac{FB}{FE}\right)^2 = \frac{FC}{FD} \times \frac{FD}{FX} = \frac{FC}{FX}$ $\qquad$ (Q.E.D.)

4



$\because \overline{AE} \parallel \overline{CD}$

$\therefore \frac{AX}{XC} = \frac{EX}{XD}$ $\qquad$ (1)

$, \because \overline{CF} \parallel \overline{AD}$ $\qquad \therefore \frac{CY}{AY} = \frac{FY}{YD}$ $\qquad$ (2)

, $\because$ AX = YC $\qquad$ (3)

, by adding XY to both sides $\qquad \therefore$ AY = XC $\qquad$ (4)

From (1) , (2) , (3) , (4) : $\therefore \frac{EX}{XD} = \frac{FY}{YD}$

$\therefore \overline{EF} \parallel \overline{XY}$ $\qquad$ (Q.E.D.)

Answers of Exercise 7

First Multiple choice questions

(1) d	(2) c	(3) a	(4) a
(5) b	(6) c	(7) c	(8) a
(9) c	(10) c	(11) a	(12) d
(13) a	(14) c	(15) c	(16) c
(17) d	(18) c	(19) b	(20) d
(21) a	(22) b	(23) b	(24) c
(25) a	(26) c	(27) c	(28) d
(29) b	(30) b	(31) d	(32) c
(33) c	(34) a	(35) c	(36) a
(37) c	(38) b	(39) c	(40) d
(41) d	(42) d	(43) c	(44) a
(45) c	(46) c	(47) d	(48) b
(49) d	(50) c	(51) b	(52) a
(53) d	(54) b	(55) c	(56) a
(57) c			

Second Essay questions

1

(1) $\because \overrightarrow{BD}$ bisects $\angle ABC$ $\quad \therefore \frac{CD}{DA} = \frac{CB}{BA}$

$\therefore \frac{X+1}{5} = \frac{X+4}{8}$ $\quad \therefore 8X + 8 = 5X + 20$

$\therefore 3X = 12$ $\quad \therefore X = 4$

(2) $\because \overrightarrow{AD}$ bisects $\angle BAC$ $\quad \therefore \frac{BD}{DC} = \frac{BA}{AC}$

$\therefore \frac{6X}{5X} = \frac{10X+4}{9X+2}$

$\therefore 6X(9X+2) = 5X(10X+4)$

$\therefore 54X^2 + 12X = 50X^2 + 20X$

$\therefore 4X^2 - 8X = 0$ $\quad \therefore 4X(X-2) = 0$

$\therefore X = 0$ (refused) or $X = 2$

2

(1) $\because m(\angle B) = m(\angle C)$ $\quad \therefore AB = AC = 7$ cm.

, $\because \overrightarrow{AD}$ bisects $\angle BAC$ $\quad \therefore \frac{BD}{DC} = \frac{AB}{AC} = 1$

$\therefore \frac{X}{4} = 1$ $\quad \therefore X = 4$

$\therefore$ The perimeter of $\Delta ABC = 7 + 7 + 8 = 22$ cm.

(2) In ΔADC which is right-angled at D

$(DC)^2 = (50)^2 - (30)^2 = 1600$

$\therefore DC = 40$ cm.

, $\because \overrightarrow{AB}$ bisects $\angle DAC$

$\therefore \frac{DB}{BC} = \frac{DA}{AC} = \frac{30}{50} = \frac{3}{5}$ $\quad \therefore \frac{DB+BC}{BC} = \frac{3+5}{5}$

$\therefore \frac{DC}{X} = \frac{8}{5}$ $\quad \therefore \frac{40}{X} = \frac{8}{5}$

$\therefore X = 25$ $\quad \therefore DB = 40 - 25 = 15$ cm.

, $AB = \sqrt{DA \times AC - BD \times BC}$

$= \sqrt{50 \times 30 - 15 \times 25} = 15\sqrt{5}$ cm.

$\therefore$ The perimeter of $\Delta ABC = 15\sqrt{5} + 50 + 25$

$= (75 + 15\sqrt{5})$ cm.

(3) $\because \overrightarrow{BD}$ bisects $\angle ABC$ $\quad \therefore \frac{AD}{DC} = \frac{AB}{BC}$

$\therefore \frac{4}{X} = \frac{6}{X+3}$ $\quad \therefore 6X = 4X + 12$

$\therefore 2X = 12$ $\quad \therefore X = 6$

$\therefore$ The perimeter of $\Delta ABC = 6 + 9 + 10 = 25$ cm.

3

(1) $\because \overrightarrow{AD}$ bisects $\angle BAC$ $\quad \therefore \frac{BD}{DC} = \frac{BA}{AC}$

$\therefore \frac{3}{4} = \frac{4}{X}$ $\quad \therefore X = 5\frac{1}{3}$

$\therefore AD = \sqrt{BA \times AC - BD \times DC}$

$= \sqrt{4 \times 5\frac{1}{3} - 3 \times 4} = \frac{2\sqrt{21}}{3}$ cm.

(2) $\because \overrightarrow{AE}$ bisects $\angle BAC$, $\overline{AD} \perp \overline{AE}$

$\therefore \overrightarrow{AD}$ bisects $\angle CAF$ $\quad \therefore \frac{BD}{CD} = \frac{BA}{CA}$

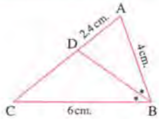
$\therefore \frac{X+10}{X+1} = \frac{12}{6} = 2$

$\therefore 2X + 2 = X + 10$ $\quad \therefore X = 8$

$\therefore AD = \sqrt{BE \times CE - BA \times AC}$

$= \sqrt{18 \times 9 - 12 \times 6} = 3\sqrt{10}$ cm.

4



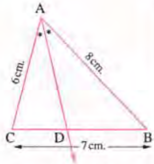
$\because \overrightarrow{BD}$ bisects $\angle ABC$

$\therefore \frac{AD}{DC} = \frac{AB}{BC}$

$\therefore \frac{2.4}{DC} = \frac{4}{6}$ $\quad \therefore DC = 3.6$ cm.

$\therefore AC = 2.4 + 3.6 = 6$ cm. (The req.)

5



$\because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore \frac{BD}{DC} = \frac{BA}{AC} = \frac{8}{6} = \frac{4}{3}$

$\therefore \frac{BD + DC}{DC} = \frac{4+3}{3}$

$\therefore \frac{BC}{DC} = \frac{7}{3}$ $\quad \therefore \frac{7}{DC} = \frac{7}{3}$ $\quad \therefore DC = 3$ cm.

$\therefore BD = 7 - 3 = 4$ cm. (The req.)

6

$\because \overrightarrow{AD}$ bisects the exterior angle at A

$\therefore \frac{AB}{AC} = \frac{DB}{DC}$ $\quad \therefore \frac{6}{8} = \frac{DB}{DC}$

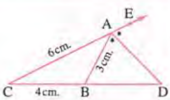
$\therefore \frac{6}{8-6} = \frac{DB}{DC - DB}$ $\quad \therefore \frac{6}{2} = \frac{DB}{5}$

$\therefore DB = \frac{6 \times 5}{2} = 15$ cm.

$\therefore AD = \sqrt{CD \times DB - AC \times AB} = \sqrt{20 \times 15 - 8 \times 6}$

$= 6\sqrt{7}$ cm. (The req.)

7



$\because \overrightarrow{AD}$ bisects $\angle BAE$

$\therefore \frac{BD}{DC} = \frac{BA}{AC}$

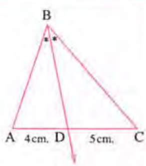
$\therefore \frac{BD}{4 + BD} = \frac{3}{6} = \frac{1}{2}$ $\quad \therefore 2\,BD = 4 + BD$

$\therefore BD = 4$ cm. $\quad \therefore CD = 8$ cm.

$\therefore AD = \sqrt{CD \times DB - CA \times AB} = \sqrt{8 \times 4 - 6 \times 3}$

$= \sqrt{14}$ cm. (The req.)

8



$\because \overrightarrow{BD}$ bisects $\angle B$

$\therefore \frac{AB}{BC} = \frac{AD}{DC} = \frac{4}{5}$

$\therefore \frac{AB + BC}{BC} = \frac{4+5}{5}$

, $\because$ the perimeter of the triangle = 27 cm. , AC = 9 cm.

$\therefore AB + BC = 27 - 9 = 18$ cm.

$\therefore \frac{18}{BC} = \frac{9}{5}$ $\quad \therefore BC = 10$ cm. $\quad \therefore AB = 8$ cm.

$\therefore BD = \sqrt{AB \times BC - AD \times DC} = \sqrt{10 \times 8 - 4 \times 5}$

$= 2\sqrt{15}$ cm.(The req.)

9

$\because \overrightarrow{AX}$ bisects $\angle BAD$ $\quad \therefore \frac{DX}{XB} = \frac{AD}{AB}$

, $\because \overline{XY} \parallel \overline{BC}$ $\quad \therefore \frac{DX}{XB} = \frac{DY}{YC}$

$\therefore \frac{DY}{YC} = \frac{AD}{AB}$ (Q.E.D.)

10

$\because \overrightarrow{DX}$ bisects $\angle ADC$ $\quad \therefore \frac{AX}{XC} = \frac{DA}{DC} = \frac{6}{9} = \frac{2}{3}$

, $\because \frac{AE}{EB} = \frac{2}{3}$ $\quad \therefore \frac{AX}{XC} = \frac{AE}{EB}$

$\therefore \overline{EX} \parallel \overline{BC}$ (Q.E.D.)

11

$\because \overrightarrow{AD}$ bisects $\angle BAC$ $\quad \therefore \frac{BD}{DC} = \frac{AB}{AC}$

, $\because \overline{ED} \parallel \overline{AC}$ $\quad \therefore \frac{BD}{DC} = \frac{BE}{EA}$

$\therefore \frac{BE}{EA} = \frac{AB}{AC}$ (First req.)

$\therefore \frac{BE}{6 - BE} = \frac{6}{9}$ $\quad \therefore 9\,BE = 36 - 6\,BE$

$\therefore 15\,BE = 36$

$\therefore BE = 2.4$ cm. , $AE = 6 - 2.4 = 3.6$ cm.

(Second req.)

12

$\because \overrightarrow{DX}$ bisects $\angle ADB$ $\quad \therefore \frac{AX}{XB} = \frac{AD}{DB}$ (1)

, $\because \overrightarrow{DY}$ bisects $\angle ADC$ $\quad \therefore \frac{AY}{YC} = \frac{AD}{DC}$ (2)

From (1) , (2) :

, $\because DB = DC$ $\quad \therefore \frac{AX}{XB} = \frac{AY}{YC}$

$\therefore \overline{XY} \parallel \overline{BC}$ (Q.E.D.)

13

$\because \overrightarrow{AX}$ bisects $\angle BAC$ $\quad \therefore \frac{AB}{AC} = \frac{BX}{XC}$ (1)

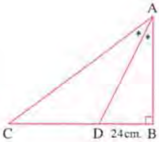
, $\because \overrightarrow{AY}$ bisects $\angle DAC$ $\quad \therefore \frac{AD}{AC} = \frac{DY}{YC}$ (2)

$\therefore$ From (1) , (2) :

, $\because AB = AD$ $\quad \therefore \frac{BX}{XC} = \frac{DY}{YC}$

$\therefore \overline{XY} \parallel \overline{BD}$ (Q.E.D.)

14



$\because \overrightarrow{AD}$ bisects $\angle BAC$

$\therefore \frac{BD}{DC} = \frac{BA}{AC} = \frac{3}{5}$

$\therefore \frac{24}{DC} = \frac{3}{5}$

$\therefore DC = 40$ cm. $\qquad \therefore BC = 40 + 24 = 64$ cm.

, $\because \frac{AB}{AC} = \frac{3}{5}$ $\qquad \therefore AB = 3$ m , $AC = 5$ m.

From phythagoras' theorem : $\therefore BC = 4$ m.

$\therefore 4\text{ m} = 64$ $\qquad \therefore m = 16$

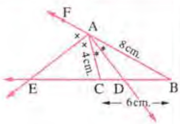
by substituting : $\therefore AB = 3 \times 16 = 48$ cm.

, $AC = 5 \times 16 = 80$ cm.

$\therefore$ The perimeter of $\Delta ABC = 80 + 48 + 64 = 192$ cm.

(The req.)

15



$\because \overrightarrow{AE}$ bisects $\angle CAF$

$\therefore \frac{BE}{CE} = \frac{BA}{AC}$

$\therefore \frac{CE + 6}{CE} = \frac{8}{4} = \frac{2}{1}$

$\therefore 2\,CE = CE + 6$ $\qquad \therefore CE = 6$ cm.

, $\because \overrightarrow{AD}$ bisects $\angle BAC$ $\qquad \therefore \frac{BA}{AC} = \frac{BD}{DC}$

$\therefore \frac{BD}{DC} = \frac{8}{4}$ $\qquad \therefore \frac{BD + DC}{DC} = \frac{8 + 4}{4}$

$\therefore \frac{BC}{DC} = \frac{12}{4}$ $\qquad \therefore \frac{6}{DC} = 3$

$\therefore DC = 2$ cm.

$\therefore DE = CE + CD = 6 + 2 = 8$ cm.

$\therefore BD = 6 - 2 = 4$ cm.

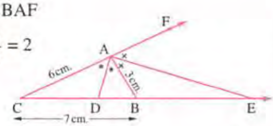
$\therefore AD = \sqrt{BA \times AC - BD \times DC} = \sqrt{8 \times 4 - 4 \times 2}$

$= 2\sqrt{6}$ cm.

, $AE = \sqrt{BE \times CE - BA \times AC} = \sqrt{12 \times 6 - 8 \times 4}$

$= 2\sqrt{10}$ cm. (The req.)

16



$\because \overrightarrow{AE}$ bisects $\angle BAF$

$\therefore \frac{CE}{EB} = \frac{AC}{AB} = \frac{6}{3} = 2$

$\therefore CE = 2\,EB$

$\therefore CB = BE$

$\therefore$ B is the midpoint of $\overline{CE}$

$\therefore \overline{AB}$ is a median of ΔACE (First req.)

$\because \overrightarrow{AD}$ bisects $\angle BAC$ $\qquad \therefore \frac{BD}{DC} = \frac{BA}{AC} = \frac{3}{6} = \frac{1}{2}$

$\therefore BD = \frac{7}{3}$ cm. , $DC = \frac{14}{3}$ cm. , $BE = 7$ cm.

$\therefore ED = \frac{7}{3} + 7 = \frac{28}{3}$ cm. , $EC = 14$ cm.

$\therefore \frac{\text{The area of } (\Delta ADE)}{\text{The area of } (\Delta ACE)} = \frac{ED}{CE} = \frac{\frac{28}{3}}{14} = \frac{2}{3}$

(because they have the same height) (Second req.)

17

(1) $\because \overrightarrow{CE}$ bisects $\angle ACB$

$\therefore \frac{BE}{AE} = \frac{BC}{CA} = \frac{6}{12} = \frac{1}{2}$

, $\because \frac{CF}{FA} = \frac{4}{8} = \frac{1}{2}$ $\qquad \therefore \frac{BE}{AE} = \frac{CF}{FA}$

$\therefore \overline{EF} \parallel \overline{BC}$ (Q.E.D.)

(2) In ΔABD : $\because \overrightarrow{BE}$ bisects $\angle ABD$

$\therefore \frac{AE}{ED} = \frac{AB}{BD}$ (1)

In ΔADC : $\because \overrightarrow{DF}$ bisects $\angle ADC$

$\therefore \frac{AF}{FC} = \frac{AD}{DC}$ (2)

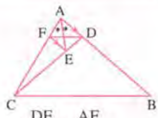
, $\because AB = AD$, $BD = DC$ (3)

From (1) , (2) , (3) : $\therefore \frac{AE}{ED} = \frac{AF}{FC}$

In ΔADC : $\therefore \overline{EF} \parallel \overline{DC}$ $\qquad \therefore \overline{EF} \parallel \overline{BC}$

(Q.E.D.)

18



$\because \overrightarrow{AE}$ bisects $\angle DAC$

$\therefore \frac{DE}{EC} = \frac{DA}{AC}$

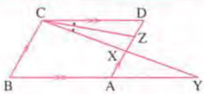
, $\because \overline{EF} \parallel \overline{DA}$ $\qquad \therefore \frac{DE}{EC} = \frac{AF}{FC}$

$\therefore \frac{DA}{AC} = \frac{AF}{FC}$

, $\because AC = DB$ $\qquad \therefore \frac{AD}{DB} = \frac{AF}{FC}$

$\therefore \overline{DF} \parallel \overline{BC}$ (Q.E.D.)

19



$\because \overline{AX} \parallel \overline{BC}$

$\therefore \frac{AY}{AB} = \frac{XY}{XC}$

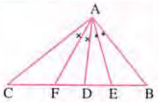
$\therefore \frac{AY}{XY} = \frac{AB}{XC}$ (1)

, ∵ $\overrightarrow{CZ}$ bisects ∠ DCX ∴ $\frac{DZ}{ZX} = \frac{DC}{CX}$ (2)

From (1) , (2) : , ∵ AB = DC

∴ $\frac{AY}{XY} = \frac{DZ}{ZX}$ (Q.E.D.)

20



∵ $\overrightarrow{AE}$ bisects ∠ BAD

∴ $\frac{BE}{ED} = \frac{BA}{AD}$ (1)

, ∵ $\overrightarrow{AF}$ bisects ∠ CAD ∴ $\frac{DF}{FC} = \frac{AD}{AC}$ (2)

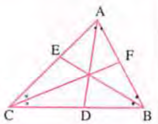
From (1) , (2) , by multiplying :

∴ $\frac{BE}{ED} \times \frac{DF}{FC} = \frac{BA}{AD} \times \frac{AD}{AC} = \frac{BA}{AC}$

, ∵ $\overrightarrow{AD}$ bisects ∠ BAC ∴ $\frac{BA}{AC} = \frac{BD}{DC}$

∴ $\frac{BE}{ED} \times \frac{DF}{FC} = \frac{BD}{DC}$ (Q.E.D.)

21



∵ $\overrightarrow{AD}$ bisects ∠ BAC

∴ $\frac{BD}{DC} = \frac{BA}{AC}$ (1)

, ∵ $\overrightarrow{BE}$ bisects ∠ ABC

∴ $\frac{CE}{EA} = \frac{CB}{BA}$ (2)

, ∵ $\overrightarrow{CF}$ bisects ∠ ACB

∴ $\frac{AF}{FB} = \frac{AC}{CB}$ (3)

From (1) , (2) , (3) : and by multiplying

∴ $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = \frac{BA}{AC} \times \frac{CB}{BA} \times \frac{AC}{CB} = 1$ (Q.E.D.)

22

∵ $\overline{XY}$ // $\overline{BC}$ ∴ $\frac{AX}{XB} = \frac{AY}{YC}$

∴ $\frac{2}{4} = \frac{AY}{3}$ ∴ AY = 1.5 cm. (First req.)

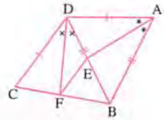
, ∵ $\overrightarrow{AE}$ bisects the exterior angle of the triangle at A

∴ $\frac{AB}{AC} = \frac{BE}{EC}$ ∴ $\frac{6}{4.5} = \frac{BE}{18}$

∴ BE = 24 cm. ∴ BC = 24 − 18 = 6 cm.

(Second req.)

23



∵ $\overrightarrow{AE}$ bisects ∠ BAD

∴ $\frac{AB}{AD} = \frac{BE}{ED}$ (1)

, ∵ $\overrightarrow{DF}$ bisects ∠ BDC

∴ $\frac{DB}{DC} = \frac{BF}{FC}$ (2)

From (1) , (2) :

, ∵ AB = BD , AD = DC

∴ $\frac{BE}{ED} = \frac{BF}{FC}$ ∴ $\overline{EF}$ // $\overline{DC}$ (Q.E.D.)

24

∵ $\overline{DE}$ // $\overline{BC}$ and $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ are two transversals.

∴ $\frac{AD}{AE} = \frac{DB}{EC} = \frac{AB}{AC}$ (1)

, ∵ $\overrightarrow{AX}$ bisects ∠ A ∴ $\frac{AD}{AE} = \frac{DX}{XE}$ (2)

From (1) , (2) : ∴ $\frac{DX}{XE} = \frac{DB}{EC}$ (Q.E.D. 1)

, ∵ $\frac{\text{the area of } (\Delta\, ADX)}{\text{the area of } (\Delta\, AEX)} = \frac{DX}{EX}$

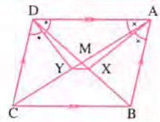
(because they have the same height)

∴ $\frac{\text{The area of } (\Delta\, ADX)}{\text{The area of } (\Delta\, AEX)} = \frac{AD}{AE}$

From (1) :

∴ $\frac{\text{The area of } (\Delta\, ADX)}{\text{The area of } (\Delta\, AEX)} = \frac{AB}{AC}$ (Q.E.D. 2)

25



In Δ ABD :

∵ $\overrightarrow{AX}$ bisects ∠ BAD

∴ $\frac{BX}{XD} = \frac{BA}{AD}$ (1)

, in Δ ACD :

∵ $\overrightarrow{DY}$ bisects ∠ ADC ∴ $\frac{CD}{DA} = \frac{CY}{YA}$ (2)

From (1) , (2) : , ∵ CD = BA

∴ $\frac{BX}{XD} = \frac{CY}{YA}$ ∴ $\overline{BC}$ // $\overline{XY}$ // $\overline{AD}$

(Q.E.D.)

26

∵ E is the midpoint of $\widehat{AB}$

∴ $\overrightarrow{DE}$ bisects ∠ ADB

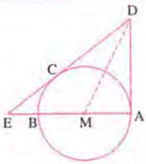
∴ $\frac{AC}{CB} = \frac{AD}{DB} = \frac{2}{3}$

, $\because \dfrac{\text{the area of } (\Delta\, ADC)}{\text{the area of } (\Delta\, BDC)} = \dfrac{\text{The area of } (\Delta\, AEC)}{\text{The area of } (\Delta\, BEC)}$
$= \dfrac{AC}{CB}$

$\therefore \dfrac{\text{The area of } (\Delta\, ADC) + \text{the area of } (\Delta\, AEC)}{\text{The area of } (\Delta\, BDC) + \text{the area of } (\Delta\, BEC)} = \dfrac{AC}{CB} = \dfrac{2}{3}$

$\therefore \dfrac{\text{The area of } (\Delta\, ADE)}{\text{The area of } (\Delta\, BDE)} = \dfrac{2}{3}$ (The req.)

27



Construction :

Draw $\overline{DM}$

Proof :

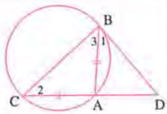
$\because \overline{DA}$, $\overline{DC}$ are two tangents segments to the circle

$\therefore \overrightarrow{DM}$ bisects $\angle$ ADC $\quad \therefore \dfrac{AM}{ME} = \dfrac{AD}{DE}$

, $\because$ AD = DC (Theorem)

$\therefore \dfrac{AM}{ME} = \dfrac{DC}{DE}$ (Q.E.D.)

28



m ($\angle$ 1) = m ($\angle$ 2)

(inscribed and tangency , angles subtended by $\widehat{AB}$)

, m ($\angle$ 2) = m ($\angle$ 3) (because AB = AC)

$\therefore$ m ($\angle$ 1) = m ($\angle$ 3) $\quad \therefore \overrightarrow{BA}$ bisects $\angle$ DBC

$\therefore \dfrac{DA}{AC} = \dfrac{DB}{BC}$

, $\because$ AB = AC $\quad \therefore \dfrac{DA}{AB} = \dfrac{DB}{BC}$

$\therefore$ DB × BA = DA × BC (Q.E.D.)

Third Higher skills

1

(1) (b) (2) (a) (3) (c) (4) (b)
(5) (a) (6) (d) (7) (b) (8) (c)
(9) (c) (10) (a) (11) (d) (12) (a)
(13) (a) (14) (d) (15) (b)

Instructions to solve 1 :

(1) In Δ ABC : $\because \overrightarrow{AD}$ bisects $\angle$ BAC

$\therefore \dfrac{AB}{AC} = \dfrac{BD}{DC} \quad \therefore \dfrac{6}{AC} = \dfrac{3}{DC} \quad \therefore \dfrac{AC}{DC} = \dfrac{6}{3} = 2$

In Δ ACD : $\because \overrightarrow{CE}$ bisects $\angle$ ACD

$\therefore \dfrac{AC}{CD} = \dfrac{AE}{ED} \quad \therefore \dfrac{AE}{ED} = \dfrac{2}{1} = 2$

(2) In Δ ABC : $\because \overrightarrow{BD}$ bisects $\angle$ ABC

$\therefore \dfrac{AB}{BC} = \dfrac{AD}{DC} \quad \therefore \dfrac{3}{BC} = \dfrac{2}{4} \quad \therefore$ BC = 6 cm.

, in Δ ABC : $\because \overrightarrow{AE}$ bisects the exterior angle at A

$\therefore \dfrac{AB}{AC} = \dfrac{BE}{EC} \quad \therefore \dfrac{3}{6} = \dfrac{BE}{BE + 6}$

$\therefore \dfrac{BE}{BE + 6} = \dfrac{1}{2} \quad \therefore$ 2 BE = BE + 6

$\therefore$ BE = 6

(3) In Δ ADC : $\because \overrightarrow{DE}$ bisects $\angle$ ADC

$\therefore \dfrac{CD}{DA} = \dfrac{CE}{EA} = \dfrac{3}{4}$ (1)

, in Δ ADB : $\because \overrightarrow{DF}$ bisects $\angle$ ADB

$\therefore \dfrac{BD}{AD} = \dfrac{BF}{FA} = \dfrac{2}{3}$ (2)

By adding (1) , (2) : $\because \dfrac{BD}{AD} + \dfrac{CD}{AD} = \dfrac{2}{3} + \dfrac{3}{4}$

$\therefore \dfrac{BD + DC}{AD} = \dfrac{17}{12} \quad \therefore \dfrac{17}{AD} = \dfrac{17}{12}$

$\therefore$ AD = 12 cm.

, from (1) : $\dfrac{CD}{12} = \dfrac{3}{4} \quad \therefore$ CD = 9 cm.

(4) In Δ ABC : $\because$ m ($\angle$ DAB) = m ($\angle$ DAC)

$\therefore \overrightarrow{AD}$ bisects $\angle$ CAB

$\therefore \dfrac{DC}{DB} = \dfrac{AC}{AB} \quad \therefore \dfrac{AC}{AB} = \dfrac{8}{4} = \dfrac{2}{1}$ (1)

, $\because$ m ($\angle$ B) = 2 m ($\angle$ DAB) = 2 m ($\angle$ DAC)

$\therefore$ m ($\angle$ B) = m ($\angle$ CAB)

$\therefore$ CA = CB = 12 cm.

, from (1) : $\therefore \dfrac{12}{AB} = \dfrac{2}{1} \quad \therefore$ AB = 6 cm.

(5) BD $= \sqrt{(3-1)^2 + (3-1)^2} = 2\sqrt{2}$ length unit

, DC $= \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}$ length unit

In Δ ABC : $\overrightarrow{AD}$ bisects $\angle$ BAC

$\therefore \dfrac{AC}{AB} = \dfrac{DC}{DB} \quad \therefore \dfrac{AC}{AB} = \dfrac{\sqrt{2}}{2\sqrt{2}} = \dfrac{1}{2}$

(6) In Δ ABC : $\because \overrightarrow{AD}$ bisects $\angle$ BAC

$\therefore \dfrac{AB}{AC} = \dfrac{BD}{DC} \quad \therefore \dfrac{AB}{AC} = \dfrac{8}{10} = \dfrac{4}{5}$ (1)

(a) If AC − AB = 5 $\quad \therefore$ AC = AB + 5

From (1) : $\therefore \dfrac{AB}{AB + 5} = \dfrac{4}{5}$

$\therefore$ 5 AB = 4 AB + 20 $\quad \therefore$ AB = 20 cm.

(b) If the perimeter of Δ ABC = 54 cm.

$\therefore$ AB + AC + BC = 54 $\therefore$ AB + AC + 18 = 54

$\therefore$ AC = 36 − AB

, from (1) : $\therefore \frac{AB}{36-AB}=\frac{4}{5}$

$\therefore 5\,AB = 144 - 4\,AB$ $\quad \therefore 9\,AB = 144$

$\therefore AB = 16$ cm.

(c) If $AD = 4\sqrt{15}$ cm. : $(AD)^2 = AB \times AC - BD \times DC$

$\therefore \left(4\sqrt{15}\right)^2 = AB \times AC - 8 \times 10$

$\therefore AB \times AC = 320$

, from (1) : $(AB \times AC) \times \left(\frac{AB}{AC}\right) = \frac{4}{5} \times 320$

$\therefore (AB)^2 = 256$ $\quad \therefore AB = 16$ cm.

$\therefore$ The answer is : Anything of the previous.

(7) In $\Delta\Delta$ ABD , ADC : $\overline{BD}$ and $\overline{DC}$ on the same straight line , and have common vertex A

$\therefore \frac{a(\Delta ABD)}{a(\Delta ADC)} = \frac{BD}{DC} = \frac{3}{5}$

$\therefore \frac{BD}{DC+BD} = \frac{3}{3+5} = \frac{3}{8}$

$\therefore \frac{BD}{8} = \frac{3}{8}$ $\quad \therefore BD = 3$ cm.

$\therefore DC = 8 - 3 = 5$ cm.

, $\because \overrightarrow{AD}$ bisects $\angle$ BAC $\quad \therefore \frac{AB}{AC} = \frac{DB}{DC} = \frac{3}{5}$

let $AB = 3x$, $AC = 5x$

$\because \Delta$ ABC is right angled triangle at $\angle$ B

$\therefore (BC)^2 = (AC)^2 - (AB)^2$

$\therefore (BC)^2 = (5x)^2 - (3x)^2$

$\therefore (8)^2 = 25x^2 - 9x^2$ $\quad \therefore 16x^2 = 64$

$\therefore x^2 = 4$ $\quad \therefore x = 2$

$\therefore AB = 3 \times 2 = 6$ cm.

(8) In Δ DBC : $\because \overrightarrow{DF}$ bisects $\angle$ BDC

$\therefore \frac{BD}{DC} = \frac{BF}{FC} = \frac{4}{8} = \frac{1}{2}$

In $\Delta\Delta$ BDF , FDC : $\overline{BF}$, $\overline{FC}$ are on the same straight line and have common vertex D

$\therefore \frac{a(\Delta BDF)}{a(\Delta FDC)} = \frac{BF}{FC} = \frac{1}{2}$ $\quad \therefore \frac{10}{a(\Delta FDC)} = \frac{1}{2}$

$\therefore a(\Delta FDC) = 20\ cm^2$

$\therefore a(\Delta CDB) = 20 + 10 = 30\ cm^2$

In $\Delta\Delta$ CDB , CDA : $\overline{DB}$, $\overline{DA}$ are on the same straight line and have common vertex C

$\therefore \frac{a(\Delta CDB)}{a(\Delta CDA)} = \frac{BD}{DA} = \frac{4}{6} = \frac{2}{3}$

$\therefore \frac{30}{a(\Delta CDA)} = \frac{2}{3}$ $\quad \therefore a(\Delta CDA) = 45\ cm^2$

In Δ ABC : $\because \overline{DE} \parallel \overline{BC}$

$\therefore \frac{BD}{BA} = \frac{CE}{CA}$ $\quad \therefore \frac{CE}{CA} = \frac{4}{10} = \frac{2}{5}$

In $\Delta\Delta$ DEC , DAC : $\because \overline{EC}$, $\overline{AC}$ are on the same straight line and have common vertex D

$\therefore \frac{a(\Delta DEC)}{a(\Delta DAC)} = \frac{EC}{AC} = \frac{2}{5}$ $\quad \therefore \frac{a(\Delta DEC)}{45} = \frac{2}{5}$

$\therefore a(\Delta DEC) = 18\ cm^2$

(9) $\because m(\widehat{BX}) = m(\widehat{XY})$

$\therefore m(\angle BCX) = m(\angle XCY)$

$\therefore \overrightarrow{CD}$ bisects $\angle$ BCA

$\therefore \frac{BC}{CA} = \frac{BD}{DA}$ $\quad \therefore \frac{BC}{CA} = \frac{2\sqrt{3}}{4\sqrt{3}} = \frac{1}{2}$

let $BC = x$, $CA = 2x$

In Δ ABC : $\because m(\angle ABC) = 90°$

$\therefore (AC)^2 - (BC)^2 = (AB)^2$

$\therefore (2x)^2 - (x)^2 = \left(6\sqrt{3}\right)^2$

$\therefore 3x^2 = 108$ $\quad \therefore x^2 = 36$ $\quad \therefore x = 6$

$\therefore BC = 6$ cm. , $CA = 12$ cm.

, $\because \overrightarrow{AB}$ is a tangent to the circle M

$\therefore (AB)^2 = AY \times AC$ $\quad \therefore \left(6\sqrt{3}\right)^2 = AY \times 12$

$\therefore AY = 9$ cm.

(10) In Δ ABC : $\because \overrightarrow{AD}$ bisects $\angle$ BAC

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$ $\quad \therefore \frac{AB}{AC} = \frac{4}{5}$

let $AB = 4x$, $AC = 5x$

, $\because (AD)^2 = AB \times AC - BD \times DC$

$\therefore \left(4\sqrt{10}\right)^2 = (4x)(5x) - 4 \times 5$

$\therefore 20x^2 - 20 = 160$ $\quad \therefore x^2 = 9$ $\quad \therefore x = 3$

$\therefore AB = 4 \times 3 = 12$ cm. , $AC = 5 \times 3 = 15$ cm.

, the perimeter of Δ ABC $= 12 + 15 + 9 = 36$ cm.

(11) In Δ ABC : $\because m(\angle BAC) = 90°$

$\therefore BC = \sqrt{(6)^2 + (8)^2} = 10$ cm.

, $a(\Delta ABC) = \frac{1}{2} \times 6 \times 8 = 24\ cm^2$

, $\because \overrightarrow{AD}$ bisects the exterior angle of Δ ABC at the vertex A

$\therefore \frac{BD}{DC} = \frac{BA}{AC}$ $\quad \therefore \frac{BD}{DC} = \frac{6}{8} = \frac{3}{4}$

$\therefore \frac{BD}{BC} = \frac{3}{1}$

In $\Delta\Delta$ ABD , ABC

$\because \overline{BD}$, $\overline{CB}$ are on the same straight line and have common vertex A

$\therefore \frac{a(\Delta ABD)}{a(\Delta ABC)} = \frac{BD}{BC} = \frac{3}{1}$

$\therefore a(\Delta ABD) = 3 \times 24 = 72 \text{ cm}^2$

(12) let ED = DC = x

In Δ ADB : $\because \overrightarrow{AC}$ bisects $\angle$ DAC

$\therefore \frac{DC}{CB} = \frac{DA}{AB}$ $\qquad \therefore \frac{x}{CB} = \frac{3}{6} = \frac{1}{2}$

$\therefore CB = 2x$

$, \because (AC)^2 = AD \times AB - CD \times CB$

$\therefore \left(\sqrt{6}\right)^2 = 3 \times 6 - x \times 2x$

$\therefore 2x^2 = 18 - 6 = 12$ $\qquad \therefore x^2 = 6$

$\therefore x = \sqrt{6}$ $\qquad \therefore DE = CD = \sqrt{6}$

$, \because DA \times DF = DE \times DC$

$\therefore 3 \times DF = \sqrt{6} \times \sqrt{6}$ $\qquad \therefore DF = 2$ cm.

(13) $\because \overrightarrow{AE}$ bisects $\angle$ BAC , $\overrightarrow{AD}$ bisects the exterior angle of Δ ABC at the vertex A

$\therefore m(\angle EAD) = 90°$

$\therefore \tan\theta = -\tan(180° - \theta) = -\tan(\angle AED)$

$= \frac{-AD}{AE} = \frac{-8}{6} = \frac{-4}{3}$

(14) In Δ ABC : $\because \overrightarrow{AD}$ bisects $\angle$ BAC

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$ $\qquad \therefore \frac{BD}{DC} = \frac{8}{16} = \frac{1}{2}$

$\therefore \frac{BD}{BC} = \frac{1}{3}$ $\qquad (1)$

, in Δ ABE : $\because \overrightarrow{AX}$ bisects $\angle$ BAE

$\therefore \frac{BX}{XE} = \frac{BA}{AE}$ $\qquad \therefore \frac{BX}{XE} = \frac{8}{4} = \frac{2}{1}$

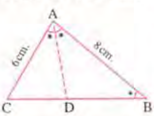
$, \because \overline{XF} \parallel \overline{EC}$

$\therefore \frac{BX}{XE} = \frac{BF}{FC} = \frac{2}{1}$ $\qquad \therefore \frac{BF}{BC} = \frac{2}{3}$ $\qquad (2)$

by subtracting (1) from (2) : $\therefore \frac{BF}{BC} - \frac{BD}{BC} = \frac{2}{3} - \frac{1}{3}$

$\therefore \frac{DF}{BC} = \frac{1}{3}$

(15) Draw $\overrightarrow{AD}$ bisects $\angle$ BAC and intersects $\overline{BC}$ at D



$, \because m(\angle A) = 2\, m(\angle B)$

$\therefore m(\angle B) = m(\angle BAD)$

$\therefore BD = DA$

let BD = DA = x

$\therefore \frac{BD}{DC} = \frac{BA}{AC}$ $\qquad \therefore \frac{x}{DC} = \frac{8}{6}$ $\qquad \therefore DC = \frac{3}{4}x$

$, \because (AD)^2 = AB \times AC - BD \times DC$

$\therefore (x)^2 = 8 \times 6 - x \times \frac{3}{4}x$

$\therefore x^2 = 48 - \frac{3}{4}x^2$ $\qquad \therefore \frac{7}{4}x^2 = 48$

$\therefore x^2 = \frac{192}{7}$ $\qquad \therefore x = \frac{8\sqrt{21}}{7}$

$\therefore BC = BD + DC = \frac{8\sqrt{21}}{7} + \frac{3}{4}\left(\frac{8\sqrt{21}}{7}\right)$

$= \frac{8\sqrt{21}}{7} + \frac{6\sqrt{21}}{7} = 2\sqrt{21}$ cm.

2

$\because AB > AC$ $\qquad \therefore BD > DC$

and it is clear that BD = BE + ED $\qquad (1)$

, CD = CE − ED $\qquad (2)$

$, \because$ E is the midpoint of $\overline{BC}$

, by subtracting (1) , (2) : $\therefore$ BD − CD = 2 ED

by adding (1) and (2) : BD + DC = BC = 2 CE

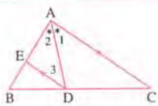
$\because \overrightarrow{AD}$ bisects $\angle$ BAC $\qquad \therefore \frac{AB}{AC} = \frac{BD}{DC}$

$\therefore \frac{AB}{BD} = \frac{AC}{DC} = \frac{AB+AC}{BD+DC} = \frac{AB-AC}{BD-DC}$

$\therefore \frac{AB+AC}{2\,EC} = \frac{AB-AC}{2\,ED}$ $\qquad \therefore \frac{AB-AC}{AB+AC} = \frac{ED}{EC}$

(Q.E.D.)

3



$\because \overrightarrow{AD}$ bisects $\angle$ BAC

$\therefore m(\angle 1) = m(\angle 2)$

$, \because \overline{AC} \parallel \overline{ED}$

$\therefore m(\angle 1) = m(\angle 3)$ (alternate angles)

$\therefore m(\angle 2) = m(\angle 3)$ $\qquad \therefore AE = DE$

$, \because \overrightarrow{AD}$ bisects $\angle$ A internally

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$ $\qquad \therefore \frac{AB}{BD} = \frac{AC}{DC} = \frac{AB+AC}{BD+DC} = \frac{AB+AC}{BC}$

$\therefore BC = \frac{DC\,(AB+AC)}{AC}$ $\qquad (1)$

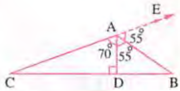
$, \because \overline{DE} \parallel \overline{AC}$

$, \because \frac{AE}{AB} = \frac{CD}{BC}$ $\qquad \therefore AE = \frac{AB \times CD}{BC}$ $\qquad (2)$

From (1) , (2) : $\therefore AE = \frac{AB \times CD \times AC}{DC\,(AB+AC)}$

$\therefore AE = \frac{AB \times AC}{AB+AC}$ $\qquad \therefore DE = \frac{AB \times AC}{AB+AC}$ $\qquad$ (Q.E.D.)

4



Construction : Draw $\overrightarrow{CA}$

, then m ($\angle$ EAB) = 55°

$\therefore \overrightarrow{AB}$ bisects $\angle$ EAD

$\therefore \frac{AD}{AC} = \frac{BD}{BC}$ $\quad \therefore AD \times BC = AC \times BD = 36 \text{ cm}^2$

$\therefore$ The area of $\Delta ABC = \frac{1}{2} BC \times AD = \frac{1}{2} \times 36 = 18 \text{ cm}^2$

(The req.)

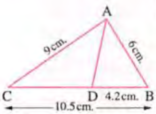
Answers of Exercise 8

First Multiple choice questions

(1) a	(2) d	(3) c	(4) c
(5) d	(6) b	(7) d	(8) b

Second Essay questions

1

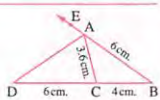


$\because \frac{AB}{AC} = \frac{6}{9} = \frac{2}{3}$

$, \frac{BD}{DC} = \frac{4.2}{10.5 - 4.2} = \frac{2}{3}$

$\therefore \frac{AB}{AC} = \frac{BD}{DC}$ $\quad \therefore \overrightarrow{AD}$ bisects $\angle$ BAC (Q.E.D.)

2



$\because \frac{BA}{AC} = \frac{6}{3.6} = \frac{5}{3}$

$, \frac{BD}{DC} = \frac{4+6}{6} = \frac{5}{3}$

$\therefore \frac{BD}{DC} = \frac{BA}{AC}$ $\quad \therefore \overrightarrow{AD}$ bisects $\angle$ CAE (Q.E.D.)

3

(1) In Δ ADC : $\because \overrightarrow{DE}$ bisects $\angle$ ADC

$\therefore \frac{CE}{EA} = \frac{CD}{DA} = \frac{28}{42} = \frac{2}{3}$

$, \because \frac{BC}{BA} = \frac{36}{54} = \frac{2}{3}$ $\quad \therefore \frac{CE}{EA} = \frac{BC}{BA}$

$\therefore \overrightarrow{BE}$ bisects $\angle$ ABC in Δ ABC (Q.E.D.)

(2) $\because \Delta$ ABC is right angled at A

$\therefore (BC)^2 = (AB)^2 + (AC)^2 = (30)^2 + (40)^2 = 2500$

$\therefore$ BC = 50 cm.

$, \because \overline{AD} \perp \overline{BC}$

$\therefore AD = \frac{BA \times AC}{BC}$ (Euclid theorem)

$\therefore$ AD = 24 cm. $\quad \therefore$ AE = 24 − 9 = 15 cm.

$, \because \Delta ABC \sim \Delta DBA$ $\quad \therefore \frac{AB}{DB} = \frac{BC}{AB}$

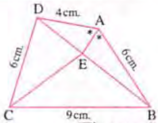
$\therefore \frac{30}{DB} = \frac{50}{30}$ $\quad \therefore$ DB = 18 cm.

$\therefore \frac{DB}{BA} = \frac{18}{30} = \frac{3}{5}$, $\frac{DE}{EA} = \frac{9}{15} = \frac{3}{5}$

$\therefore \frac{DB}{BA} = \frac{DE}{EA}$

$\therefore \overrightarrow{BE}$ bisects $\angle$ ABC (Q.E.D.)

4



$\because \overrightarrow{AE}$ bisects $\angle$ DAB

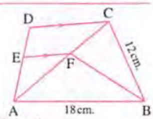
$\therefore \frac{AB}{AD} = \frac{BE}{ED}$

$\therefore \frac{BE}{ED} = \frac{6}{4} = \frac{3}{2}$ (First req.)

$, \because \frac{BC}{CD} = \frac{9}{6} = \frac{3}{2}$ $\quad \therefore \frac{BE}{ED} = \frac{CB}{CD}$

$\therefore \overrightarrow{CE}$ bisects $\angle$ BCD (Second req.)

5



In Δ ACD : $\because$ 2 AE = 3 ED

$\therefore \frac{AE}{ED} = \frac{3}{2}$

$, \because \overrightarrow{EF} \parallel \overline{DC}$ $\quad \therefore \frac{AF}{FC} = \frac{AE}{ED} = \frac{3}{2}$

$\because \frac{AB}{BC} = \frac{18}{12} = \frac{3}{2}$ $\quad \therefore \frac{AF}{FC} = \frac{AB}{BC}$

$\therefore \overrightarrow{BF}$ bisects $\angle$ ABC in Δ ABC (Q.E.D.)

6

$\because \overrightarrow{DE}$ bisects $\angle$ ADB $\quad \therefore \frac{DA}{DB} = \frac{AE}{EB}$ (1)

$, \because \overline{EF} \parallel \overline{BC}$ $\quad \therefore \frac{AE}{EB} = \frac{AF}{FC}$ (2)

From (1) , (2) : , $\because$ BD = DC $\quad \therefore \frac{DA}{DC} = \frac{AF}{FC}$

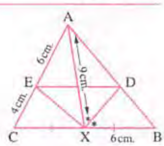
$\therefore \overrightarrow{DF}$ bisects $\angle$ ADC (Q.E.D. 1)

$, \because \overrightarrow{DE}$ bisects $\angle$ ADB , $\overrightarrow{DF}$ bisects $\angle$ ADC

$, D \in \overleftrightarrow{BC}$

$\therefore \overline{ED} \perp \overline{DF}$ (Q.E.D. 2)

7



$\because \overrightarrow{XD}$ bisects $\angle$ AXB

$\therefore \frac{AD}{DB} = \frac{AX}{XB} = \frac{9}{6}$

$= \frac{3}{2}$ (First req.)

$, \because \frac{AE}{EC} = \frac{6}{4} = \frac{3}{2}$ $\quad \therefore \frac{AD}{DB} = \frac{AE}{EC}$

$\therefore \overline{DE} \parallel \overline{BC}$ (Second req.)

$, \because \frac{AX}{XB} = \frac{3}{2}$, $XB = XC$ $\quad \therefore \frac{AX}{XC} = \frac{3}{2}$

$, \because \frac{AE}{EC} = \frac{3}{2}$ $\quad \therefore \frac{AX}{XC} = \frac{AE}{EC}$

$\therefore \overrightarrow{XE}$ bisects $\angle AXC$ (Third req.)

8

$\because \overrightarrow{BX}$ bisects $\angle ABC$ $\quad \therefore \frac{AB}{BC} = \frac{AX}{XC}$

$, \because \overline{XY} \parallel \overline{CD}$ $\quad \therefore \frac{AX}{XC} = \frac{AY}{YD}$

$\therefore \frac{AB}{BC} = \frac{AY}{YD}$

$, \because AB = AC$, $BC = CD$ $\quad \therefore \frac{AC}{CD} = \frac{AY}{YD}$

$\therefore \overrightarrow{CY}$ bisects $\angle ACD$ (Q.E.D.)

9

$\because \overrightarrow{AE}$ bisects $\angle BAC$ $\quad \therefore \frac{AC}{AB} = \frac{CE}{EB}$

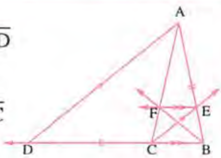
$, \because \overline{EF} \parallel \overline{BD}$ $\quad \therefore \frac{EC}{EB} = \frac{CF}{FD}$

$\therefore \frac{AC}{AB} = \frac{CF}{FD}$

$, \because AB = AD$ $\quad \therefore \frac{AC}{AD} = \frac{CF}{FD}$

$\therefore \overrightarrow{AF}$ bisects $\angle CAD$ (Q.E.D.)

10



In ΔABD : $\because \overrightarrow{CE} \parallel \overline{AD}$

$\therefore \frac{BC}{CD} = \frac{BE}{EA}$ (1)

In ΔABC : $\because \overrightarrow{EF} \parallel \overline{BC}$

$\therefore \frac{CF}{FA} = \frac{BE}{EA}$ (2)

From (1) , (2) : $\therefore \frac{BC}{CD} = \frac{CF}{FA}$

$, \because AB = CD$ $\quad \therefore \frac{BC}{AB} = \frac{CF}{FA}$

$\therefore \overrightarrow{BF}$ bisects $\angle ABC$ in ΔABC (Q.E.D.)

11

$\because \overrightarrow{BM}$ bisects $\angle B$ $\quad \therefore \frac{BD}{BC} = \frac{DM}{MC}$ (1)

$, \because \overrightarrow{AM}$ bisects $\angle A$ $\quad \therefore \frac{AD}{AC} = \frac{DM}{MC}$ (2)

From (1) , (2) : $\therefore \frac{BD}{BC} = \frac{AD}{AC} = \frac{AB}{BC + AC}$

$\therefore \frac{BD}{16} = \frac{AD}{8} = \frac{12}{24}$

$\therefore AD = 4$ cm. (The req.)

12

$\because \overrightarrow{ZM}$ bisects $\angle XZL$, $\overrightarrow{YM}$ bisects $\angle XYL$

$\therefore$ M is the point of intersection of the interior angles of the triangle.

$\therefore \overrightarrow{XM}$ bisects $\angle ZXY$ $\quad \therefore \frac{ZL}{LY} = \frac{XZ}{XY}$

$\therefore \frac{ZL}{YL} = \frac{5}{8}$ $\quad \therefore 8\ ZL = 5\ YL$ (Q.E.D.)

13

$\because \frac{AC}{AB} = \frac{15}{9} = \frac{5}{3}$, $\frac{DC}{BD} = \frac{10}{6} = \frac{5}{3}$

$\therefore \frac{AC}{AB} = \frac{DC}{BD}$ $\quad \therefore \overrightarrow{AD}$ bisects $\angle BAC$ (Q.E.D.)

14

$\because \frac{AC}{AB} = \frac{10}{5} = 2$

$, \frac{CD}{DB} = \frac{6}{3} = 2$

$\therefore \frac{AC}{AB} = \frac{CD}{DB}$

$\therefore \overrightarrow{AD}$ bisects $\angle BAC$ (First req.)

$\because A \in \overleftrightarrow{FC}$, $\overrightarrow{AD}$ bisects $\angle CAB$

$, \overline{AD} \perp \overline{AE}$

$\therefore \overrightarrow{AE}$ bisects $\angle FAB$ $\quad \therefore \frac{AB}{AC} = \frac{BE}{EC}$

$\therefore \frac{5}{10} = \frac{BE}{9 + BE}$ $\quad \therefore 45 + 5\ BE = 10\ BE$

$\therefore BE = 9$ cm. (Second req.)

15

In ΔABD : $\because \overrightarrow{BM}$ bisects $\angle DBX$

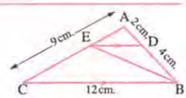
$\therefore \frac{DM}{MA} = \frac{DB}{BA}$

, in ΔACD : $\because \overrightarrow{CM}$ bisects $\angle DCY$

$\therefore \frac{DM}{MA} = \frac{DC}{CA}$ $\quad \therefore \frac{DB}{BA} = \frac{DC}{CA}$

$\therefore \frac{DB}{DC} = \frac{BA}{AC}$ $\quad \therefore \overrightarrow{AM}$ bisects $\angle BAC$ (Q.E.D.)

16



$\because \overline{DE} \parallel \overline{BC}$

$\therefore \frac{AE}{AC} = \frac{AD}{AB}$

$\therefore \frac{AE}{9} = \frac{2}{6}$ $\quad \therefore AE = 3$ cm. (First req.)

$\therefore \frac{AE}{EC} = \frac{3}{6} = \frac{1}{2}$, $\frac{AB}{BC} = \frac{6}{12} = \frac{1}{2}$

$\because \frac{AE}{EC} = \frac{AB}{BC}$ $\qquad \therefore \overrightarrow{BE}$ bisects $\angle ABC$

(Second req.)

17

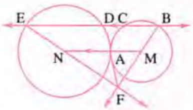
$\because \overline{ED} // \overline{XY} // \overline{BC}$ $\qquad \therefore \frac{EX}{BX} = \frac{DY}{CY}$ $\qquad$ (1)

, $\because AD \times BX = AC \times EX$ $\qquad \therefore \frac{EX}{BX} = \frac{AD}{AC}$ $\qquad$ (2)

From (1) , (2) : $\therefore \frac{DY}{CY} = \frac{AD}{AC}$

$\therefore \overrightarrow{AY}$ bisects $\angle CAD$ $\qquad$ (Q.E.D.)

18



In ΔBFE : $\because \overline{MN} // \overleftrightarrow{BE}$

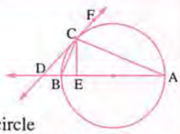
$\therefore \frac{BM}{MF} = \frac{EN}{FN}$

, $\because BM = MA$, $EN = AN$

$\therefore \frac{MA}{MF} = \frac{AN}{FN}$ $\qquad \therefore \frac{MA}{AN} = \frac{MF}{FN}$

$\therefore \overrightarrow{FA}$ bisects $\angle MFN$ $\qquad$ (Q.E.D.)

19



$\because \frac{DB}{BE} = \frac{DC}{CE}$

$\therefore \overrightarrow{CB}$ bisects $\angle DCE$ $\quad$ (1)

, $\because \overline{AB}$ is a diameter of the circle

$\therefore m(\angle ACB) = 90°$ $\qquad \therefore \overline{AC} \perp \overline{BC}$ $\qquad$ (2)

From (1) , (2) : $\therefore \overrightarrow{CA}$ bisects $\angle FCE$

(angle bisectors are perpendicular) $\qquad$ (Q.E.D. 1)

$\therefore \frac{DA}{AE} = \frac{DC}{CE}$

, $\because \frac{DB}{BE} = \frac{DC}{CE}$ $\qquad \therefore \frac{AD}{AE} = \frac{DB}{BE}$

$\therefore \frac{DA}{DB} = \frac{AE}{BE}$ $\qquad$ (Q.E.D. 2)

Third | Higher skills

In ΔABC : $\because CD = 10 - 4 = 6$ cm.

$\therefore \frac{BD}{DC} = \frac{4}{6} = \frac{2}{3}$, $\frac{BA}{AC} = \frac{6}{9} = \frac{2}{3}$

$\therefore \frac{BD}{DC} = \frac{BA}{AC}$ $\qquad \therefore \overrightarrow{AD}$ bisects $\angle BAC$ $\quad$ (First req.)

In ΔABF : $\because \overrightarrow{AE}$ bisects $\angle A$, $\overline{AE} \perp \overline{BF}$

From the congruence of $\Delta\Delta$ AEB , AEF

$\therefore \Delta ABF$ is an isosceles triangle

$\therefore AB = AF = 6$ cm. $\qquad \therefore CF = 9 - 6 = 3$ cm.

, $\because \Delta\Delta$ BAF , BCF have a common vertex B , $F \in \overline{AC}$

$\therefore \frac{\text{Area of } (\Delta ABF)}{\text{Area of } (\Delta CBF)} = \frac{AF}{FC} = \frac{6}{3} = 2$ $\qquad$ (Second req.)

Answers of Exercise 9

First | Multiple choice questions

(1) c	(2) b	(3) a	(4) d
(5) a	(6) a	(7) d	(8) a
(9) c	(10) a	(11) d	(12) d
(13) d	(14) c	(15) c	(16) b
(17) c	(18) c	(19) b	(20) c
(21) d	(22) a	(23) a	(24) c
(25) c	(26) b	(27) c	(28) d
(29) a	(30) b	(31) c	(32) d
(33) a	(34) c	(35) b	(36) b
(37) b	(38) c	(39) c	(40) c
(41) b			

Second | Essay questions

1

(1) 63 $\qquad$ (2) zero $\qquad$ (3) 1

2

(1) $\because P_M(A) = -36 < 0$ $\therefore$ A lies inside the circle.

, $\because P_M(A) = (MA)^2 - r^2$

$\therefore -36 = (MA)^2 - 100$ $\qquad \therefore (AM)^2 = 64$

$\therefore AM = 8$ cm.

(2) $\because P_M(B) = 96 > 0$ $\qquad \therefore$ B lies outside the circle.

, $\because P_M(B) = (MB)^2 - r^2$ $\qquad \therefore 96 = (MB)^2 - 100$

$\therefore (MB)^2 = 196$ $\qquad \therefore BM = 14$ cm.

(3) $\because P_M(C) = 0$ $\qquad \therefore$ C lies on the circle.

$\therefore MC = r = 10$ cm.

3

$P_M(A) = (MA)^2 - r^2$ $\qquad \therefore 400 = 625 - r^2$

$\therefore r^2 = 225$ $\qquad \therefore r = 15$ cm. $\quad$ (The req.)

4

$\because \overline{AD}$ is a tangent to the circle at D

$\therefore AD = \sqrt{P_M(A)}$

$\therefore P_M(A) = (AD)^2 = (8)^2 = 64$ (The req.)

5

$\because$ A lies outside the circle , $\overline{AB}$ is a tangent to the circle at B

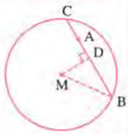
$\therefore AB = \sqrt{P_M(A)} = \sqrt{81} = 9$ cm. (First req.)

, $\because P_M(A) = (MA)^2 - r^2$ $\quad \therefore 81 = (MA)^2 - 144$

$\therefore (MA)^2 = 225$ $\quad \therefore MA = 15$

$\therefore AC = 15 - 12 = 3$ cm. (Second req.)

6



$\because P_M(A) = (MA)^2 - r^2$

$= (23)^2 - (31)^2 = -432$

, $\because P_M(A) = -AB \times AC$

$\therefore -432 = -AB \times AC$ $\quad \therefore 432 = AB \times AC$

, $\because AB = 3\,AC$ $\quad \therefore 432 = 3\,AC \times AC$

$\therefore (AC)^2 = 144$ $\quad \therefore AC = 12$ cm.

$\therefore AB = 36$

$\therefore BC = 36 + 12 = 48$ cm. (First req.)

assuming that the distance between the chord $\overline{BC}$ and the centre of the circle is $\overline{MD}$, where :

$\overline{MD} \perp \overline{BC}$

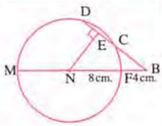
$\therefore$ D is the midpoint of $\overline{BC}$

, $\because P_M(D) = (MD)^2 - r^2 = -BD \times DC$

$\therefore (MD)^2 - (31)^2 = -24 \times 24$ $\quad \therefore (MD)^2 = 385$

$\therefore MD \simeq 19.6$ cm. (Second req.)

7



$\because P_M(B) = (NB)^2 - r^2$

$= (12)^2 - (8)^2 = 80$

, $\because P_N(B) = BC \times BD$

$\therefore 80 = BC \times BD$

, $\because BC = CD$ $\quad \therefore 80 = CD \times 2\,CD$

$\therefore CD = 2\sqrt{10}$ cm. (First req.)

assuming that the distance between chord $\overline{CD}$ and the centre of the circle is NE where : $\overline{NE} \perp \overline{CD}$

$\therefore$ E is the midpoint of $\overline{CD}$

, $\because P_N(E) = (EN)^2 - r^2 = -EC \times ED$

$\therefore (EN)^2 - (8)^2 = -\sqrt{10} \times \sqrt{10}$

$\therefore NE = 3\sqrt{6}$ cm. (Second req.)

8

$\because P_M(C) = CD \times CA = 16 \times 25 = 400$

, $\because$ C lies outside the circle

, $\overline{CB}$ is a tangent to the circle at B

$\therefore CB = \sqrt{P_M(C)} = \sqrt{400} = 20$ cm.

$\therefore (AB)^2 = (AC)^2 - (CB)^2 = (25)^2 - (20)^2 = 225$

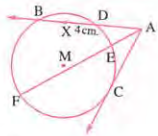
$\therefore AB = 15$ cm.

$\therefore AM = r = 7.5$ cm. (First req.)

, the area of $\Delta\,ABC = \frac{1}{2} \times 15 \times 20 = 150\ cm^2$.

(Second req.)

9



$\because$ A lies outside the circle

, $\overrightarrow{AC}$ is a tangent to the circle at C

$\therefore AC = \sqrt{P_M(A)} = \sqrt{144} = 12$ cm.

, $\because P_M(A) = AD \times AB$ $\quad \therefore 144 = 8 \times AB$

$\therefore AB = 18$ cm. $\quad \therefore DB = 10$ cm.

, $\because P_M(A) = AE \times AF$ $\quad \therefore 144 = AE \times (AE + 18)$

$\therefore 144 = (AE)^2 + 18\,AE$

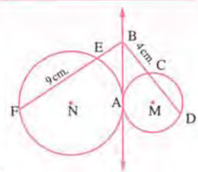
$\therefore (AE)^2 + 18\,AE - 144 = 0$

$\therefore (AE + 24)(AE - 6) = 0$

$\therefore AE = 6$ cm. (First req.)

$\therefore P_M(X) = -DX \times XB = -4 \times 6 = -24$ (Second req.)

10



$\because$ A lies on the circle M

, A lies on the circle N

$\therefore P_M(A) = P_N(A) = 0$

, $\because \overleftrightarrow{BA}$ is a tangent to the circle M at A

$\therefore P_M(B) = (AB)^2$

, $\because \overleftrightarrow{AB}$ is a tangent to the circle N at A

$\therefore P_N(B) = (AB)^2$ $\therefore P_M(B) = P_N(B)$

$\therefore \overleftrightarrow{AB}$ is the principle axis of the two circles M , N

(First req.)

$\because P_M(B) = BC \times BD$ $\therefore 36 = 4 \times BD$

$\therefore BD = 9$ cm. $\therefore CD = 5$ cm.

, $\because \overleftrightarrow{AB}$ is a tangent to the circle M.

$\therefore AB = \sqrt{P_M(B)} = \sqrt{36} = 6$ cm.

, $\because P_M(B) = P_N(B)$, $\because P_N(B) = BE \times BF$

$\therefore 36 = BE \times (9 + BE)$ $36 = (BE)^2 + 9\ BE$

$\therefore (BE)^2 + 9\ BE - 36 = 0$ $\therefore (BE + 12)(BE - 3) = 0$

$\therefore BE = 3$ cm. (Second req.)

11

$\because$ A lies on the circle M , A lies on the circle N

$\therefore P_M(A) = P_N(A) = 0$

Similarly : $P_M(B) = P_N(B) = 0$

$\therefore \overleftrightarrow{AB}$ is a principle axis of the two circles M , N

, $\because C \in \overleftrightarrow{AB}$

$\therefore \overleftrightarrow{BC}$ is the principle axis of the two circles M , N

(First req.)

$\because P_N(C) = CA \times CB$ $\therefore 64 = CA \times (CA + 12)$

$\therefore 64 = (CA)^2 + 12\ CA$ $\therefore (CA)^2 + 12\ CA - 64 = 0$

$\therefore (CA + 16)(CA - 4) = 0$ $\therefore CA = 4$ cm.

, $\because C \in$ the principle axis of the two circles.

$\therefore P_M(C) = P_N(C)$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle M at D

$\therefore CD = \sqrt{P_M(C)} = \sqrt{64} = 8$ cm. (Second req.)

12

$\because$ A lies on the circle M , A lies on the circle N

$\therefore P_M(A) = P_N(A) = 0$

Similarly : $P_M(B) = P_N(B)$

$\therefore \overleftrightarrow{AB}$ is the principle axis of the two circles M , N

(First req.)

, $\because X \in \overleftrightarrow{AB}$, $\because P_M(X) = P_N(X)$

, $\because P_M(X) = XD \times XC$

, $\because XD = 2\ DC$ $\therefore 144 = 2\ DC \times 3\ DC$

$\therefore (DC)^2 = 24$ $\therefore DC = 2\sqrt{6}$ cm.

$\therefore XC = 6\sqrt{6}$ cm.

, $\because P_N(X) = XF \times XE$ $\therefore 144 = XF \times (XF + 10)$

$\therefore 144 = (XF)^2 + 10\ XF$

$\therefore (XF)^2 + 10\ XF - 144 = 0$ $\therefore (XF + 18)(XF - 8) = 0$

$\therefore XF = 8$ cm. (Second req.)

$\because P_M(X) = P_N(X)$ $\therefore XD \times XC = XF \times XE$

$\therefore$ Figure CDFE is a cyclic quadrilateral. (Third req.)

13

(1) $15° = \frac{1}{2}[x - 60°]$ $\therefore 30° = x - 60°$

$\therefore x = 90°$

$\therefore y = 360° - (130° + 60° + 90°) = 80°$

$\therefore z = \frac{1}{2}[130° - 80°] = 25°$

(2) $y = 360° - 2x$, $x = \frac{1}{2}[(360° - 2x) - 2x]$

$\therefore 2x = 360° - 4x$ $\therefore 6x = 360°$

$\therefore x = 60°$ $\therefore y = 240°$

(3) $\because m(\angle A) = \frac{1}{2}(8x - 4x)$

, $m(\angle A) = \frac{1}{2}(5x - 20°)$

$\therefore 8x - 4x = 5x - 20°$ $\therefore x = 20°$

14

$\because m(\angle BDC) = 70°$

$\therefore m(\angle CDX) = 180° - 70° = 110°$

, $\because m(\angle CDX) = \frac{1}{2}[m(\widehat{CX}) + m(\widehat{AB})]$

$110° = \frac{1}{2}[100° + m(\widehat{XY}) + 94°]$

$\therefore 220° = (100° + m(\widehat{XY}) + 94°)$

$\therefore m(\widehat{XY}) = 26°$ (First req.)

, $m(\widehat{BC}) = 2\ m(\angle BAC) = 66°$

$\therefore m(\widehat{AX}) = 360° - (94° + 66° + 100° + 26°) = 74°$

(Second req.)

$\therefore m(\angle BEC) = \frac{1}{2}[m(\widehat{BC}) - m(\widehat{XY})]$

$= \frac{1}{2}[66° - 26°] = 20°$ (Third req.)

15

$\because$ AB = BC = CD = DE = AE

(properties of regular pentagon)

$m(\widehat{AB}) = m(\widehat{BC}) = m(\widehat{CD}) = m(\widehat{DE})$

$= m(\widehat{AE}) = \frac{360°}{5} = 72°$ (First req.)

, $m(\widehat{ACE}) = 360° - 72° = 288°$

$\therefore m(\angle AXE) = \frac{1}{2}\left[m(\widehat{ACE}) - m(\widehat{AE})\right]$

$= \frac{1}{2}[288° - 72°] = 108°$ (Second req.)

Third Higher skills

(1) d (2) c

Instructions to solution :

(1) $\because \overline{AB}$ is a diameter in the circle.

$\therefore m(\widehat{AE}) + m(\angle EB) = 180°$ (1)

, $\because m(\angle ECD) = 150°$ $\therefore m(\angle ECA) = 30°$

$\therefore \frac{1}{2}[m(\widehat{AE}) - m(\widehat{EB})] = 30°$

$\therefore m(\widehat{AE}) - m(\widehat{EB}) = 60°$ (2)

by adding the two equations (1) , (2) :

$2\,m(\widehat{AE}) = 240°$ $\therefore m(\widehat{AE}) = 120°$

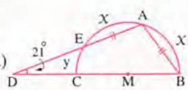
and so $\theta = \frac{1}{2} \times 120° = 60°$

(2) $\because \overline{BC}$ is a diameter in the circle

$\therefore 2X + y = 180°$ (1)

, $\because m(\angle D) = 21°$

$\therefore \frac{1}{2}[X - y] = 21°$ (2)



$\therefore X - y = 42°$

By adding (1) , (2) :

$\therefore 3X = 222°$ $\therefore X = 74°$ $\therefore y = 32°$

$\therefore m(\angle B) = \frac{1}{2}[74° + 32°] = 53°$

In Δ ABD : $m(\angle A) = 180° - (53° + 21°) = 106°$

Answers of Life Applications on Unit Four

1

$\because m(\angle B) = m(\angle D) = 90°$

and they are alternate angles.

$\therefore \overline{AB} \,/\!/\, \overline{DC}$ $\therefore \frac{AE}{AC} = \frac{BE}{BD}$

$\therefore \frac{60}{AC} = \frac{45}{150}$ $\therefore$ AC = 200 m.

$\therefore$ The distance between the location C and the location A = 200 m. (The req.)

2

$\because \overline{BE} \,/\!/\, \overline{CD}$ $\therefore \frac{AB}{BC} = \frac{AE}{ED}$ $\therefore \frac{AB}{33} = \frac{130}{39}$

$\therefore$ AB = 110 m.

$\therefore$ The length of the oil spot = 110 m. (The req.)

3

Yes , Yousef's division of the strip is correct

$\because$ The perpendicular distance between each two lines of the paper is equal.

$\therefore$ When he placed the two ends of the paper on two lines of this paper and the edge of the paper as a secant of the lines , then the included parts are equal in length.

4

$\because \overline{AD} \,/\!/\, \overline{BE} \,/\!/\, \overline{CF}$ $\therefore \frac{AB}{AC} = \frac{ED}{DF}$

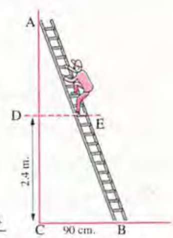
$\therefore \frac{1.2}{AC} = \frac{0.8}{12.8}$ $\therefore$ AC = 19.2 m.

$\therefore$ The length of the tube $\simeq$ 19 m. (The req.)

5

In Δ ABC

which is right in C :



$\because (AC)^2 = (AB)^2 - (BC)^2$

$= (4.1)^2 - (0.9)^2 = 16$

$\therefore$ AC = 4 m

, $\because \overline{ED} \,/\!/\, \overline{BC}$

$\therefore \frac{BE}{AB} = \frac{CD}{AC}$ $\therefore \frac{BE}{4.1} = \frac{2.4}{4}$

$\therefore$ BE = 2.46 m.

$\therefore$ The distance which a man ascends on the ladder = 2.46 m. (The req.)

6

$\because AB : BC : CD = 5 : 4 : 3$

$\therefore \frac{AB}{5} = \frac{BC}{4} = \frac{CD}{3}$ $\qquad \therefore \frac{180}{5} = \frac{BC}{4} = \frac{CD}{3}$

$\therefore BC = 144$ cm. , $CD = 108$ cm.

, $\because \overline{AE} \parallel \overline{BF} \parallel \overline{CX} \parallel \overline{DY}$

$\therefore \frac{EY}{EF} = \frac{AD}{AB}$ $\qquad \therefore \frac{EY}{200} = \frac{432}{180}$

$\therefore EY = 480$ cm. (The req.)

7

In ΔABE : $\because AB = BE$, $m(\angle B) = 90°$

$\therefore m(\angle BAE) = 45°$ (1)

$\because m(\angle BAD) = 90°$ $\qquad \therefore m(\angle DAX) = 45°$ (2)

From (1) , (2) : $\therefore m(\angle BAX) = m(\angle DAX)$

$\therefore \overrightarrow{AX}$ bisects $\angle A$ in ΔABD

$\therefore \frac{BX}{XD} = \frac{BA}{AD} = \frac{42}{56} = \frac{3}{4}$

$\therefore \frac{BX}{BX + XD} = \frac{3}{3+4}$ $\qquad \therefore \frac{BX}{BD} = \frac{3}{7}$

$\because \Delta ABX$, ΔABD have the same height.

$\therefore \frac{\text{The area of } (\Delta ABX)}{\text{The area of } (\Delta ABD)} = \frac{BX}{BD} = \frac{3}{7}$

$\therefore$ The area of $(\Delta ABX) = \frac{3}{7} \times$ the area of (ΔABD)

$= \frac{3}{7} \times \frac{1}{2} \times 42 \times 56$

$= 504 \text{ m}^2$ (First req.)

In the right-angled triangle BAD at A

$\therefore (BD)^2 = (AB)^2 + (AD)^2 = (42)^2 + (56)^2 = 4900$

$\therefore BD = 70$ m.

, $\because \frac{BX}{BD} = \frac{3}{7}$ $\qquad \therefore \frac{BX}{70} = \frac{3}{7}$

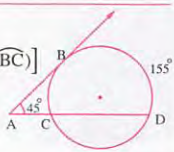
$\therefore BX = 30$ m. $\qquad \therefore XD = 70 - 30 = 40$ m.

$\therefore AX = \sqrt{BA \times AD - BX \times XD}$

$= \sqrt{42 \times 56 - 30 \times 40}$

$= 24\sqrt{2}$ m. (Second req.)

8



$\because m(\angle A) = \frac{1}{2}\left[m(\widehat{BD}) - m(\widehat{BC})\right]$

$\because 45° = \frac{1}{2}\left(155° - m(\widehat{BC})\right)$

$\therefore 90° = 155° - m(\widehat{BC})$

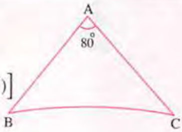
$\therefore m(\widehat{BC}) = 65°$

$\therefore m(\widehat{DC}) = 360° - (155° + 65°) = 140°$

$\therefore$ Length of $(\widehat{DC}) = \frac{140°}{360°} \times 2 \times 10 \times \pi$

$\simeq 24.4$ cm. (The req.)

9



$\because m(\angle A)$

$= \frac{1}{2}\left[(360° - m(\widehat{BC})) - m(\widehat{BC})\right]$

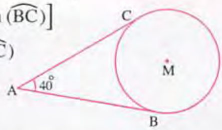
$\therefore 80° = \frac{1}{2}\left[360° - 2\, m(\widehat{BC})\right]$

$\therefore 160° = 360° - 2\, m(\widehat{BC})$

$\therefore 2\, m(\widehat{BC}) = 200°$ $\qquad \therefore m(\widehat{BC}) = 100°$

(The req.)

10



$\because m(\angle A)$

$= \frac{1}{2}\left[\left(360° - m(\widehat{BC})\right) - m(\widehat{BC})\right]$

$\therefore 40° = \frac{1}{2}\left[360° - 2\, m(\widehat{BC})\right]$

$\therefore 80° = 360° - 2\, m(\widehat{BC})$

$\therefore 2\, m(\widehat{BC}) = 280°$

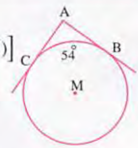
$\therefore m(\widehat{BC}) = 140°$

$\therefore m(\widehat{BC})$ major $= 360° - 140° = 220°$

$\therefore$ Length of $(\widehat{BC})$ major $= \frac{220°}{360°} \times 2 \times 9 \times \pi$

$\simeq 34.56$ cm. (The req.)

11



$m(\angle A) = \frac{1}{2}\left[m(\widehat{BC}) \text{ major} - m(\widehat{BC})\right]$

$= \frac{1}{2}\left[360° - 54° - 54°\right]$

$= 126°$ (First req.)

$\because$ Length $(\widehat{BC}) = \frac{54°}{360°} \times 2\pi r$

$\therefore 6011 = \frac{54°}{360°} \times 2 \times \pi \times r$

$\therefore r = \frac{6011 \times 360°}{54° \times 2 \times \pi} \simeq 6378$ km. (Second req.)